This answer is at the bottom

1. prove f(n) = log(n!) = O(nlogn) = Ω (nlogn)
2. Calculate time complexity

```
int fun(int n){
    int sum = 0;                      1
    for(int i=1;i<=n;i*=2)            1+n+1+2n
        sum += i*i;                   3n
    return i;                         1
}
```

$1+3n+2n+3n+1 = 6n+4 = O(n)$

$f(\frac{n}{2}) = \begin{cases} 1 & n \le 1 \\ f(\frac{n}{4})+1+1 & n>1 \end{cases}$

$f(n) = f(\frac{n}{2^k}) + k$

when $\frac{n}{2^k} > 1$

$n > 2^k$

$\log_2 n > k$

$T(n) = 1+\log_2 n = O(\log n)$

```
int fun(int n)
    if( n <= 1 )
        return 1;
    return fun( n/2 ) + n*n;
}
```

$f(n) = \begin{cases} 1 & n \le 1 \\ f(\frac{n}{2})+1 & n>1 \end{cases}$

3. Analyze the recursion, find out the recurrence relation between cost functions of different input and solve the recurrence relation:

```
int fun(int n) {
    int a,b;
    if(n<=4)          4
      return 0;       1     = O(1)
    else {
      a = n/2;        1
      b = fun(n/4);
      return a+b;     1
    }
}
```

$f(n) = \begin{cases} O(1) & n \le 4 \\ f(\frac{n}{4})+2 & n>4 \end{cases}$

$f(\frac{n}{4}) = f(\frac{n}{16})+2+2$

$f(n) = f(\frac{n}{4^k})+2k \quad n>4$

$\frac{n}{4^k} > 4 \rightarrow n > 4^{k+1}$

$\log_4 n > k+1 \quad k < \log_4 n + 1$

$T(n) = O(\log n) \quad | \quad T(n) = O(\log n)$

4.For each pair of f(n) and g(n) below, decide if f(n) = O(g(n)), f(n) = Ω (g(n)), or f(n) = Θ (g(n)). Justify your answer. Note that more than one of these relations may hold for a given pair; list all correct ones.

(a) $f(n) = (\log_3 n)^2$ and $g(n) = log_2(n^3)$.

(b) $f(n) = 2^n$ g(n)=n!

5. find out the relationship between the pairs of functions. Determine whether *f* is O(g), Ω(g) or Θ(g).

1. $f(n) = \log\sqrt{n}, \ g(n) = \sqrt{\log n}.$
2. $f(n) = 2n^2 - n + 6, \ g(n) = n^3 + 8.$
3. $f(n) = \begin{cases} 10^9, \text{if } n \text{ is even,} \\ n, \text{if } n \text{ is odd.} \end{cases} \quad g(n) = \begin{cases} n, \text{if } n \ge 1000, \\ n^2, \text{if } n < 1000. \end{cases}$

$\log(n!) = \log 1 + \log 2 + \cdots + \log n$

$$\frac{n}{2}\cdot\log\left(\frac{n}{2}\right)\geq\frac{n}{2}\cdot\frac{\log n}{2}=\frac{n\log n}{4}$$

$c=\frac{1}{4}$ $\quad n_0=4 \quad n\geq n_0$

$$\log(n!)\geq c\cdot n\log n$$

thus, $\log(n!)=\Omega(n\log n)$

$$f(n)=\log(n!)=O(n\log n)=\Omega(n\log n)$$

4. a $g(n) \log_2(n)^3 = 3\log_2^n = \frac{3\log_3^n}{\log_3^2} = \frac{3}{\log_3^2} \cdot \log_3^n$

$= O(\log^n)$

$f(n) = O(\log n^2)$ so $f(n) = \Omega(g(n))$

b. when $n = k+1$ $(k+1) = (k+1) \times k!$

$2^n = 2^{k+1} = 2 \times 2^k$

if $k \geq 4$ $(k+1) \times k! > (k+1) \times 2^k$

$2^k \times (k+1) > 2 \times 2^k = 2^{k+1}$

$\therefore k! > 2^k$

$\therefore f(n) = \Omega(g(n))$

5 $\log\sqrt{n} = \log n^{\frac{1}{2}} = \frac{1}{2}\log^n$

$t = \log^n$ $f(n) = \frac{1}{2}t$ $g(n) = \sqrt{t}$

$\frac{1}{2}t = \sqrt{t}$ $t = 4$

when $\frac{1}{2}t \geq \sqrt{t}$ $t \geq 4$

$f(n) = \frac{1}{2}g(n)$

$f(n) = \Omega(g(n))$.

if $f(n) = O(g(n))$ $\longrightarrow$ $\frac{1}{2}\log n \leq C\sqrt{\log n}$

is not real

6

2. $f(n) = O(n^2)$ $g(n) = O(n^3)$

$n^3 > n^2$

$f(n) = O(g(n))$

$f(n) = \Omega(g(n))$: $2n^2 - n + 6 > C(n^3+8)$

$n \to +\infty$ $Cn^3 >> 2n^2$ not real

3 $n \geq 1000$ $g(n) = n$

if n is odd $f(n) = n = g(n)$

$f(n) \leq 1 \cdot g(n)$

if n is even $f(n) = 10^9$ $f(n) \leq 1 \cdot g(n)$

$c = 1$ $f(n) \leq cg(n)$ $f(n) = O(g(n))$

$f(n) = \Omega(g(n))$ $c > 0$ $f(n) \geq Cg(n)$

n is even $f(n) = 10^9$ $g(n) = n$

is not real

$f(n) = O(g(n))$