Bandits on stochastic block-model graphs

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Problem definition

N arms, T steps.

1. The environment chooses losses for every arm noted $l_{t,i}$ for the arm i at the step t.
2. Following the algorithm we hope would minimize as much as possible the regret the player draws an arm I_t.
3. The player receives the loss l_{t,I_t}.
4. We define $(O_t)_{i\in[N]}$ as the indicative function of observed loss at step t. We have:

$$O_{t,I_t} = 1 \qquad \forall i \neq I_t,\ O_{t,i} \sim B(r)$$

   $(O_t)_{i\in[N]}$ corresponds to the value of the logic expression *i is neighbor of* I_t in the Erdös-Rényi random graph drawn at step t.
5. For all i such that $O_{t,i} = 1$ the player can observe the loss $l_{t,i}$.

Duplex tricks

Definition geometrical random variables

$$M_t^* = \min\{1 \leq i < N : O'_{t-1,i} = 1\} \cup \{N\}$$
$$G_{t,i} = \min\left(K_{t,i}, M_t\right)$$

Independence

$$p_{t+2,i} \propto w_{t+2,i} = \frac{1}{N} \exp\left(-\eta_{t+2} \hat{L}_{t,i}\right)$$

Generalizing

ESTIMATE_R gives a safe lower bounding on r.

If $\underline{r} = 0$, we run vanilla EXP3.

If $\underline{r} \geq \frac{\log T}{N}$, we run vanilla DUPLEX.

If $0 < \underline{r} < \frac{\log T}{N}$, $A = \left\lceil \frac{\log T}{N\underline{r}} \right\rceil$ and run DUPLEX grouping A steps.

Random graph with Stochastic Block-Model

ZE figure!

Probability of side information

n clusters. We know in which cluster is every vertex.
$R = (r_{ij})_{1 \leq i,j \leq n}$ the matrix representing the probability of having side information.
r_{ij} represents the probability that a vertex of the cluster j reveals his loss to a given vertex of the cluster i

Adapt M_t sampling

Lower bounding R

Generalized algorithm

If $\underline{R} = 0$, we run vanilla EXP3.

If $\min_{i,j,\ \underline{r}_{i,j}>0} \left(\underline{r}_{ij}\, N_j\right) \geq \log T$, we run adapted DUPLEX.

Else,

$$A = \left\lceil \frac{\log T}{r_N*} \right\rceil \text{ where}$$

$$r_N* = \min_{i,j,\ \underline{r}_{i,j}>0} \left(\underline{r}_{ij}\, N_j\right)$$

and run DUPLEX grouping A steps.

Heavily unbalanced graph

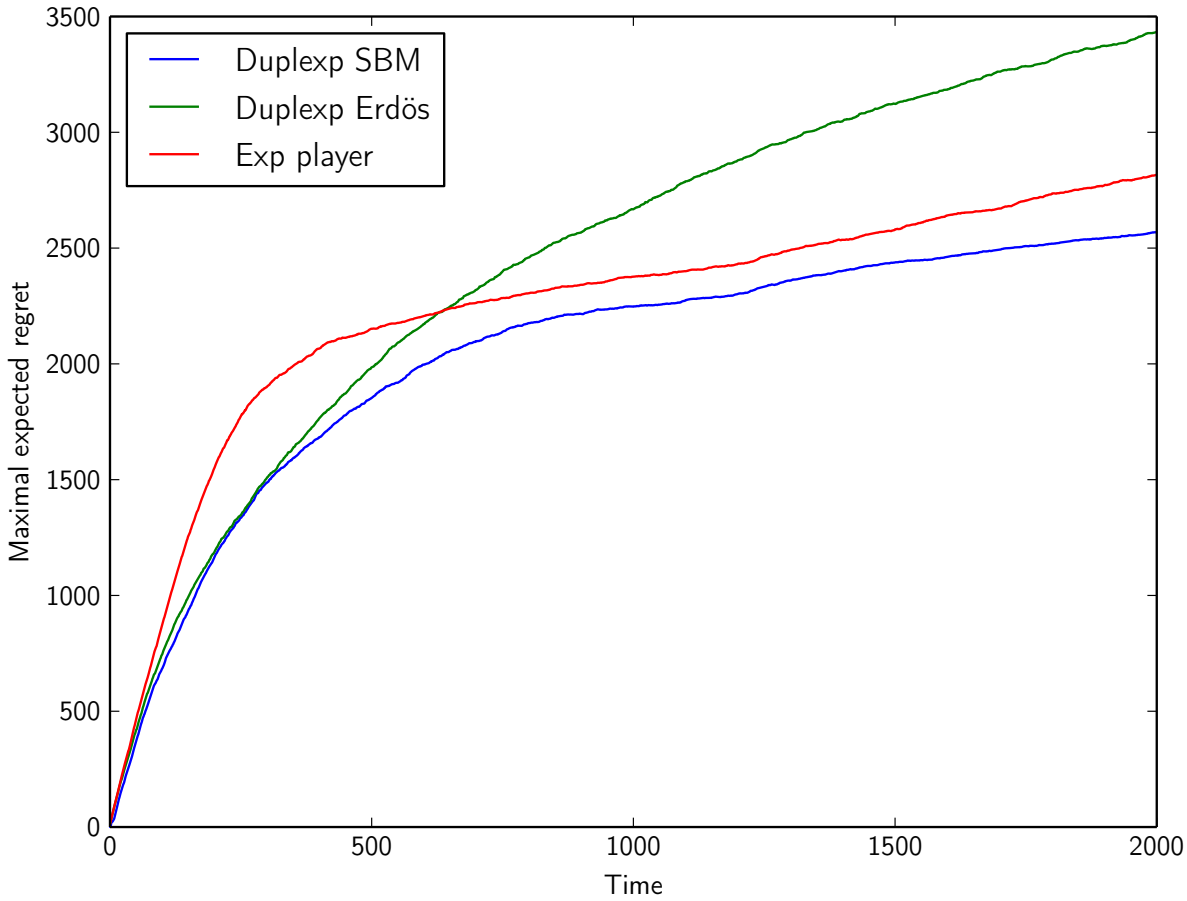

Figure : Outperformance of our algorithm

Graph 'close' to Erdös-Rényi

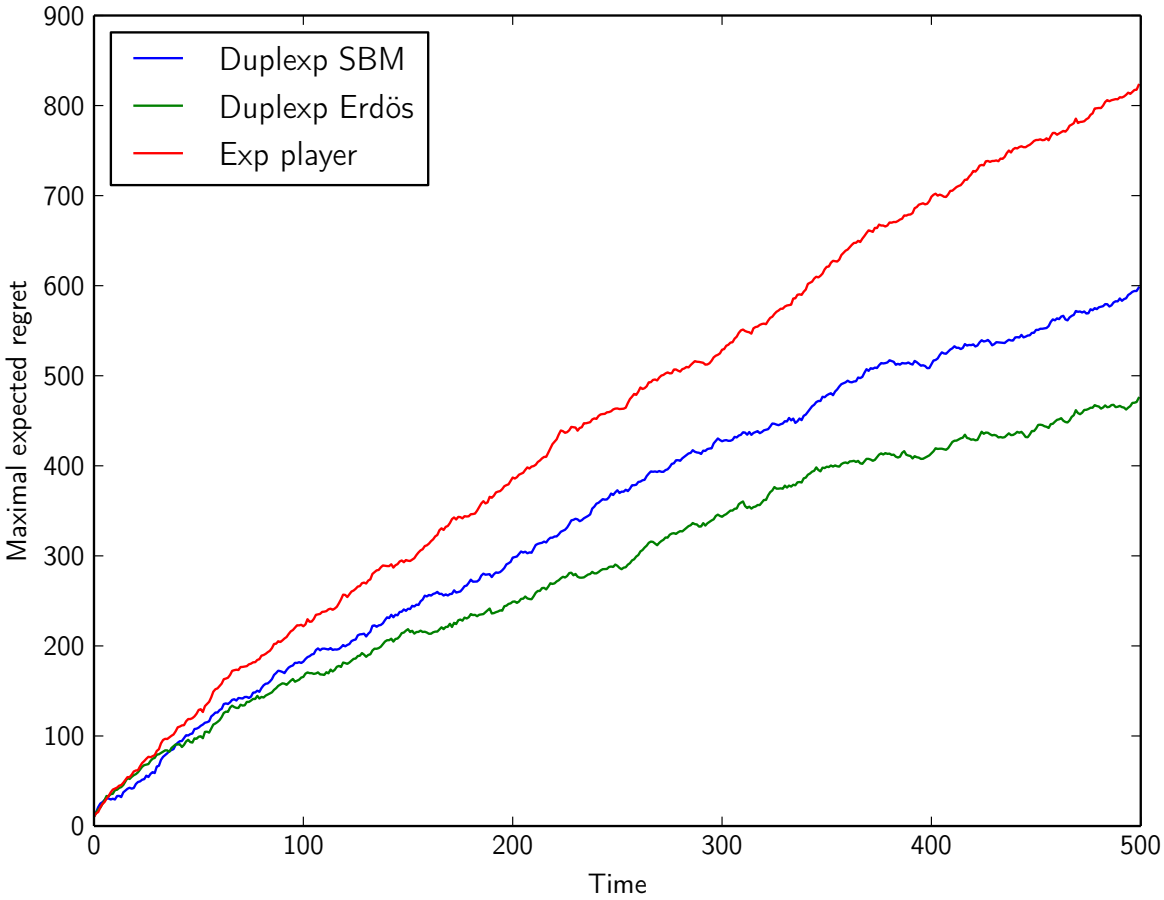

Figure : Orginal algorithm outperforms our algorithm