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6.976
High Speed Communication Circuits and Systems
Lecture 16
Noise in Integer-N Frequency Synthesizers

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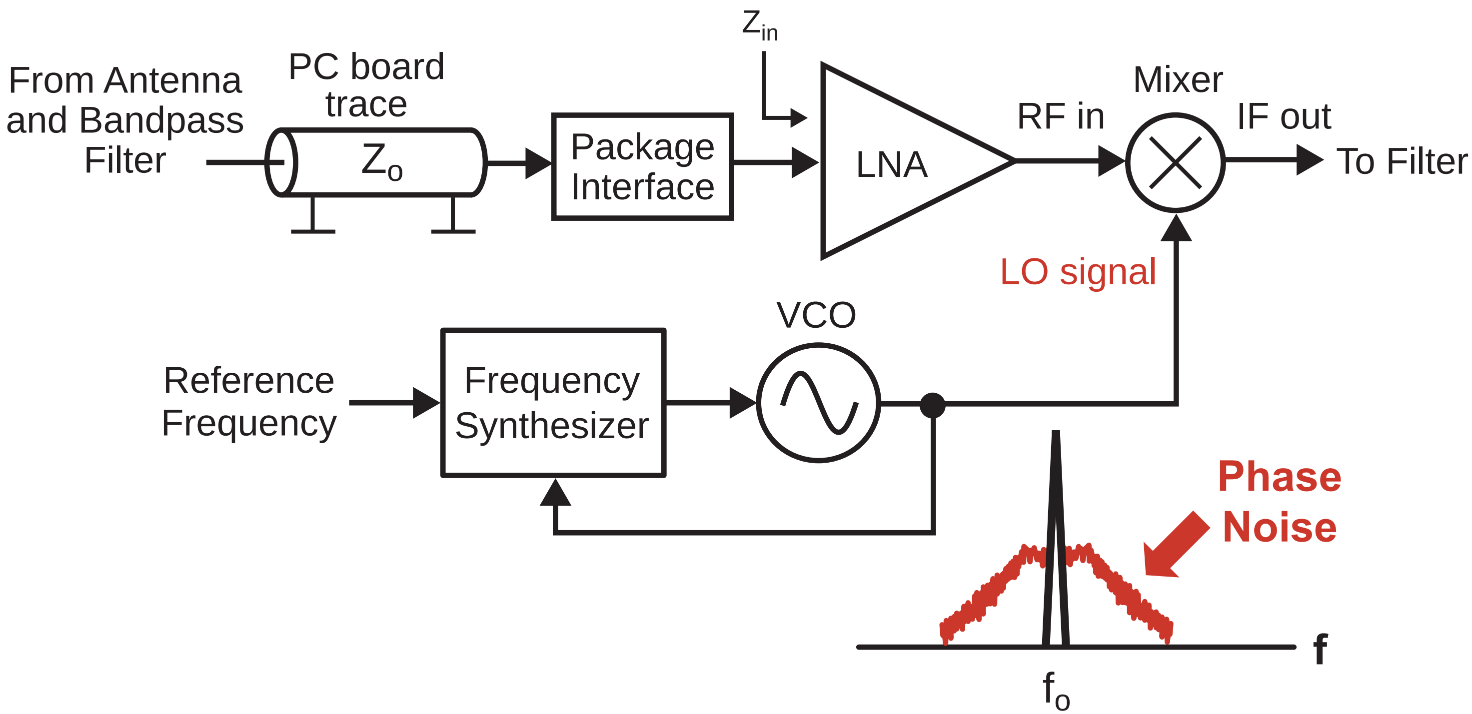
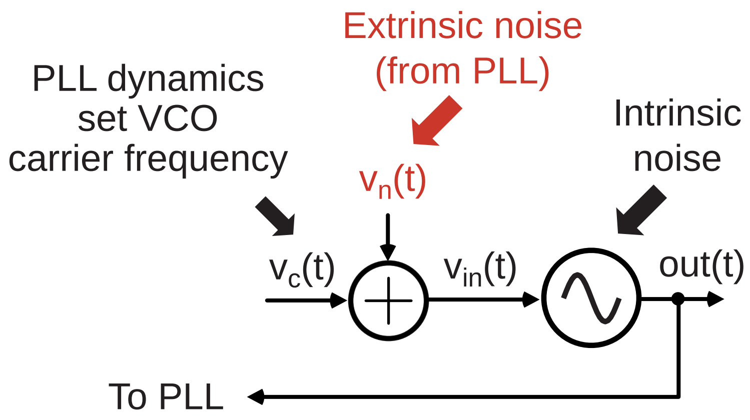
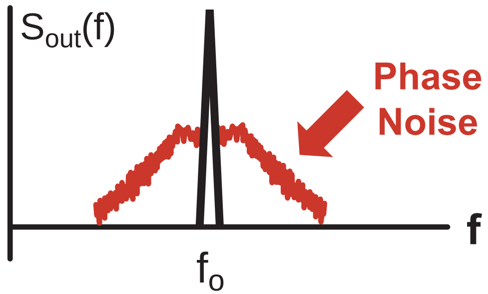
Frequency Synthesizer Noise in Wireless Systems



- Synthesizer noise has a negative impact on system
  - Receiver – lower sensitivity, poorer blocking performance
  - Transmitter – increased spectral emissions (output spectrum must meet a mask requirement)
- Noise is characterized in frequency domain

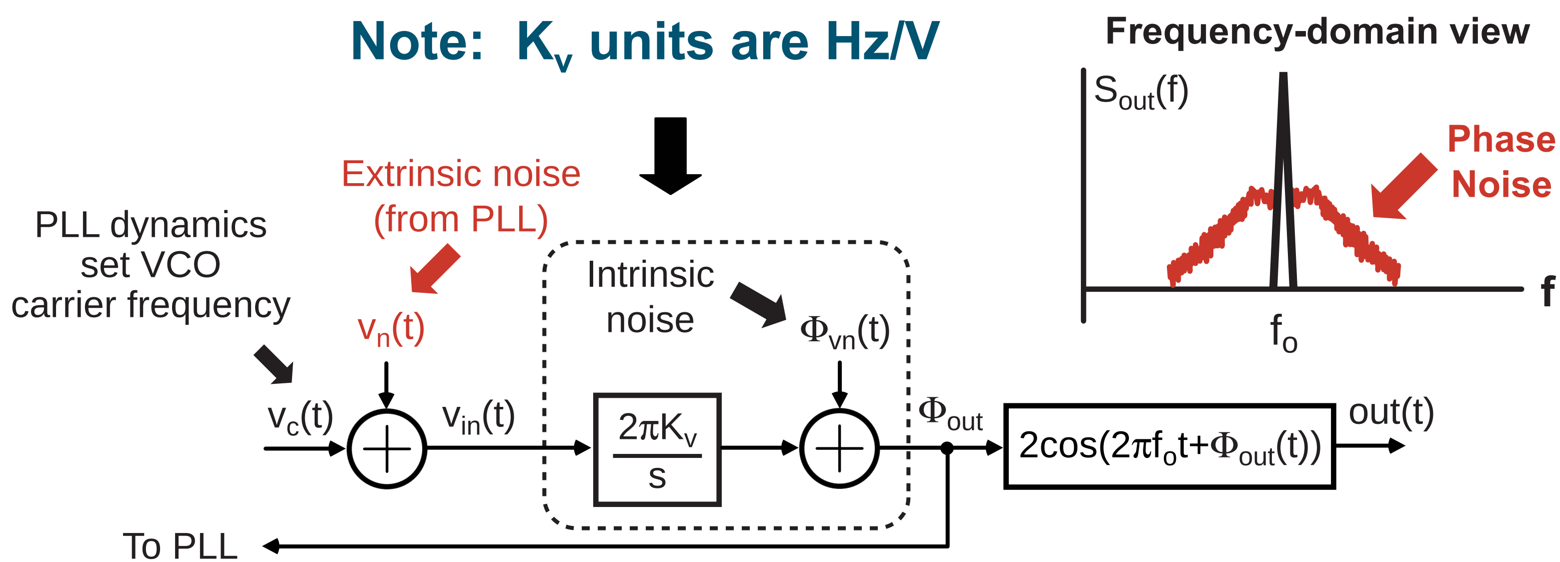
Noise Modeling for Frequency Synthesizers





- **PLL has an impact on VCO noise in two ways**
  - **Adds extrinsic noise from various PLL circuits**
  - **Highpass filters VCO noise through PLL feedback dynamics**
- **Focus on modeling the above based on phase deviations**
  - **Simpler than dealing directly with PLL sine wave output**

Phase Deviation Model for Noise Analysis



- **Model the impact of noise on instantaneous phase**
  - **Relationship between PLL output and instantaneous phase**

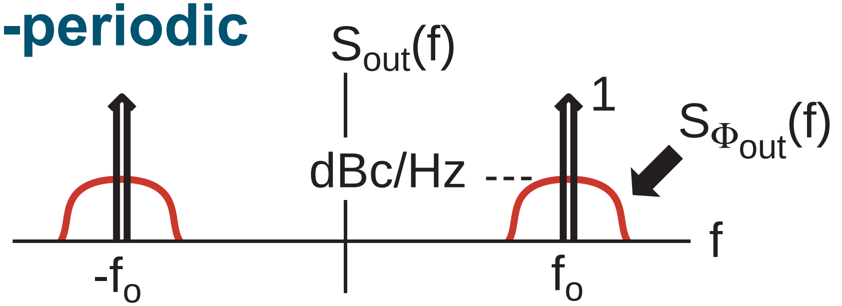
$$out(t) = 2\cos(2\pi f_o t + \underline{\Phi_{out}(t)})$$

  - **Output spectrum (from Lecture 12)**

$$\boxed{S_{out}(f) = S_{sin}(f) + S_{sin}(f) * S_{\Phi_{out}}}$$

Phase Noise Versus Spurious Noise

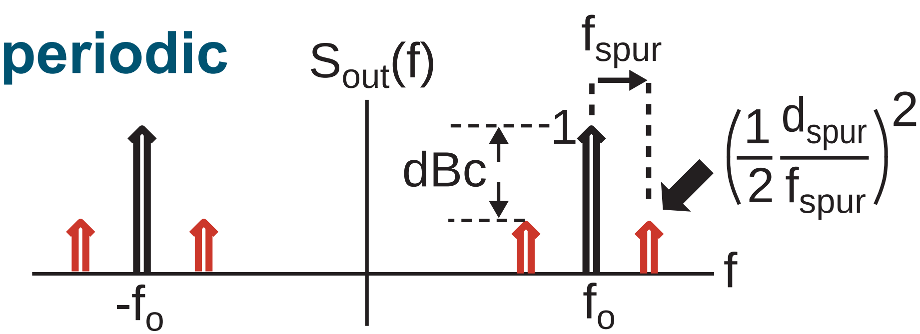
- **Phase noise is non-periodic**



  - **Described as a spectral density relative to carrier power**

$$L(f) = 10\log(S_{\Phi_{out}}(f))\ \mathrm{dBc/Hz}$$

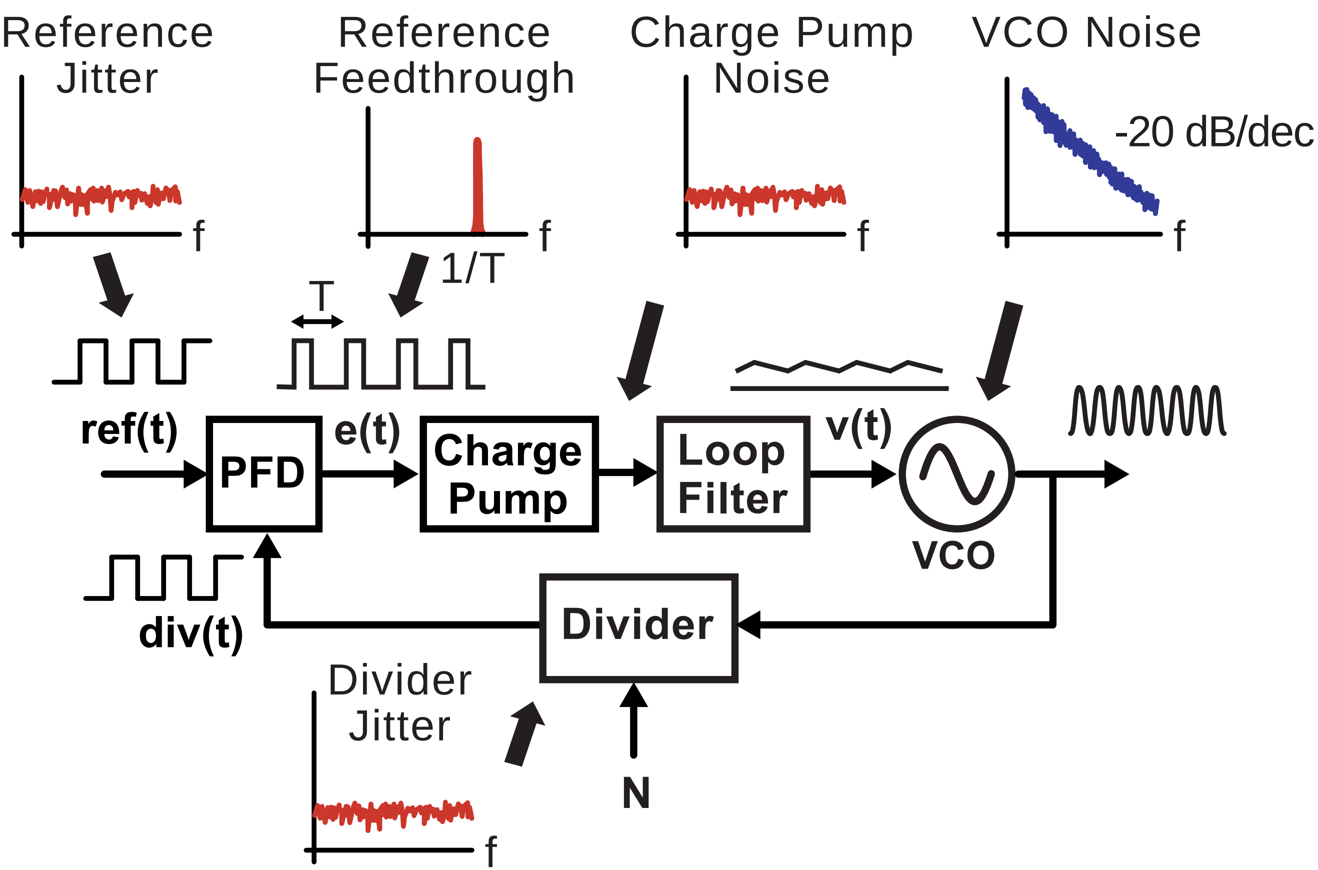
- **Spurious noise is periodic**



  - **Described as tone power relative to carrier power**

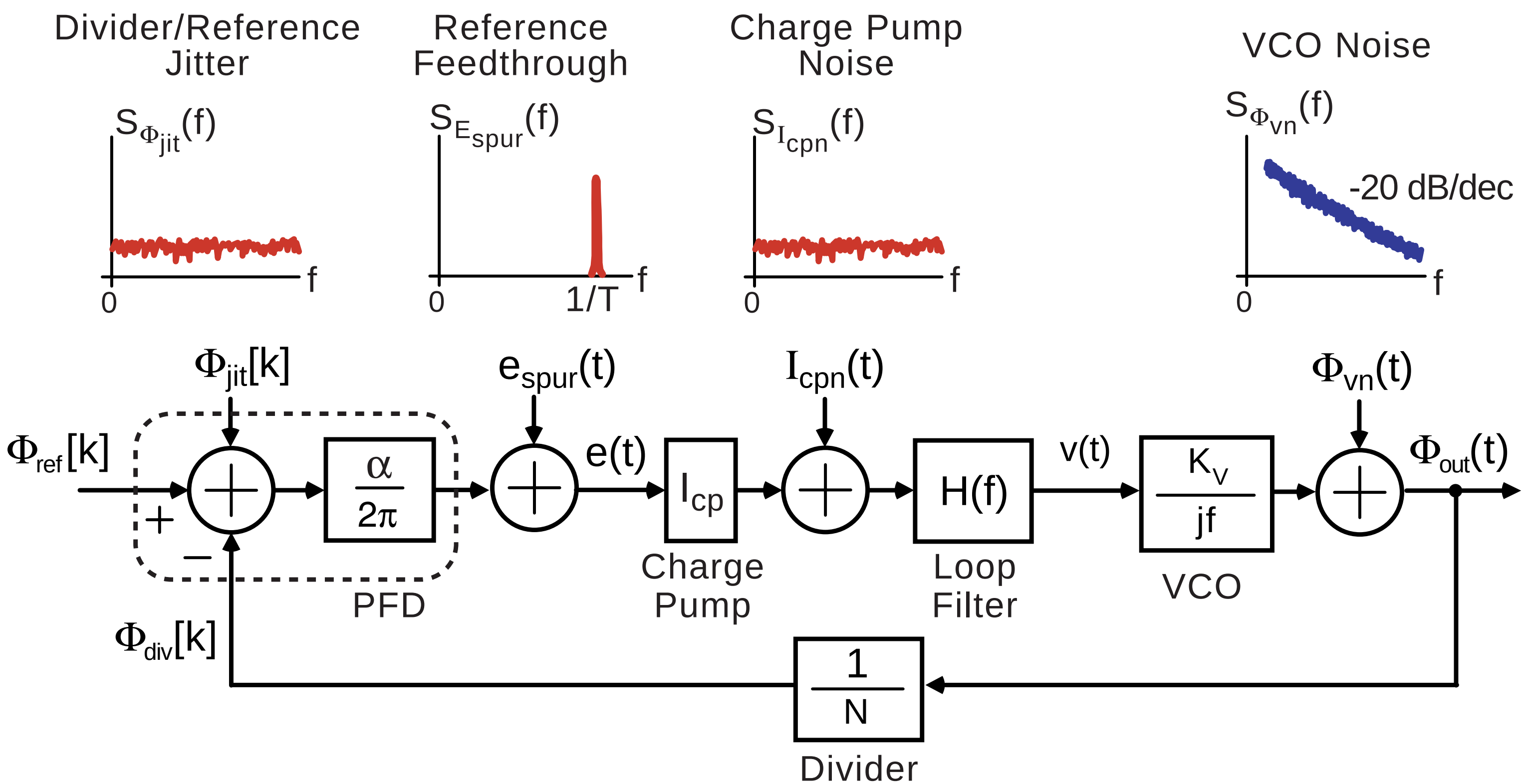
$$20\log\left(\frac{d_{spur}}{2f_{spur}}\right)\ \mathrm{dBc}$$

Sources of Noise in Frequency Synthesizers



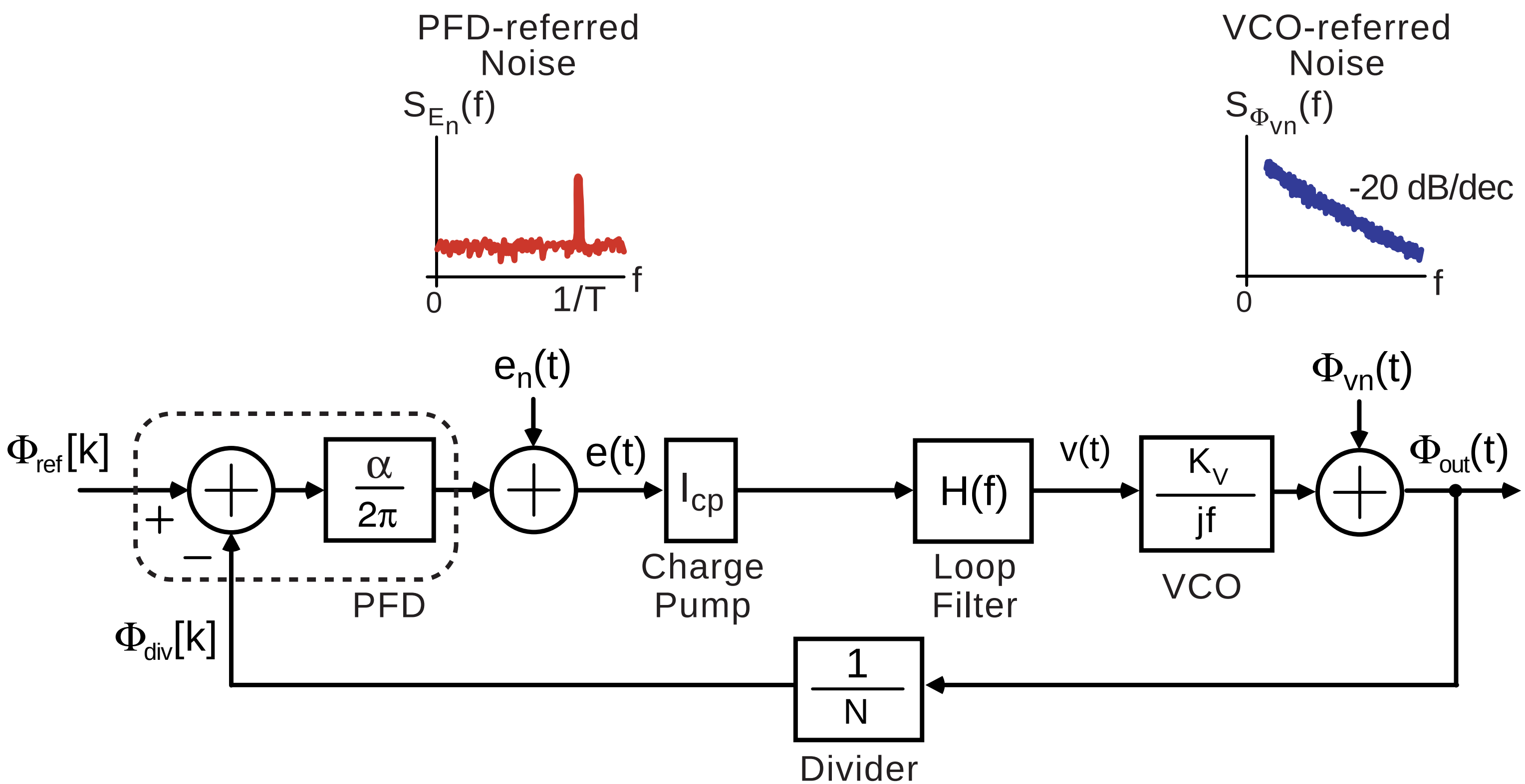
- **Extrinsic noise sources to VCO**
  - **Reference/divider jitter and reference feedthrough**
  - **Charge pump noise**

Modeling the Impact of Noise on Output Phase of PLL



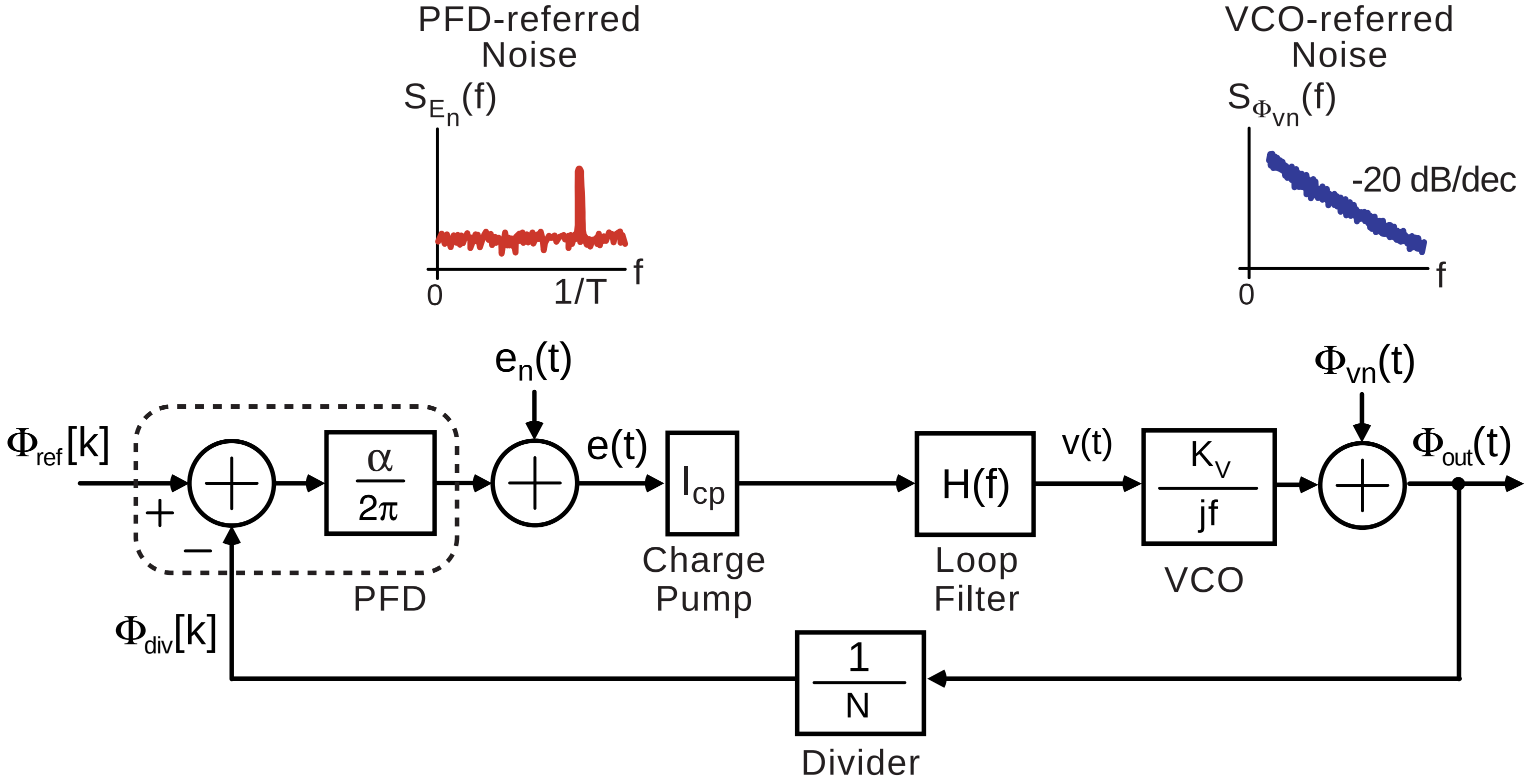
- Determine impact on output phase by deriving transfer function from each noise source to PLL output phase
  - There are a lot of transfer functions to keep track of!

Simplified Noise Model



- **Refer all PLL noise sources (other than the VCO) to the PFD output**
  - **PFD-referred noise corresponds to the sum of these noise sources referred to the PFD output**

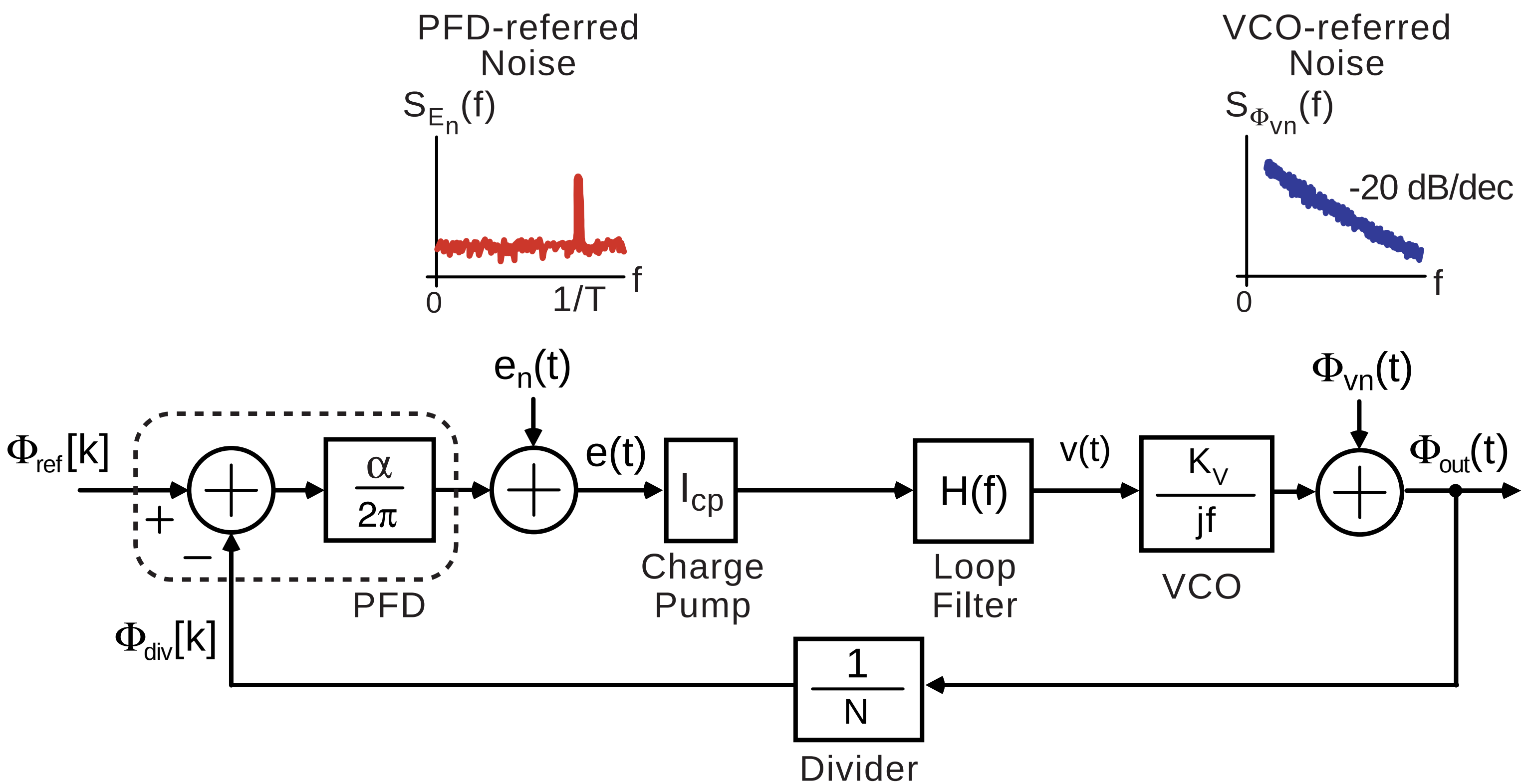
Impact of PFD-referred Noise on Synthesizer Output



- **Transfer function derived using Black's formula**

$$\frac{\Phi_{out}}{e_n} = \frac{I_{cp}H(f)K_v/(jf)}{1+\alpha/(2\pi)I_{cp}H(f)K_v/(jf)(1/N)}$$

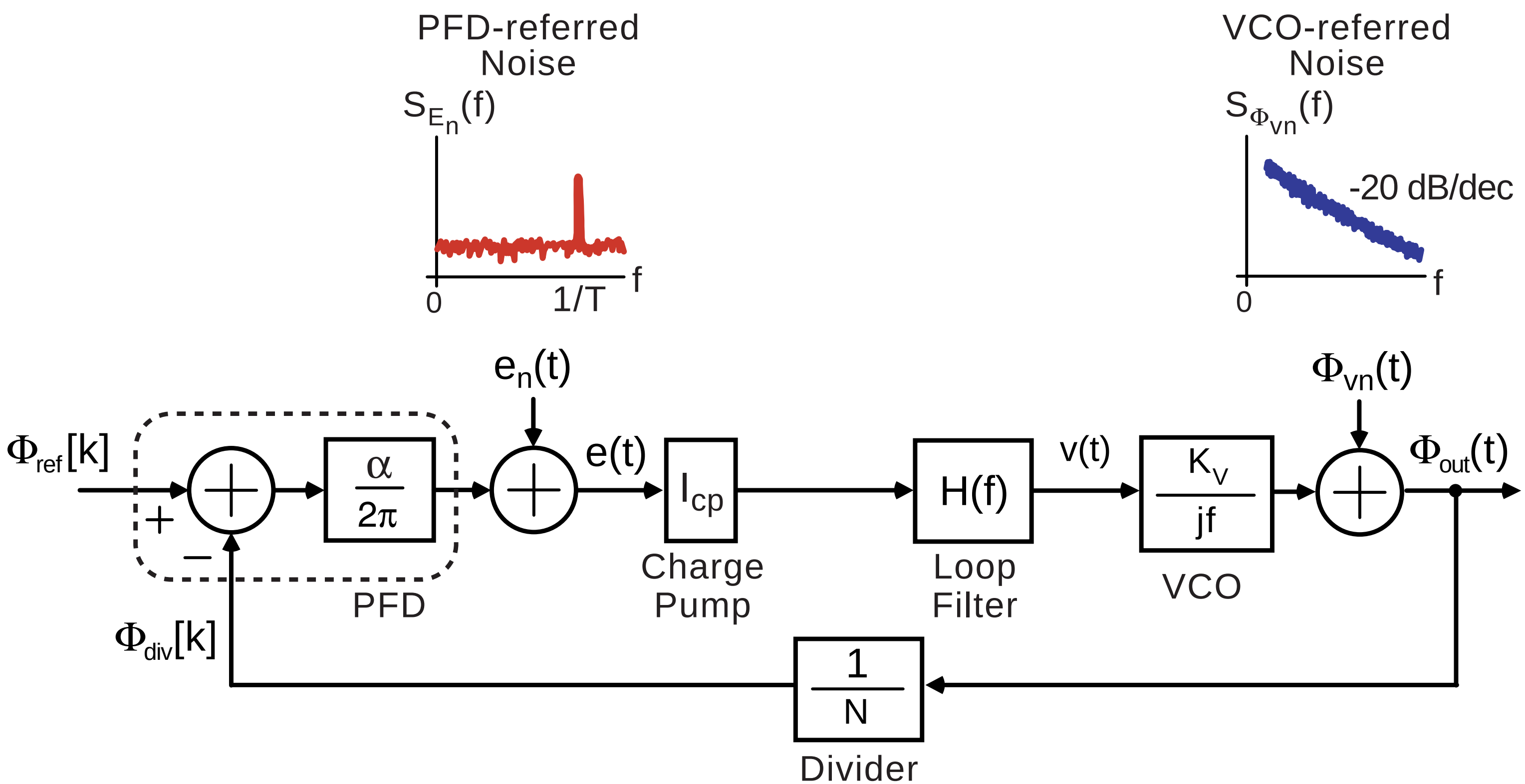
Impact of VCO-referred Noise on Synthesizer Output



- **Transfer function again derived from Black's formula**

$$\frac{\Phi_{out}}{e_n} = \frac{1}{1 + \alpha/(2\pi) I_{cp} H(f) K_v/(jf)(1/N)}$$

A Simpler Parameterization for PLL Transfer Functions



- **Define G(f) as**

$$\boxed{G(f) = \frac{A(f)}{1 + A(f)}}$$

Always has a gain of one at DC

  - **A(f) is the open loop transfer function of the PLL**

$$A(f) = \alpha/(2\pi) I_{cp} H(f) K_v/(jf)(1/N)$$

Parameterize Noise Transfer Functions in Terms of G(f)

- **PFD-referred noise**

$$\frac{\Phi_{out}}{e_n} = \frac{I_{cp}H(f)K_v/(jf)}{1+\alpha/(2\pi)I_{cp}H(f)K_v/(jf)(1/N)}$$

$$= \frac{2\pi}{\alpha}N\frac{\alpha/(2\pi)I_{cp}H(f)K_v/(jf)(1/N)}{1+\alpha/(2\pi)I_{cp}H(f)K_v/(jf)(1/N)}$$

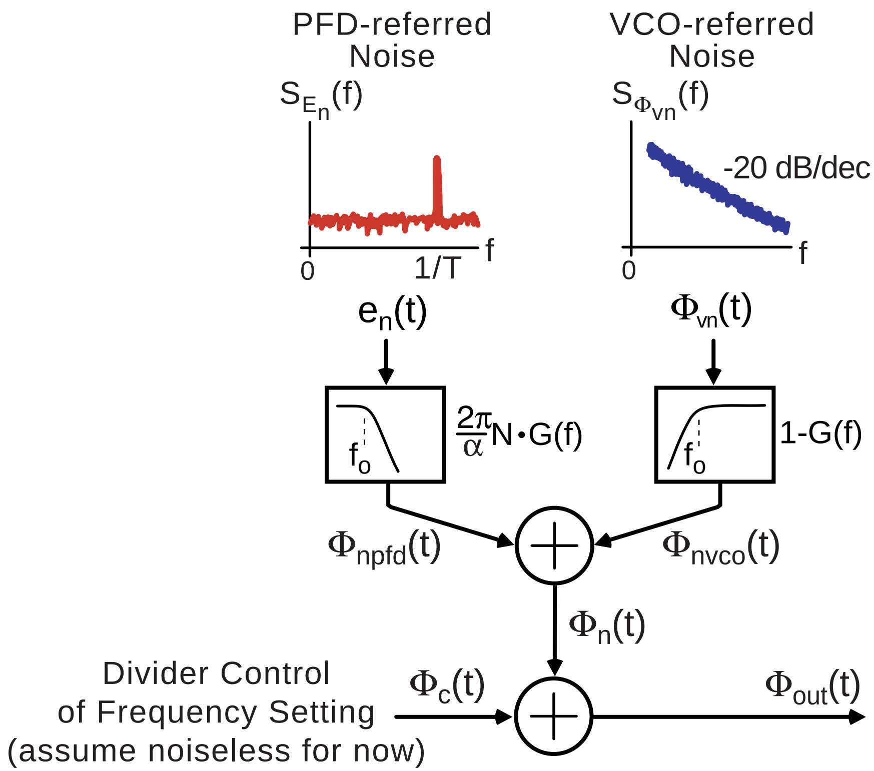
$$= \frac{2\pi}{\alpha}N\frac{A(f)}{1+A(f)} = \boxed{\frac{2\pi}{\alpha}NG(f)}$$

- **VCO-referred noise**

$$\frac{\Phi_{out}}{\Phi_{vn}} = \frac{1}{1+\alpha/(2\pi)I_{cp}H(f)K_v/(jf)(1/N)}$$

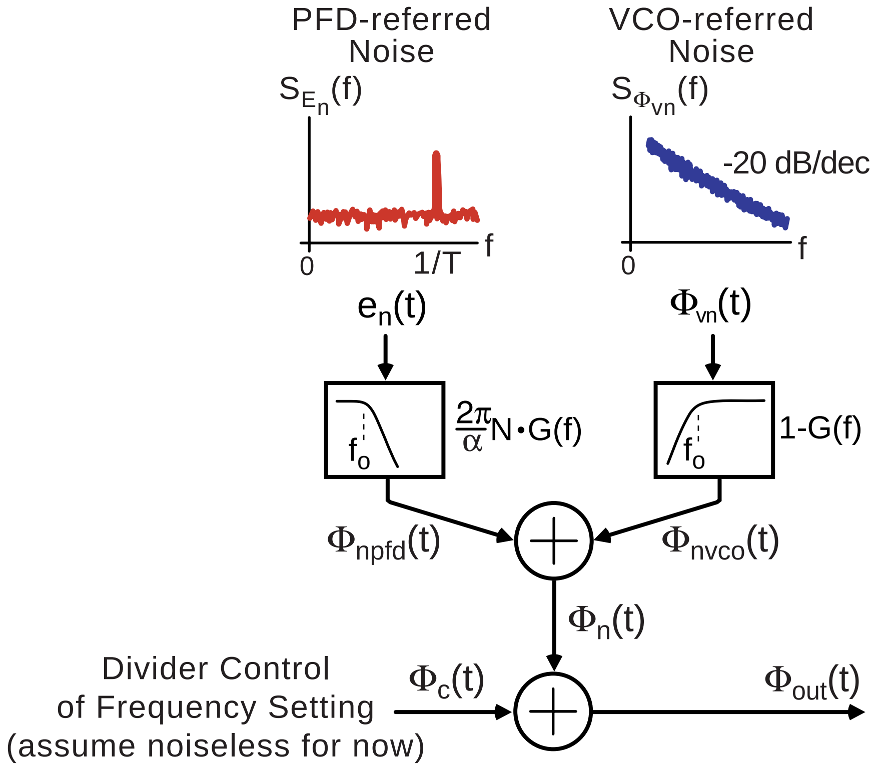
$$= \frac{1}{1+A(f)} = 1 - \frac{A(f)}{1+A(f)} = \boxed{1-G(f)}$$

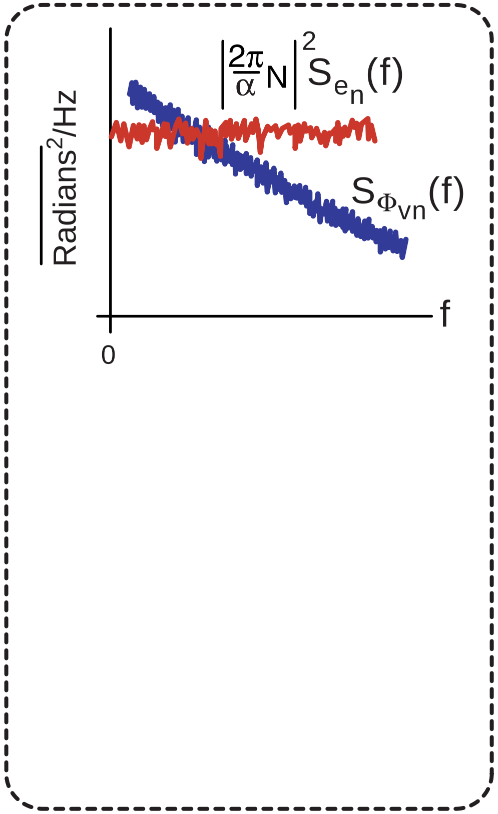
Parameterized PLL Noise Model



- PFD-referred noise is lowpass filtered
- VCO-referred noise is highpass filtered
- Both filters have the same transition frequency values
  - Defined as f_o

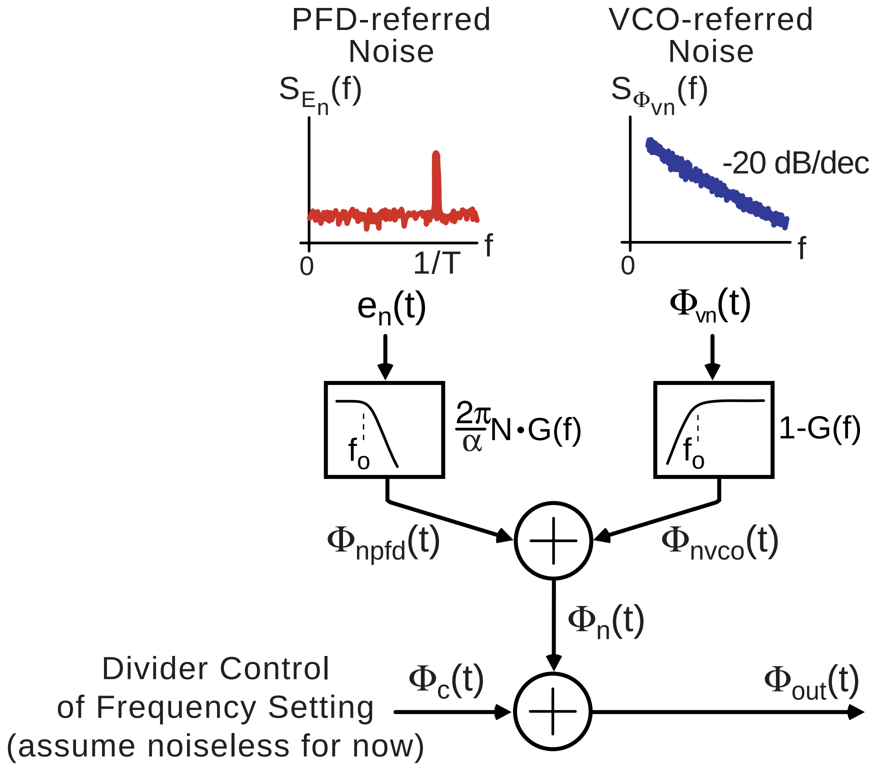
Impact of PLL Parameters on Noise Scaling

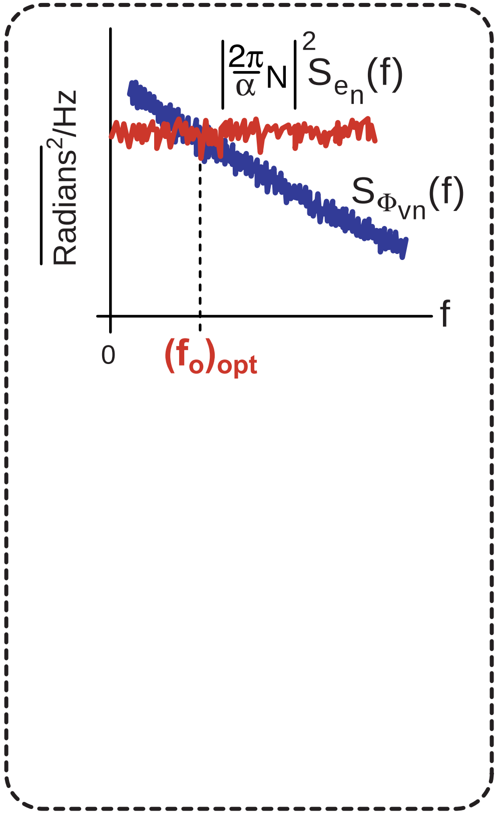




- **PFD-referred noise is scaled by square of divide value and inverse of PFD gain**
  - **High divide values lead to large multiplication of this noise**
- **VCO-referred noise is not scaled (only filtered)**

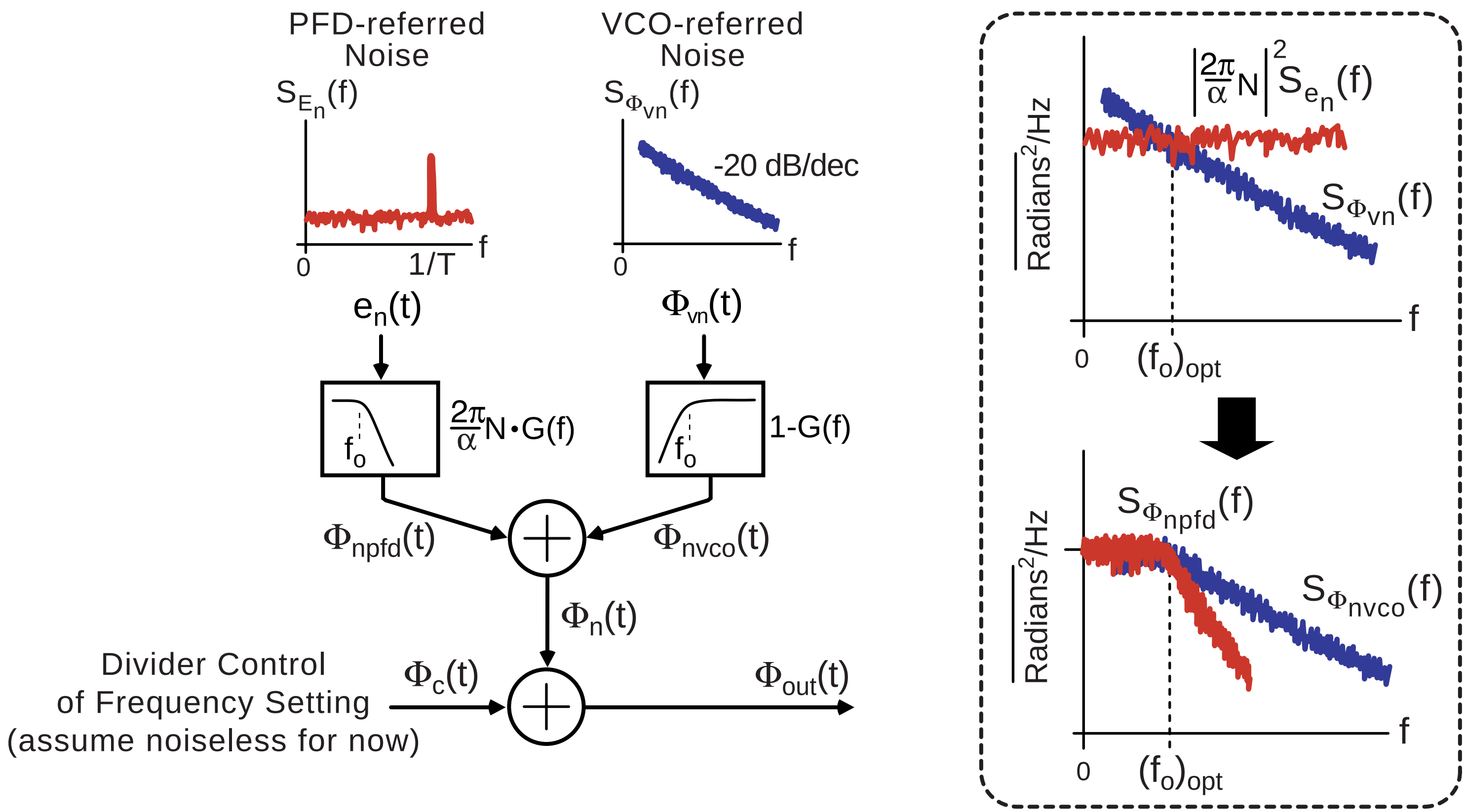
Optimal Bandwidth Setting for Minimum Noise





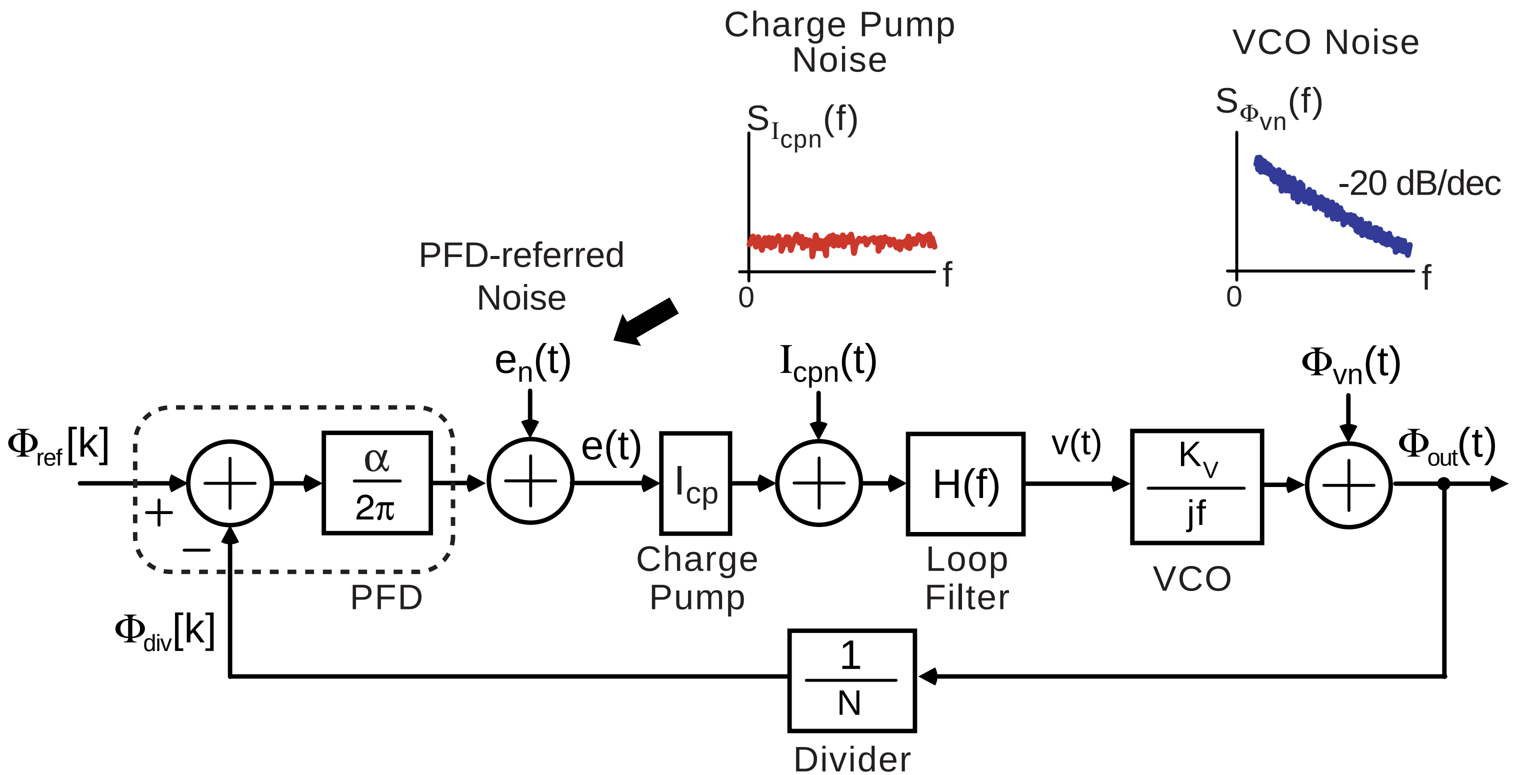
- Optimal bandwidth is where scaled noise sources meet
  - Higher bandwidth will pass more PFD-referred noise
  - Lower bandwidth will pass more VCO-referred noise

Resulting Output Noise with Optimal Bandwidth



- **PFD-referred noise dominates at low frequencies**
  - **Corresponds to close-in phase noise of synthesizer**
- **VCO-referred noise dominates at high frequencies**
  - **Corresponds to far-away phase noise of synthesizer**

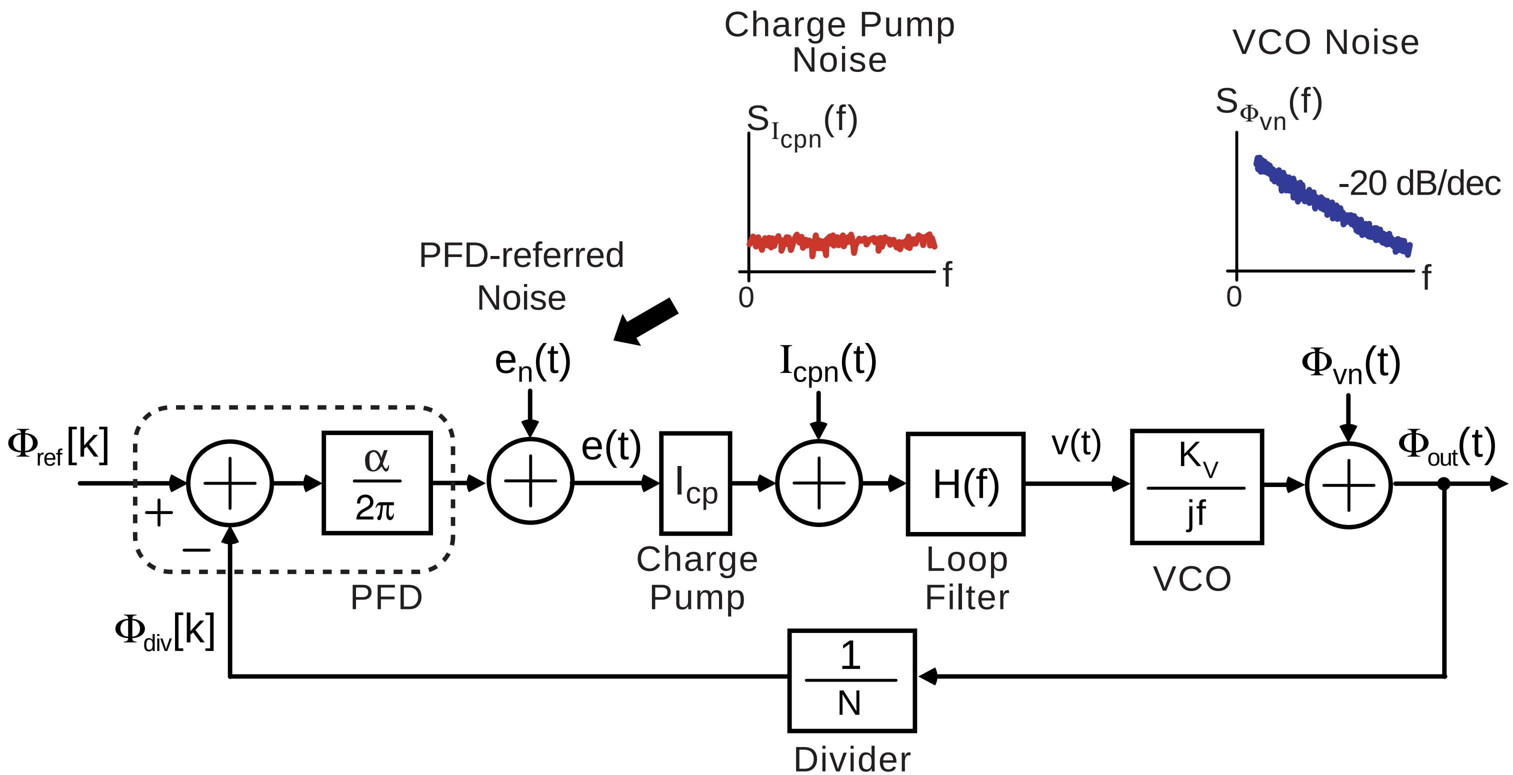
Analysis of Charge Pump Noise Impact



- **We can refer charge pump noise to PFD output by simply scaling it by $1/I_{cp}$**

$$\frac{\Phi_{out}}{I_{cpn}} = \left(\frac{1}{I_{cp}}\right)\frac{\Phi_{out}}{e_n} = \left(\frac{1}{I_{cp}}\right)\frac{2\pi}{\alpha}NG(f)$$

Calculation of Charge Pump Noise Impact

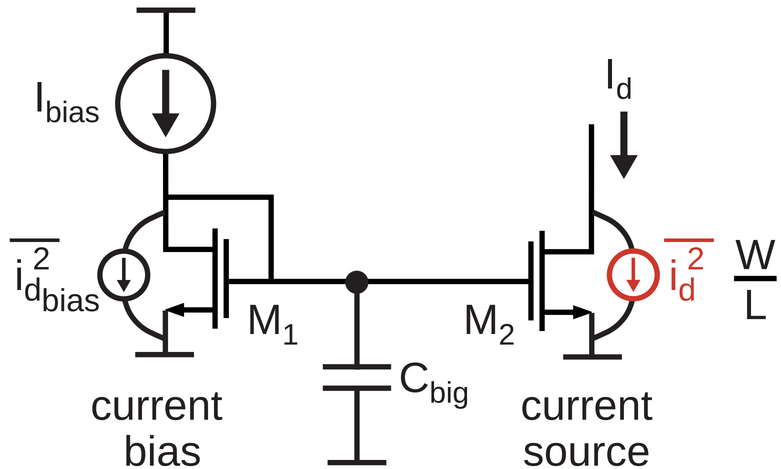


- **Contribution of charge pump noise to overall output noise**

$$S_{\Phi_{out}}(f) = \left(\frac{1}{I_{cp}}\right)^2 \left(\frac{2\pi}{\alpha}N\right)^2 |G(f)|^2 S_{I_{cpn}}(f) + \text{other sources}$$

  - **Need to determine impact of I_{cp} on $S_{Icpn}(f)$**

Impact of Transistor Current Value on its Noise



- **Charge pump noise will be related to the current it creates as**

$$S_{I_{cpn}}(f) \propto \frac{\overline{I_d^2}}{\Delta f} = 4kT\gamma g_{do}$$

- **Recall that g_{do} is the channel resistance at zero V_{ds}**
  - **At *a fixed current density*, we have**

$$g_{do} \propto W \propto I_d \Rightarrow \boxed{\overline{I_d^2} \propto I_d}$$

Impact of Charge Pump Current Value on Output Noise

- **Recall**

$$S_{\Phi_{out}}(f) = \left(\frac{1}{I_{cp}}\right)^2 \left(\frac{2\pi}{\alpha}N\right)^2 |G(f)|^2 S_{I_{cpn}}(f) + \text{other sources}$$

- **Given previous slide, we can say**

$$S_{I_{cpn}}(f) \propto I_{cp}$$

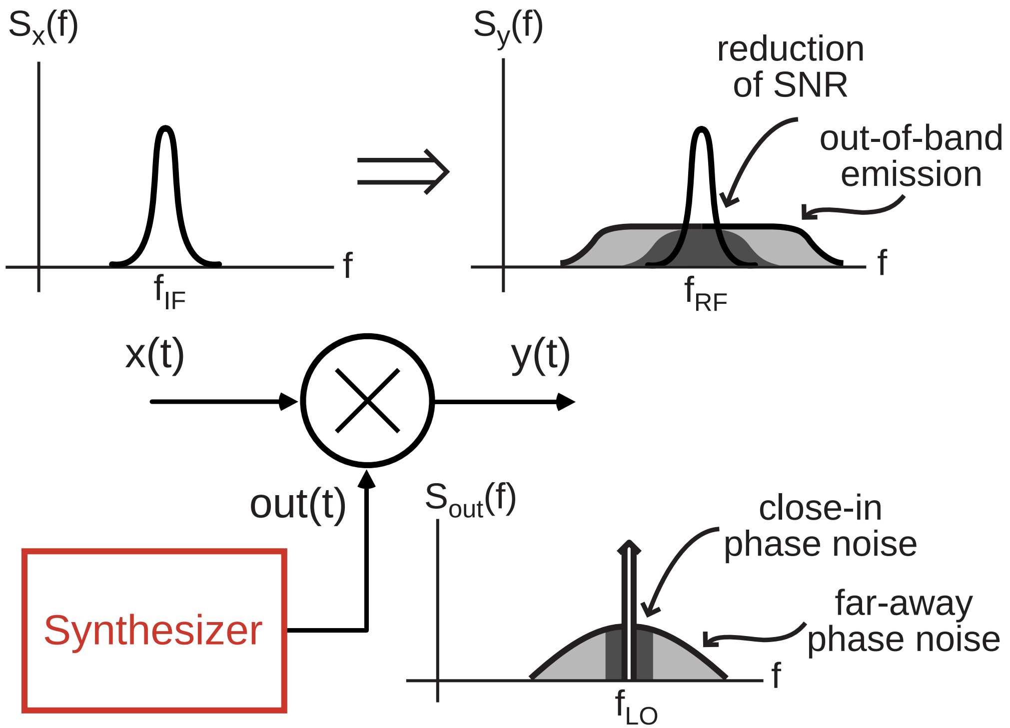
  - **Assumes a fixed current density for the key transistors in the charge pump as I_{cp} is varied**

- **Therefore**

$$S_{\Phi_{out}}(f)\Big|_{\text{charge pump}} \propto \frac{1}{I_{cp}}$$

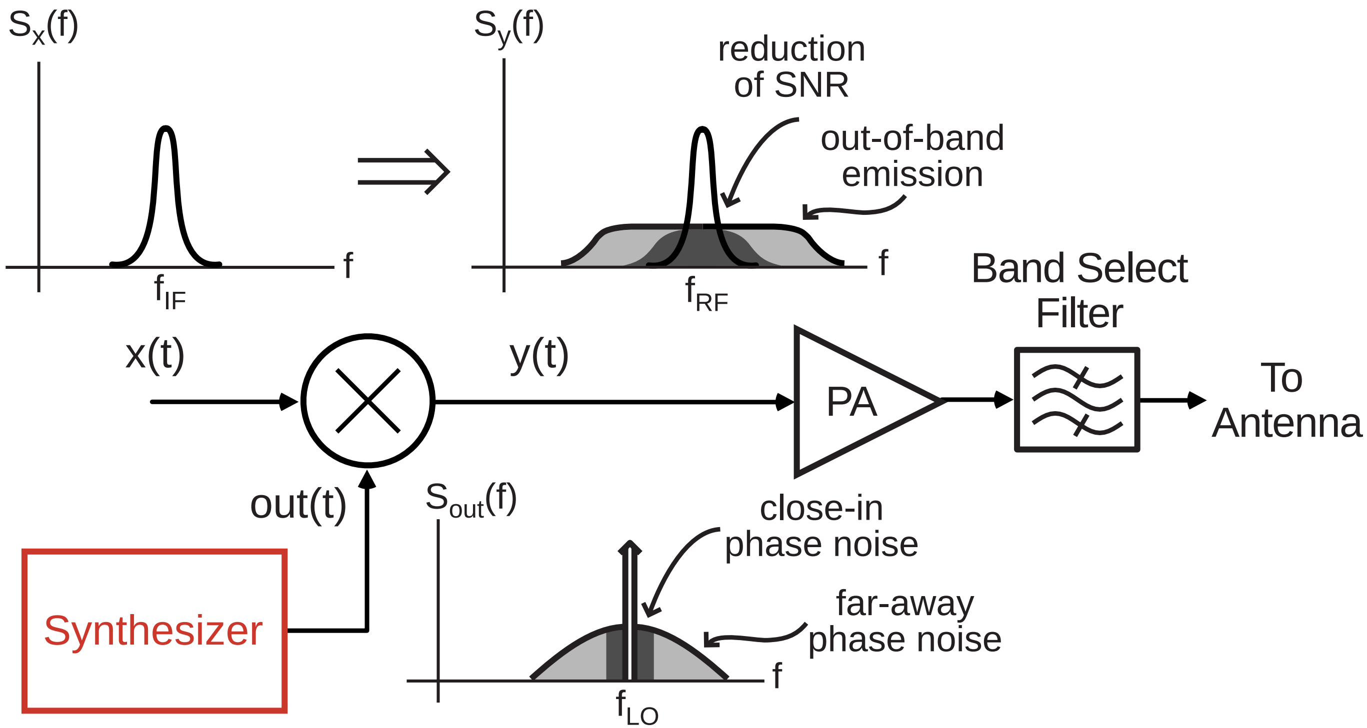
  - **Want high charge pump current to achieve low noise**
  - **Limitation set by power and area considerations**

Impact of Synthesizer Noise on Transmitters



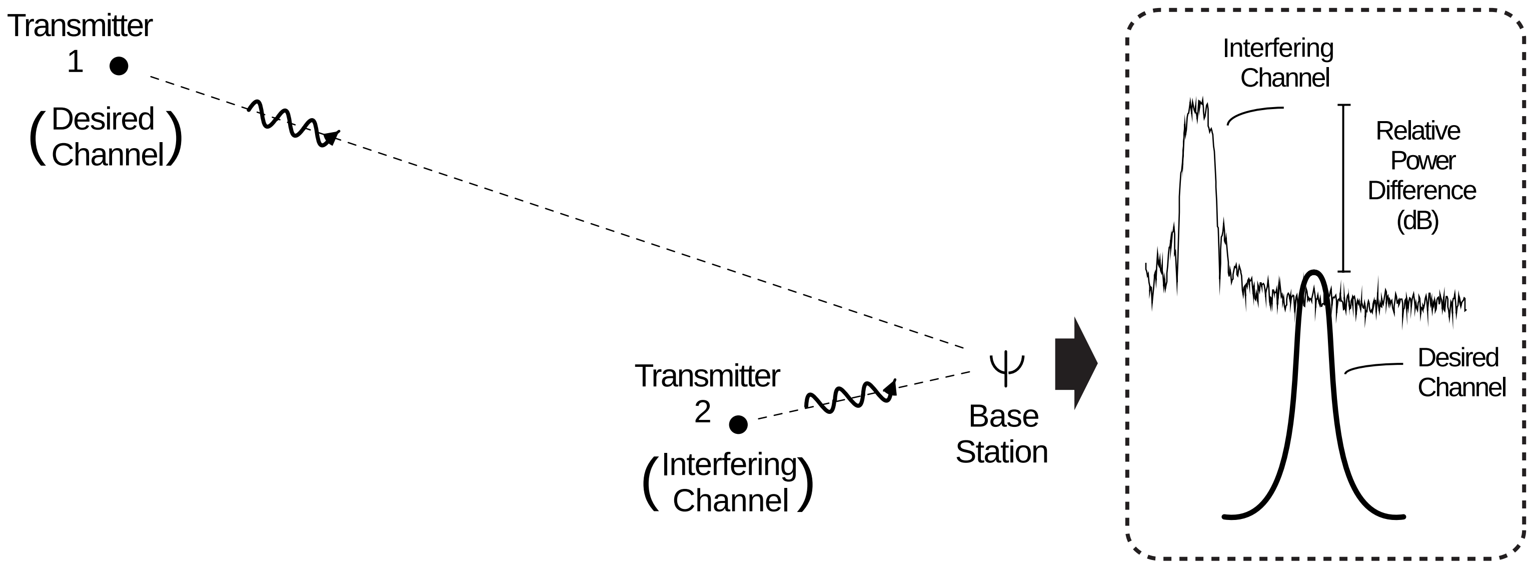
- **Synthesizer noise can be lumped into two categories**
  - **Close-in phase noise: reduces SNR of modulated signal**
  - **Far-away phase noise: creates spectral emissions outside the desired transmit channel**
    - This is the critical issue for transmitters

Impact of Remaining Portion of Transmitter



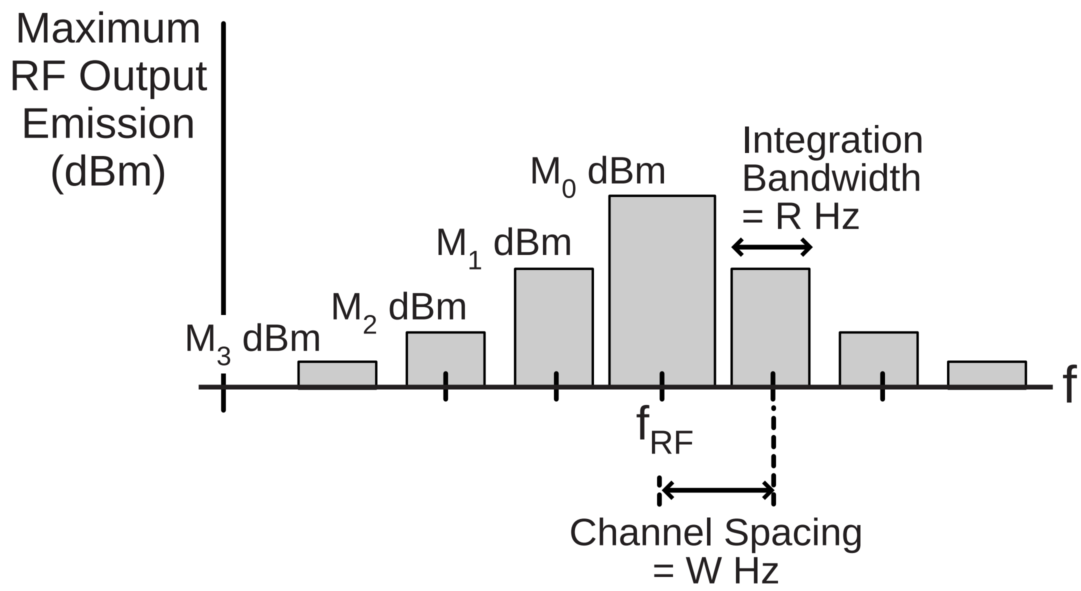
- **Power amplifier**
  - **Nonlinearity will increase out-of-band emission and create harmonic content**
- **Band select filter**
  - **Removes harmonic content, but not out-of-band emission**

Why is Out-of-Band Emission A Problem?



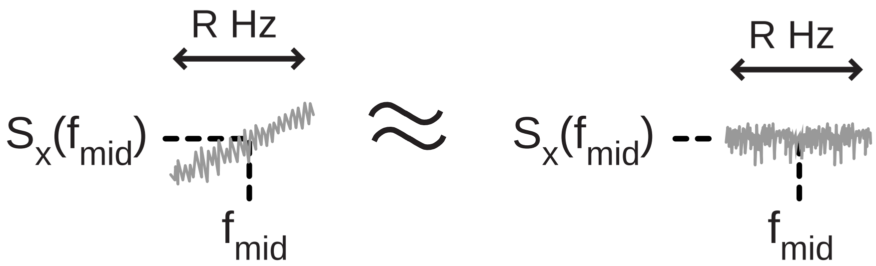
- **Near-far problem**
  - **Interfering transmitter closer to receiver than desired transmitter**
  - **Out-of-emission requirements must be stringent to prevent complete corruption of desired signal**

Specification of Out-of-Band Emissions



- **Maximum radiated power is specified in desired and adjacent channels**
  - **Desired channel power: maximum is M_0 dBm**
  - **Out-of-band emission: maximum power defined as integration of transmitted spectral density over bandwidth R centered at midpoint of each channel offset**

Calculation of Transmitted Power in a Given Channel



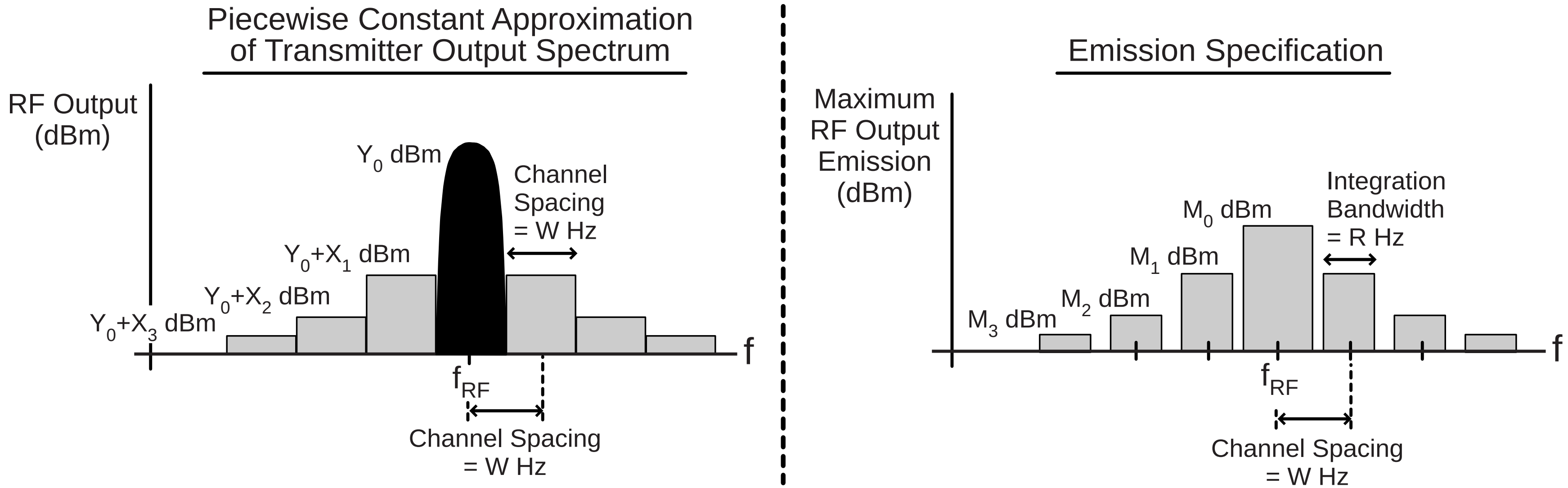
- **For simplicity, assume that the spectral density is flat over the channel bandwidth**
  - **Actual spectral density of signal often varies with frequency over the bandwidth of a given channel**
- **Resulting power calculation (single-sided $S_x(f)$)**

$$P_x = \int_{f_{mid}-R/2}^{f_{mid}+R/2} S_x(f)df \approx RS_x(f_{mid})$$

- **Express in dB (Note: dB(x) = 10log(x))**

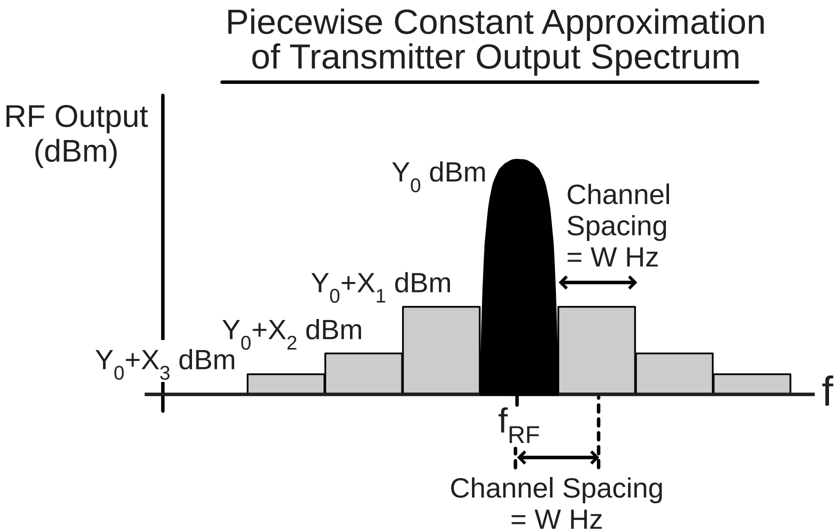
$$\mathrm{dB}(P_x) \approx \mathrm{dB}(RS_x(f_{mid})) = \boxed{\mathrm{dB}(S_x(f_{mid})) + \mathrm{dB}(R)}$$

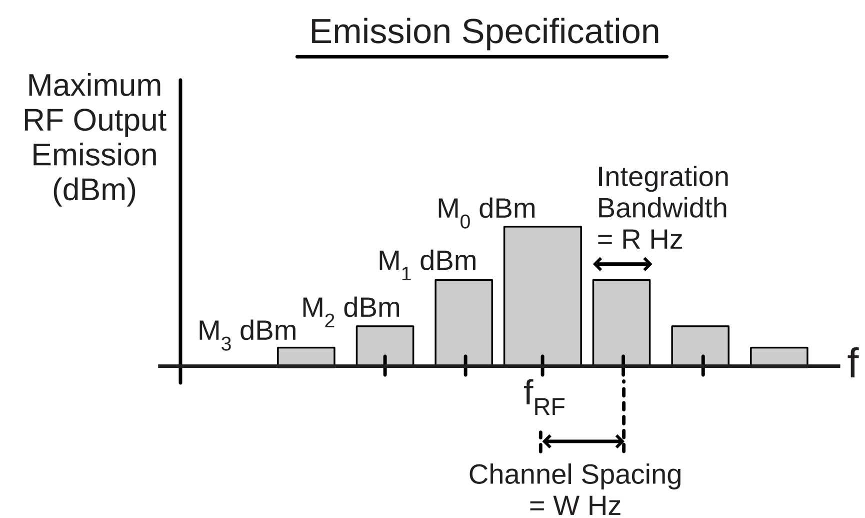
Transmitter Output Versus Emission Specification



- **Assume a piecewise constant spectral density profile for transmitter**
  - **Simplifies calculations**
- **Issue: emission specification is measured over a narrower band than channel spacing**
  - **Need to account for bandwidth discrepancy when doing calculations**

Correction Factor for Bandwidth Mismatch





- **Calculation of maximum emission in offset channel 1**

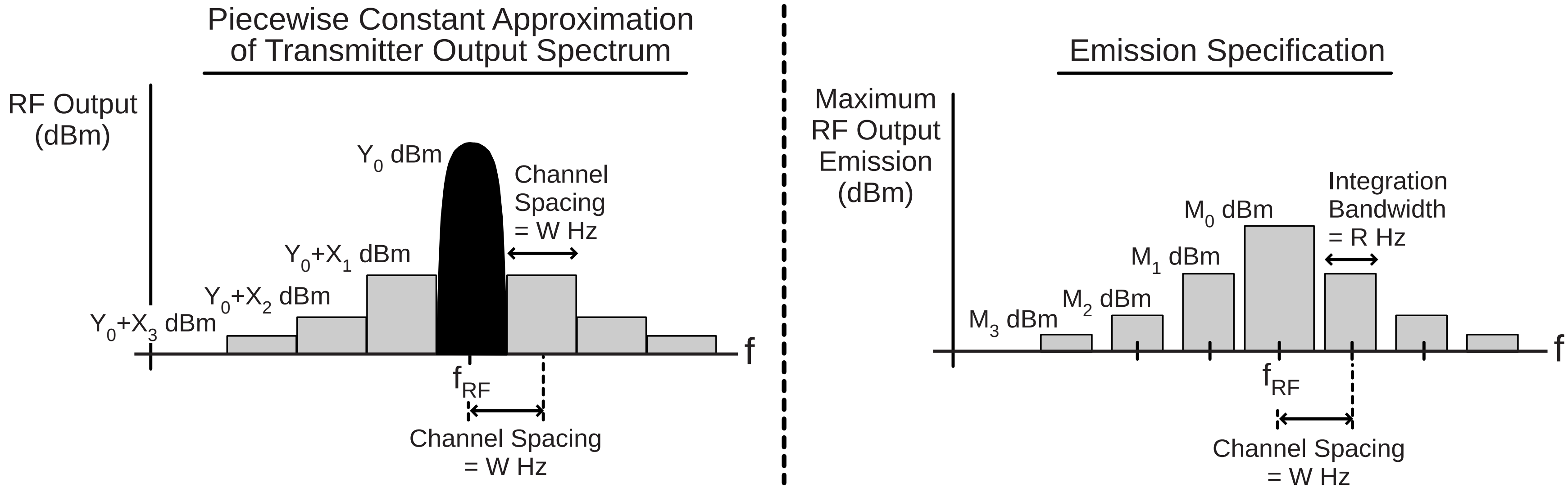
$$\mathrm{dB}(S_{(Y_0+X_1)}R) \leq M_1$$

$$\Rightarrow \mathrm{dB}\left(S_{(Y_0+X_1)}W\frac{R}{W}\right) \leq M_1$$

$$\Rightarrow \mathrm{dB}\left(S_{(Y_0+X_1)}W\right) + \mathrm{dB}\left(\frac{R}{W}\right) \leq M_1$$

$$\Rightarrow Y_0+X_1+\mathrm{dB}\left(\frac{R}{W}\right) \leq M_1 \quad \Rightarrow \boxed{X_1 \leq M_1 - Y_0 + \mathrm{dB}\left(\frac{W}{R}\right)}$$

Condition for Most Stringent Emission Requirement



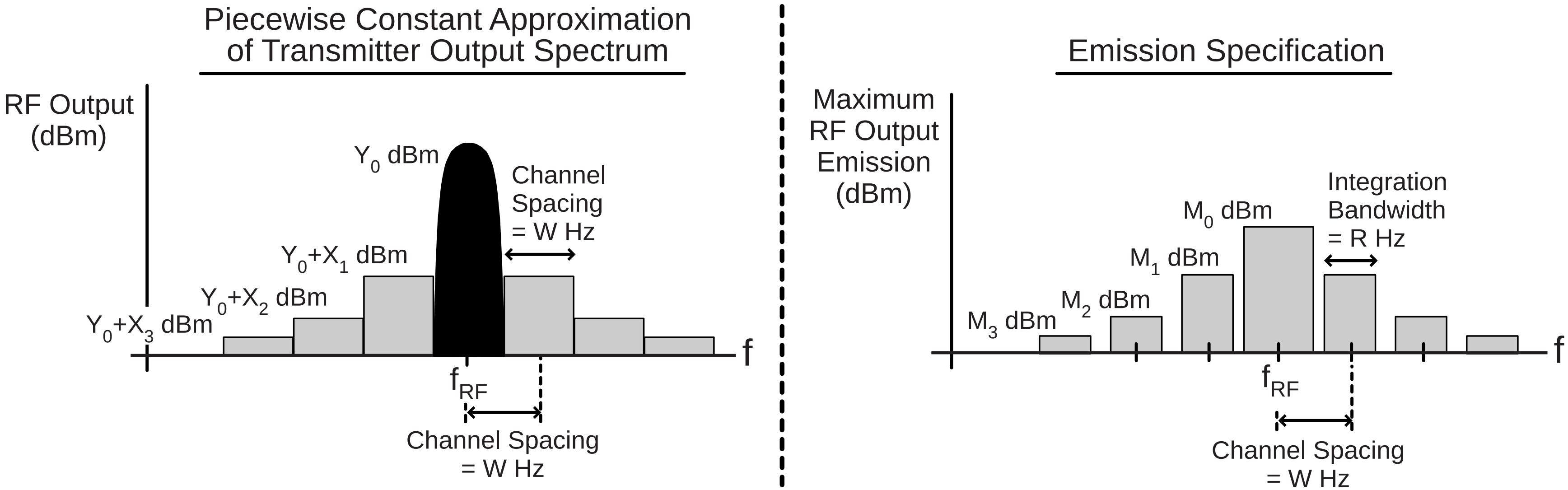
- **Out-of-band emission requirements are function of the power of the signal in the desired channel**
  - **For offset channel 1 (as calculated on previous slide)**

$$X_1 \leq M_1 - Y_0 + \mathrm{dB}\left(\frac{W}{R}\right)$$

  - **Most stringent case is when Y_0 maximum**

$$\Rightarrow Y_0 = M_0$$

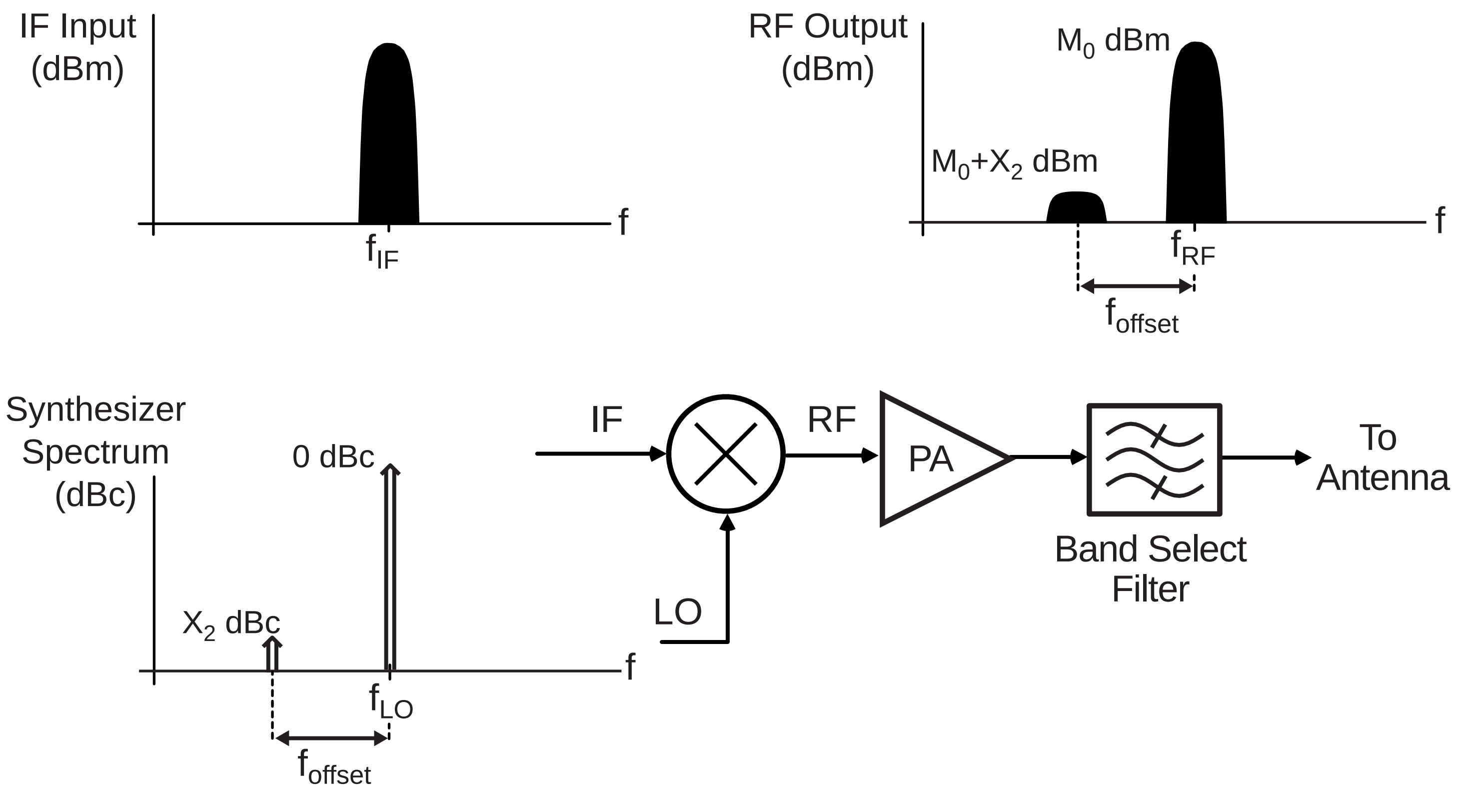
Table of Most Stringent Emission Requirements



Channel Offset	Mask Power	Emission Requirements (Most Stringent)
0	M_0 dBm	$Y_0 = M_0$ (for most stringent case)
1	M_1 dBm	$X_1 = M_1 - M_0 + \text{dB}(W/R)$ dB
2	M_2 dBm	$X_2 = M_2 - M_0 + \text{dB}(W/R)$ dB
3	M_3 dBm	$X_3 = M_3 - M_0 + \text{dB}(W/R)$ dB

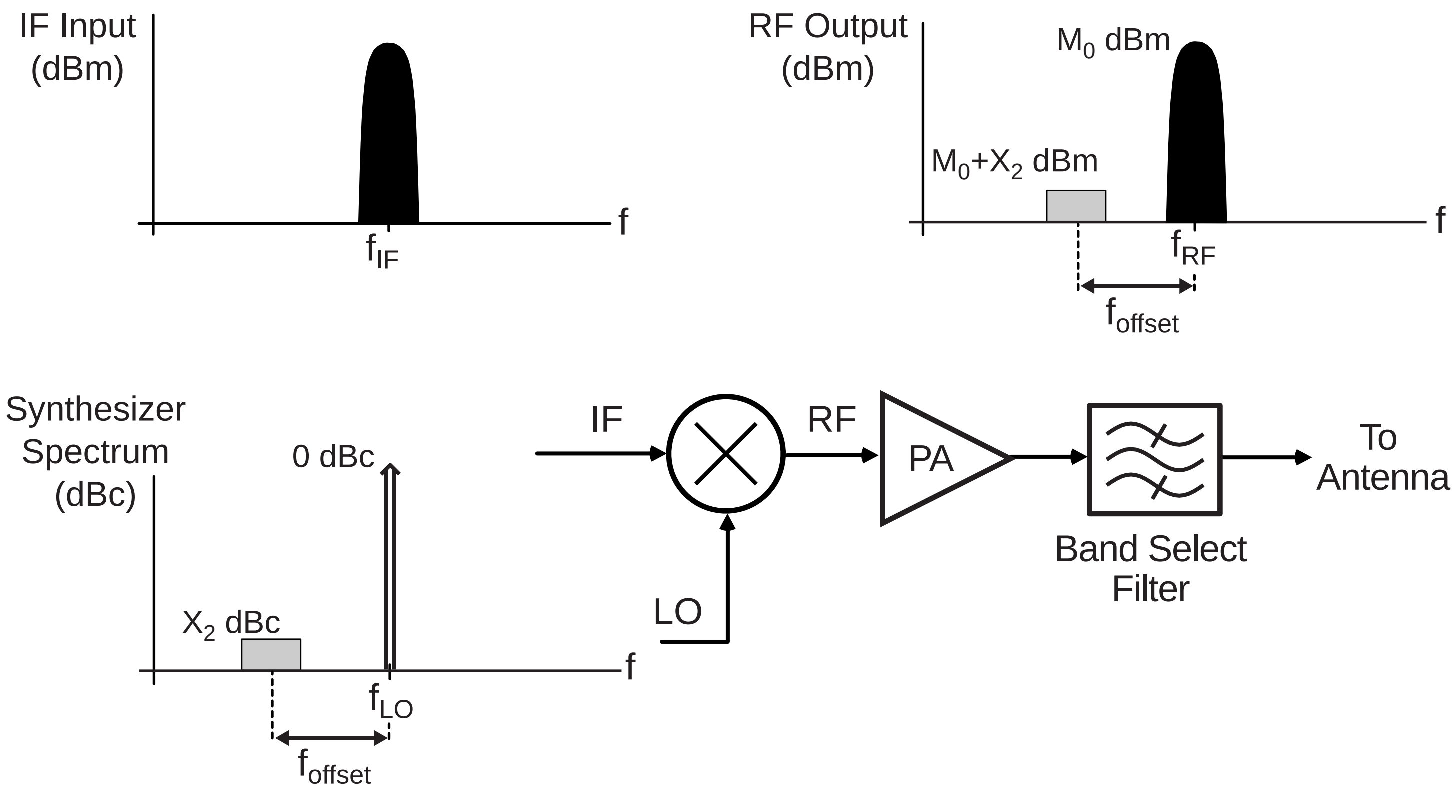
$$(\text{Note}: \quad \text{dB}(W/R) = 10\log(W/R))$$

Impact of Synthesizer Noise on Transmitter Output



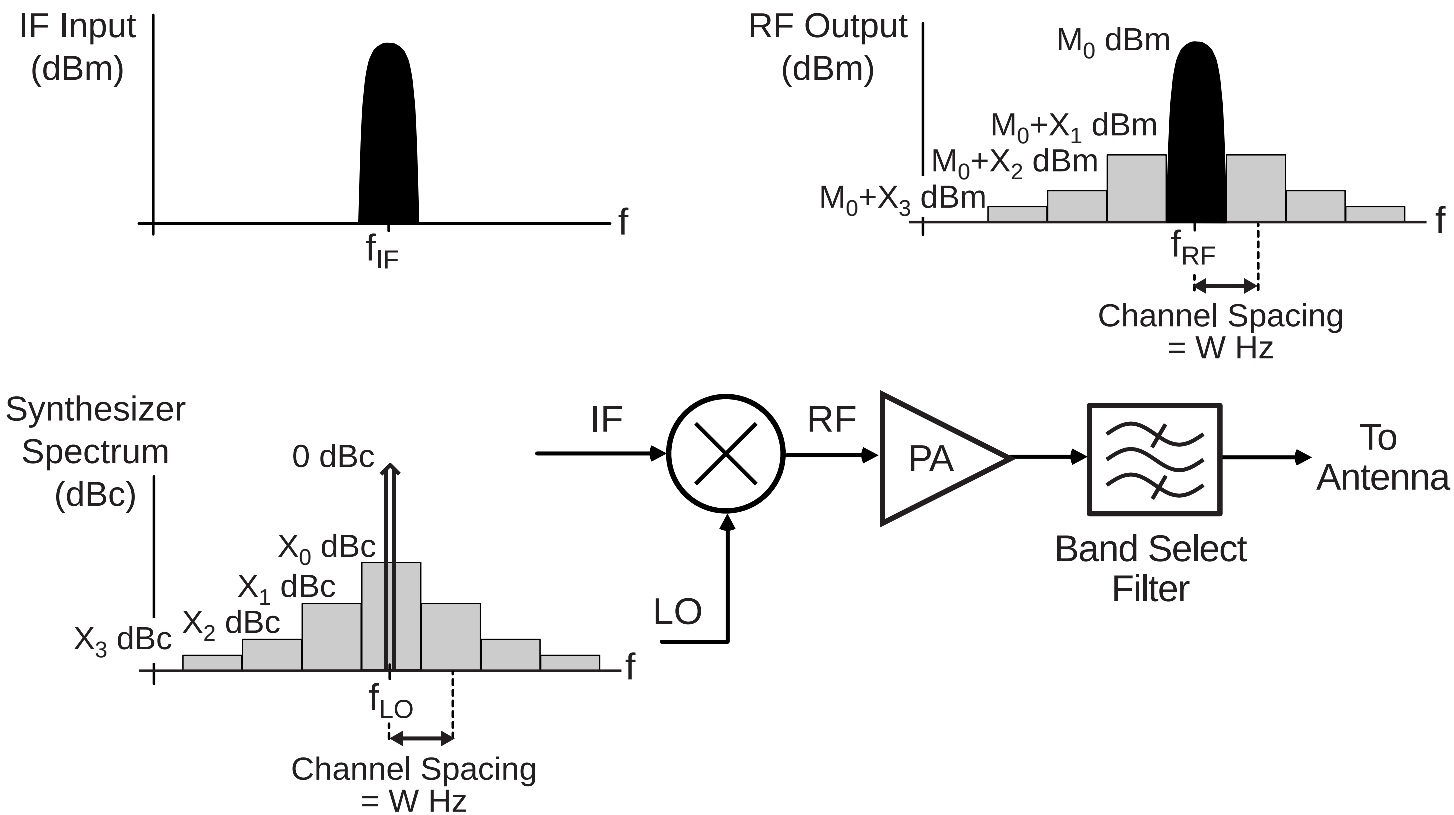
- **Consider a spurious tone at a given offset frequency**
  - **Convolution with IF signal produces a replica of the desired signal at the given offset frequency**

Impact of Synthesizer Phase Noise (Isolated Channel)



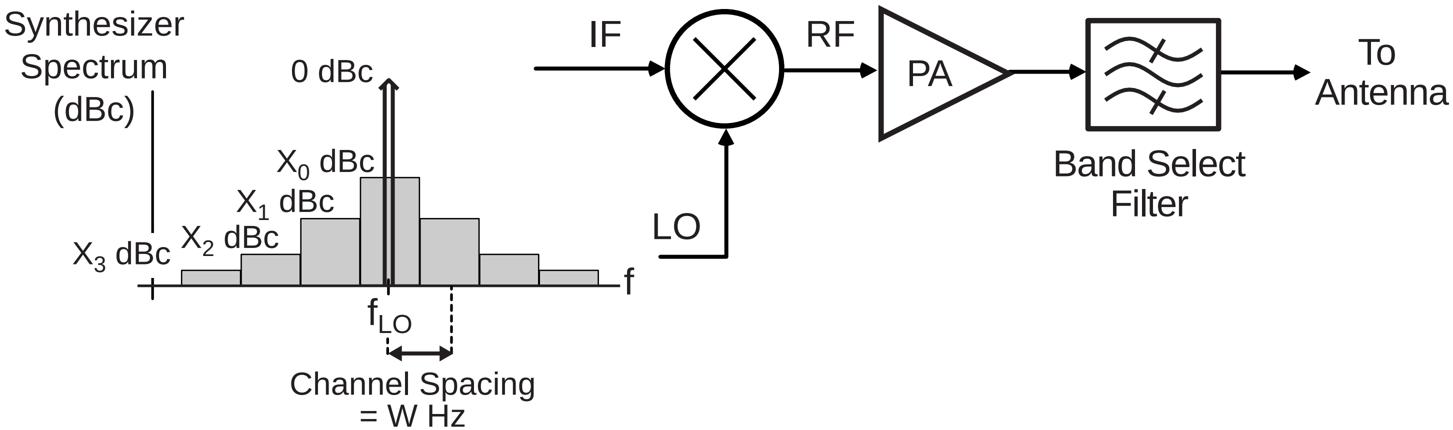
- **Consider phase noise at a given offset frequency**
  - **Convolution with IF signal produces a smeared version of the desired signal at the given offset frequency**
    - For simplicity, approximate smeared signal as shown

Impact of Synthesizer Phase Noise (All Channels)



- **Partition synthesizer phase noise into channels**
  - **Required phase noise power (dBc) in each channel is related directly to spectral mask requirements**
    - Exception is X_0 – set by transmit SNR requirements

Synthesizer Phase Noise Requirements



- **Impact of channel bandwidth (offset channel 1)**

$$\mathrm{dB}(S_{X_1}W) \le X_1 \text{ dBc} \Rightarrow \boxed{\mathrm{dB}(S_{X_1}) \le X_1 - \mathrm{dB}(W) \text{ dBc/Hz}}$$

- **Overall requirements (most stringent, i.e., $Y_0 = M_0$)**

Channel Offset	Emission Requirements (Most Stringent)	Maximum Synth. Phase Noise (Most Stringent)
0	$Y_0 = M_0$	set by required transmit SNR
1	$X_1 = M_1 - M_0 + \mathrm{dB}(W/R)$ dB	$X_1 - \mathrm{dB}(W)$ dBc/Hz
2	$X_2 = M_2 - M_0 + \mathrm{dB}(W/R)$ dB	$X_2 - \mathrm{dB}(W)$ dBc/Hz
3	$X_3 = M_3 - M_0 + \mathrm{dB}(W/R)$ dB	$X_3 - \mathrm{dB}(W)$ dBc/Hz

Example – DECT Cordless Telephone Standard

- Standard for many cordless phones operating at 1.8 GHz
- Transmitter Specifications
  - Channel spacing: W = 1.728 MHz
  - Maximum output power: M_o = 250 mW (24 dBm)
  - Integration bandwidth: R = 1 MHz
  - Emission mask requirements

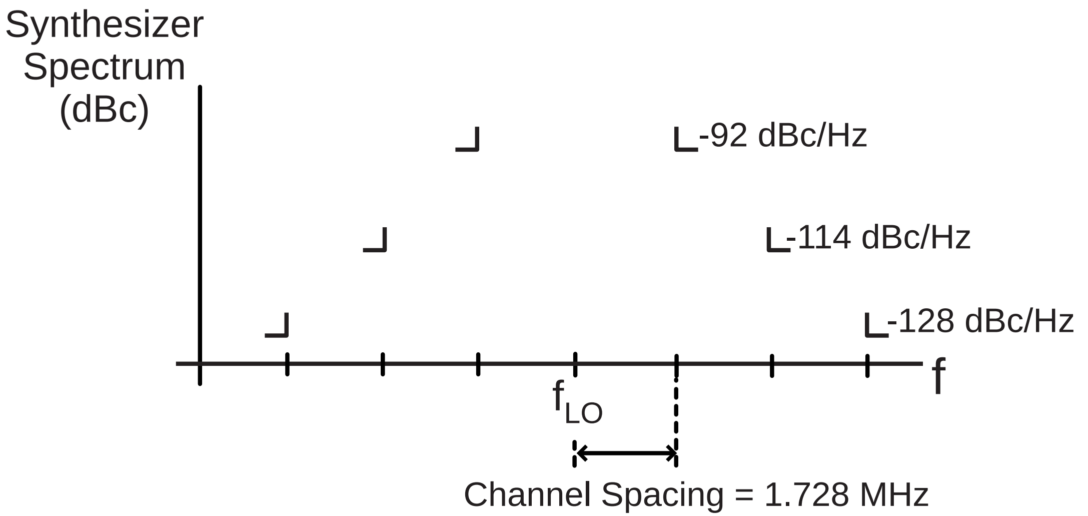
f_{offset} (MHz)	Emission Mask (dBm)
0	M_0 = 24 dBm
1.728	M_1 = -8 dBm
3.456	M_2 = -30 dBm
5.184	M_3 = -44 dBm

Synthesizer Phase Noise Requirements for DECT

- **Using previous calculations with DECT values**

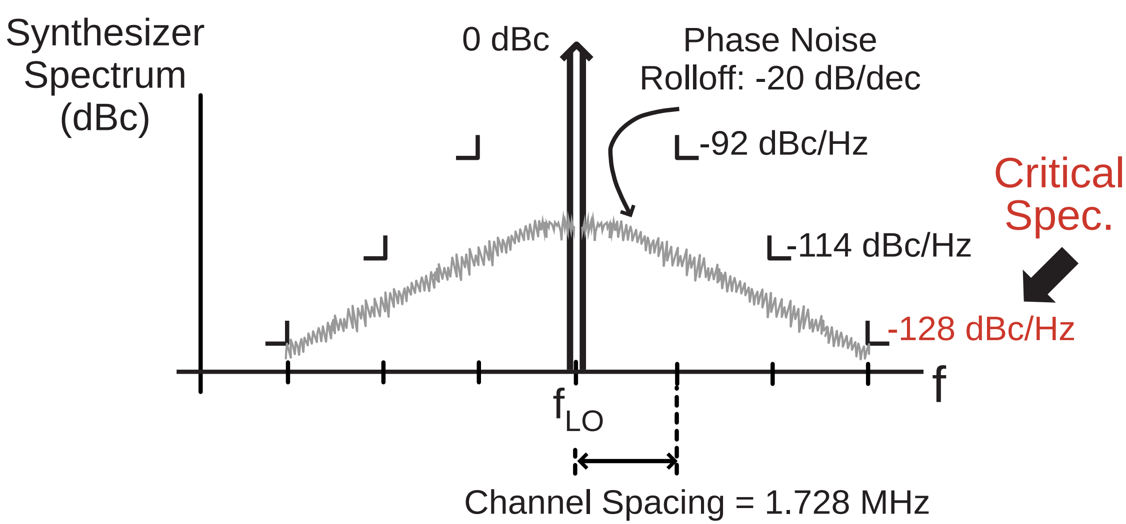
Channel Offset	Mask Power	Maximum Synth. Noise Power in Integration BW	Maximum Synth. Phase Noise at Channel Offset
0	24 dBm	set by required transmit SNR	
1.728 MHz	-8 dBm	X_1 = -29.6 dBc	-92 dBc/Hz
3.456 MHz	-30 dBm	X_2 = -51.6 dBc	-114 dBc/Hz
5.184 MHz	-44 dBm	X_3 = -65.6 dBc	-128 dBc/Hz

- **Graphical display of phase noise mask**

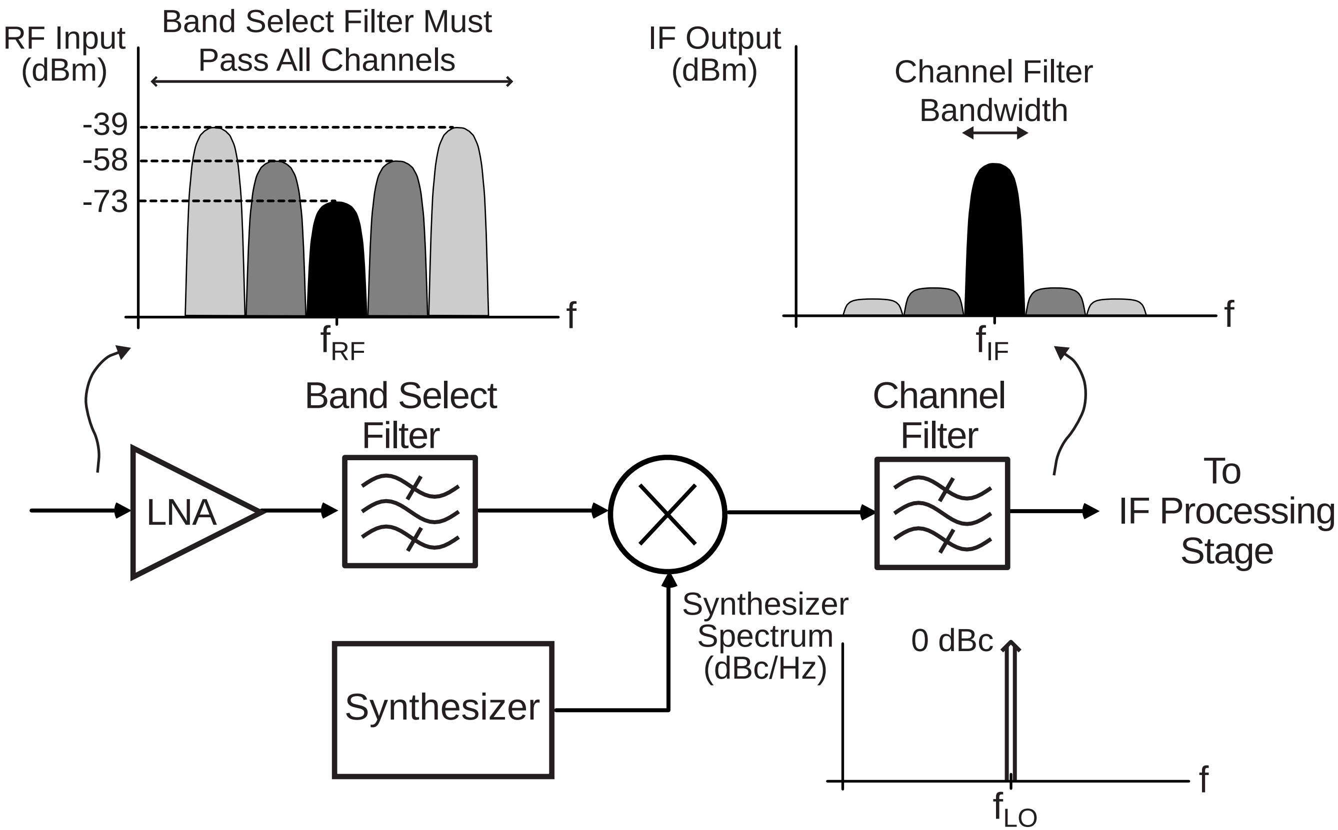


Critical Specification for Phase Noise

- Critical specification is defined to be the one that is hardest to meet with an assumed phase noise rolloff
  - Assume synthesizer phase noise rolls off at -20 dB/decade
    - Corresponds to VCO phase noise characteristic
- For DECT transmitter synthesizer
  - Critical specification is -128 dBc/Hz at 5.184 MHz offset

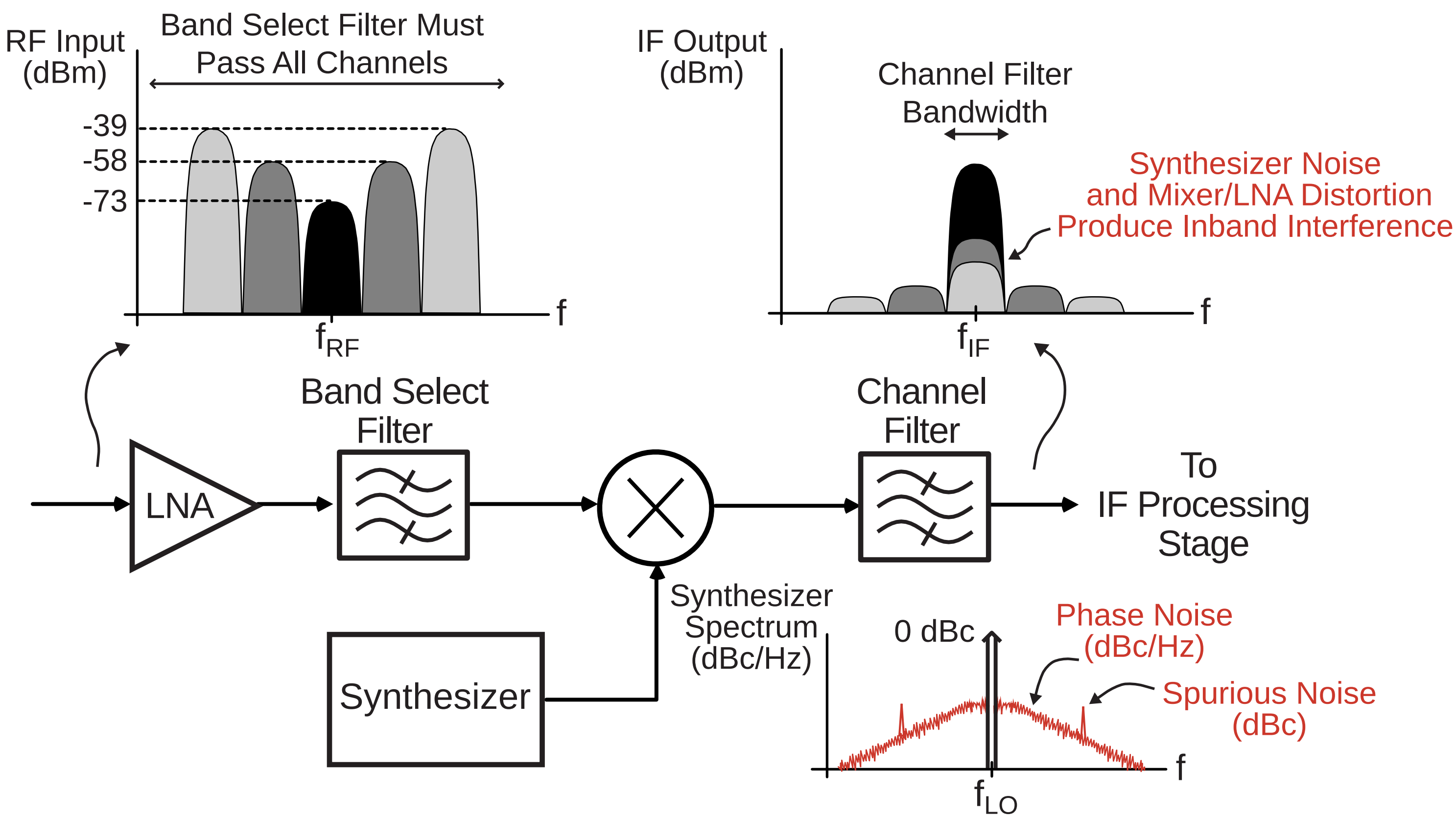


Receiver Blocking Performance



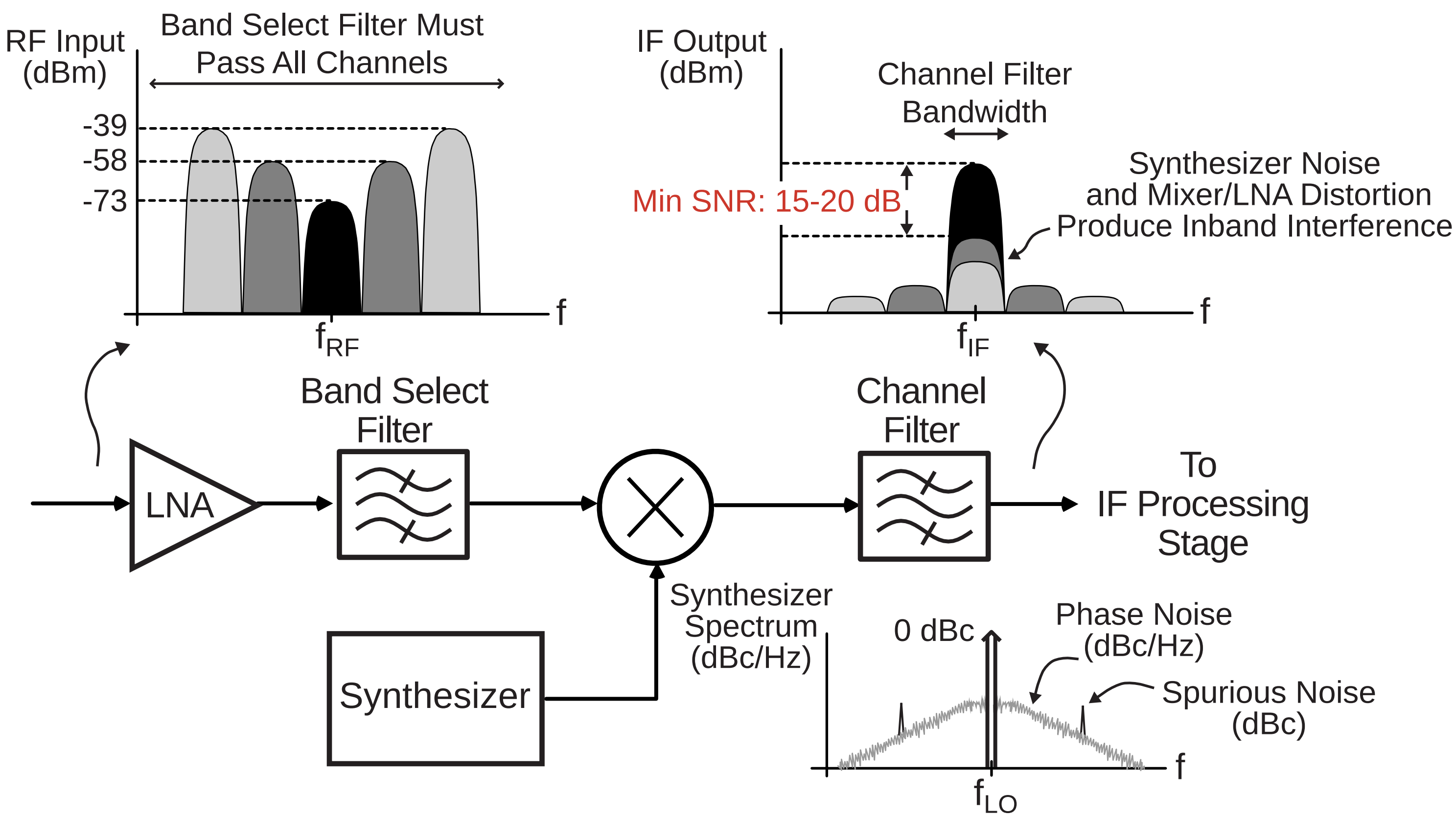
- **Radio receivers must operate in the presence of large interferers (called blockers)**
- **Channel filter plays critical role in removing blockers**
  - Passes desired signal channel, rejects interferers

Impact of Nonidealities on Blocking Performance



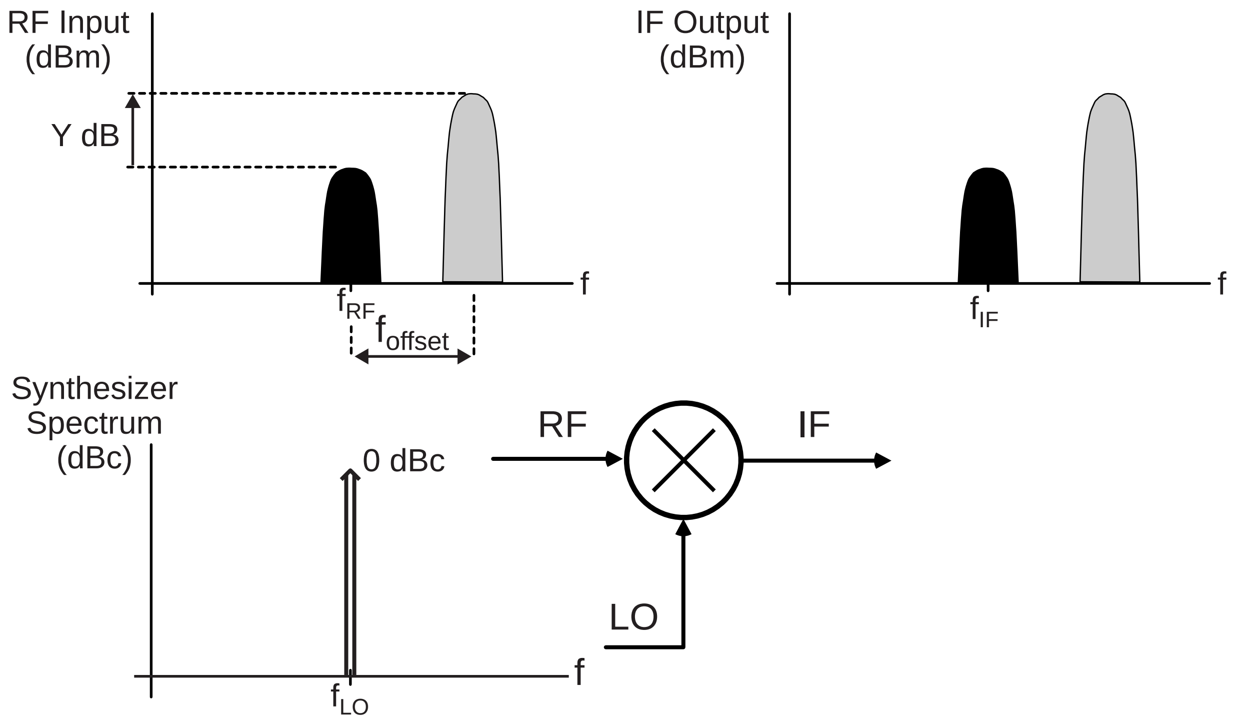
- **Blockers leak into desired band due to**
  - **Nonlinearity of LNA and mixer (IIP3)**
  - **Synthesizer phase and spurious noise**
- **In-band interference cannot be removed by channel filter!**

Quantifying Tolerable In-Band Interference Levels



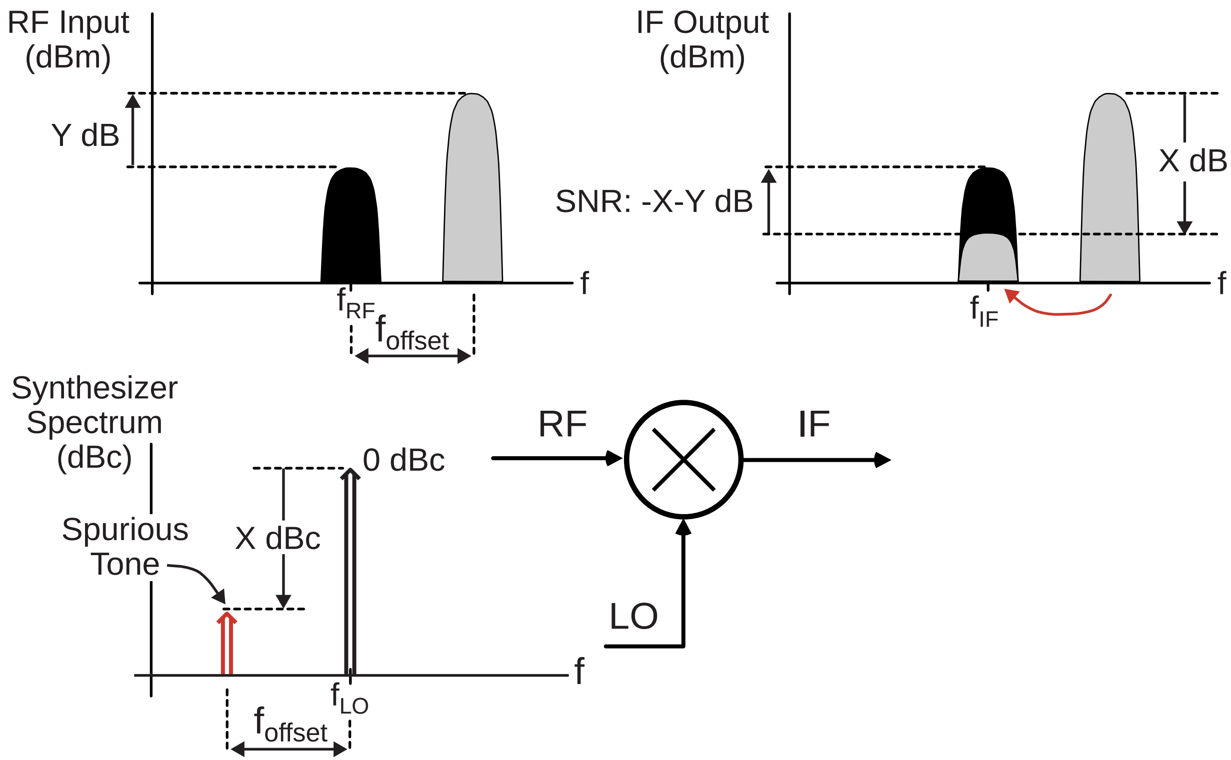
- **Digital radios quantify performance with bit error rate (BER)**
  - **Minimum BER often set at 1e-3 for many radio systems**
  - **There is a corresponding minimum SNR that must be achieved**
- **Goal: design so that SNR with interferers is above SNR_{min}**

Impact of Synthesizer on Blockers



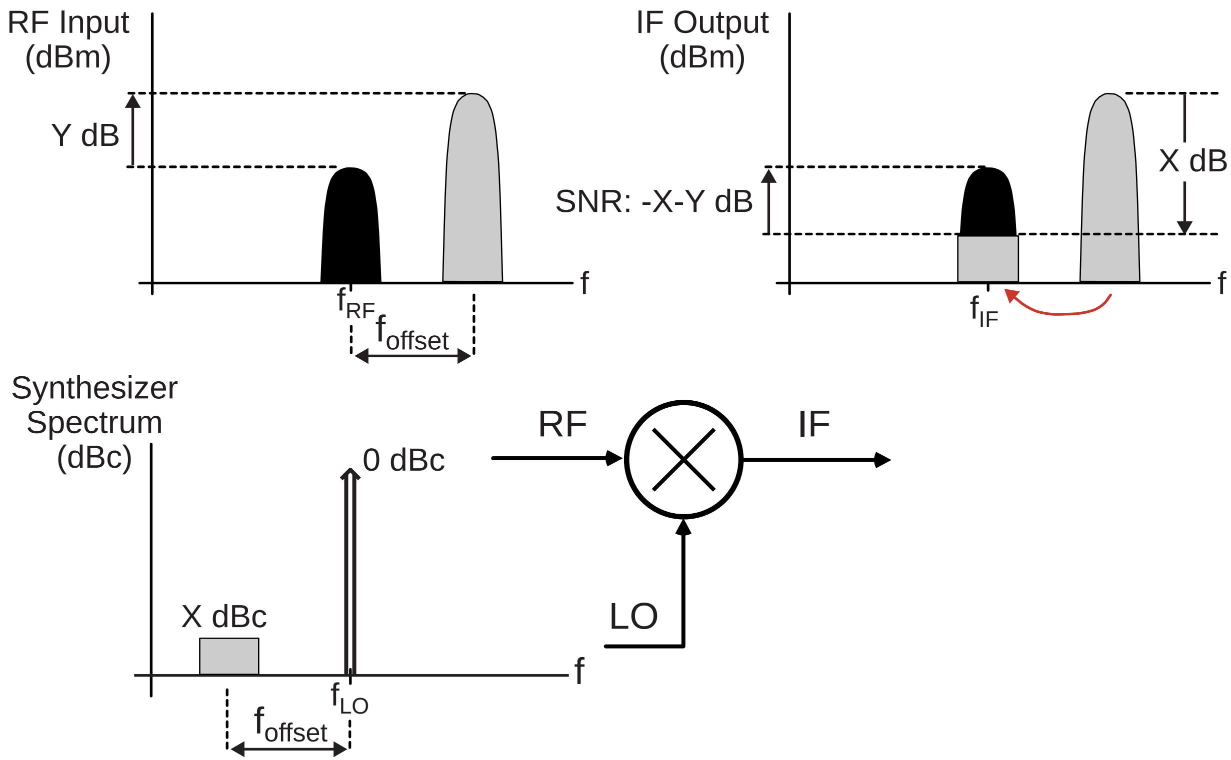
- **Synthesizer passes desired signal and blocker**
  - **Assume blocker is Y dB higher in signal power than desired signal**

Impact of Synthesizer Spurious Noise on Blockers



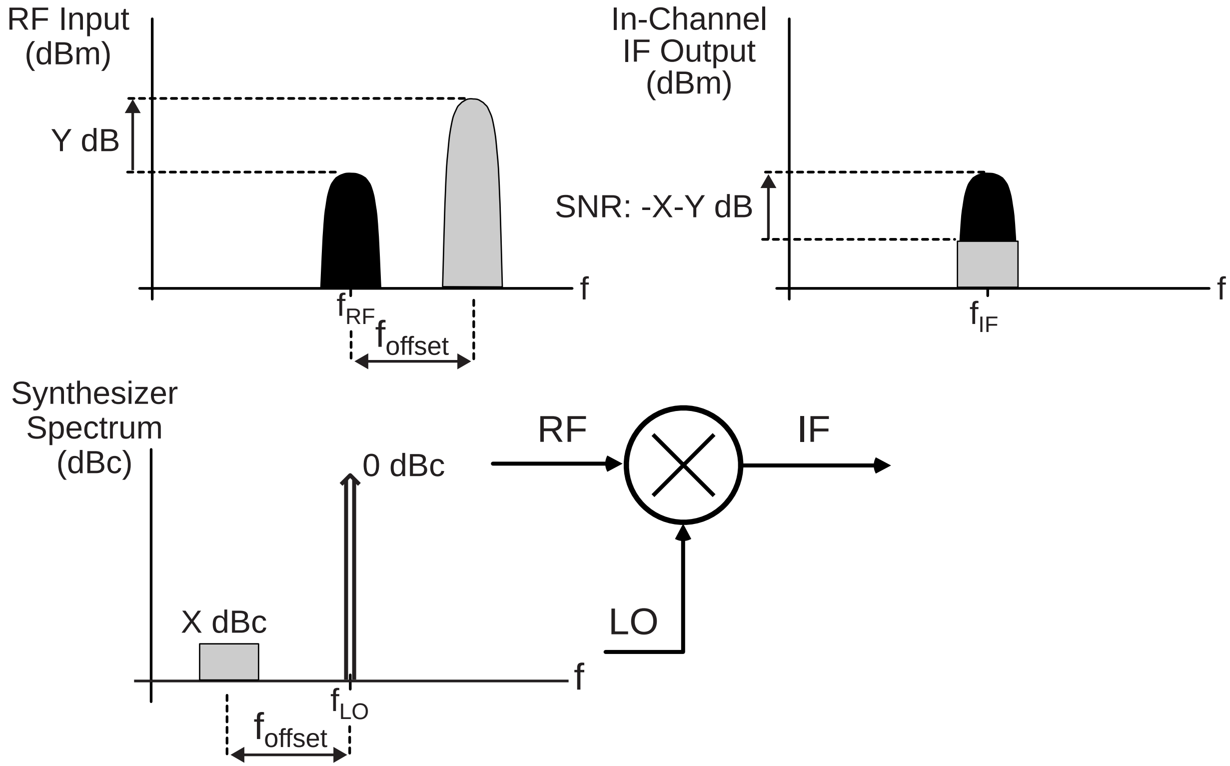
- **Spurious tones cause the blocker (Y dB) (and desired) signals to "leak" into other frequency bands**
  - **In-band interference occurs when spurious tone offset frequency is same as blocker offset frequency**
  - **Resulting SNR = -X-Y dB with spurious tone (X dBc)**

Impact of Synthesizer Phase Noise on Blockers



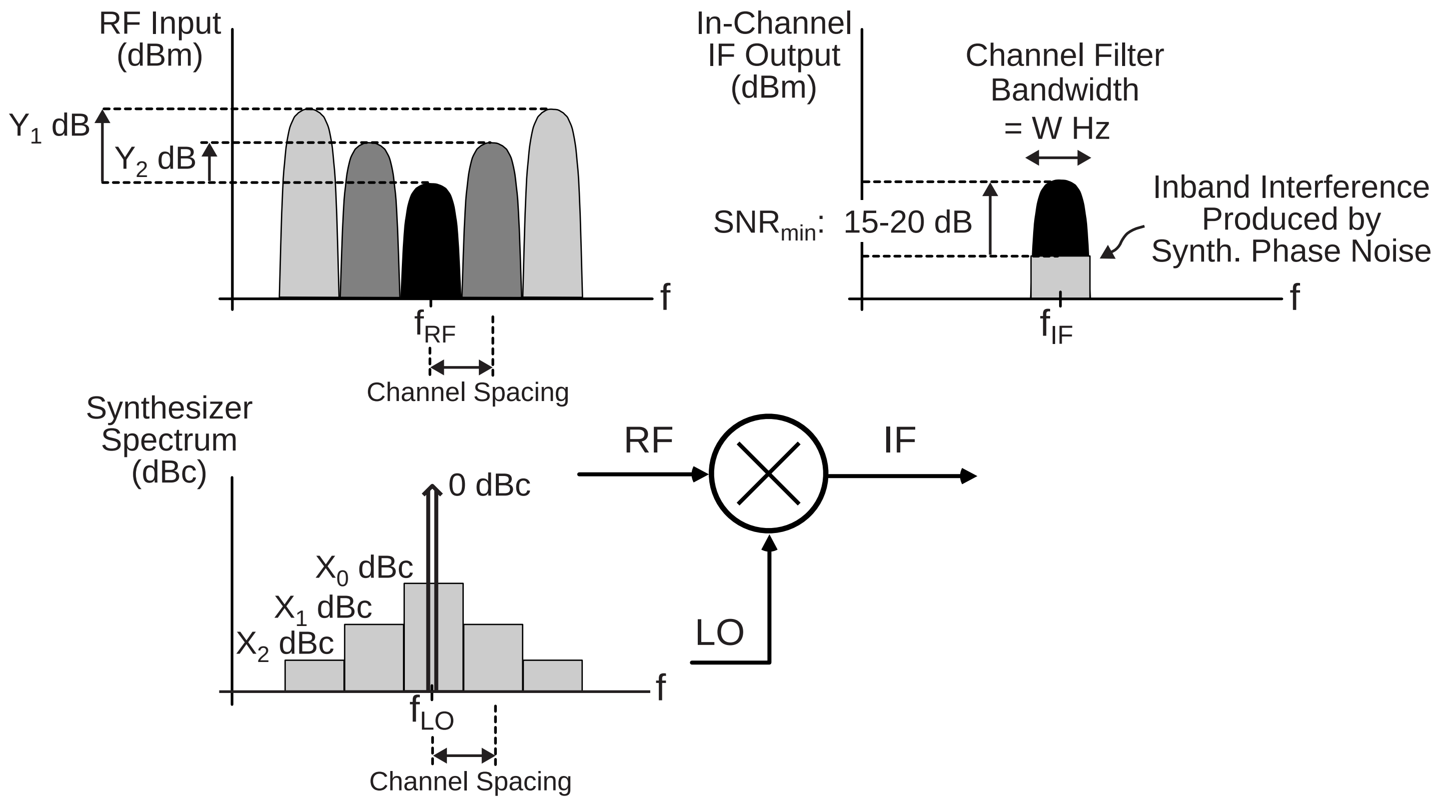
- **Same impact as spurious tone, but blocker signal is “smeared” by convolution with phase noise**
  - **For simplicity, ignore “smearing” and approximate as shown above**

Blocking Performance Analysis (Part 1)



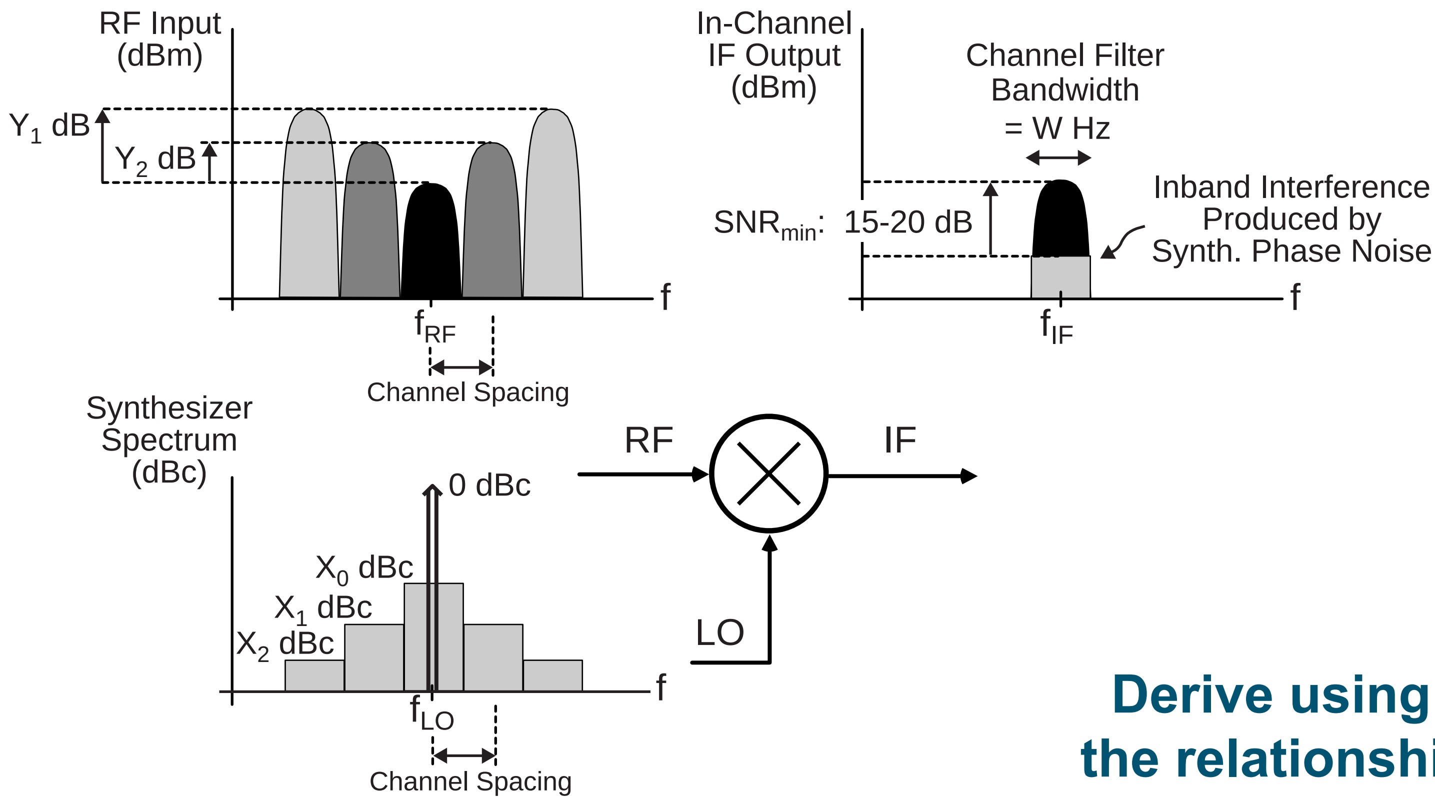
- Ignore all out-of-band energy at the IF output
  - Assume that channel filter removes it
  - Motivation: simplifies analysis

Blocking Performance Analysis (Part 2)



- Consider the impact of blockers surrounding the desired signal with a given phase noise profile
  - SNR_{min} must be maintained
  - Evaluate impact on SNR one blocker at a time

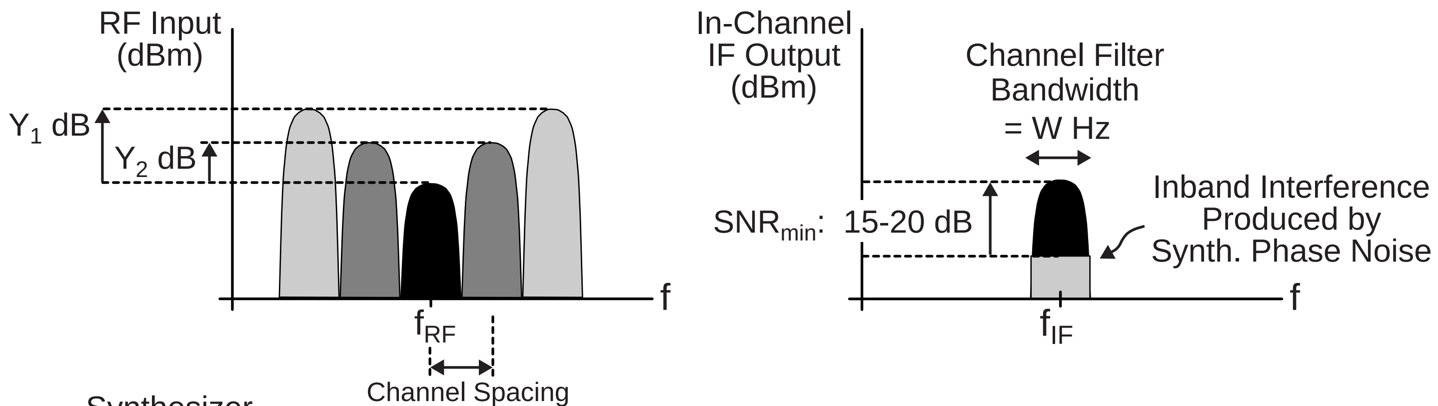
Blocking Performance Analysis (Part 3)

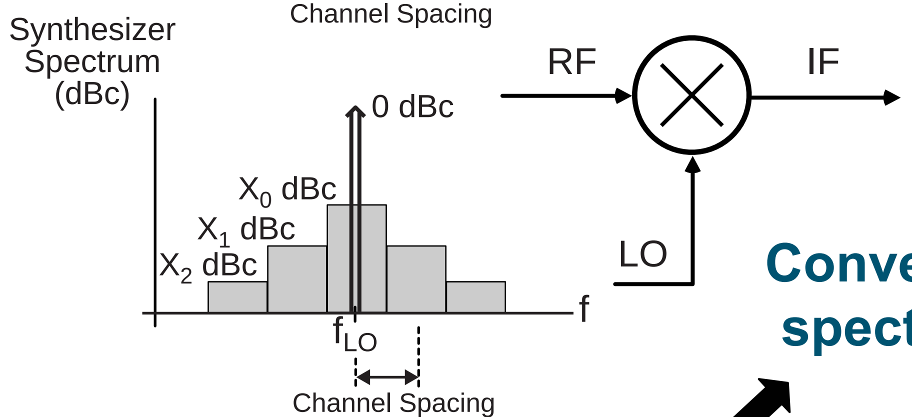


Derive using the relationship $SNR = -X-Y$ dB $>= SNR_{min}$

Channel Offset	Relative Blocking Power	Maximum Synth. Noise Power at Channel Offset
0	0 dB	$X_0 = -SNR_{min}$ dBc
1	Y_1 dB	$X_1 = -SNR_{min} - Y_1$ dBc
2	Y_2 dB	$X_2 = -SNR_{min} - Y_2$ dBc
3	Y_3 dB	$X_3 = -SNR_{min} - Y_3$ dBc

Blocking Performance Analysis (Part 4)





Convert power to spectral density

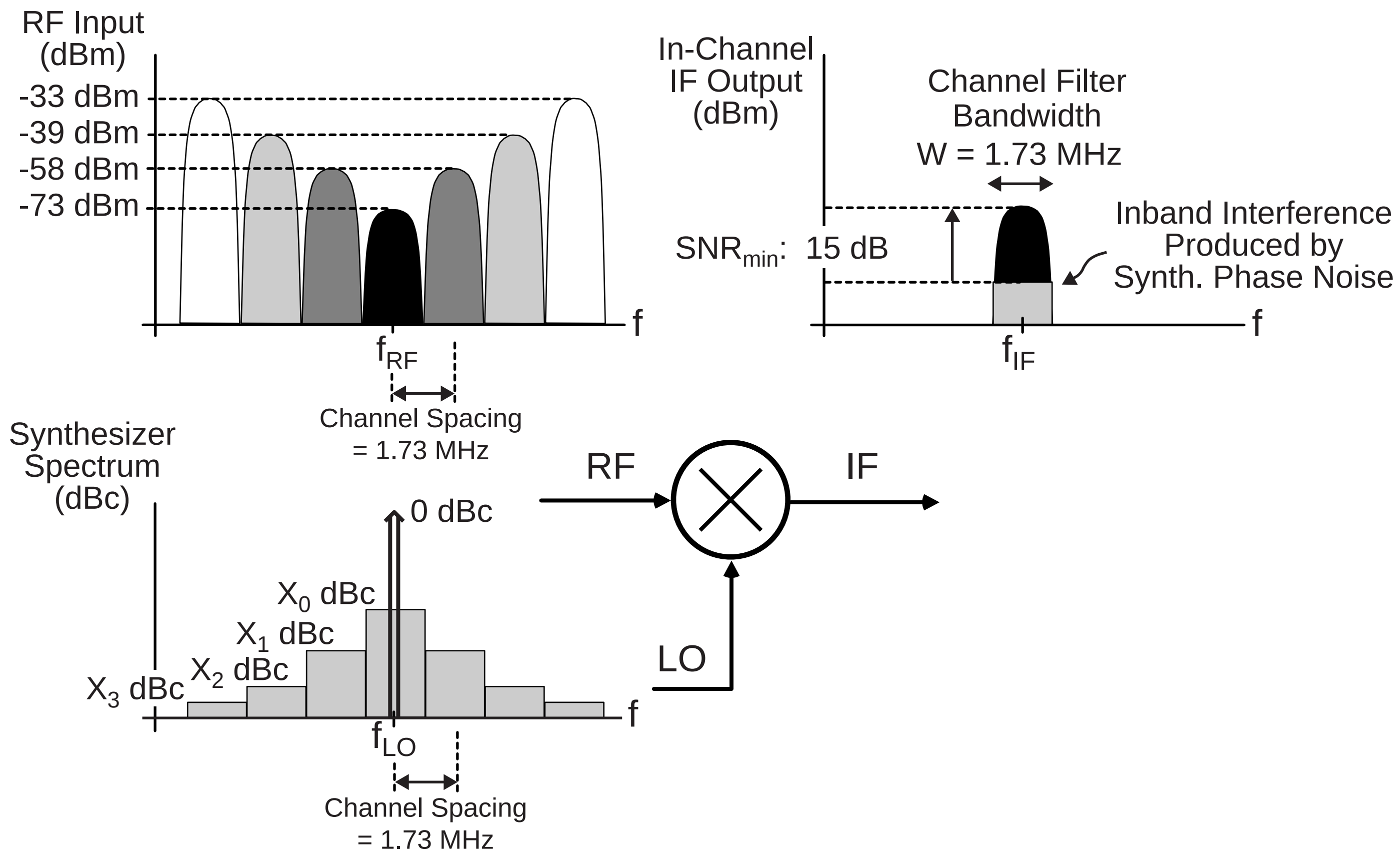
Channel Offset	Relative Blocking Power	Maximum Synth. Noise Power at Channel Offset	Maximum Synth. Phase Noise at Channel Offset
0	0 dB	$X_0 = -SNR_{min}$ dBc	X_0 - dB(W) dBc/Hz
1	Y_1 dB	$X_1 = -SNR_{min} - Y_1$ dBc	X_1 - dB(W) dBc/Hz
2	Y_2 dB	$X_2 = -SNR_{min} - Y_2$ dBc	X_2 - dB(W) dBc/Hz
3	Y_3 dB	$X_3 = -SNR_{min} - Y_3$ dBc	X_3 - dB(W) dBc/Hz

Example – DECT Cordless Telephone Standard

- **Receiver blocking specifications**
  - **Channel spacing: W = 1.728 MHz**
  - **Power of desired signal for blocking test: -73 dBm**
  - **Minimum bit error rate (BER) with blockers: 1e-3**
    - Sets the value of SNR_{min}
      - Perform receiver simulations to determine SNR_{min}
    - Assume SNR_{min} = 15 dB for calculations to follow
  - **Strength of interferers for blocking test**

f_{offset} (MHz)	Blocker Power (dBm)	Relative Strength
1.728	-58 dBm	$Y_1 = 15$ dB
3.456	-39 dBm	$Y_2 = 34$ dB
5.184	-33 dBm	$Y_3 = 40$ dB

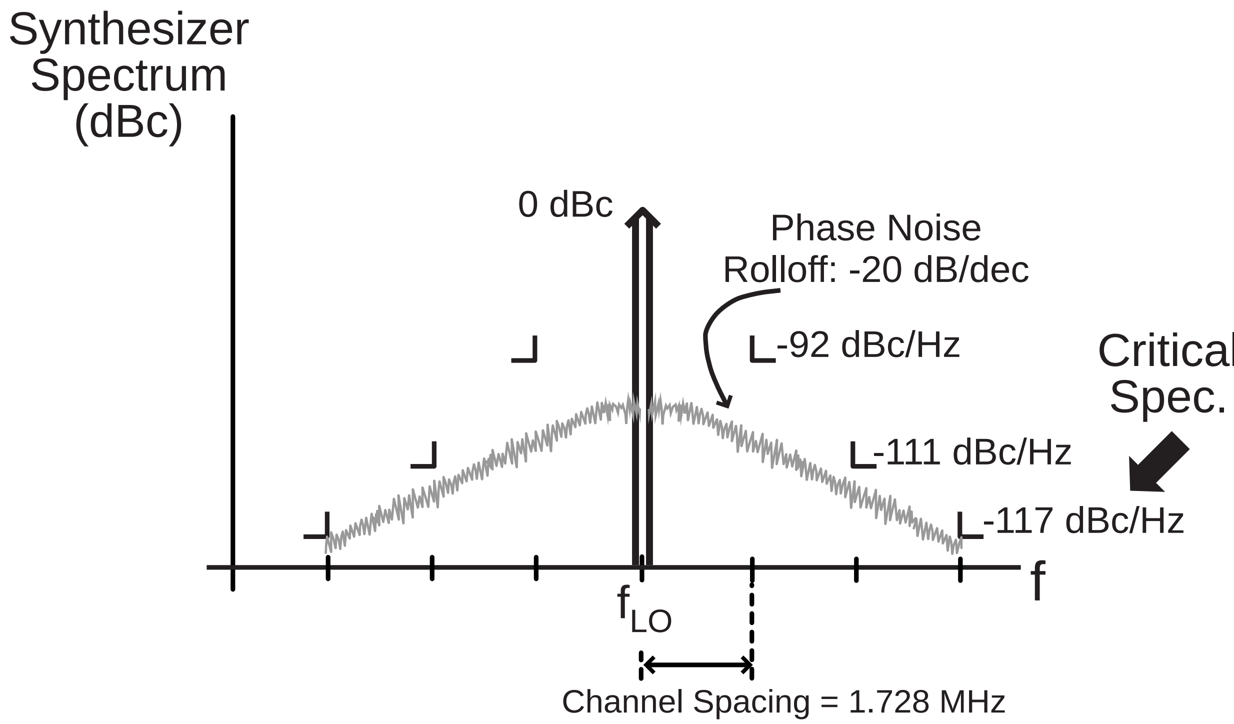
Synthesizer Phase Noise Requirements for DECT



Channel Offset	Relative Blocking Power	Maximum Synth. Noise Power at Channel Offset	Maximum Synth. Phase Noise at Channel Offset
0	0 dB	X_0 = -15 dBc	-77 dBc/Hz
1.728 MHz	Y_1 = 15 dB	X_1 = -30 dBc	-92 dBc/Hz
3.456 MHz	Y_2 = 34 dB	X_2 = -49 dBc	-111 dBc/Hz
5.184 MHz	Y_3 = 40 dB	X_3 = -55 dBc	-117 dBc/Hz

Graphical Display of Required Phase Noise Performance

- **Mark phase noise requirements at each offset frequency**



- **Calculate critical specification for receive synthesizer**
  - **Critical specification is -117 dBc/Hz at 5.184 MHz offset**
    - Lower performance demanded of receiver synthesizer than transmitter synthesizer in DECT applications!