LECTURE 060 -PLL DESIGN EQUATIONS AND **MEASUREMENTS INTRODUCTION**

Objective

The objective of this presentation is

- 1.) To provide a summary of relationships and equations that can be used to design PLLs.
- 2.) Illustrate the design of a DPLL frequency synthesizer
- 3.) Show how to make measurements on PLLs

Outline

- PLL design equations
- PLL design example
- PLL measurements
- Summary

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Lecture 060 – PLL Design Equations and Measurements (09/01/03)

Page 060-2

PLL DESIGN EQUATIONS

Introduction

The following design equations are to be used in designing PLLs and apply both to LPPLs and DPLLs with the following definitions:

LPLLs:
$$N = 1$$
 and $\beta = 1$

where N is the divider in the feedback loop and β is the loop expansion factor determined by the type of PFD.

Loop gain =
$$K_v = \frac{K_d K_o F(0)}{N}$$

Goal of these equations:

Permit the basic design of an LPLL or DPLL.

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Type – I, First-Order Loop (F(0) = 1)

Crossover frequency (frequency at which the loop gain is 1 or 0dB):

$$\omega_c = K_v \text{ (radians/sec.)}$$

-3dB Bandwidth (frequency at which the closed-loop gain is equal to -3dB):

Closed loop transfer function = $\frac{K_v}{s + K_v}$ $\rightarrow \omega_{-3\text{dB}} = K_v \text{ (radians/sec.)}$

Noise Bandwidth:

$$B_n = \int_0^\infty |H(j2\pi f)|^2 df = \int_0^\infty \frac{K_v^2}{K_v^2 + (2\pi f)^2} df = \frac{K_v}{2\pi} \int_0^\infty \frac{K_v}{K_v^2 + (2\pi f)^2} d(2\pi f) = \frac{K_v}{2\pi} \frac{\pi}{2} = \frac{K_v}{4} \text{ (Hz)}$$

Hold Range:

$$\Delta\omega_H = \beta N K_{v}$$

Lock (Capture) Range:

$$\Delta\omega_L = \Delta\omega_H = \beta N K_v$$

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Page 060-4

Type-I, First-Order Loop (F(0) = 1) - Continued

Steady-State Phase Error:

For a sinusoidal phase detector, $\varepsilon_{ss} = \lim_{t \to \infty} \theta(t) = \sin^{-1} \left(\frac{\Delta \omega_{osc}}{NK_v} \right)$

For a nonsinusoidal (digital) phase detector, $\varepsilon_{ss} = \lim_{t \to \infty} \theta(t) = \frac{\Delta \omega_{osc}}{NK_v} \le \beta$

The steady-state error is never larger than β . A larger error indicates a failure to lock. Frequency Acquisition Time:

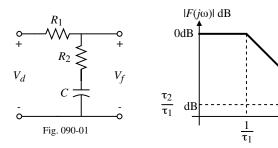
$$T_a = \frac{1}{K_v} \text{ (sec.)}$$

For a Type-I loop,

Lock Range and Acquisition Time = Hold Range and Acquisition Time.

Type-I, Second-Order Loop

This type of loop is generally implemented with a lag-lead filter as shown below.



Filter Transfer Function:

$$F(s) = \frac{1 + \tau_2 s}{1 + \tau_1 s}$$
 where $\tau_1 = (R_1 + R_2)C$ and $\tau_2 = R_2C$

(Note: The definition for $\tau_1 = (R_1 + R_2)C$ which is different from that in Lecture 050)

System Parameters:

$$\omega_n = \sqrt{\frac{K_o K_d}{\tau_1}} = \sqrt{\frac{K_v}{\tau_1}} \quad \text{and} \quad \left[\zeta = \frac{\omega_n}{2} \left(\tau_2 + \frac{1}{K_o K_d} \right) = \frac{1}{2} \sqrt{\frac{1}{K_v \tau_1}} \left(1 + \tau_2 K_v \right) \right]$$

Note that because $\tau_2 < \tau_1$, we see that

$$\frac{\omega_n}{2K_v} < \zeta < \frac{K_v^2 + \omega_n^2}{2\omega_n K_v}$$

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Page 060-6

Type-I, Second-Order Loop - Continued

Crossover Frequency:

The general close-loop frequency response for high-gain loops is,

$$H(s) = \frac{2s\zeta\omega_n + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{1}{1 + \frac{s^2}{2\zeta\omega_n s + \omega_n^2}} = \frac{1}{1 + \text{Loop Gain}}$$

The crossover frequency, ωc , is the frequency when the loop gain is unity.

$$\therefore \frac{\omega_c^4}{\omega_n^4 + 4\xi^2 \omega_n^2 \omega_c^2} = 1 \quad \Rightarrow \quad \omega_c^4 - (4\xi^2 \omega_n^2) \omega_c^2 - \omega_n^4 = 0$$

Solving for ω_c gives,

$$\omega_c = \omega_n \sqrt{2\xi^2 + \sqrt{4\xi^4 + 1}}$$

3dB Bandwidth:

$$\omega_{-3\text{dB}} = \omega_n \sqrt{b + \sqrt{b^2 + 1}}$$
 where $b = 2\zeta^2 + 1 - \frac{\omega_n}{K_v} \left(4\zeta - \frac{\omega_n}{K_v} \right)$

Noise Bandwidth:

$$B_n = \frac{\omega_n}{2} \left(\zeta + \frac{1}{4\zeta} \right) \quad (Hz)$$

Type-I, Second-Order Loop - Continued

Hold Range:

$$\Delta\omega_H = \beta N K_v$$

Lock Range:

$$\Delta\omega_L = \frac{\tau_2}{\tau_1} \Delta\omega_H = \frac{\tau_2}{\tau_1} \beta N K_v$$

Lock Time:

The lock time is set by the loop natural frequency, ω_n and is

$$T_L = \frac{2\pi}{\omega_n}$$

Pull-in Range:

$$\Delta\omega_P = N\beta\sqrt{2}\sqrt{2\zeta\omega_n K_\nu F(0) - \omega_n^2}$$

This formula is only valid for moderate or high loop gains, i.e. $K_{\nu}F(0) \le 0.4\omega_n$.

Pull-in Time:

$$T_{P} \approx \frac{4\left(\frac{\Delta f_{osc}}{N}\right)^{2}}{B_{n}^{3}} \approx \frac{\pi^{2} \Delta \omega_{osc}^{2}}{16 \zeta \omega_{n}^{3}} \quad \text{Note that } \Delta \omega_{H} \leq \Delta \omega_{osc} \leq \Delta \omega_{P}$$

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Page 060-8

Type-I, Second-Order Loop - Continued

Frequency Acquisition Time:

$$T_a = T_P + T_L$$

If the frequency step is within the lock limit (one beat), then the pull-in time is zero. Steady-State Phase Error to a frequency step of $\Delta\omega_{osc}$:

For a sinusoidal phase detector,
$$\varepsilon_{ss} = \lim_{t \to \infty} \theta(t) = \sin^{-1} \left(\frac{\Delta \omega_{osc}}{NK_v} \right)$$

For a nonsinusoidal (digital) phase detector,
$$\varepsilon_{ss} = \lim_{t \to \infty} \theta(t) = \frac{\Delta \omega_{osc}}{NK_v} \le \beta$$

The steady-state error is $\leq \beta$. A larger error indicates a failure to lock.

Maximum Sweep Rate of the Input Frequency:

Largest sweep rate of the input frequency for which the loop remains in lock.

$$\frac{d(\Delta\omega)}{dt} = \omega_n^2 \left(\frac{\text{radians/sec}}{\text{sec}} \right)$$

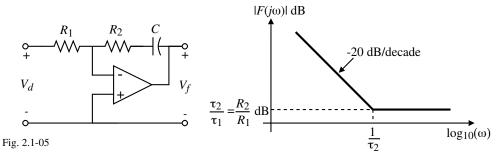
Maximum Sweep Rate for Aided Acquisition:

This is the case when the VCO is externally swept to speed up acquisition.

$$\frac{d(\Delta\omega)}{dt} = \frac{\omega_n^2}{2} \left(\frac{\text{radians/sec}}{\text{sec}} \right)$$

Type-2, Second-Order Loop

This type of PLL system generally uses the active PI filter as shown below.



Filter Transfer Function:

$$F(s) = -\frac{1 + s\tau_2}{s\tau_1} = -\left(\frac{\tau_2}{\tau_1}\right)\left(\frac{s + 1/\tau_2}{s}\right) = -\left(\frac{R_2}{R_1}\right)\left(\frac{s + 1/\tau_2}{s}\right) \quad \text{where} \quad \tau_1 = R_1C \text{ and } \tau_2 = R_2C$$

System Parameters:

$$\omega_n = \sqrt{\frac{K_v}{\tau_1}}$$
 and $\zeta = \frac{1}{2} \sqrt{K_v \tau_2 \frac{R_2}{R_1}} = \frac{1}{2} \tau_2 \sqrt{\frac{K_v}{\tau_1}} = \frac{\tau_2 \omega_n}{2}$

3dB Bandwidth:

$$\omega_{-3\text{dB}} = \omega_n \sqrt{2\xi^2 + 1 + \sqrt{(2\xi^2 + 1)^2 + 1}}$$

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Page 060-10

Type-2, Second-Order Loop - Continued

Noise Bandwidth:

$$B_n = \frac{\omega_n}{2} \left(\xi + \frac{1}{4\xi} \right)$$

Hold Range:

Limited by the dynamic range of the loop components.

Lock (Capture) Range:

$$\Delta\omega_H = \beta N 2 \zeta \omega_n$$

Lock (Capture) Time:

$$T_L = \frac{2\pi}{\omega_n}$$

Pull-in Range:

The pull-in range is the frequency range beyond the lock (capture) range over which the loop will lock after losing lock (skipping cycles).

- The pull-in range for a 2nd or higher order, type-2 loop is theoretically infinite and limited by the amplifier and phase detector offsets and by the dynamic range of the loop.
- A system with large offsets and a large frequency error may never lock.

Type-2, Second-Order Loop - Continued

Pull-in Time:

$$T_P = \tau_2 \left(\frac{\Delta \omega_{osc}}{\frac{N}{K_v R_1}} - \sin \theta_o \right)$$

where θ_o is the initial phase difference between the reference and VCO signals. Assume $\sin \theta_o = -1$ for the worst case.

Pull-out Range:

$$\Delta\omega_{PO} \approx 1.8N\beta\omega_n(1+\zeta)$$

Frequency Acquisition Time:

$$T_a = T_P + T_L$$

If the frequency step is within the lock limit (one beat), then the pull-in time is zero. Steady-state Phase Error:

The steady-state phase error of a type-2 system is zero for both a phase step and a frequency step.

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Page 060-12

Type-2, Second-Order Loop - Continued

Steady-state Phase Error – Continued:

The steady-state phase error due to a frequency ramp of $\Delta\omega_{osc}$ radians/sec./sec. is,

For a sinusoidal phase detector, $\varepsilon_{ss} = \lim_{t \to \infty} \theta(t) = \sin^{-1} \left[\left(\frac{R_1}{R_2} \right) \frac{\tau_2 \frac{d\Delta \omega_{osc}}{dt}}{NK_v} \right]$

For a nonsinusoidal (digital) phase detector, $\varepsilon_{ss} = \lim_{t \to \infty} \theta(t) = \left[\frac{R_1}{R_2} \frac{\tau_2 \frac{d\Delta\omega_{osc}}{dt}}{NK_v} \right] \le \beta$

The steady-state error is $\leq \beta$. A larger error indicates a failure to lock.

Maximum Sweep Rate of Input Frequency:

Largest sweep rate of the input frequency for which the loop remains in lock.

$$\frac{d(\Delta\omega_{in})}{dt} = \beta\omega_n^2 \left(\frac{\text{radians/sec}}{\text{sec}}\right)$$

Maximum Sweep Rate for Aided Acquisition:

This is the case when the VCO is externally swept to speed up acquisition.

$$\frac{d(\Delta\omega_{osc})}{dt} = \frac{N\beta}{2\tau_2} \left(4B_n - \frac{1}{\tau_2} \right) \left(\frac{\text{radians/sec}}{\text{sec}} \right)$$

DESIGN OF A 450-475 MHz DPLL FREQUENCY SYNTHESIZER

Specifications

Design a DPLL frequency synthesizer that meets the following specifications:

Frequency Range: 450 – 475 MHz

Channel Spacing: 25 kHz

Modulation: FM from 300 to 3000 Hz

Modulation Deviation: ±5kHz Loop Type: Type 2

Loop Order: Second order

VCO Gain: $K_0 = 1.25 \text{MHz/V} = 7.854 \text{ Mradians/sec./V}$

Phase Detector Type: PFD $(\beta = 2\pi)$

Phase Detector Gain: $K_d = 0.796 \text{ V/radian}$

(This example will be continued later in more detail concerning phase noise and spurs)

Note on channel spacing:

Carson's rule \rightarrow BW of an FM signal is $\approx 2[\Delta f_c + f_m(\text{max})] = 2(\pm 5\text{kHz} + 3\text{kHz}) = 16\text{kHz}$

If we assume a 9 kHz guard band, then

Channel Spacing = 9 kHz + 16 kHz = 25 kHz

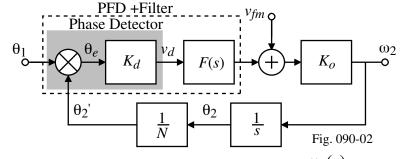
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Page 060-14

PLL System

Block Diagram:



The pertinent transfer function for this problem is given as $\frac{\omega_2(s)}{V_f(s)}$ which is found as

$$\omega_2(s) = K_o \left[V_f(s) + F(s) K_d \left(\theta_1 - \frac{\omega_2(s)}{sN} \right) \right] = K_o \left[V_f(s) + F(s) K_d \theta_1 - \frac{F(s) K_d}{sN} \omega_2(s) \right]$$

Setting $\theta_1 = 0$ gives

$$\frac{\omega_2(s)}{V_f(s)} = \frac{K_o}{1 + \frac{F(s)K_dK_o}{sN}}$$

PLL System - Continued

The filter has a transfer function given as

$$F(s) = \frac{1 + \tau_2 s}{\tau_1 s}$$

The final form of the closed-loop transfer is given as

$$\frac{\omega_2(s)}{V_f(s)} = \frac{K_o}{1 + \frac{(1 + \tau_2 s)K_d K_o}{s^2 N \tau_1}} = \frac{s^2 K_o}{s^2 + \frac{K_d K_o \tau_2}{N \tau_1} s + \frac{K_d K_o}{N \tau_1}} = \frac{s^2 K_o}{s^2 + 2 \zeta \omega_n s + \omega_n^2}$$

where,

$$\omega_n = \sqrt{\frac{K_d K_o}{N \tau_1}}$$
 and $\zeta = \frac{\tau_2}{2} \sqrt{\frac{K_d K_o}{N \tau_1}}$

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Page 060-16

Finding the Loop Parameters

1.) Division Ratio

$$N_{min} = \frac{450 \text{ MHz}}{25 \text{ kHz}} = 18,000$$
 and $N_{max} = \frac{475 \text{ MHz}}{25 \text{ kHz}} = 19,000$

2.) Loop Bandwidth

To pass the 300Hz lower frequency limit, we require that the maximum -3dB frequency is 300Hz. Therefore, $B_L = 300$ Hz.

3.) Damping Constant

For reasons discussed previously, we select $\zeta = 0.707$. Let us check to see if this is consistent with the design.

We know that,

$$\zeta = \frac{\tau_2}{2} \sqrt{\frac{K_d K_o}{N \tau_1}} \rightarrow \zeta = \frac{k}{\sqrt{N}}$$

$$\therefore \quad \xi_{max} = \frac{k}{\sqrt{N_{min}}} \quad \text{and} \quad \xi_{min} = \frac{k}{\sqrt{N_{max}}} \quad \rightarrow \quad \xi_{max} = \xi_{min} \cdot \sqrt{\frac{N_{max}}{N_{min}}} = 1.0274 \xi_{min}$$

Also, $\xi = \sqrt{\xi_{max} \cdot \xi_{min}} = 0.707$, which gives

$$\zeta_{min}^2(1.0274) = 0.5 \rightarrow \zeta_{min} = 0.6976$$
 and $\zeta_{max} = 1.0274 \cdot 0.6976 = 0.7167$

Finding the Loop Parameters - Continued

4.) Natural frequency, ω_n

$$\omega_{-3\text{dB}} = \omega_n \sqrt{2\xi^2 + 1 + \sqrt{(2\xi^2 + 1)^2 + 1}} \ \to \ \omega_n = \frac{\omega_{-3\text{dB}}}{\sqrt{2\xi^2 + 1 + \sqrt{(2\xi^2 + 1)^2 + 1}}}$$

The maximum ω_n will occur at the minimum value of N and the minimum damping factor. Therefore,

$$\omega_n(\text{max}) = \frac{\omega_{-3\text{dB}}}{\sqrt{2\xi_{\text{min}}^2 + 1 + \sqrt{(2\xi_{\text{min}}^2 + 1)^2 + 1}}}$$

$$= \frac{2\pi \cdot 300}{\sqrt{2(0.6976)^2 + 1 + \sqrt{(2(0.6976)^2 + 1)}}} = 980 \text{ radians/sec.}$$

$$\omega_n(\text{min}) = \frac{\omega_{-3\text{dB}}}{\sqrt{2\xi_{\text{max}}^2 + 1 + \sqrt{(2\xi_{\text{max}}^2 + 1)^2 + 1}}} = 910 \text{ radians/sec.}$$

$$\therefore \omega_n = \sqrt{\omega_n(\text{max}) \cdot \omega_n(\text{min})} = 944$$

Loop Parameter Summary:

Frequency (MHz)	N	ω_n (rads./sec.)	ζ	Bandwidth (Hz)
450.00	18,000	910	0.7167	300
475.00	19,000	980	0.6976	300

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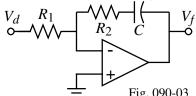
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Page 060-18

Design of the Loop Filter

The loop filter selected is the active PI using the single-ended realization below.



The transfer function is,

$$F(s) = \frac{V_f(s)}{V_d(s)} = \frac{sR_2C + 1}{sR_1C} = \frac{s\tau_2 + 1}{s\tau_1} \rightarrow \tau_1 = R_1C$$
 and $\tau_2 = R_2C$

1.) Time constants

We will use the date for N = 18,000 to design the filter.

$$\tau_1 = \frac{K_d K_o}{N\omega_n^2} = \frac{0.796 \cdot 7.854 \times 10^6}{18,000(910)^2} = 0.419 \text{ ms}$$

$$\tau_2 = \frac{2\zeta}{\omega_n} = \frac{2 \cdot 0.7167}{910} = 1.575 \text{ ms}$$

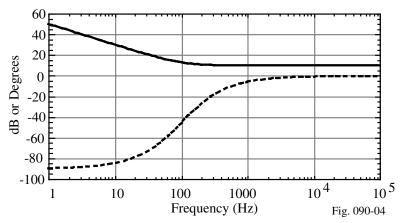
2.) Loop filter design

Select $R_1 = 2.4k\Omega$ which gives

$$C = \frac{\tau_1}{R_1} = \frac{0.419 \times 10^{-3}}{2.4 \times 10^3} = 0.175 \,\mu\text{F}$$
 and $R_2 = \frac{\tau_2}{C} = \frac{1.575 \times 10^{-3}}{0.175 \times 10^{-6}} = 9.0 \,\text{k}\Omega$

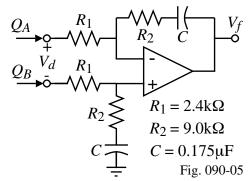
Design of the Loop Filter - Continued

3.) Simulated response of the filter.



4.) Differential version of the filter.

It is necessary to use a differential input in order to interface with the PFD.



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Page 060-20

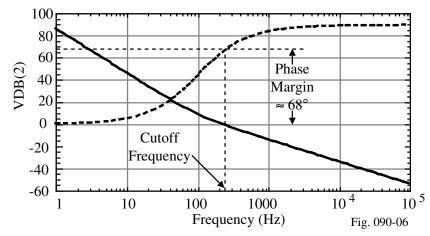
Loop Stability

1.) Loop Gain.

The loop gain for N = 18,000 is given by

$$LG(s) = \frac{K_d K_o F(s)}{Ns} = \frac{K_v (s\tau_2 + 1)}{s^2 N \tau_1} = \frac{7.854 \times 10^6 \cdot 0.796 \ (1 + 1.575 \times 10^{-3} s)}{0.419 \times 10^{-3} \cdot 18,000 \ s^2}$$
$$= \frac{828.83 \times 10^3 (1 + 1.575 \times 10^{-3} s)}{s^2}$$

2.) Bode Plot

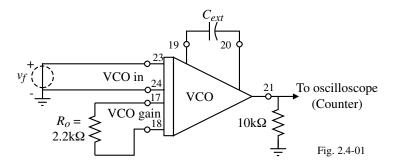


(We will continue this example later.)

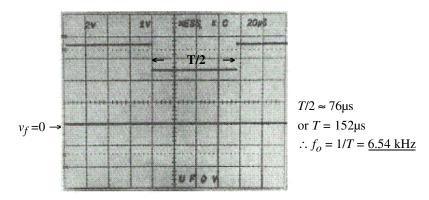
MEASUREMENT OF PLL PERFORMANCE

(The device under test in this section is the Exar XR-S200.)

Measurement of Center Frequency, f_o



Results:



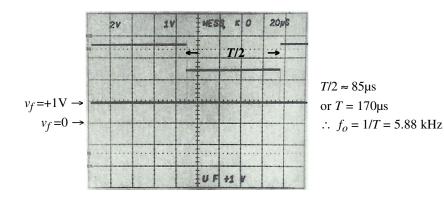
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Page 060-22

Measurement of the VCO Gain, Ko

Use the same measurement configuration as for f_o . Vary v_f and measure the output frequency of the VCO.



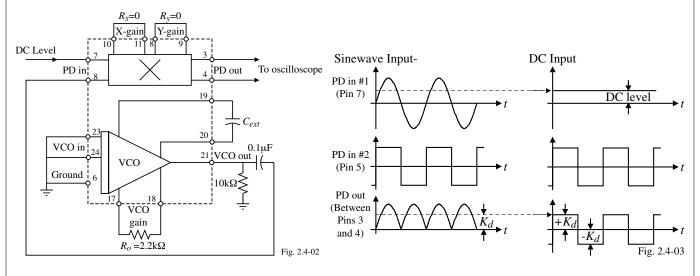
Calculation of K_o .

$$K_o = \frac{\Delta\omega}{\Delta v_f} = \frac{2\pi (6.54-5.88)}{1-0} \times 10^3 = 4.13 \times 10^3 \frac{1}{\text{V-sec}}$$

Measurement of the Phase Detector Gain, K_d

Test circuit:

Measurement Principle:



The above measurement assumes that $\theta_e = 90^\circ$ so that $\overline{v_d} = K_d \sin 90^\circ = K_d$

For a sinewave input, the dc output of the PD is $2/\pi$ of the peak sinusoidal voltage.

If a dc voltage is applied at the PD input of $(\sqrt{2}/1)x(2/\pi)$ $V_{peak} \approx 0.9V_{peak}$, then K_d is simply 1/2 of the peak-to-peak output of the phase detector.

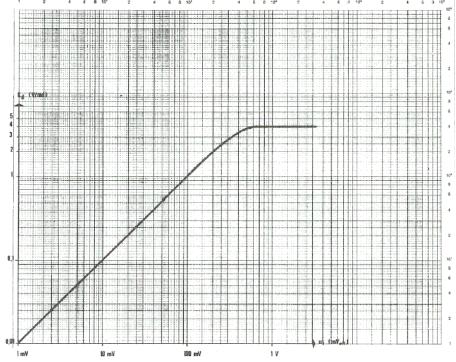
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Page 060-24

Measurement of K_d - Continued

$$v_1 = 10, 20, 30 \text{ and } 40 \text{ mV (rms)}$$

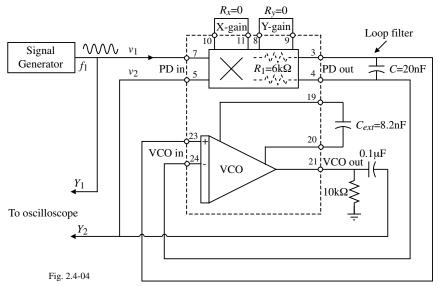


More values indicate that the PD saturates at 0.4V (rms)

Measurement of the Hold Range, $\Delta\omega_H$, and the Pull In Range, $\Delta\omega_p$

Measurement Circuit:

The measurement requires a full functional PLL. The loop filter must be added which in this case consists of both on-chip and external components.



- 1.) To measure Δf_H , start with a value of f_1 where the loop is locked and slowly vary f_1 to find the upper and lower values where the system unlocks.
- 2.) To measure Δf_p , start with f_1 at approximately the center frequency, then increase f_1 until the loop locks out. Decrease f_1 until the loop pulls in. The difference between this value of f_1 and f_o is Δf_p .

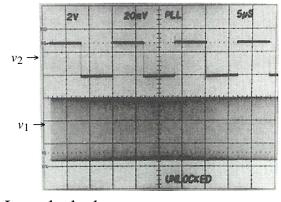
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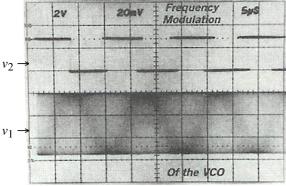
Page 060-26

Measurement of $\Delta\omega_H$ and $\Delta\omega_D$ – Continued

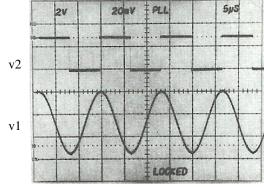
Loop out of lock



Loop on the threshold of lock

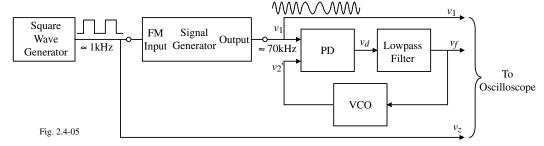


Loop locked

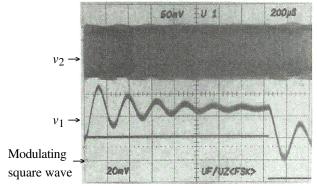


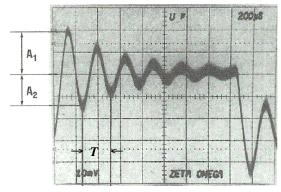
Measurement of ω_n , ξ , and the Lock Range $\Delta\omega_L$

Test circuit:



Waveforms:





Parameter extraction:

With
$$A_1$$
=1.9, A_2 =1.4, $\zeta = \frac{\ln(A_1/A_2)}{\sqrt{\pi^2 + [\ln(A_1/A_2)]^2}}$ and $\omega_n = \frac{2\pi}{T\sqrt{1-\zeta^2}} \Rightarrow \zeta = 0.8$ and $f_n = 4.1$ kHz

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Page 060-28

Measurement of ω_n , ξ , and the Lock Range $\Delta\omega_L$ – Continued

Measurement of $\Delta\omega_L$:

1.) The signal generator is adjusted to generate two frequencies, ω_{high} and ω_{low} such that,

$$\omega_{high}>\omega_o+\Delta\omega_p$$

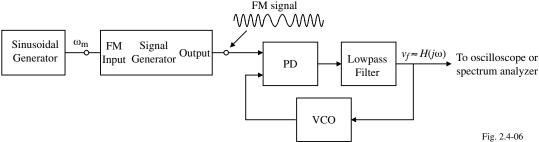
- 2.) Set $\omega_{low} = \omega_{high}$ (the amplitude of the square wave generator will be zero)
- 3.) Decrease ω_{low} .
- 4.) When $\omega_{low} \approx \omega_o + \Delta \omega_L$, the PLL will lock.

$$\therefore \Delta \omega_L \approx \omega_{low} - \omega_o$$

Measurement of the Phase Transfer Function, $H(j\omega)$

Since most signal generators are not phase modulated, use a frequency modulated signal generator instead as follows.

Test circuit:



Principle:

$$\omega_{1} = \omega_{o} + \Delta\omega \sin\omega_{m}t \quad \Rightarrow \quad \theta_{1}(t) = \int_{0}^{t} \omega_{1} dt = -\frac{\Delta\omega}{\omega_{m}} \cos\omega_{m}t \quad \Rightarrow \quad |\theta_{1}(j\omega)| = \frac{\Delta\omega}{\omega_{m}}$$

$$H(j\omega) = \frac{\theta_{2}(j\omega)}{\theta_{1}(j\omega)} \quad \text{and} \quad \theta_{2}(j\omega) = \frac{K_{o}}{j\omega_{m}} V_{f}(j\omega_{m})$$

$$\therefore \quad |H(j\omega)| = \frac{K_{o}V_{f}(j\omega_{m})}{\Delta\omega}$$

What about $\Delta\omega$?

As long as $\Delta \omega$ is small enough, the PD operates in its linear region and $v_f(t)$ is an undistorted sinewave (see next slide).

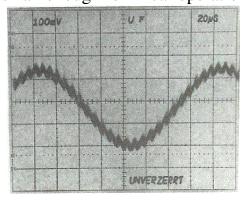
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Lecture 060 - PLL Design Equations and Measurements (09/01/03)

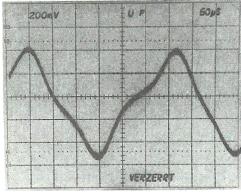
Page 060-30

Measurement of $H(j\omega)$ – Continued

 $\Delta\omega$ small enough for linear operation.



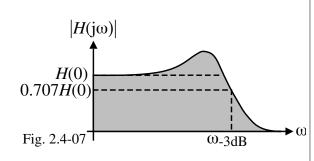
 $\Delta\omega$ too large for linear operation.



Implementation:

- 1.) Can plot the frequency response point-by-point.
- 2.) Use a spectrum analyzer
 - Sweep generator rate << Spectrum analyzer sweep rate
 - Watch out that resonance peaks in the response don't cause nonlinear operation.

Typical results: →

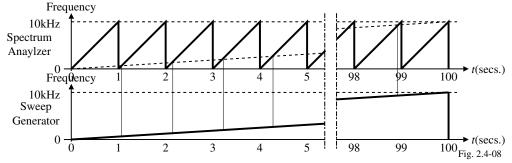


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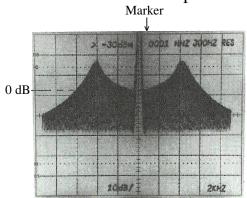
Measurement of $H(j\omega)$ – Continued

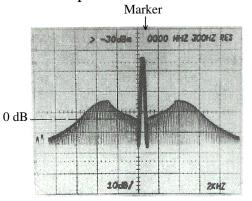
Timing relationship between the sweep generator and the spectrum analyzer:



Basically, f_m should approximate a constant during one sweep period of the analyzer.

Problem due to resonance peaks that cause nonlinear operation:





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Page 060-32

SUMMARY

• PLL Design Equations

Basic design equations for

- Type-I, first-order loop
- Type-I, second-order loop
- Type-II, second-order loop
- Design of a 450-475 MHz DPLL Frequency Synthesizer

PFD plus Charge Pump

Design of active PI filter

Stability

Measurements of PLL Performance

How to experimentally measure the various performance parameters of a PLL