LECTURE 110 – PHASE NOISE
INTRODUCTION

Objective

The objective of this presentation is to understand and model phase noise found in PLLs

Outline

- Jitter and Phase Noise in PLLs
- Spurious Sidebands in PLLs
- Linear Time Invarient Models of VCO Phase Noise
- Linear Time Varying Model of VCO Phase Noise
- Phase Noise in Differential LC Oscillators
- Jitter and Phase Noise in Ring Oscillators
- Finding the Impulse Sensitivity Function
- Amplitude Noise
- Summary

JITTER AND PHASE NOISE IN PLLs

Phase Noise in Oscillators

Causes of spectral purity degradation (phase noise):

1.) Random noise in the reference input, the PFD, loop filter and VCO (also dividers if the PLL is a frequency synthesizer)

2.) Spurious sidebands – high energy sidebands with no harmonic relationship to the generated output signal. It is systematic in origin.

Why is spectral purity important?

Phase noise can degrade the sensitivity of a receiver due to reciprocal mixing.

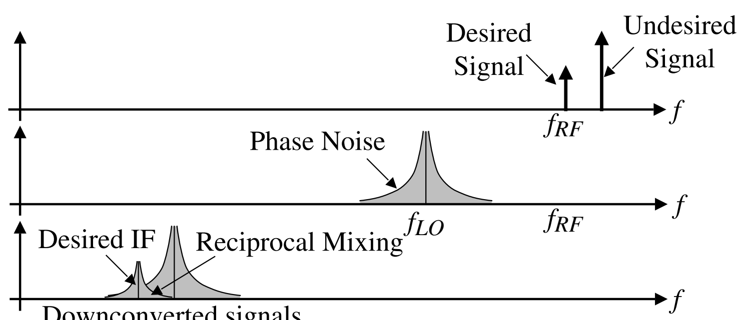


Fig. 12.4-1

Phase noise produces adjacent channel interference in the transmitter.

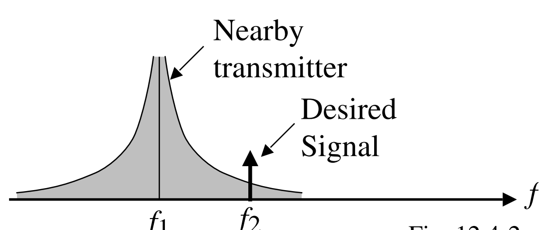


Fig. 12.4-2

Single Sideband Noise Spectral Density

An oscillator's short term instabilities can be characterized in the frequency domain in terms of the single sideband noise spectral density.

This single sideband noise spectral density is given by,

$$\mathcal{L}(\Delta\omega) = 10log\left[\frac{P_{sideband}(\omega_o+\Delta\omega,1\text{Hz})}{P_{carrier}}\right]$$

where

$P_{sideband}(\omega_o+\Delta\omega,1\text{Hz})$ = the single sideband power at a frequency offset of $\Delta\omega$ from the carrier in a measurement bandwidth of 1Hz.

$P_{carrier}$ = total power under the power spectrum.

Advantage of $\mathcal{L}(\Delta\omega)$ is its ease of measurement.

Disadvantage of $\mathcal{L}(\Delta\omega)$ is that is shows the sum of both amplitude and phase variations. It is often important to know both the amplitude and phase noise separately because they behave differently in a circuit.

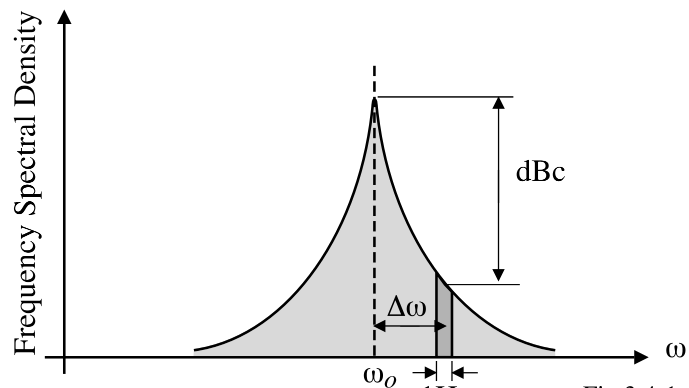


Fig.3.4-1

Signal-to-Noise Ratio (SNR) and Phase Noise

The previous definition can be used to estimate the phase noise required to achieve a desired signal-to-noise ratio.

The in-band noise relative to the carrier is found as,

$$P_{noise} = \int_{\Delta f_{min}}^{\Delta f_{max}} \mathcal{L}(\Delta f)d(\Delta f)$$

where Δf_{min} and Δf_{max} are the offsets from the center of the channel to the edges of the adjacent channel.

Assuming that the phase noise has a $1/f^2$ slope ($\mathcal{L}(\Delta f) = k/\Delta f^2$) gives

$$P_{noise} = \int_{\Delta f_{min}}^{\Delta f_{max}} \frac{k}{\Delta f^2}d(\Delta f) = \frac{k(\Delta f_{max}-\Delta f_{min})}{(\sqrt{\Delta f_{min}\Delta f_{max}})^2}$$

$$= \mathcal{L}(\sqrt{\Delta f_{min}\Delta f_{max}})\ (\Delta f_{max}-\Delta f_{min})$$

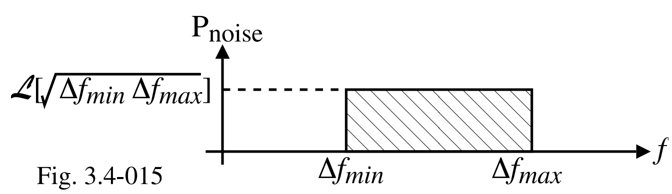


Fig. 3.4-015

This result implies that the phase noise is equivalent to a constant phase noise of $\mathcal{L}(\sqrt{\Delta f_{min}\Delta f_{max}})$ between Δf_{min} and Δf_{max}.

The minimum SNR is given as

$$10log(SNR_{min}) = 10log(P_{signal})-10log(P_{interferer})-10log(P_{noise})-10log\ (\Delta f_{max}-\Delta f_{min})$$

Example 1 – Phase Noise

Find the maximum allowable phase noise at a 100kHz offset for a channel spacing of 200kHz and Δf_{min} = 100kHz and Δf_{max} = 300kHz if an adjacent interferer is 40dB stronger than the desired signal and for a minimum *SNR* of 20dB.

Solution

The maximum allowable phase noise can be calculated using the previous relationship as,

$$10log(P_{noise}) = -20\text{dB} - 40\text{dB} - 10\log(200\text{kHz}) = -113\text{dBc}$$

This phase noise corresponds to a frequency offset of $\sqrt{\Delta f_{min}\Delta f_{max}}$ = 173kHz

The equivalent phase noise at an offset of 100kHz assuming a $1/f^2$ slope is –108dBc.

The frequency shift from 173kHz to 100kHz is accomplished by,

$$P(f_1) = P(f_2) + 20\log_{10}(f_2/f_1) \rightarrow P(f_1) = P(f_2) + 20\log_{10}(1.73) = P(f_2) + 4.76\text{dB}$$

$$\therefore\ P(100\text{kHz}) = P(173\text{kHz}) + 4.76\text{dB} = -113\text{dBc} + 4.76\text{dB} = -108.24\text{dBc}$$

Phase Noise in VCOs

In most oscillators, $\mathcal{L}_{total}(\Delta\omega)$ is dominated by the phase noise, $\mathcal{L}_{phase}(\Delta\omega)$ which will be denoted as simply $\mathcal{L}(\Delta\omega)$.

Typical phase noise plot for a free running oscillator:

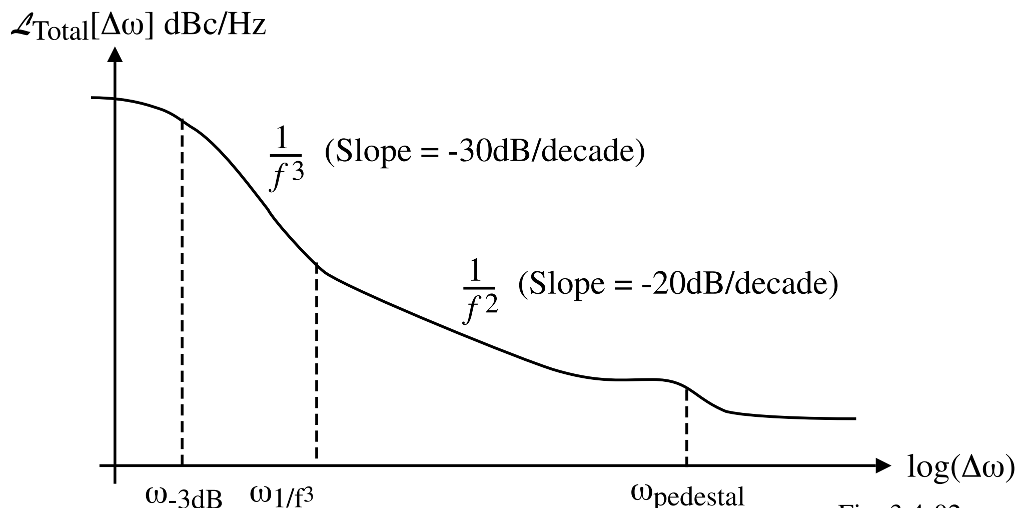


Fig. 3.4-02

The mechanisms responsible for the various regions will be discussed in the material that follows.

VCO Noise

Assume that the phase noise of the VCO is dominated by its phase noise in the $1/f^2$ region.

An equivalent model for the VCO is:

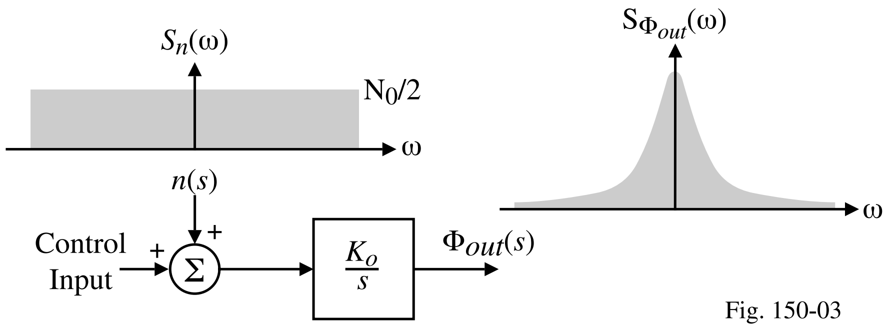


Fig. 150-03

where $n(s)$ is a white noise source with the double sideband power spectral density of $N_0/2$.

Since the VCO acts as an ideal integrator, the output power spectrum can expressed as,

$$S_{\Phi_{out}}(\omega) = \left|\frac{K_v}{j\omega}\right|^2 S_n(\omega) = \frac{K_v{}^2 N_0/2}{\omega^2}$$

Note that $N_0/2$ is chosen in such a way that $S_{\Phi_{out}}(\omega)$ corresponds to the phase noise of the VCO in the $1/f^2$ region.

Phase Noise in PLLs

Assuming the PLL is a linear time-invarient system, we can model the noise sources in a PLL as,

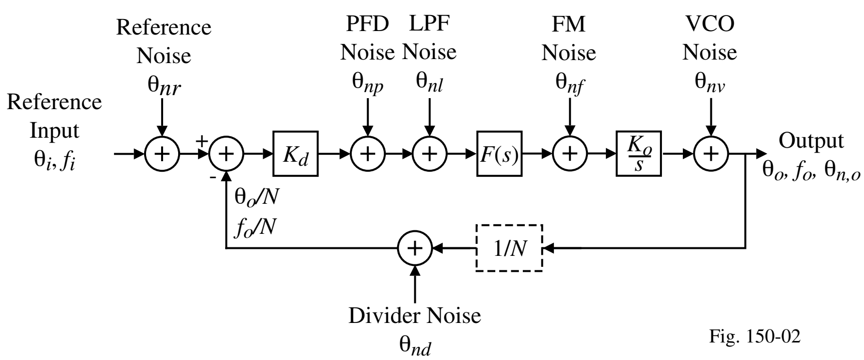


Fig. 150-02

The various transfer functions from each noise source to the output can be found as,

$$T_1(\mathrm{s}) = \frac{\theta_{no}(s)}{\theta_{nr}(s)} = \frac{\dfrac{K_dK_oF(s)}{s}}{1 + \dfrac{K_dK_oF(s)}{Ns}} = \frac{NO(s)}{1 + O(s)}$$

$$T_2(\mathrm{s}) = \frac{\theta_{no}(s)}{\theta_{np}(s)} = \frac{\theta_{no}(s)}{\theta_{nl}(s)} = \frac{\dfrac{K_oF(s)}{s}}{1 + \dfrac{K_dK_oF(s)}{Ns}} = \frac{N}{K_d}\frac{O(s)}{1 + O(s)}$$

Phase Noise in PLLs – Continued

$$T_3(s) = \frac{\theta_{no}(s)}{\theta_{nf}(s)} = \frac{\frac{K_o}{s}}{1+\frac{K_dK_oF(s)}{Ns}} = \frac{N}{K_dF(s)}\frac{O(s)}{1+O(s)}$$

$$T_4(s) = \frac{\theta_{no}(s)}{\theta_{nd}(s)} = \frac{\frac{K_dK_oF(s)}{s}}{1+\frac{K_dK_oF(s)}{Ns}} = \frac{NO(s)}{1+O(s)} \qquad T_5(s) = \frac{\theta_{no}(s)}{\theta_{nv}(s)} = \frac{1}{1+\frac{K_dK_oF(s)}{Ns}} = \frac{1}{1+O(s)}$$

where $O(s)$ is the open-loop gain given by $O(s) = \frac{K_dK_oF(s)}{Ns} = \frac{K_vF(s)}{Ns}$

The total output phase noise contributed by each source can be written as,

$$\theta_{no}^2 = N^2(\theta_{nr}^2+\theta_{n,eq}^2)\left(\frac{O(s)}{1+O(s)}\right)^2 + \theta_{nv}^2\left(\frac{1}{1+O(s)}\right)^2$$

where $\theta_{n,eq}^2 = \frac{1}{K_d^2}(\theta_{np}^2+\theta_{nl}^2) = \frac{1}{K_d^2F(s)^2}\theta_{nf}^2 + \theta_{nd}^2$

Note that, $\frac{O(s)}{1+O(s)} = \frac{\frac{K_dK_oF(s)}{N}}{s+\frac{K_dK_oF(s)}{N}}$ and $\frac{1}{1+O(s)} = \frac{s}{s+\frac{K_dK_oF(s)}{N}}$

Phase Noise in PLLs – Continued

Interpretation of the above results:

1.) Since $F(s)$ is either unity or low pass, the PLL functions as a low pass filter for phase noise arising in the reference signal, PFD, low-pass filter and frequency divider.

2.) However, the PLL functions as a high-pass filter for phase noise generated in the VCO.

Therefore:

1.) To minimize the output noise due to the VCO, the loop bandwidth must be as large as possible.

2.) To achieve minimum phase noise within the loop bandwidth, the in-band noise contributed by the other loop components should be kept to a minimum.

3.) However, the loop bandwidth must be less than the input reference frequency to keep the loop stable and suppress the spurs at the output due to the reference leakage signal.

Phase Noise and Jitter in First Order Loops

Use the following equivalent first order PLL model with only VCO noise:

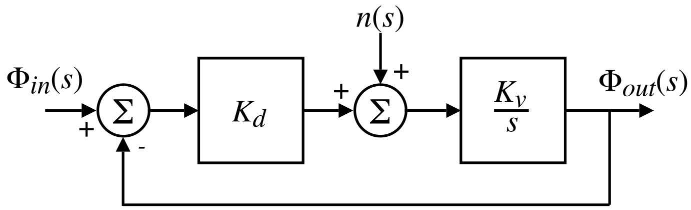


Fig. 3.4-05

In this model, the noise sources $\Phi_{in}(s)$ and $n(s)$, are the only noise sources that influence the phase noise of the output, $\Phi_{out}(s)$. $n(s)$ is white noise to model the VCO noise.

Assuming that these noise sources are uncorrelated (a reasonably good assumption) the phase noise power spectrum at the output can be calculated using superposition.

I.e.,

$$\text{Total } S_{\Phi_{out}}(\omega) = S_{\Phi_{out}}(\omega) \text{ due to VCO} + S_{\Phi_{out}}(\omega) \text{ due to the Input}$$

Output Phase Noise Spectrum with a Noiseless Input

$$\frac{\Phi_{out}(s)}{n(s)} = \frac{sK_o}{s+K_oK_d} \quad \rightarrow \quad S_{\Phi_{out}}(\omega) = \frac{N_o}{2}\frac{K_o^2}{\omega^2+(K_dK_o)^2} = \frac{N_o}{2}\frac{K_o^2}{\omega^2+K_v^2}$$

Illustration:

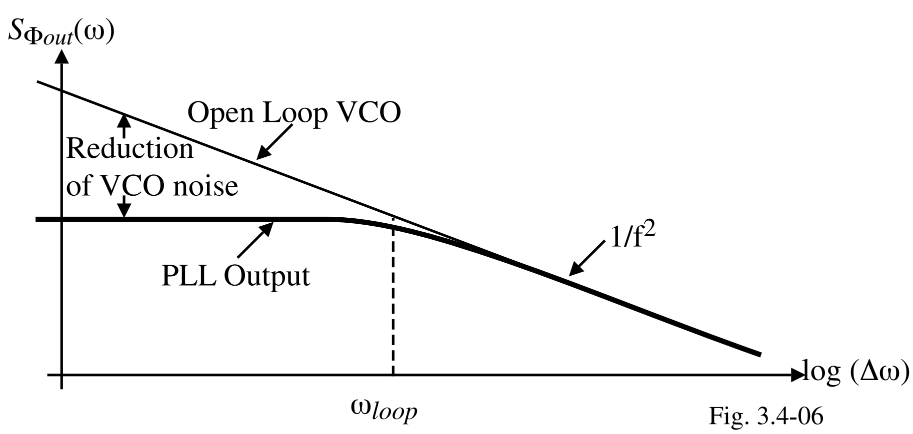


Fig. 3.4-06

For offset frequencies below ω_{loop}, the PLL can react to the noise variation of the VCO and compensate for slow random variations less than ω_{loop}. For offset frequencies above ω_{loop}, the PLL is unable to react fast enough to fast random changes in the VCO output and they appear directly on the output.

Jitter in Oscillators

Illustration of jitter:

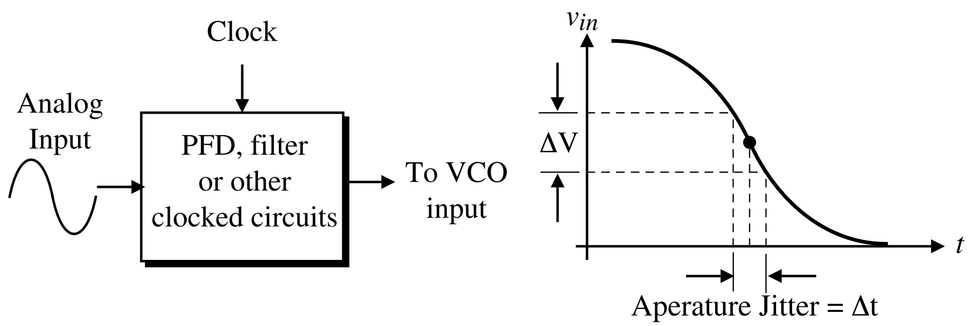


Fig.12.4-2A

If we assume that $v_{in}(t) = V_p sin\omega t$, then the maximum slope is equal to ωV_p.

Therefore, the value of ΔV is given as

$$\Delta V = \left|\frac{dv_{in}}{dt}\right| \Delta t = \omega V_p \Delta t \ .$$

The rms value of this noise is given as

$$\Delta V(\text{rms}) = \left|\frac{dv_{in}}{dt}\right| \Delta t = \frac{\omega V_p \Delta t}{2\sqrt{2}} \ .$$

This ΔV(rms) will lead to an uncertainty in the output frequency when applied on top of the VCO input.

Jitter in Oscillators - Continued

Comparator Noise:

Noise of a comparator is modeled as if the comparator were biased in the transition region.

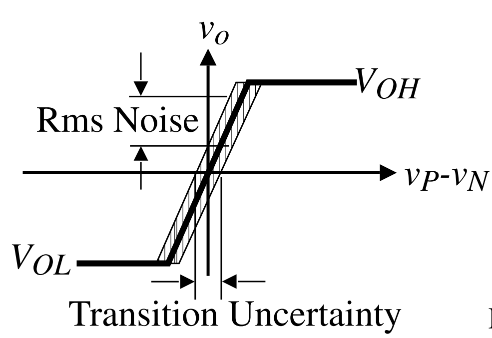


Fig. 8.1-8

Noise leads to an uncertainty in the transition region that causes jitter or phase noise.

Timing Jitter

Clock jitter increases with the time delay between the reference and the observed transition as shown below.

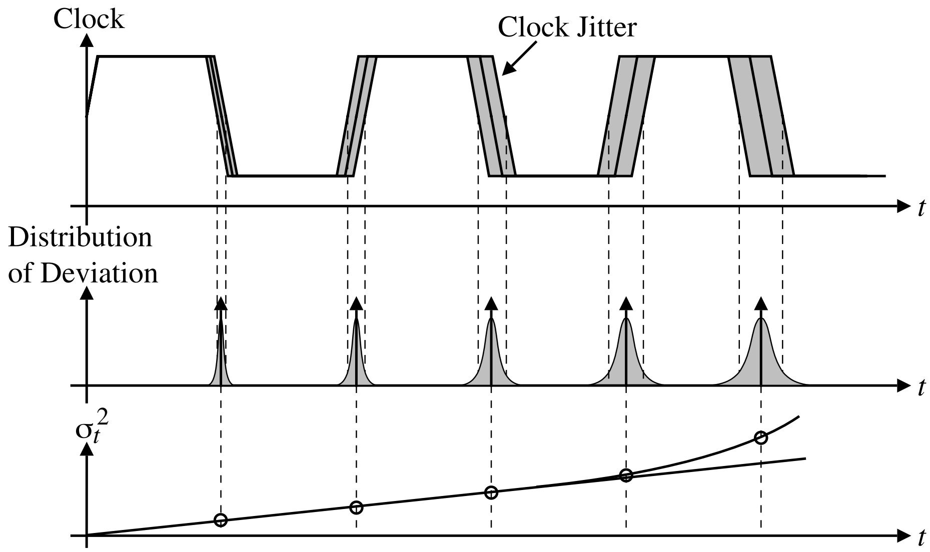


Fig. 3.4-07

Note that the variance, σ_t^2, will increase as the time between reference and the observed transition increases. This is called "jitter accumulation".

Jitter Accumulation

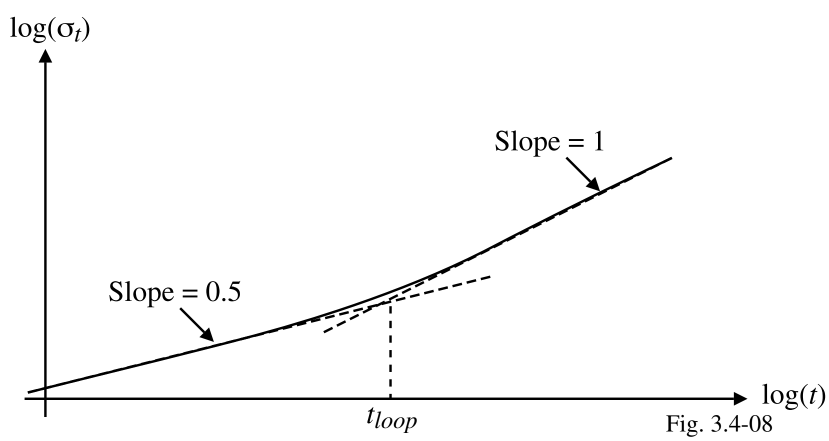


Fig. 3.4-08

Various regions of behavior:

1.) Short time-

$$\sigma_t \propto \sqrt{t}$$

2.) Long time-

$$\sigma_t \propto t$$

Relationship between Jitter and Phase Noise

Start by observing that jitter is the standard deviation of the timing uncertainty,

$$\sigma_\tau^2 = \frac{1}{\omega_o^2} E\{[\phi(t+\tau) - \phi(t)]^2\} = \frac{E[\phi(t)^2]}{\omega_o^2} + \frac{E[\phi(t+\tau)]^2}{\omega_o^2} - \frac{2E[\phi(t)\phi(t+\tau)]^2}{\omega_o^2}$$

where $E\{x\}$ is the expected value of x.

Next, the autocorrelation function of $\phi(t)$ is expressed as

$$R_\phi(\tau) = E\{\phi(t)\phi(t+\tau)\} \quad \rightarrow \quad \sigma_\tau^2 = \frac{2}{\omega_o^2}[R_\phi(0) - R_\phi(\tau)]$$

Using Khinchin's theorem†,

$$R_\phi(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_\phi(\omega)e^{j\omega\tau}d\omega \quad \rightarrow \quad \sigma_\tau^2 = \frac{2}{\omega_o^2}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} S_\phi(\omega)d\omega - \frac{1}{2\pi}\int_{-\infty}^{\infty} S_\phi(\omega)e^{j\omega\tau}d\omega\right]$$

$$\therefore \quad \sigma_\tau^2 = \frac{1}{\pi\omega_o^2}\int_{-\infty}^{\infty} S_\phi(\omega)[1-e^{j\omega\tau}]d\omega = \frac{1}{\pi\omega_o^2}\int_{-\infty}^{\infty} S_\phi(\omega)\sin^2\left(\frac{\omega\tau}{2}\right)d\omega$$

Therefore, knowing the phase noise, $S_\phi(\omega)$, one can find the jitter noise, σ_τ^2.

† W.A. Gardner, *Introduction to Random Processes*, McGraw-Hill Book Co., New York, 1990.

Jitter in First Order Loops with a Noiseless Input

In general, for a first-order PLL with a loop bandwidth of ω_{loop}, we can use the previous relationship to find the timing jitter as,

$$\sigma_t^2 = \frac{2\pi^2 NK_o^2}{\omega_o^2}\frac{1}{\omega_{loop}}(1- e^{-\omega_{loop}t})$$

where ω_o is center frequency of the VCO. If $t < t_{loop}$, then

$$\sigma_t^2 \approx \frac{2\pi^2 NK_o^2}{\omega_o^2}t$$

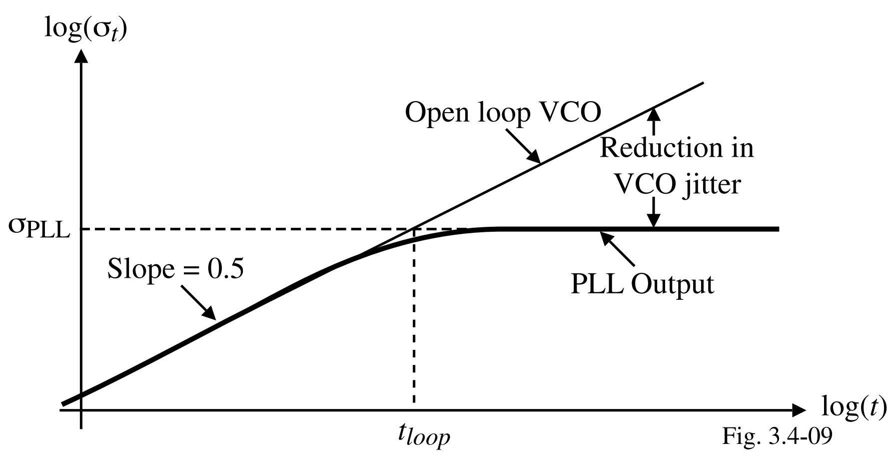


Fig. 3.4-09

Output Phase Noise Spectrum with a Noiseless VCO

In this case, the noise will be due to phase variations in the input reference frequency. The transfer function from the input to output based on the simple first-order PLL model is

$$\frac{\Phi_{out}(s)}{\Phi_{in}(s)} = \frac{K_oK_d}{s + K_oK_d}$$

Assuming input oscillator phase noise in the $1/f^{\,2}$ region, we can express its power spectrum as,

$$S_{\Phi in}(\omega) = \frac{\alpha}{\omega^2} \qquad \rightarrow \qquad S_{\Phi out}(\omega) = \frac{\alpha}{\omega^2}\,\frac{(K_oK_d)^2}{\omega^2 + (K_dK_o)^2} = \frac{\alpha}{\omega^2}\,\frac{K_v{}^2}{\omega^2 + K_v{}^2}$$

Illustration:

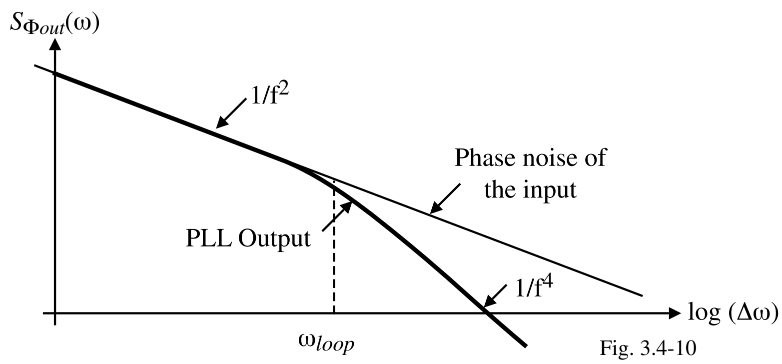


Fig. 3.4-10

Output Timing Jitter with a Noiseless VCO

Corresponding time domain behavior.

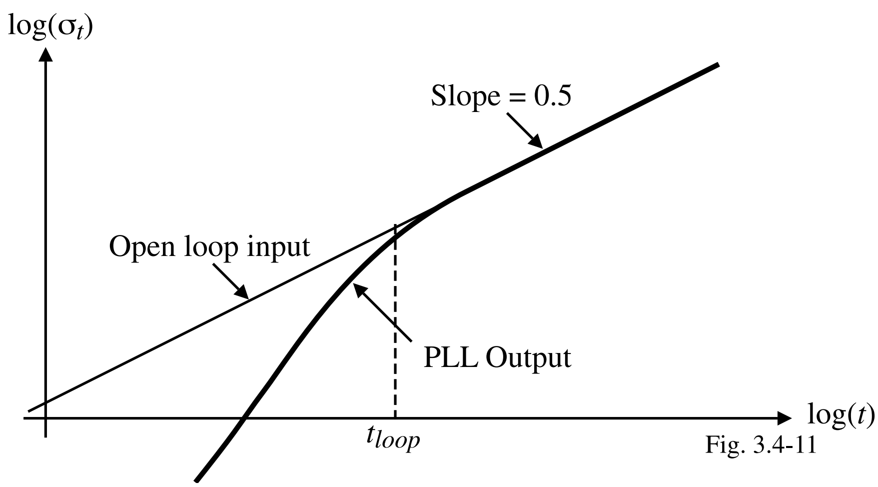


Fig. 3.4-11

Output Phase Noise Spectrum with a Low Noise Input

This circumstance is typical of a microprocessor clock distribution scheme or frequency synthesis.

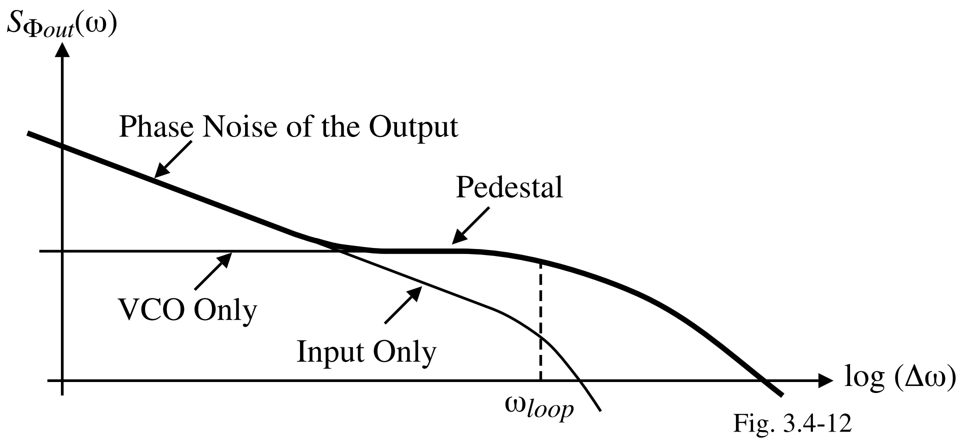


Fig. 3.4-12

The phase noise is dominated by the input phase noise for small offset frequencies and by the VCO noise for large offset frequencies.

The phase noise pedestal shown is common in frequency synthesizers.

Output Timing Jitter with a Low Noise Input

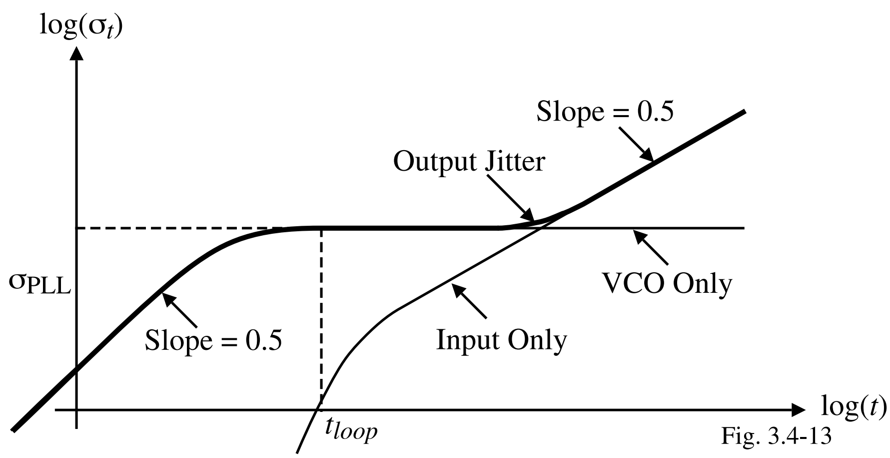


Fig. 3.4-13

Note that if the input signal has better frequency stability compared to the internal time base used in the phase noise/jitter measurement system, phase noise at low offsets (jitter at large delay times) will be dominated by the phase noise (jitter) of the measurement system.

Output Phase Noise Spectrum with a Input Noiser than the VCO

This is typical of clock recovery applications.

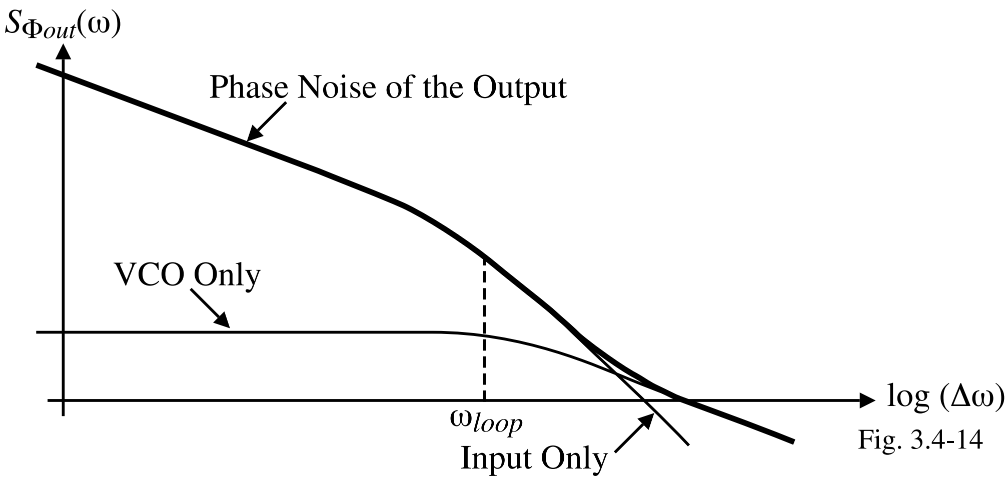


Fig. 3.4-14

Output Timing Jitter with a Input Noiser than the VCO

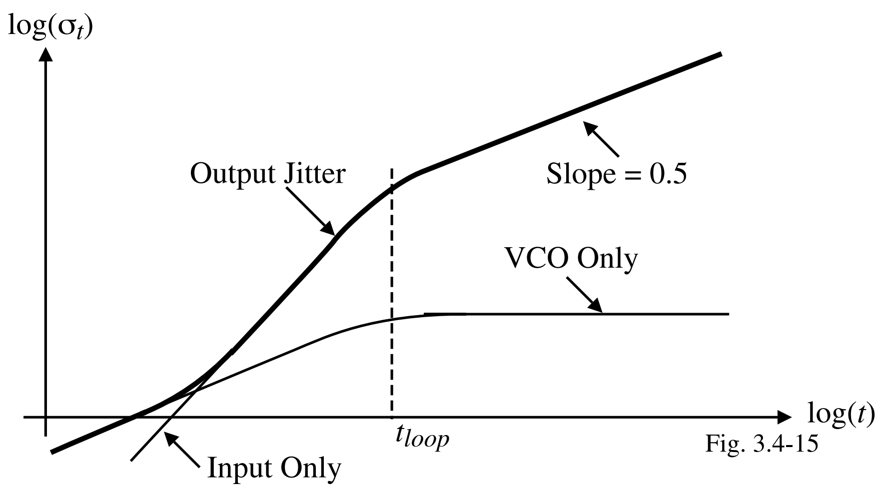


Fig. 3.4-15

Second-Order Charge Pump PLLs

Typical charge pump PFD:

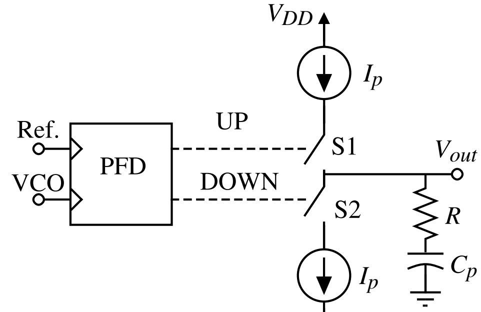


Fig. 3.4-16

The filter transfer function that corresponds to the charge pump PFD using a compensation zero is,

$$K_dF(s) = \frac{I_p}{2\pi C_p}\frac{s\tau_z + 1}{s}$$

Putting this into,

$$\frac{\Phi_{out}(s)}{\Phi_{in}(s)} = \frac{K_dF(s)K_o}{s + K_dF(s)K_o} \quad \rightarrow \quad \frac{\Phi_{out}(s)}{\Phi_{in}(s)} = \frac{s\tau_z + 1}{\dfrac{s^2}{\left(\dfrac{K_oI_p}{2\pi C_p}\right)} + s\tau_z + 1}$$

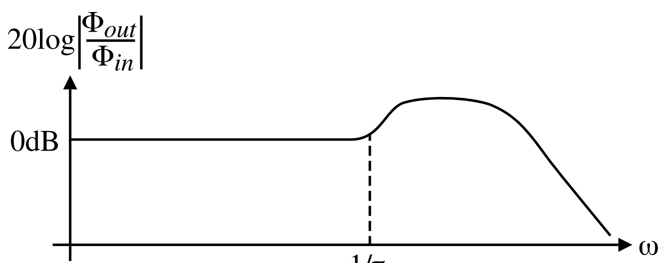


Fig. 3.4-17

Note the zero steady state error typical of charge pumps.

Output Phase Noise of a Charge Pump PLL with a Low Noise Input

Consider the case of a charge pump with a compensation zero that is described by the phase function,

$$\frac{\Phi_{out}(s)}{n(s)} = \frac{2\pi C_p}{I_p}\,\frac{s}{\dfrac{s^2}{\left(\dfrac{K_oI_p}{2\pi C_p}\right)} + s\tau_z + 1} \quad \rightarrow \quad S_{\Phi out}(\omega) = \frac{N_o}{2}\,\frac{\omega^2}{\left[1 + \left(\dfrac{\omega^2}{\dfrac{K_oI_p}{2\pi C_p}}\right)^2\right]^2 + (\tau_z\omega)^2}$$

Output phase noise:

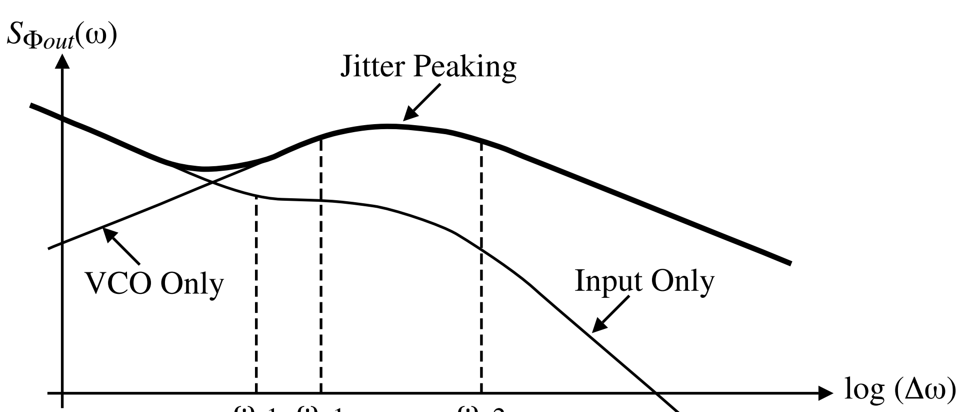


Fig. 3.4-18

A PLL with A Frequency Divider in the Feedback Path

Block diagram:

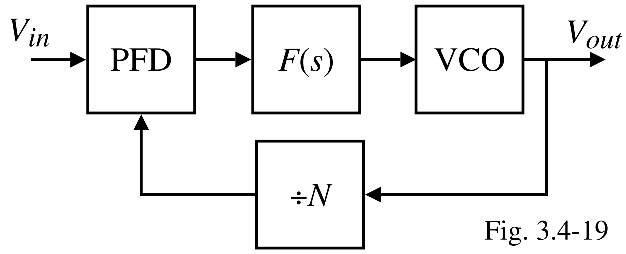


Fig. 3.4-19

The closed-loop transfer function for a charge pump PLL with a divide by N in the feedback path is found to be,

$$\frac{\Phi_{out}(s)}{\Phi_{in}(s)} = \frac{s\tau_z + 1}{\frac{s^2}{K} + \frac{s\tau_z + 1}{N}}$$

Note, when $s \to 0$, the transfer function reduces to N whereas the transfer function without the divider reduces to 1. Thus the low frequency input phase variations get multiplied by N. For $s \to \infty$, both transfer functions reduce to $K\tau_z/s$.

Output phase noise $\rightarrow$

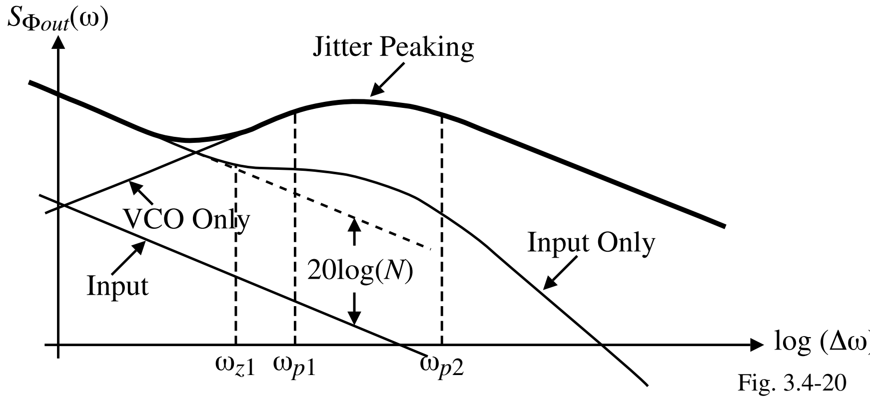


Fig. 3.4-20

SPURIOUS SIDEBANDS IN PLLs

Spurious Sidebands In Oscillators

Spurious sidebands are the undesired systematic variation of the oscillator frequency as a function of time.

These spurious sidebands appear as unwanted sidebands close to the carrier signal and are significantly higher than the noise floor.

The major source spurious sidebands is the reference input frequency and its harmonics that couple through the phase/frequency detector (recall that the phase/frequency detectors are highly nonlinear).

The amplitude of the output spurs can be calculated using the theory of FM modulation:

$$\text{Output amplitude of spurs} = 20 \log\left(\frac{\Delta f_{rms}}{f_m\sqrt{2}}\right)$$

where

$\Delta f_{rms} = K_v V_{rms}$

K_v = PLL bandwidth

V_{rms} = amplitude level of the harmonics of the reference frequency

f_m = frequency deviation from the carrier and modulating frequency.

To reduce the spurious sidebands, a higher-order loop filter can be used to suppress the reference frequency allowing a much smaller loop bandwidth to be used.

Spurs From the Phase Detector

We have seen that one source of spurs comes from the modulation (AM, PM or FM) of the VCO. The phase detector can also generate spurious responses.

How does the phase detector generate spurs?

Answer - The spurs from the PD pass through the filter and if they are strong enough, they will phase modulate the VCO and generate undesired sidetones.

Consider the following phase detector:

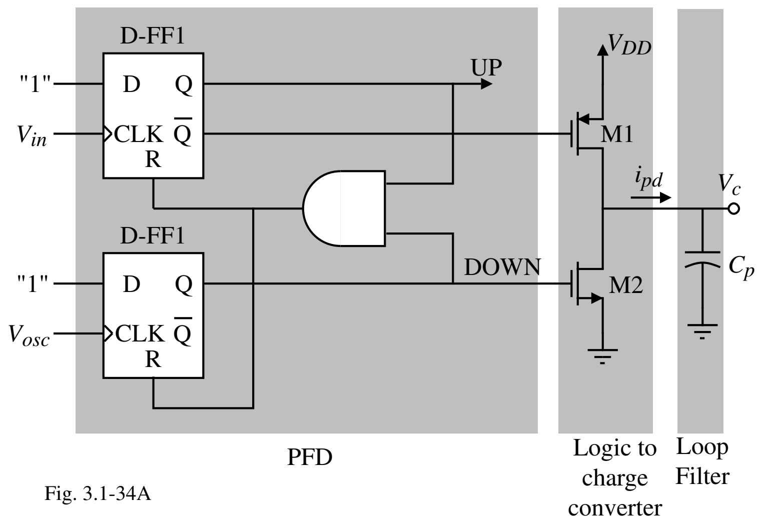


Fig. 3.1-34A

Spurs due to PDs – Continued

Assume the steady-state phase error, θ_e, is positive and fixed which means that V_{in} is always on for a fixed period of time before V_{osc} is on. The duty cycle is

$$d = \frac{\theta_e}{2\pi}T \approx 0.1\%$$

for typical technology and transistor sizes where $T = 1/f_{in}$.

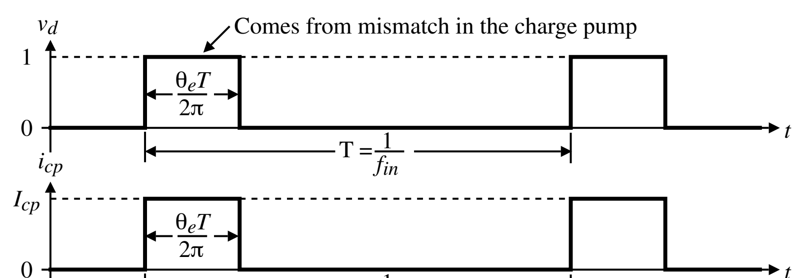


Fig. 3.1-35A

The Fourier series expansion of $i_{cp}(t)$ is given as,

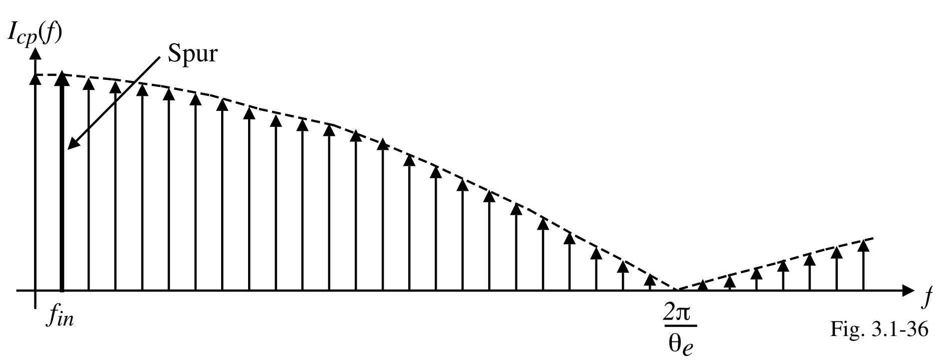


Fig. 3.1-36

The frequency component closest to the origin is the most damaging.

$$\therefore \quad i_{spur} \approx \overline{i_{cp}}\ \sin\omega_{in}t = (\theta_e/2\pi)I_{cp}\sin\omega_{in}t = d\, I_{cp}\sin\omega_{in}t$$

Spurs due to PDs – Continued

The spur can be expressed in the phase domain as

$$\theta_{spur}(t) = d(2\pi)\sin\omega_{in}t$$

Now,

$$\frac{i_{spur}}{\theta_{spur}} = \frac{d\, I_{cp}\sin\omega_{in}t}{d(2\pi)\sin\omega_{in}t} = \frac{I_{cp}}{2\pi}$$

but,

$$I_{cp} = 2\pi K_d \quad \Rightarrow \quad \frac{i_{spur}}{\theta_{spur}} = K_d$$

The sinusoidal phase change at the output of the VCO, θ_o, can be expressed as

$$\theta_o = H\frac{i_{spur}}{K_d}$$

where H is the transfer function from the phase detector to the VCO output.

If the VCO output is assumed to be $v_o(t) = 0.5A_c \sin\omega_o t$, then if the phase modulation of θ_o is narrowband modulation (which is the case when θ_{spur} and i_{spur} are small), then the VCO output will consist of the original frequency at f_o and two new sidetones at $f_o \pm f_{in}$.

$$\therefore \quad v_o(t) = 0.5A_c \sin\omega_o t + 0.5\theta_o A_c \sin(\omega_o+\omega_{in})t + 0.5\theta_o A_c \sin(\omega_o-\omega_{in})t$$

Thus, $$\frac{\text{Spur Amplitude}}{\text{Carrier Amplitude}} = \theta_o = H\frac{i_{spur}}{K_d}$$

Example 2 – Spur Calculations

Suppose the PLL is given as shown. H can be found as,

$$H(s) = \frac{K_vF(s)}{1+\dfrac{K_vF(s)}{sN}} \quad \text{where } K_v = K_dK_o$$

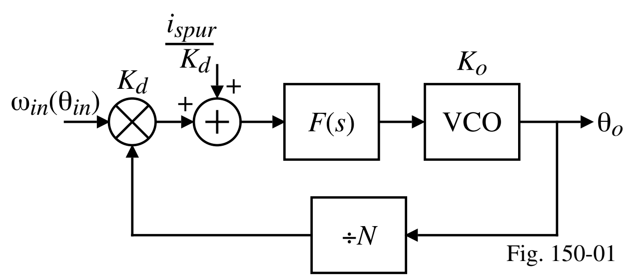


Fig. 150-01

From the previous slide, we can write,

$$\text{Spur (dBc)} = 10\log_{10}\left(\frac{\text{Spur Power}}{\text{Carrier Power}}\right) = 10\log_{10}\theta_o^2 = 20\log_{10}\theta_o = 20\log_{10}\left(H\frac{i_{spur}}{K_d}\right)$$

If $F(s) = \dfrac{s+z_1}{s(s+p_3)}$, then at low frequencies (spur offsets) $|H| \approx \dfrac{NK_vp_3}{\omega_{spur}^2}$.

Thus,

$$\text{Spur (dBc)} = 20\log_{10}\left(\left|\frac{NK_vp_3}{\omega_{spur}^2}\times\frac{\text{Amplitude of } i_{spur}}{K_d}\right|\right) = 20\log_{10}\left(\left|\frac{NK_vp_3}{\omega_{spur}^2}\times\frac{d\cdot I_{cp}}{I_{cp}/2\pi}\right|\right)$$

$$\text{Spur (dBc)} = 20\log_{10}\left(\left|\frac{NK_vp_32\pi d}{\omega_{spur}^2}\right|\right)$$

If $N = 1099$, $p_3 = 972$Krads/s, $K_v = 324$ Krads/s, $d = 0.001$, and $f_{spur} = f_{in} = 1.728$MHz,

$$\text{Spur (dBc)} = 20\log_{10}\left(\left|\frac{1099\cdot 324\text{K}\cdot 972\text{K}}{2\pi(1.728\text{M})^2}0.001\right|\right) = -34.7 \text{ dBc}$$

Methods of Reducing Spurious Sidebands

- Use a higher order loop filter to suppress the reference frequency with the loop bandwidth smaller than the reference frequency
- Reduce the leakage current that occurs due to the charge pump, loop filter components, varactor diodes and other components
- Use a fully differential configuration
- Use a higher reference frequency (as done in fractional-N synthesis techniques)

LINEAR TIME INVARIENT MODELS OF VCO PHASE NOISE

Amplifier Phase Noise (A Two-Port Approach)

Consider the oscillator as an amplifier with feedback as shown.

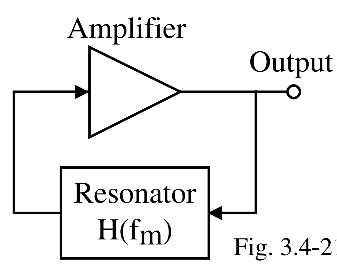


Fig. 3.4-21

Let us examine the phase noise added to an amplifier that has a noise factor of F where

$$F = \frac{(S/N)_{in}}{(S/N)_{in}}$$

If the amplifier has a power gain of G, then

$$N_{out} = FGkT\Delta f \qquad \text{and} \qquad N_{in} = FkT\Delta f$$

The input phase noise in a 1 Hz bandwidth at a frequency of $f_o + f_m$ from the carrier is given by,

$$\theta_{rms1} = \frac{V_{Nrms}}{\sqrt{2}\cdot V_{Srms}} = \sqrt{\frac{FkT}{2P_s}}$$

where

V_{Nrms} = the *rms* noise voltage at the input
V_{Srms} = the *rms* signal voltage at the input
F = noise factor
P_s = input signal power

Amplifier Phase Noise - Continued

Since a correlated random phase deviation, θ_{rms2}, exists at $f_o - f_m$, the total phase noise deviation becomes,

$$\theta_{rms} = \sqrt{\theta_{rms1}{}^2 + \theta_{rms2}{}^2} = \sqrt{\frac{FkT}{P_s}}$$

Now, the phase noise spectral density of the noise contributed by the amplifier, $N(j\omega)$, can be written as,

$$S_\theta(f_m) = \theta_{rms}{}^2 = \frac{FkT}{P_s}$$

In addition to the above thermal noise, we can include the flicker or 1/f noise. The phase noise spectral density is given as,

$$S_\theta(f_m) = \frac{FkT}{P_s}\left(1 + \frac{f_c}{f}\right)$$

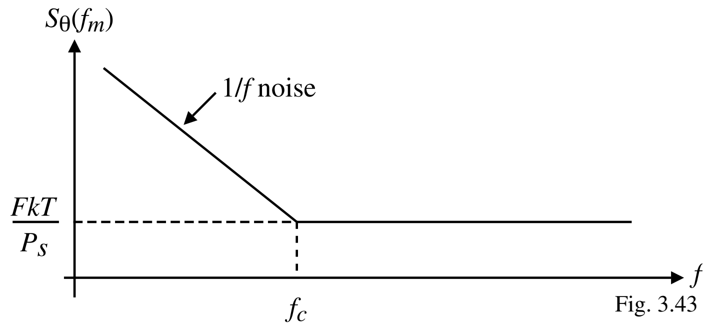


Fig. 3.43

Linear Time Invarient Model for the Phase Noise of an Oscillator using a Resonator

$N(j\omega)$ = phase noise contributed by the amplifier

Solving for $Y(j\omega)$:

$$Y(j\omega) = \frac{1}{1 - H(j\omega)} N(j\omega) + \frac{1}{1 - H(j\omega)} X(j\omega)$$

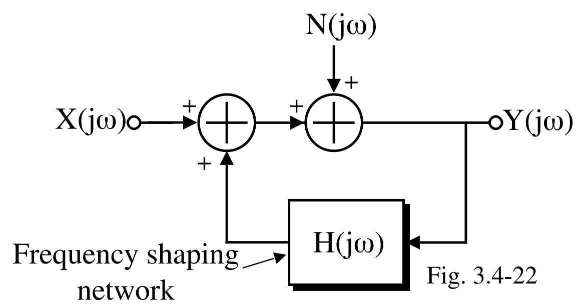


Fig. 3.4-22

Let $S_{o\theta}(f_m)$ = the output phase noise spectral density in (volts^2/Hz) of the oscillator

f_o = oscillator frequency

f_m = frequency deviation about f_o

$S_\theta(f_m)$ = phase noise spectral density of $N(j\omega)$

$$S_{o\theta}(f_m) = \left|\frac{1}{1 - H(f_m)}\right|^2 S_\theta(f_m)$$

Assume that $H(j\omega)$ is a bandpass function.

$$\therefore \quad H(j\omega) = \frac{\frac{j\omega\omega_o}{Q}}{(j\omega)^2 + \frac{j\omega\omega_o}{Q} + \omega_o^2} = \frac{\frac{jf}{f_oQ}}{1 - \left(\frac{f}{f_o}\right)^2 + \frac{jf}{f_oQ}} = \frac{1}{1 + jQ\left(\frac{f_o}{f} - \frac{f}{f_o}\right)}$$

Noise Transfer Function of a Resonator Oscillator - Continued

If $f = f_o + f_m$, then

$$H(f_m) = \frac{1}{1 + jQ\left(\frac{f_o}{f_o+f_m} - \frac{f_o+f_m}{f_o}\right)} = \frac{1}{1 - jQ\left(1 + \frac{f_m}{f_o} - \frac{1}{1+f_m/f_o}\right)} \approx \frac{1}{1 - jQ\left(\frac{2f_m}{f_o}\right)}$$

Substituting this expression in $S_{o\theta}(f_m) = \left|\frac{1}{1 - H(f_m)}\right|^2 S_\theta(f_m)$, gives

$$S_{o\theta}(f_m) = \left|\frac{1}{1 - H(f_m)}\right|^2 S_\theta(f_m) = \left|\frac{1/H(f_m)}{1/H(f_m) - 1}\right|^2 S_\theta(f_m) = \left|\frac{1 - jQ\left(\frac{2f_m}{f_o}\right)}{-jQ\left(\frac{2f_m}{f_o}\right)}\right|^2 S_\theta(f_m)$$

$$\boxed{S_{o\theta}(f_m) \approx \left|\frac{1}{-jQ\left(\frac{2f_m}{f_o}\right)}\right|^2 S_\theta(f_m) = \left[\frac{1}{4Q^2}\left(\frac{f_o}{f_m}\right)^2\right] S_\theta(f_m)} \quad \leftarrow \text{Leeson's equation}$$

Comments:

- The further away f_m is from f_o, the smaller the phase noise.
- The larger the open-loop Q, the smaller the phase noise.

Phase Noise of an Ideal LC Oscillator (Two-Terminal Approach)†

In general, an LC oscillator can be modeled as,

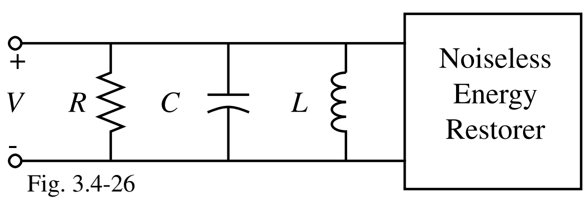


Fig. 3.4-26

The energy stored is,

$$E_{stored} = \frac{1}{2} C V_{peak}^2$$

Assuming a sinusoidal signal, the mean square carrier voltage is,

$$\overline{V_{sig}^2} = \frac{E_{stored}}{C}$$

The total mean square noise is,

$$\overline{V_n^2} = 4kTR \int_0^\infty \left|\frac{Z(f)}{R}\right|^2 df = 4kTR\left(\frac{1}{4RC}\right) = \frac{kT}{C}$$

where $|Z(f)/R|$ is the bandwidth of the resonator.

† T.H. Lee and A. Hajimiri, "Oscillator Phase Noise: A Tutorial," *IEEE J. of Solid-State Circuits*, Vol. 35, No. 3, March 2000.

Phase Noise of an Ideal LC Oscillator (Two-Terminal Approach) - Continued

The "noise-to-signal" ratio is given as,

$$\frac{N}{S} = R\frac{\overline{V_n^2}}{V_{sig}^2} = \frac{kT}{E_{stored}}$$

which confirms that one needs to maximize the signal level to reduce the noise-to-carrier ratio.

Power consumption and Q can be brought into the relationship via the definition of Q,

$$Q = \frac{\text{Energy stored}}{\text{Energy dissipated per cycle}} = \frac{\omega E_{stored}}{P_{diss}}$$

Therefore,

$$\frac{N}{S} = \frac{\omega kT}{QP_{diss}}$$

These relationships are reasonably valid for real oscillators and encourage the use of large values of Q to achieve low phase noise (which is not the total picture).

Phase Noise of an Ideal LC Oscillator (Two-Terminal Approach) - Continued

Assume that the only source of noise is the thermal (white) noise of the resistor. Therefore, the noise can be represented by a current source in parallel with the LC tank with a mean-square spectral density of,

$$\frac{\overline{i_n^2}}{\Delta f} = 4kTG$$

The noise voltage is more useful and is found by multiplying the noise current times the tank impedance. However, if the energy restoring circuit perfectly cancels the positive resistance, we have an ideal LC impedance at resonance. For relatively small deviations from resonance, $\Delta\omega$, we have,

$$Z(\omega_o+\Delta\omega) \approx \mathrm{j}\frac{\omega_o L}{2\frac{\Delta\omega}{\omega_o}}$$

A more useful form is achieved using the expression for the unloaded Q of the LC tank.

$$Q = \frac{R}{\omega_o L} = \frac{1}{\omega_o GL} \qquad \rightarrow \qquad |Z(\omega_o+\Delta\omega)| \approx \frac{1}{G}\frac{\omega_o}{2Q\Delta\omega}$$

The spectral density of the mean-square noise voltage can be expressed as,

$$\frac{\overline{v_n^2}}{\Delta f} = \frac{\overline{i_n^2}}{\Delta f}|Z|^2 = 4kTR\left(\frac{\omega_o}{2Q\Delta\omega}\right)^2 \qquad \rightarrow \qquad 1/f^{\,2} \text{ behavior}$$

Phase Noise of an Ideal LC Oscillator (Two-Terminal Approach) - Continued

In the ideal model, the thermal noise affects both amplitude and phase which is represented by the previous expression. When in equilibrium, these two noise contribution are equal.

In an amplitude limited system, the limiting mechanism removes the amplitude noise so that the spectral density of the mean-square noise voltage for an *LC* tank with an amplitude limiting mechanism is equal to half of the previous result,

$$\frac{\overline{v_n^2}}{\Delta f} = \frac{\overline{i_n^2}}{\Delta f}|Z|^2 = 2kTR\left(\frac{\omega_o}{2Q\Delta\omega}\right)^2$$

Normalizing the voltage noise by the rms signal voltage ($P_s = V_s^2/R$), gives the single-sideband noise spectral density as,

$$\mathcal{L}(\Delta\omega) = 10\log\left[\frac{2kT}{P_s}\left(\frac{\omega_o}{2Q\Delta\omega}\right)^2\right]$$

Interpretation:

- Phase noise improves as Q increases (the *LC* tank's impedance falls off as $1/Q\Delta\omega$).
- Phase noise improves as the carrier power increases (thermal noise stays constant)

Phase Noise of an Ideal LC Oscillator (Two-Terminal Approach) - Continued

Results of simplifying assumptions:

- Although real spectra possess a $1/f^2$ region, the magnitudes are larger than that predicted above.
- There are other noise sources besides the tank loss (i.e., the energy restorer).
- The measured spectra will eventually flatten out for large frequency offsets (this is due to noise such as the output buffers or even the limitation of the measurement equipment itself).
- There is almost always a $1/(\Delta\omega)^3$ region at small offsets.

A Modification to the Single-Sideband Phase Noise (Leeson)

A modification to the single-sideband phase noise by Leeson is as follows:

$$\mathcal{L}(\Delta\omega) = 10\log\left(\frac{1}{2}S_{o\theta}(f_m)\right) = 10\log\left[\frac{2FkT}{P_s}\left(1+\frac{1}{4Q^2}\left(\frac{f_o}{f_m}\right)^2\right)\left(1+\frac{f_c}{f}\right)\right]$$

The modifications are:

1.) To include a factor, F, to account for the increased noise in the $1/(\Delta\omega)^2$ region.

2.) Include a 1 inside the brackets to include the flattening out of the spectra.

3.) Include a multiplicative term to provide a $1/(\Delta\omega)^3$ region at small offset frequencies.

Typical result:

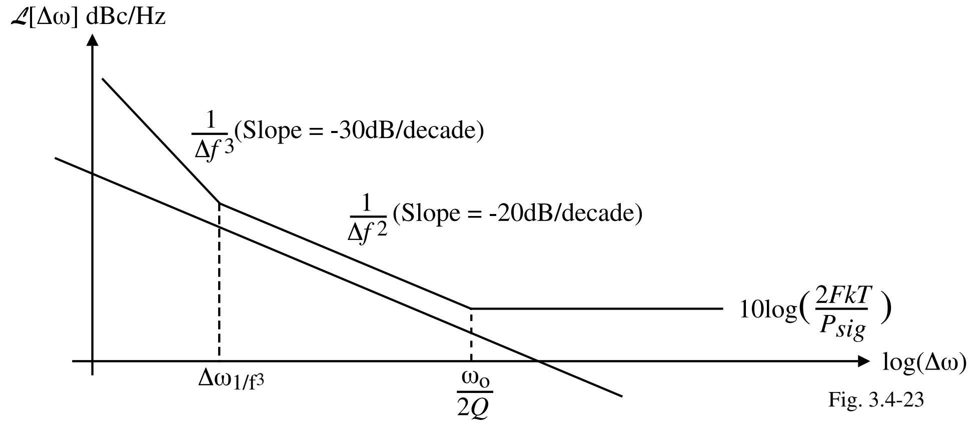


Fig. 3.4-23

Example 3 – Linear Time Invarient Phase Noise Model

A model for single sideband noise using the time-invarient theory is given by

$$\mathcal{L}\{f_m\} = 10\log\left\{\frac{2FkT}{P_s}\left[1+\frac{1}{4Q^2}\left(\frac{f_o}{f_m}\right)^2\right]\left(1+\frac{f_c}{f_m}\right)\right\}$$

(a.) Describe each term in this equation and give the units of the term.

(b.) If F = 2dB, what is the noise floor if the carrier power is 10 dBm at room temperature (27°C) and $k = 1.381\text{x}10^{-23}$ Joules/K°?

(c.) Make an approximate sketch of $\mathcal{L}\{f_m\}$ in dBc as a function of log10(f_m) and identify the various regions.

Solution

(a.)

F = the noise figure or factor depending upon terminology. It is unitless.
k = Boltsmann's constant and is equal to $1.381\text{x}10^{-23}$ Joules/K°.
T = temperature in °K
P_s = power in the carrier in watts.
Q = open-loop Q of the oscillator. It is unitless.
f_o = carrier frequency in Hz.
f_m = deviation frequency from the carrier in Hz.
f_c = corner frequency in Hz associate where the $1/f$ noise is no longer significant.

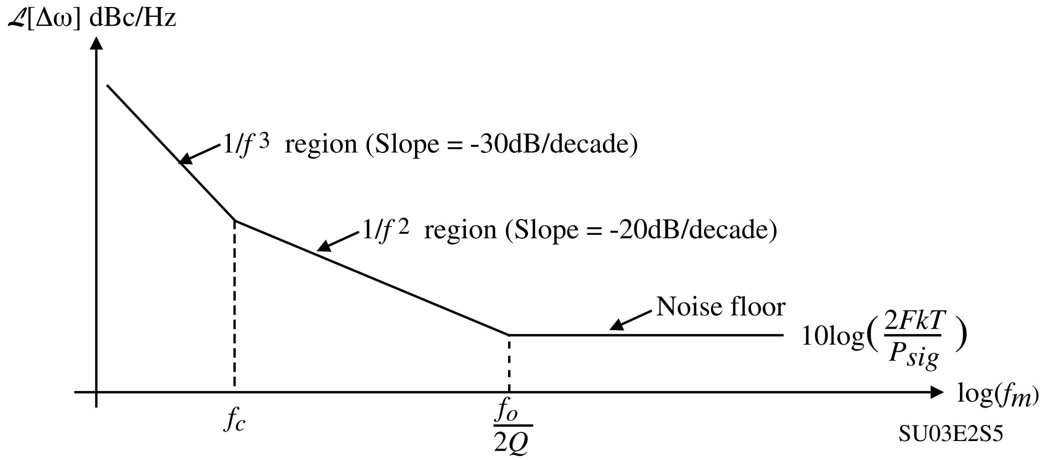
Example 3 - Continued

(b.) The noise floor is 10 log $\left(\frac{2FkT}{P_s}\right)$. We need to perform some "preprocessing" first before using the equation.

$$F = 2\text{dB} \rightarrow F = 10^{2/10} = 1.585 \quad \text{and} \quad P_s = 10\text{dBm} \rightarrow P_s = 10^{10/10} = 10\text{mW}$$

$$\mathcal{L}\{f_m\} = 10\log\left(\frac{2\cdot 1.585\cdot 1.381\text{x}10^{-23}\cdot 300}{10\text{x}10^{-3}}\right) = -178.8 \text{ dBc}$$

(c.)



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LINEAR TIME VARYING NOISE MODEL FOR VCO PHASE NOISE

Linear Time Varying Noise Model

In reality, most oscillators are time varying systems and the previous time-invarient analysis needs to be modified to account for time variance. (Linearity is still a reasonable assumption, however.)

How are oscillators time varying?

Consider the LC oscillator shown excited by a current pulse:

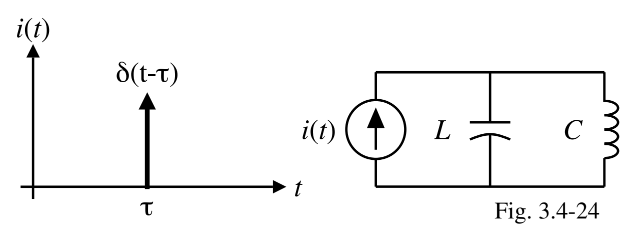


Fig. 3.4-24

Assume that the oscillator is oscillating with some constant amplitude. The following shows the impulse response of the oscillator at two different times and demonstrates time variance.

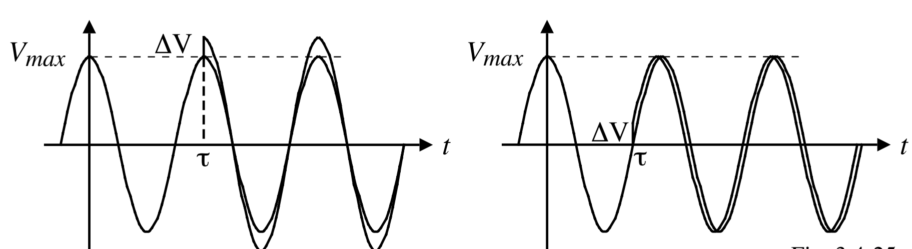


Fig. 3.4-25

Impulse Sensitivity Function

The impulse response completely characterizes the oscillator since linearity still remains a good assumption. Therefore, let us find the single-sideband phase noise using the impulse response approach.

The impulse response for a step change in the phase may be written as,

$$h_\phi(t,\tau) = \frac{\Gamma(\omega_o\tau)}{q_{\max}}\, u(t\text{-}\tau)$$

where $u(t)$ is a unit step function and $\Gamma(x)$ is called the *impulse sensitivity function* (ISF) and $q_{\max}$ is the maximum charge displacement across the capacitor.

ISF is a dimensionless, frequency and amplitude independent function periodic in 2π. It encodes information about the sensitivity of the oscillator to an impulse injected at phase $\omega_o\tau$.

The following are some examples of the ISF:

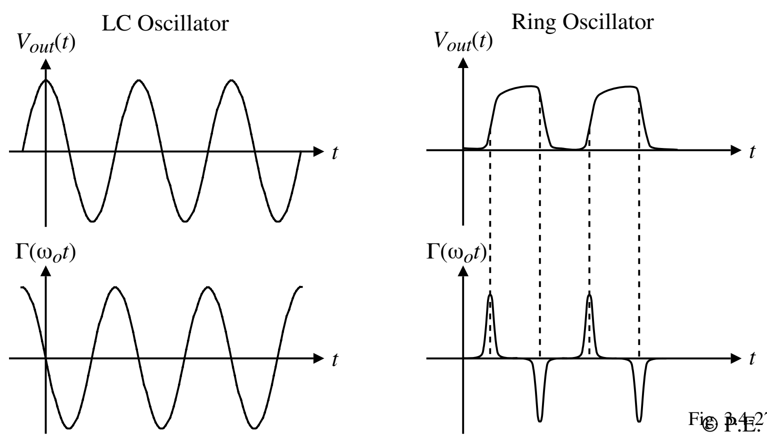


Fig. 3.4-27

Excess Phase using the ISF

Once the ISF has been determined (many means are possible but simulation is probably the best), we may compute the excess phase through the use of the superposition integral:

$$\phi(t) = \int_{-\infty}^{\infty} h_\phi(t,\tau) i(\tau) d\tau = \frac{1}{q_{\max}} \int_{-\infty}^{t} \Gamma(\omega_o\tau) i(\tau) d\tau$$

Illustration of this computation:

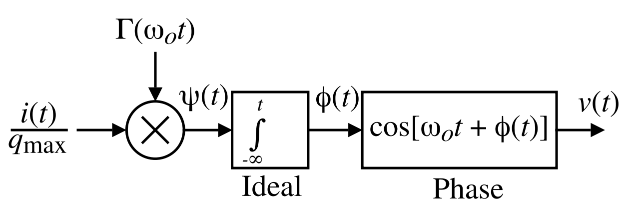


Fig. 3.4-28

This process involves the modulation of the normalized input noise current injected into the node of interest by a periodic function (ISF), followed by an ideal integration and a nonlinear phase modulation that converts phase to voltage.

Excess Phase using the ISF – Continued

To put the above equation in a more practical form, note that the ISF is periodic and therefore can be represented by a Fourier series as,

$$\Gamma(\omega_o\tau) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_o\tau + \theta_n)$$

where the coefficients, c_n, are real and θ_n is the phase of the n-th harmonic of the ISF.

In the following, we shall assume that the noise components are uncorrelated so their relative phase is unimportant and θ_n can be ignored. If the series converges rapidly, then the ISF is well-approximated by only the first few terms.

Substituting the Fourier expansion of the ISF into the previous work gives the excess phase as,

$$\phi(t) = \frac{1}{q_{\max}}\left[c_0 \int_{-\infty}^{t} i(\tau)d\tau + \sum_{n=1}^{\infty} c_n \int_{-\infty}^{t} i(\tau)\cos(n\omega_o\tau)\, d\tau\right]$$

Excess Phase using the ISF – Continued

Illustration of the ISF decomposition:

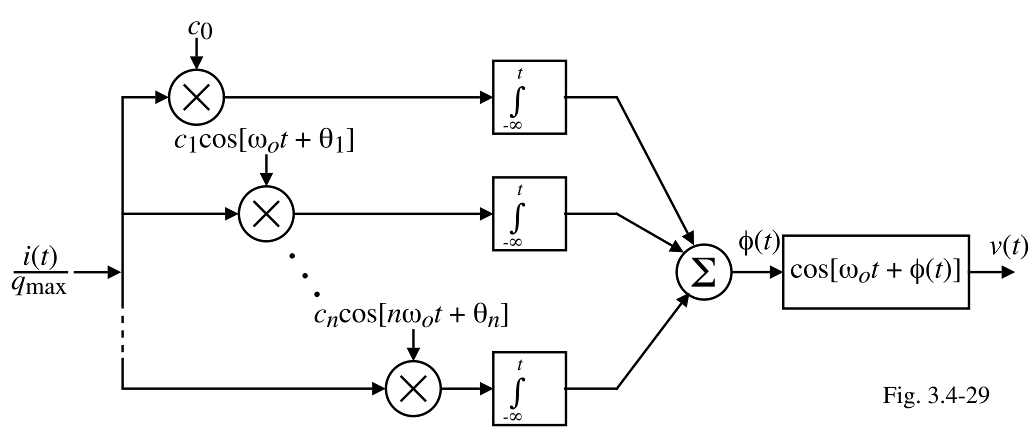


Fig. 3.4-29

The block diagram contains elements that are analogous to those of a superheterodyne receiver. The normalized noise current is analogous to a broadband "RF" signal whose Fourier components undergo simultaneous down-conversion by a "local oscillator" at all harmonics of the oscillation frequency.

Sidebands of Excess Phase

The previous analogy can be used to show that the excess phase noise has two equal sidebands at $\pm\Delta\omega$ even though injection occurs near some integer multiple of ω_o.

Consider a sinusoidal current that is injected at a frequency $\Delta\omega$, where $\Delta\omega << \omega_o$.

$$i(t) = I_n \cos(\Delta\omega t)$$

Substitute this expression into the previous expression for $\phi(t)$ with n =0, gives the following

$$\phi(t) \approx \frac{I_0 c_0}{q_{max}} \int_{-\infty}^{t} \cos(\Delta\omega t) d\tau$$

$$= \frac{I_0 c_0 \sin(\Delta\omega t)}{q_{max}\Delta\omega}$$

where there is a negligible contribution to the integral by terms other than n = 0.

Therefore, the spectrum of $\phi(t)$ consists of two equal sidebands at $\Delta\omega$ even though the injection occurred near ω =0.

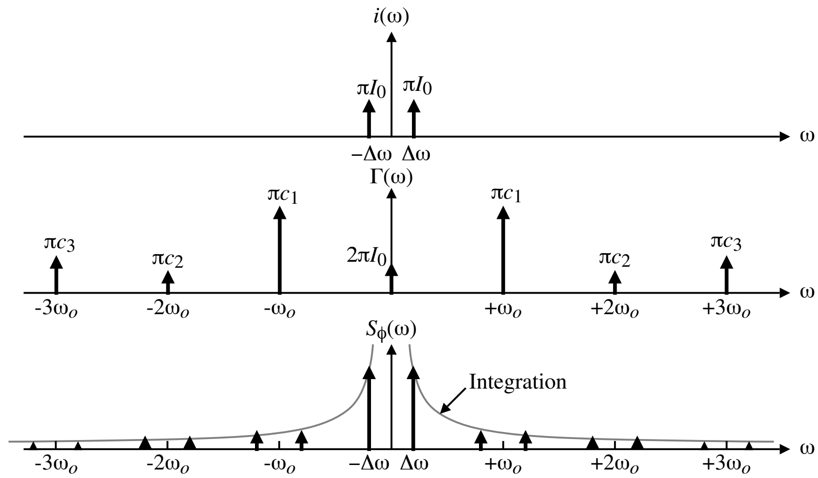


Fig. 3.4-30

Sidebands of Excess Phase - Continued

Consider a sinusoidal current that is injected at a frequency which is close to the oscillation frequency,

$$i(t) = I_1 \cos[\omega_o + \Delta\omega)t]$$

where $\Delta\omega << \omega_o$.

Substitute this expression into the previous expression for $\phi(t)$ gives the following

$$\phi(t) \approx \frac{I_1 c_1 \sin(\Delta\omega t)}{2q_{max}\Delta\omega}$$

where there is a negligible contribution to the integral by terms other than n=1.

Again, the spectrum of $\phi(t)$ consists of two equal sidebands at $\Delta\omega$ even though the injection occurred near ω_o.

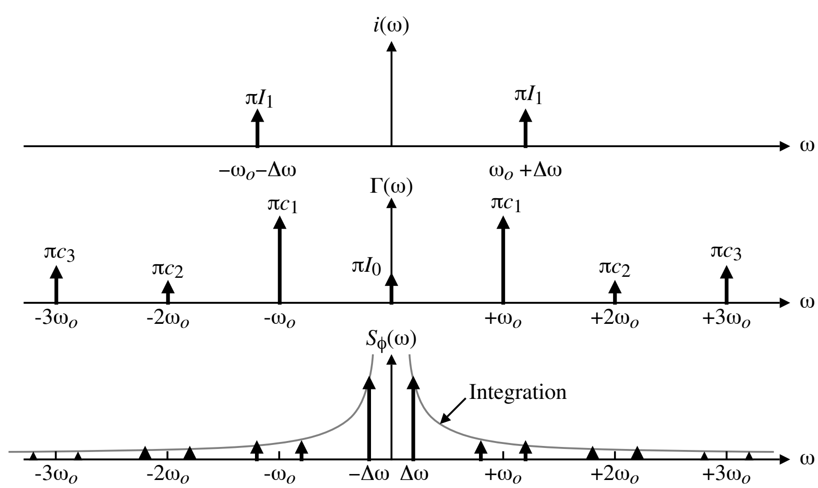


Fig. 3.4-31

Sidebands of Excess Phase - Continued

In general, consider a sinusoidal current that is injected at a frequency near an integer n of the oscillation frequency,

$$i(t) = I_n \cos\,[n\omega_o + \Delta\omega)t]$$

where $\Delta\omega << \omega_o$.

Substitute this expression into the previous expression for $\phi(t)$ gives the following

$$\phi(t) \approx \frac{I_n c_n \sin(\Delta\omega t)}{2q_{\max}\Delta\omega}$$

where there is a negligible contribution to the integral by terms other than n.

Therefore, the spectrum of $\phi(t)$ consists of two equal sidebands at $\Delta\omega$ even though the injection occurred near some integer multiple of ω_o.

Single-Sideband Noise using the LTV Model

How is the excess phase noise linked to spectrum of the output voltage of the oscillator?

Consider the following equation,

$$v_{out}(t) = \cos[\omega_o t + \phi(t)]$$

which acts like a phase-to-voltage converter.

Expanding $v_{out}(t)$ gives,

$$v_{out}(t) = \cos(\omega_o t)\cos[\phi(t)] - \sin(\omega_o t)\sin[\phi(t)] \approx \cos(\omega_o t) - \phi(t)\sin(\omega_o t)$$

for small values of $\phi(t)$.

Substituting the value of $\phi(t)$ from the previous slide gives

$$v_{out}(t) = \cos(\omega_o t) - \frac{I_n c_n \sin(\Delta\omega t)}{2q_{\max}\Delta\omega}\sin(\omega_o t)$$

Therefore, the single-sideband power relative to the carrier is given as,

$$P_{dBc}(\Delta\omega) = \left(\frac{I_n c_n}{4q_{\max}\Delta\omega}\right)^2 \qquad \rightarrow \qquad P_{dBc}(\Delta\omega) = \left(\frac{\frac{\overline{i_n^2}}{\Delta f}\sum_{n=0}^{\infty} c_n{}^2}{4q_{\max}{}^2\Delta\omega^2}\right)$$

for white noise[†].

[†] A. Hajimiri and T. Lee, "Design issues in CMOS differential LC oscillators," *IEEE J. Solid-State Cricuts*, vol. 34, no. 5, May 1999, pp. 716-724.

Conversion of Noise to Phase Fluctuations and Phase-Noise Sidebands

The previous expressions for $P_{dBc}(\Delta\omega)$ imply both an upward and downward conversion of phase noise onto the noise near the carrier as illustrated below.

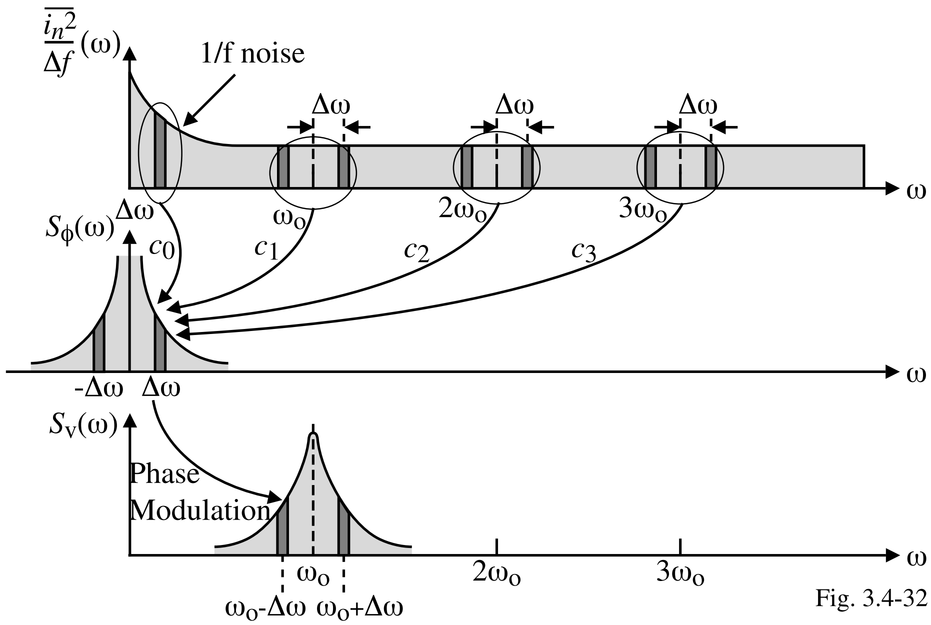


Fig. 3.4-32

Components of the noise near integer multiples of the carrier frequency all fold into noise near the carrier itself.

Single-Sideband Phase Noise of the LTV Model

The total single-sideband phase noise spectral density due to one noise source at an offset frequency of $\Delta\omega$ is given by the sum of the powers in the previous figure and is

$$L(\Delta\omega) = 10 \log \left(\frac{\frac{\overline{i_n^2}}{\Delta f} \sum_{n=0}^{\infty} c_n^2}{4 q_{max}^2 \Delta\omega^2} \right)$$

According to Parseval's relation,

$$\sum_{n=0}^{\infty} c_n^2 = \frac{1}{\pi} \int_0^{2\pi} |\Gamma(x)|^2 dx = 2\Gamma_{rms}^2$$

where Γ_{rms} is the rms value of $\Gamma(x)$.

Therefore,

$$L(\Delta\omega) = 10 \log \left(\frac{\Gamma_{rms}^2}{q_{max}^2} \quad \frac{\overline{i_n^2}/\Delta f}{2\Delta\omega^2} \right)$$

This equation is rigorous equation for the $1/f^2$ region and no empirical curve-fitting parameters are needed.

Single-Sideband Phase Noise of the LTV Model – Continued

The close-in phase noise can be modeled by assuming the current noise behaves as follows in the 1/f region,

$$\overline{i_{n,1/f}^{2}} = \overline{i_n^2}\,\frac{\omega_{1/f}}{\Delta\omega}$$

Using the previous results for white noise, we obtain the following,

$$L(\Delta\omega) = 10\log\left(\frac{\dfrac{\overline{i_n^2}}{\Delta f}\,c_0^{\,2}}{8q_{max}^{\,2}\Delta\omega^2}\,\frac{\omega_{1/f}}{\Delta\omega}\right)$$

which describes the behavior in the $1/f^3$ region.

Equating the above to the single-sideband phase noise in the $1/f^2$ region gives,

$$\Delta\omega_{1/f} = \omega_{1/f}\,\frac{c_0^{\,2}}{4\Gamma_{rms}^2} = \omega_{1/f}\left(\frac{\Gamma_{dc}}{\Gamma_{rms}^{\,2}}\right)^2$$

where Γ_{dc} is the dc value of Γ.

Note that the $1\,f^3$ is not necessarily the same as the $1/f$ circuit noise corner and is generally lower.

Pre-Summary

So what does all this mean?

To reduce the phase noise in PLLs due to VCO's:

1.) Make the tank Q or resonator Q as large as possible.

2.) Maximize the signal power.

3.) Minimize the ISF.

4.) Force the energy restoring circuit to function when the ISF is at a minimum and to deliver it's energy in the shortest possible time.

5.) The best oscillators will possess symmetry which leads to small Γ_{dc} for minimum up-conversion of 1/f noise.

PHASE NOISE IN LC OSCILLATORS

LC Oscillator Example using the LTV Theory – Colpitts Oscillator

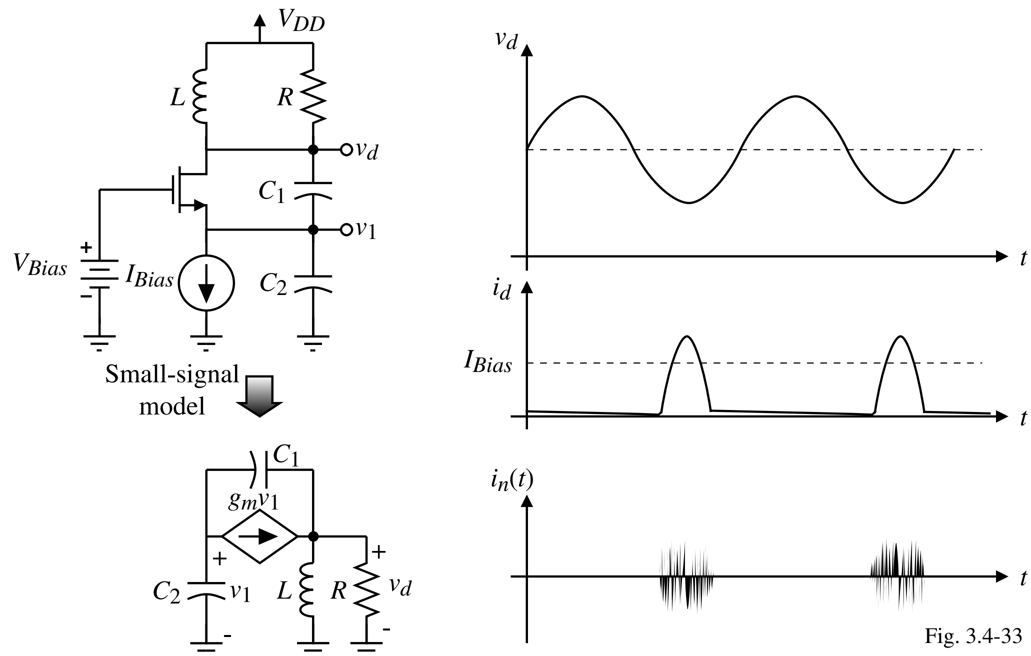


Fig. 3.4-33

Note that i_d only flows during a short interval coincident with minimum ISF.

Colpitts LC Oscillator Example – Continued

Find the conditions for oscillation by equating the determinant of the circuit equal to zero:

Nodal eqs:

$$g_mV_1 + sC_2V_1 + sC_1(V_1\text{-}V_d) = 0 \quad \rightarrow \quad (g_m + sC_1 + sC_2)V_1 - sC_1V_d = 0$$

$$-g_mV_1 + sC_1(V_d\text{-}V_1) + (G_L + 1/sL)V_d = 0 \quad \rightarrow \quad -(g_m + sC_1)V_1 + (G_L + sC_1 + 1/sL)V_d = 0$$

$$\text{Det}\{\ \} = [g_m + s(C_1+C_2)][G_L + sC_1 + 1/sL] - sC_1(g_m + sC_1) = 0$$

$$g_mG_L + sg_mC_1 + \frac{g_m}{sL} + sG_L(C_1+C_2) + s^2C_1{}^2 + s^2C_1C_2 + \frac{C_1+C_2}{L} - sg_mC_1 - s^2C_1{}^2 = 0$$

Replacing s by $j\omega$ and eliminating canceling terms gives,

$$\left[g_mG_L - \omega^2C_1C_2 + \frac{C_1+C_2}{L}\right] + j\omega\left[-\frac{g_m}{\omega^2L} + G_L(C_1+C_2)\right] = 0 + j0$$

Assuming that $g_mL << R_L(C_1+C_2)$, gives

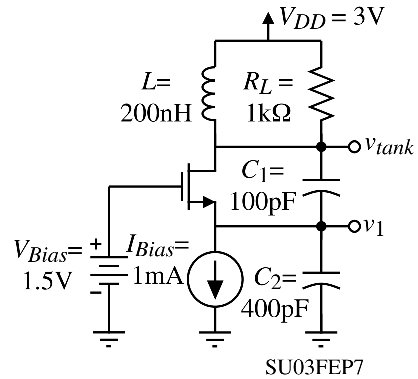
$$\omega_{osc} = \sqrt{\frac{1}{LC_1} + \frac{1}{LC_2}} = \frac{1}{\sqrt{\frac{LC_1C_2}{C_1+C_2}}} \qquad \text{and} \qquad g_mR = \frac{(C_1+C_2)^2}{C_1C_2} = \frac{C_1}{C_2}\left(1+\frac{C_2}{C_1}\right)^2$$

It can be shown by the LTV theory that the best phase noise occurs when C_2 is approximately 4 to 5 times C_1.

Example 4 – LTV Noise Theory Applied to an LC Oscillator

An LC oscillator is shown (a.) What is the frequency of oscillation of this oscillator? (b.) At room temperature, assume that the noise of R_L is dominant over all other noise sources and calculate the single sideband phase noise resulting from the resistor's noise using the linear time varying theory if $\Gamma_{rms}^2 = 0.5$. Note that the tank voltage, v_{tank} can be approximated as

$$v_{tank} \approx 2I_{Bias}R_L\left(1-\frac{C_1}{C_1+C_2}\right)$$



(c.) At an offset of 200kHz, what is the phase noise in dBc?

Solution

(a.) The frequency of oscillation is

$$\omega_{osc} = \frac{1}{\sqrt{L\frac{C_1C_2}{C_1+C_2}}} = \frac{1}{\sqrt{200\text{nH}\cdot 80\text{pF}}} = 250 \text{ Mrads/sec.} \quad \rightarrow \quad f_{osc} = 39.79 \text{ MHz}$$

$$\boxed{\omega_{osc} = 250 \text{ Mrads/sec and } f_{osc} = 39.79\text{MHz}}$$

(b.) The LTV noise theory gives the SSB noise as,

$$\mathcal{L}\{\Delta\omega\} = 10\log_{10}\left[\frac{\frac{\overline{i_n^2}}{\Delta f}\,\Gamma_{rms}^2}{2q_{max}^2\Delta\omega^2}\right]$$

Example 4 - Continued

The noise due to R_L is found as

$$\frac{\overline{i_n^2}}{\Delta f} = \frac{4kT}{R_L} = \frac{4\cdot 1.381\text{x}10^{-23}\cdot 300}{10^3} = 1.6572\text{x}10^{-23} \text{ A}^2/\text{Hz}$$

q_{max} is the maximum charge across the equivalent tank capacitance (80pF) given as

$$q_{max} = C_{eq}\, v_{max}$$

Let us assume that v_{max} is equal to v_{tank} which is given from the above expression as

$$v_{tank} \approx 2I_{Bias}R_L\left(1-\frac{C_1}{C_1+C_2}\right) = 2\cdot 1\text{mA}\cdot 1\text{k}\Omega\left(1-\frac{100}{500}\right) = 1.6\text{V}$$

$$\therefore\ q_{max} = 80\text{pF}\cdot 1.6\text{V} = 128 \text{ pC}$$

$$\boxed{\mathcal{L}\{\Delta\omega\} = 10\log_{10}\left[\frac{0.000253}{(\Delta\omega)^2}\right]}$$

(c.) $$\mathcal{L}\{\Delta\omega\} = 10\log_{10}\left[\frac{1.6572\text{x}10^{-23}}{4(128\text{x}10^{-12})^2(0.4\pi\text{x}10^6)^2}\right] = 10\log_{10}(6.405\text{x}10^{-16}) = -151.9\text{dBc}$$

$$\boxed{\mathcal{L}\{\Delta\omega\} = -151.9\text{dBc at an offset of 200kHz}}$$

An Optimum Low-Phase Noise LC Oscillator

The Colpitts LC oscillator suffers from the fact the tank voltage cannot exceed power supply. The Clapp LC oscillator, shown previously, avoids this problem with a tapped resonator. A common implementation of the Clapp oscillator is the differential version shown.[1 2 3]

This circuit uses an automatic amplitude control circuit to force the value of loop gain needed to provide contant oscillation amplitude.

Impulse sensitivity function modeling was used to optimize the noise performance. The optimum tapping ratio ($1+C_4/C_1$) was found to be 4.5.

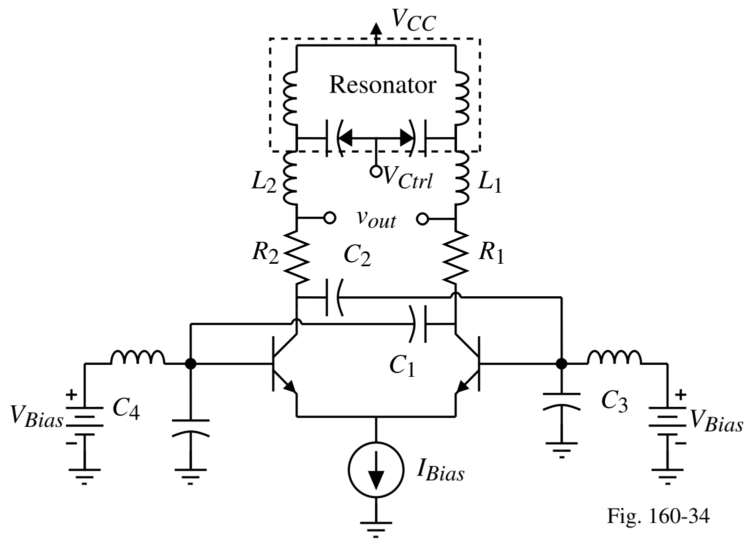


Fig. 160-34

[1] J. Craninckx and M. Steyaert, "A 1.8 GHz CMOS low-phase-noise voltage-controlled oscillator with prescaler," *IEEE J. of Solid-State Circuits*, Vol. 30, pp. 1474-1482, Dec. 1995.

[2] T.I. Ahrens and T. H. Lee, "A 1.4 GHz, 3mW CMOS *LC* low phase noise VCO using tapped bond wire inductance," *Proc. ISLPED*, Aug. 1998, pp. 16-19.

[3] M. Margarit, J. Tham, R. Meyer, and M. Deen, "A low-noise, low-power VCO with automatic amplitude control for wireless applications," *IEEE J. of Solid-State Circuits*, Vol. 34, pp. 761-771, June 1999.

Symmetrical LC Oscillator

This configuration exploits importance in the LTV theory of symmetry.

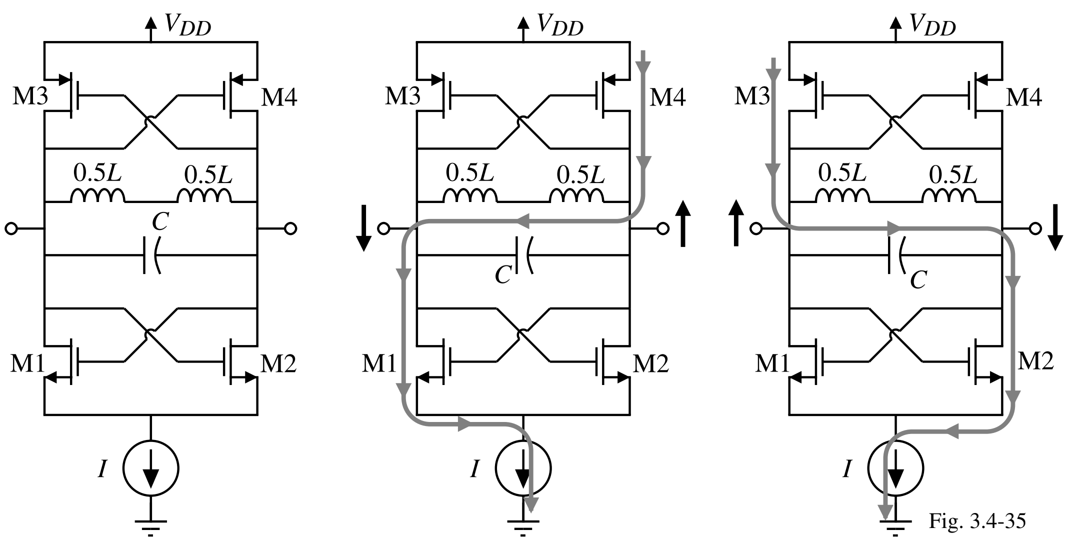


Fig. 3.4-35

- Select the relative widths of the PMOS and NMOS to minimize the dc value of the ISF which will minimize the upconversion of 1/f noise.
- The bridge arrangement of transistors allows for greater signal swings.
- 0.25µm CMOS gives –121dBc/Hz at 600kHz offset at 1.8 GHz dissipating 6mW[†]

[†] A. Hajimiri and T. Lee, "Design issues in CMOS differential *LC* oscillators," *IEEE J. Solid-State Circutis*, vol. 34, pp. 716-724, May 1999.

Finding the ISF

1.) Direct Method

Apply an impulse to the oscillator and measure the steady state perturbation. Repeat the application of the impulse throughout the entire cycle of the oscillator

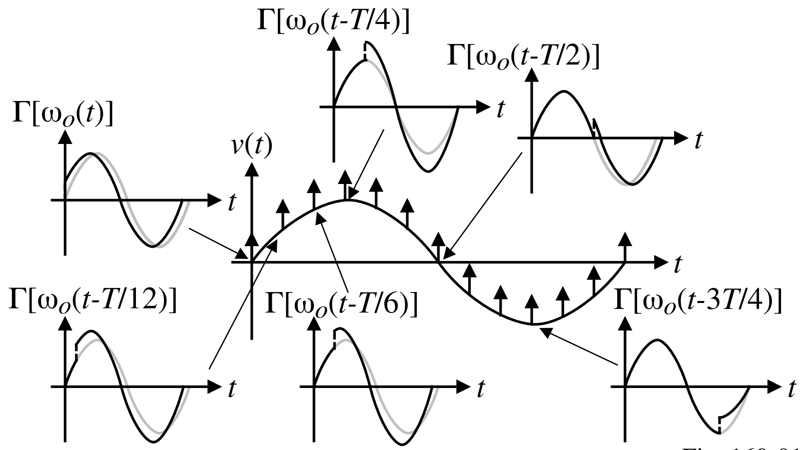


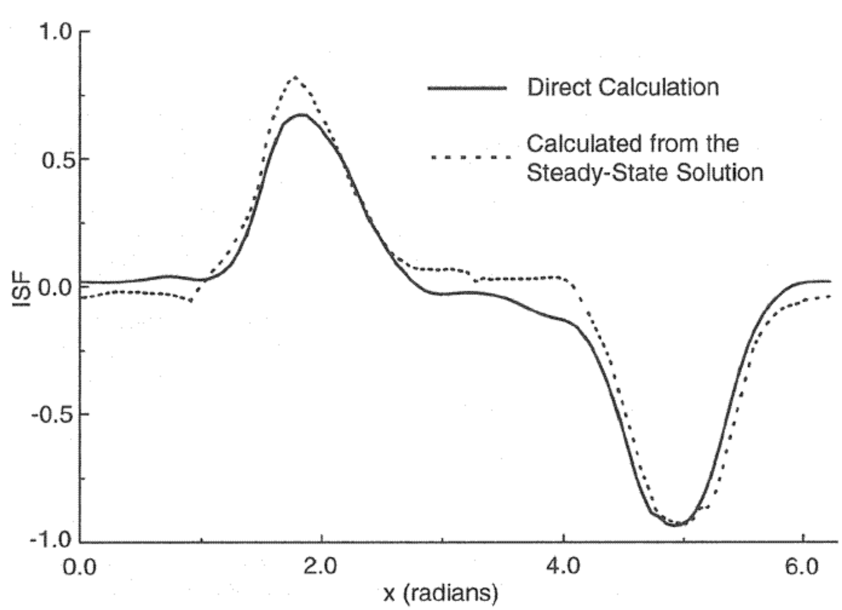
Fig. 160-01

Caution: The impulse amplitude must be small enough to insure the assumption of linearity is valid. One can check by increasing or decreasing the impulse amplitude and see if the response scales linearly.

Finding the ISF – Continued

2.) Steady-State Method.

Simulate the oscillator limit cycle upon the application of a small perturbation. The phase shift is given by the change in time to transverse the new limit cycle.



Does not take into account the AM-to-PM conversion that occurs in the oscillator.

Amplitude Noise (versus Phase Noise)

The close-in sideband are dominated by the phase noise whereas the far out sidebands are more affected by amplitude noise.

Unlike phase noise, amplitude noise will decay with time because of the amplitude restoring mechanisms present in all oscillators. The excess amplitude may decay slowly as in the case of a harmonic oscillator or quickly as in the case of a ring oscillator.

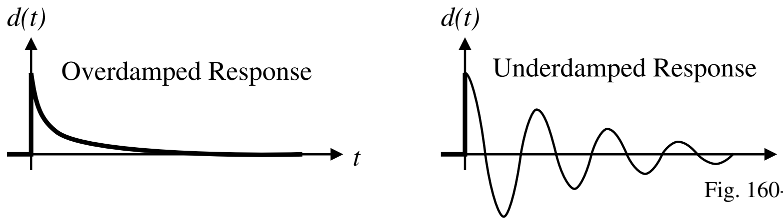


Fig. 160-03

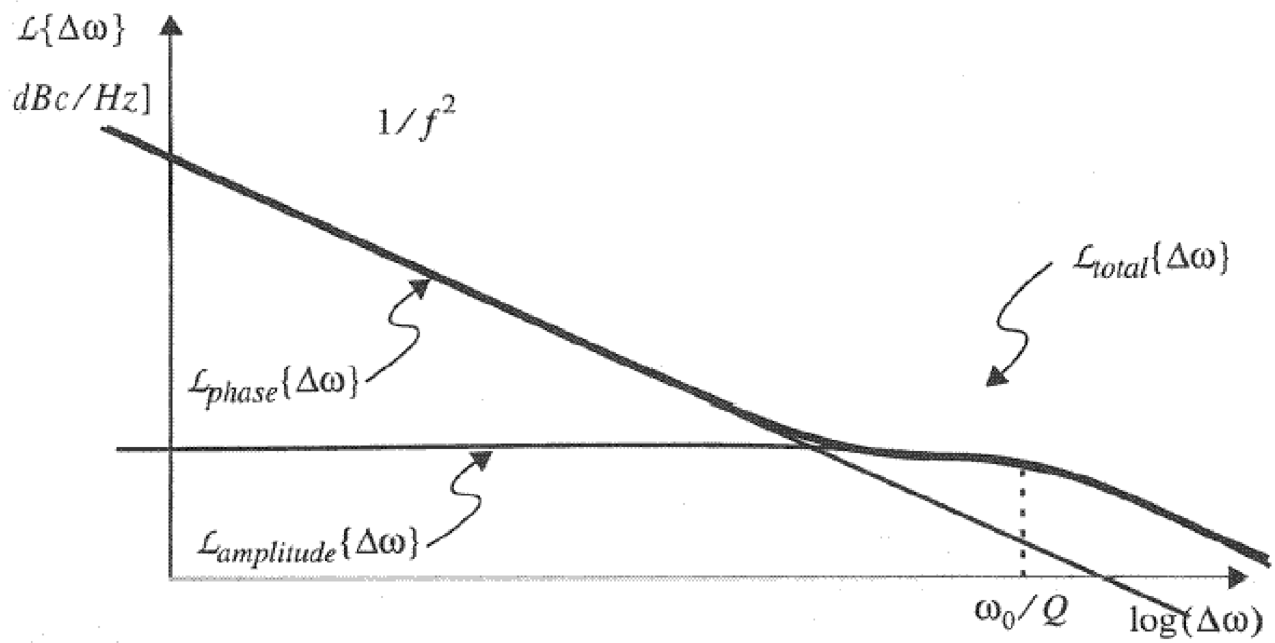
If the current impulse that causes an instantaneous voltage change on the capacitor is a white noise source with a power spectral density of $\overline{i_n^2}/\Delta f$, then the single sideband noise can be found as,

$$\mathcal{L}_{\text{amplitude}}\{\Delta\omega\} = \frac{\Lambda(\omega_o\tau}{q_{max}^2}\,\frac{\overline{i_n^2}/\Delta f}{2\left(\frac{\omega_o^2}{Q^2}+\Delta\omega^2\right)}$$

where $\Lambda(\omega_o t)$ is a periodic function that determines the sensitivity of each point of the waveform to an impulse and is called the amplitude impulse sensitivity function.

Amplitude Noise – Continued

The amplitude and phase noise and total output sideband power for the overdamped exponentially decaying amplitude response is shown below.



JITTER AND PHASE NOISE IN RING OSCILLATORS

Ring Oscillators

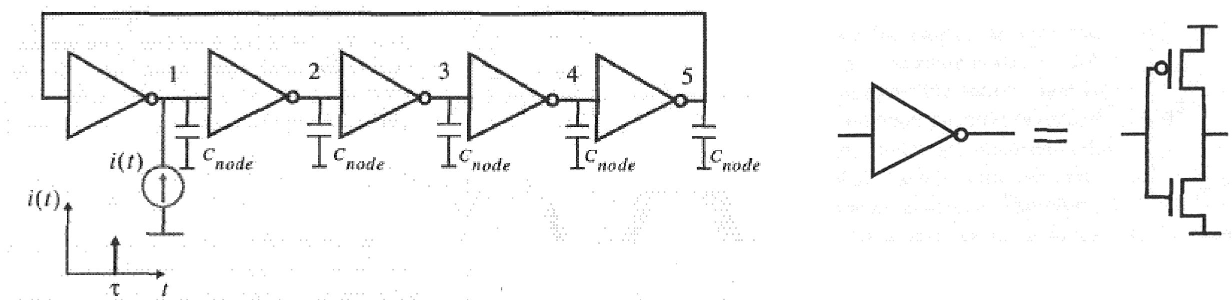
Problems:

1.) The Q is low (energy stored in the capacitor is discharged every cycle)

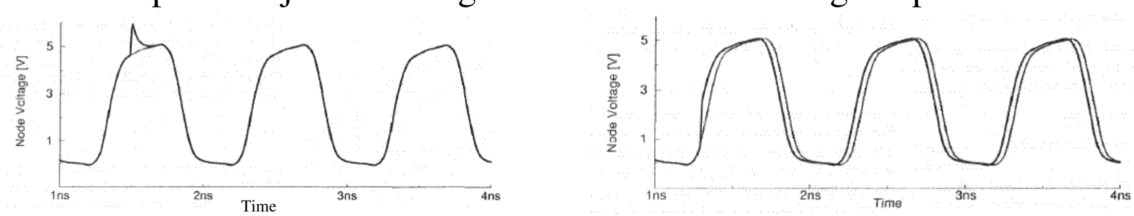
2.) The energy is stored at the rising/falling edges rather than at voltage maximums

However, the ring oscillator achieves better phase noise in a mixed signal environment.

Five-stage inverter-chain ring oscillator with a current impulse injected:

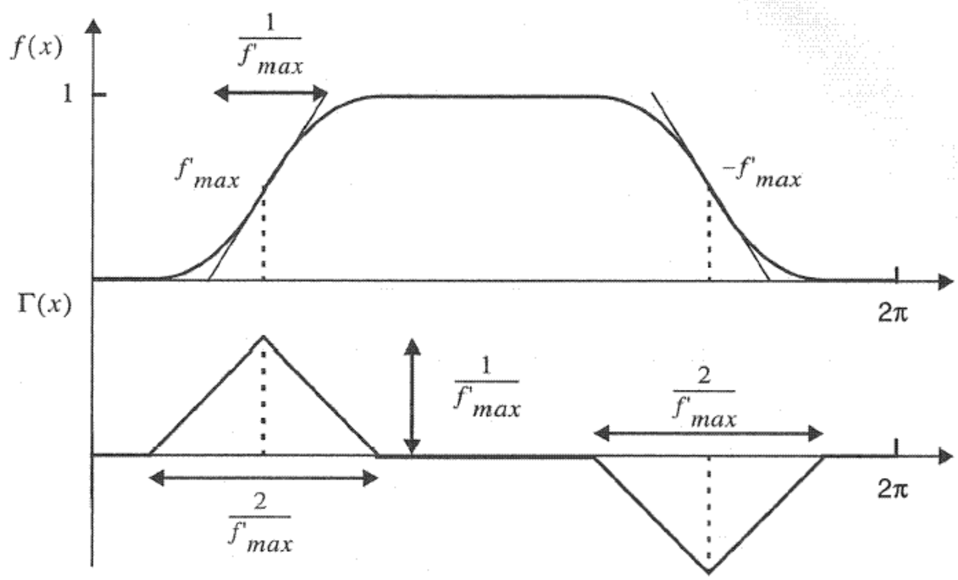


Effect of impulses injected during the transition and during the peak.



Impulse Sensitivity Function for Single-Ended Ring Oscillators

Approximate waveform and ISF for a single-ended ring oscillator:

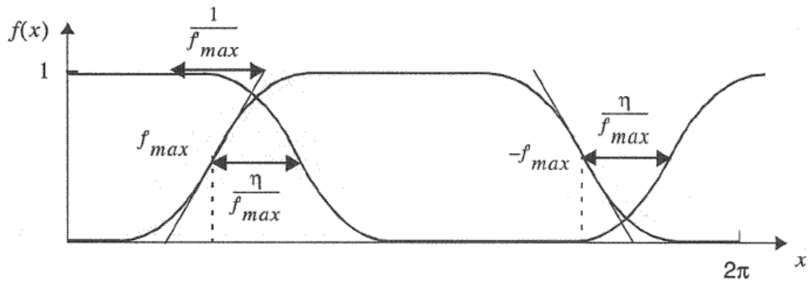


The approximate *rms* value of Γ is,

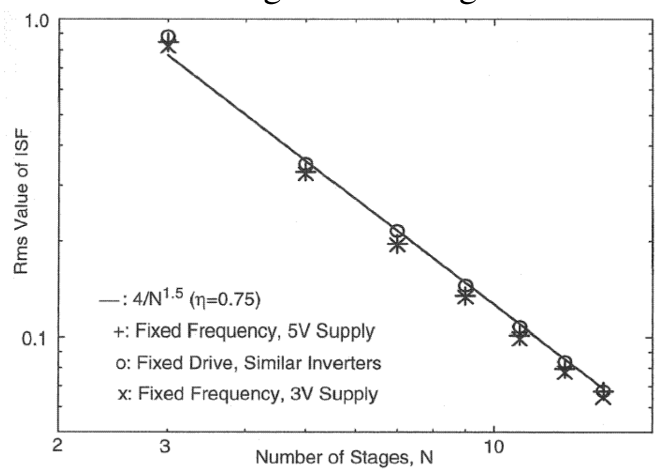
$$\Gamma_{rms} \approx \sqrt{\frac{2\pi^2}{3\eta^3}\frac{1}{N^{1.5}}}$$

Impulse Sensitivity Function for Single-Ended Ring Oscillators - Continued

The relationship between risetime and delay (definition of η):



RMS values of the ISFs for various single-ended ring oscillators versus no. of stages:



Phase Noise and Jitter of the Single-Ended Ring Oscillator

(We should not conclude from the previous result that the phase noise will decrease with the number of stages.)

Assuming $V_{TN} = |V_{TP}|$, the maximum total channel current noise from the inverter is

$$\frac{\overline{i_n^2}}{\Delta f} = \left(\frac{\overline{i_n^2}}{\Delta f}\right)_N = \left(\frac{\overline{i_n^2}}{\Delta f}\right)_p = 4kT\gamma\mu_{eff}C_{ox}\frac{W_{eff}}{L}\Delta V$$

where

ΔV = the gate overdrive in the middle of the transition = $0.5V_{DD} - V_T$

γ = 2/3 for long channel devices in saturation and 1.5 to 2 for shorter channel devices in saturation

$$\mu_{eff} = \frac{\mu_n W_n + \mu_p W_p}{W_n + W_p}$$

$$W_{eff} = W_n + W_p$$

Phase Noise and Jitter for Single-Ended Ring Oscillators - Continued

Assumptions –

Thermal noise sources of the different devices are uncorrelated.

The waveform (hence the ISF) of all the nodes are the same except for a phase shift.

The resulting phase noise and jitter is given as,

$$L\{\Delta\omega\} \approx \frac{8}{3\eta}\frac{kT}{P}\frac{V_{DD}}{V_{char}}\frac{\omega_o^2}{\Delta\omega^2}$$

$$\sigma_\tau = \sqrt{\frac{8}{3\eta}}\sqrt{\frac{kT}{P}\frac{V_{DD}}{V_{char}}}\sqrt{\tau}$$

where

$$V_{char} = \frac{(V_{DD}/2) - V_T}{\gamma}$$

$$P = 2\eta N V_{DD} q_{max} f_o$$

$$f_o = \frac{1}{Nt_D} = \frac{1}{\eta N(t_r+t_f)} \approx \frac{\mu_{eff}W_{eff}C_{ox}\Delta V^2}{8\eta N L q_{max}}$$

Note that $L\{\Delta\omega\}$ and σ_τ are independent of N. Why?

The increase in the number of stages adds more noise and counters the decrease in the ISF with N.

Phase Noise and Jitter of a Differential Ring Oscillator

Consider the following differential MOS ring oscillator with a resistive load.

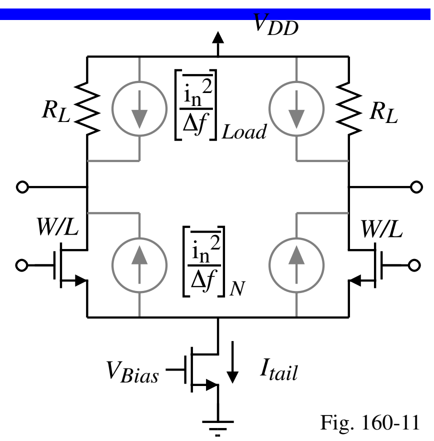


Fig. 160-11

Total power dissipation:

$$P = NI_{tail}V_{DD}$$

The frequency of oscillation:

$$f_o = \frac{1}{Nt_D} = \frac{1}{2\eta N t_r} \approx \frac{I_{tail}}{2\eta N q_{max}}$$

Characteristics of phase noise in differential ring oscillators:

1.) Tail current noise in the vicinity of f_o does not affect the phase noise.

2.) Tail current noise influences the phase noise at low frequencies and at *even* multiples of f_o.

3.) Tail current at low frequencies can be reduced by exploiting symmetry.

4.) Tail current at even multiples of f_o, can be reduced by harmonic traps.

Therefore, the total current noise on each single-ended node is

$$\frac{\overline{i_n^2}}{\Delta f} = \left(\frac{\overline{i_n^2}}{\Delta f}\right)_N + \left(\frac{\overline{i_n^2}}{\Delta f}\right)_{Load} = 4kTI_{tail}\left(\frac{1}{V_{char}} + \frac{1}{R_L I_{tail}}\right)$$

Phase Noise of a Differential Ring Oscillator - Continued

Making the following assumptions simplifies the expressions for phase noise and jitter:

1.) No correlation among the various noise sources.

2.) The ISF for the differential pair transistor noise sources and the ISF for the load resistors noise are the same.

To further simplify the expressions, assume that the phase noise and jitter due to all 2N noise sources is 2N times the values of the individual phase and noise sources. Thus,

$$L\{\Delta\omega\} \approx 2N\left(\frac{\Gamma_{rms}{}^2}{q_{max}{}^2}\frac{\overline{i_n{}^2}/\Delta f}{2\Delta\omega^2}\right) = \frac{2N}{q_{max}{}^2}\left(\frac{2\pi^2}{3\eta^3N^3}\right)\frac{\overline{i_n{}^2}/\Delta f}{2\Delta\omega^2} = \frac{1}{6q_{max}{}^2\eta^3N^2f_o{}^2}\left(\frac{\overline{i_n{}^2}}{\Delta f}\right)\frac{\omega_o{}^2}{2\Delta\omega^2}$$

Substituting for f_o gives,

$$L\{\Delta\omega\} \approx \frac{4\eta^2N^2q_{max}{}^2}{6q_{max}{}^2\eta^3N^2I_{tail}{}^2}\left(\frac{\overline{i_n{}^2}}{\Delta f}\right)\frac{\omega_o{}^2}{\Delta\omega^2} = \frac{4}{6\eta I_{tail}{}^2}\left(\frac{\overline{i_n{}^2}}{\Delta f}\right)\frac{\omega_o{}^2}{\Delta\omega^2}$$

Substituting for $\overline{i_n{}^2}/\Delta f$ and replacing I_{tail} in terms of P gives,

$$\boxed{L\{\Delta\omega\} \approx \frac{8NkT}{3\eta P}\left(\frac{V_{DD}}{V_{char}} + \frac{V_{DD}}{R_LI_{tail}}\right)\frac{\omega_o{}^2}{\Delta\omega^2}}$$

where

$$V_{char} = \frac{V_{GS}\text{-}V_T}{\gamma} \text{ for long channel and } V_{char} = \frac{E_cL}{\gamma} \text{ for short channels}$$

Jitter Noise of a Differential Ring Oscillator

Assuming as before that the jitter noise due to all 2N noise sources is 2N times the values of the individual jitter noise sources gives,

$$\sigma_\tau \approx \sqrt{2N}\,\frac{\Gamma_{rms}}{q_{max}\omega_o}\sqrt{\frac{1}{2}\frac{\overline{i_n{}^2}}{\Delta f}}\sqrt{\tau}$$

$$= \frac{\sqrt{2N}}{q_{max}}\left(\sqrt{\frac{2\pi^2}{3\eta^3}}\frac{1}{N\sqrt{N}}\right)\left(\frac{2\eta Nq_{max}}{2\pi I_{tail}}\right)\sqrt{2kTI_{tail}\left(\frac{1}{V_{char}}+\frac{1}{R_LI_{tail}}\right)}\sqrt{\tau}$$

$$\sigma_\tau \approx \sqrt{\frac{4}{3\eta N}}\sqrt{\frac{2kT}{I_{tail}}\left(\frac{1}{V_{char}}+\frac{1}{R_LI_{tail}}\right)}\sqrt{\tau}$$

Replacing the first I_{tail} in terms of P gives,

$$\boxed{\sigma_\tau \approx \sqrt{\frac{8}{3\eta}}\sqrt{\frac{kTN}{P}\left(\frac{1}{V_{char}}+\frac{1}{R_LI_{tail}}\right)}\sqrt{\tau}}$$

Comparison of the Single-Ended and Differential Ring Oscillator Noise

Note, that both $L\{\Delta\omega\}$ and σ_τ for the differential ring oscillator will increase with N.

Why? The answer is in the way the two oscillators dissipate power.

Differential Ring Oscillator:

The dc current from the supply is independent of the number and slope of the transition.

Single-Ended Ring Oscillator:

This oscillator dissipate power mainly on a per transition basis and therefore have better phase noise for a given power dissipation.

However, a differential topology may still be preferred in an IC implementation because of the lower sensitivity to substrate and supply noise, as well as lower noise injection into other circuits on the chip.

Optimum Number of Stages for Ring Oscillators

Single-ended ring oscillators:

What is the optimum number of stages for an inverter (single-ended) ring oscillator for best jitter and phase noise for a given frequency, f_o, and power, P?

Observations:

1.) The phase noise and jitter in the $1/f^2$ region are not strong functions of N.

2.) If the symmetry criteria is not well satisfied and/or the process has a large $1/f$ noise, then a larger N will reduce the jitter.

Result:

The number of stages for an inverter ring oscillator depends on $1/f$ noise, the maximum frequency of oscillation, and the influence of external noise sources such as supply and substrate which may not scale with N.

Differential ring oscillators:

Jitter and phase noise will increase with increasing N. Therefore, if the $1/f$ noise corner is not large and/or proper symmetry measures have been taken, then the minimum number of stage gives the best results (N = 3 or 4).

Ring Oscillators with Correlated Noise Sources

Noise analysis of ring oscillators:

1.) Assume that all noise sources are strongly correlated (i.e. substrate noise and power supply noise).

2.) If all noise sources in the inverters are the same, then

$$\phi(t) = \frac{1}{q_{max}} \int_{-\infty}^{t} i(\tau)\left[\sum_{n=0}^{N-1} \Gamma\left(\omega_o\tau + \frac{2\pi n}{N}\right)\right] d\tau$$

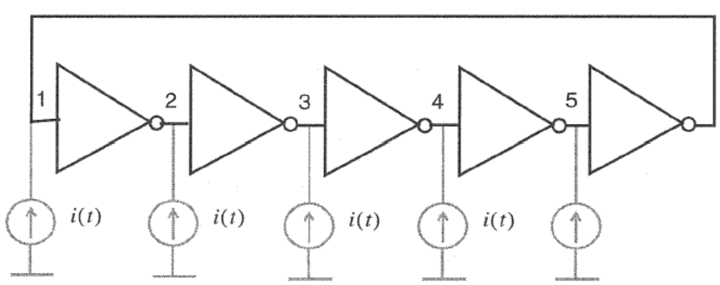
However, the term $\left[\sum_{n=0}^{N-1} \Gamma\left(\omega_o\tau + \frac{2\pi n}{N}\right)\right]$ is zero except at dc and multiples of $N\omega_o$.

$$\therefore \quad \phi(t) = \frac{N}{q_{max}} \sum_{n=0}^{N-1} c_{(nN)} \int_{-\infty}^{t} i(\tau) cos(nN)\omega_o\tau\, d\tau$$

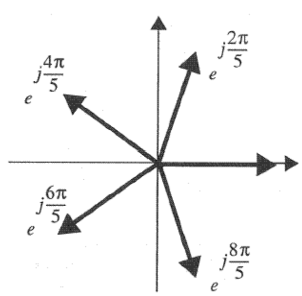
Phase Noise of Ring Oscillators – Continued

The previous expression implies that for fully correlated sources, only noise in the vicinity of integer multiples of $N\omega_o$ affects the phase noise. $\Rightarrow$ Therefore, every effort should be made to *maximize the correlations* of noise arising from the substrate and supply perturbations.

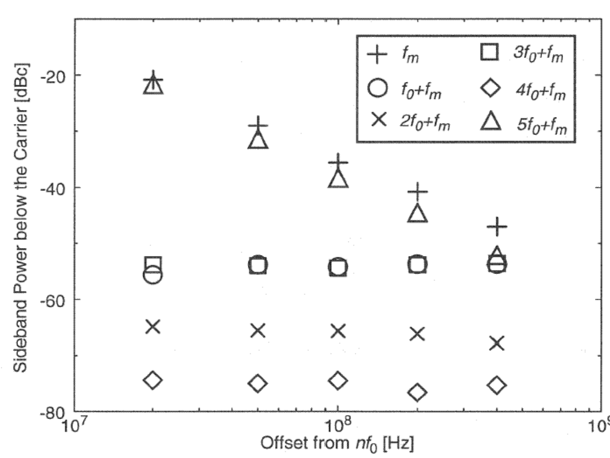
Example of a 5-stage ring oscillator with correlated noise sources:



Phasors for the noise contributions from each source:



Sideband power below carrier for fully correlated injection at nf_o+f_m:



Minimizing the Correlated Noise of Ring Oscillators

Methods of minimizing the phase noise in ring oscillators:

1.) Make the stages identical.

2.) The physical orientation of all stages should be the same.

3.) Layout the stages close together.

4.) Interconnect wires between stages should be the same length and shape.

5.) A common supply line should feed all inverter stages.

6.) The loading of each stage should be identical – use dummy stages.

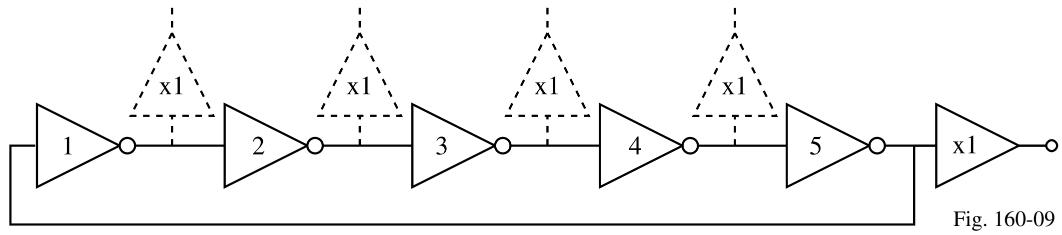


Fig. 160-09

7.) Use the largest number of stages consistent with the oscillator.

8.) If the low frequency portion of the substrate and supply noise dominates, exploit symmetry to minimize Γ_d.

SUMMARY

- Phase noise and jitter are key parameters in characterizing the spectral purity of periodic waveforms
- Two important phase noise models are the LTI and LTV models
- To reduce the phase noise in LC oscillators:
  1.) Make the tank Q or resonator Q as large as possible.
  2.) Maximize the signal power.
  3.) Minimize the ISF.
  4.) Force the energy restoring circuit to function when the ISF is at a minimum and to deliver it's energy in the shortest possible time.
  5.) The best oscillators will possess symmetry which leads to small Γ_{dc} for minimum upconversion of 1/f noise.
- To reduce the phase and jitter noise in ring oscillators
  1.) The inverter ring oscillator does not depend strongly upon N so reduction of noise depends more on the noise sources than the number of stages.
  2.) Use differential ring oscillators to minimize substrate and supply noise.
  3.) If the noise sources are correlated, use matching to reduce the noise.

For further information on applying these principles to oscillators, see Hajimiri and Lee, *The Design of Low Noise Oscillators*, Kluwer Academic Publishers, 1999.