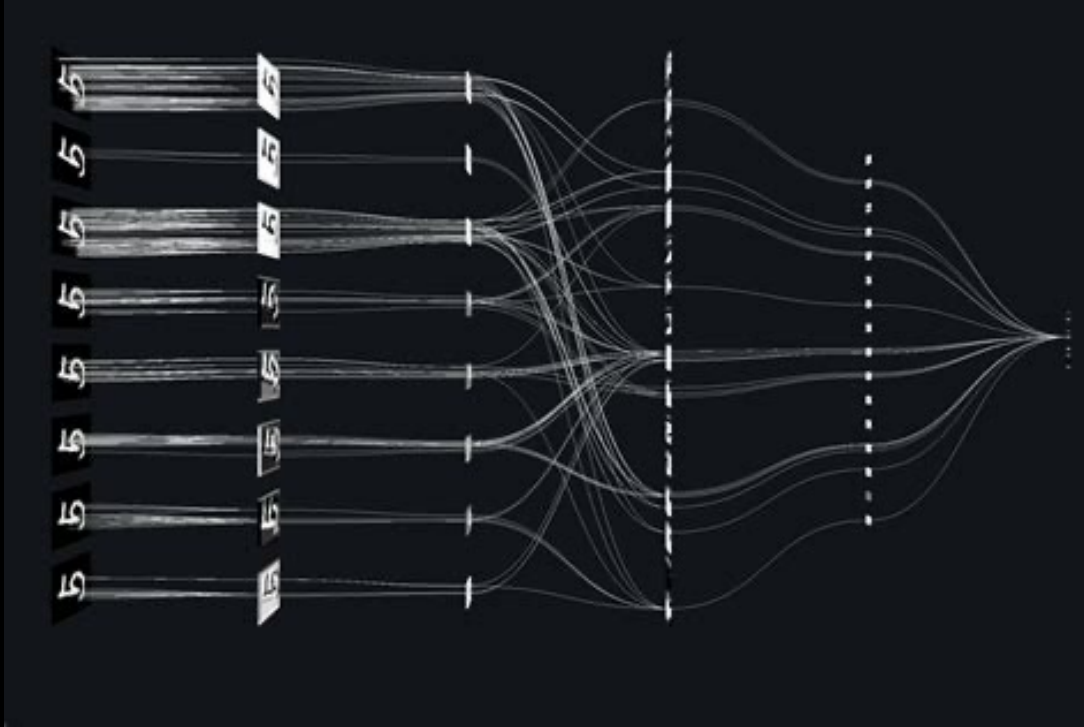


Explainable ML - XML

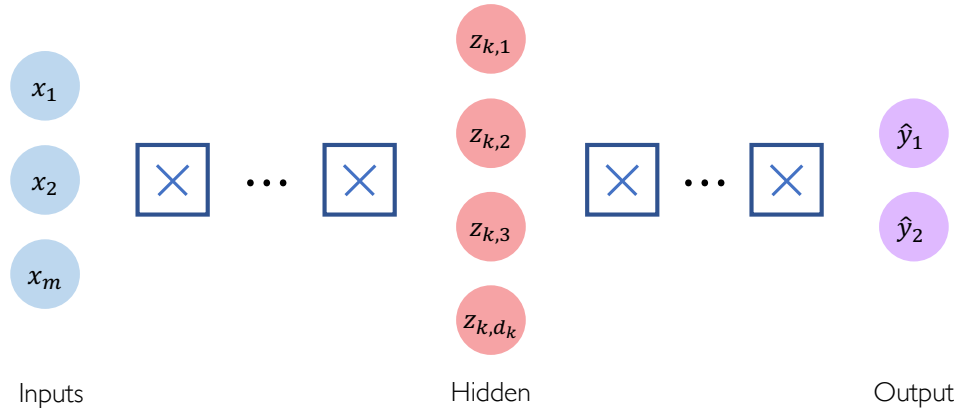
Pierre Gentine – Columbia University



The problem

Major advances with the deep learning

Deep Neural Network



$$z_{k,i} = w_{0,i}^{(k)} + \sum_{j=1}^{d_{k-1}} g(z_{k-1,j}) w_{j,i}^{(k)}$$

Many (many) weights \rightarrow black box

How to open up the black box?

Interpreting what is in the gut of deep NN/CNN

Explainable ML

Also helps build trustworthiness in models

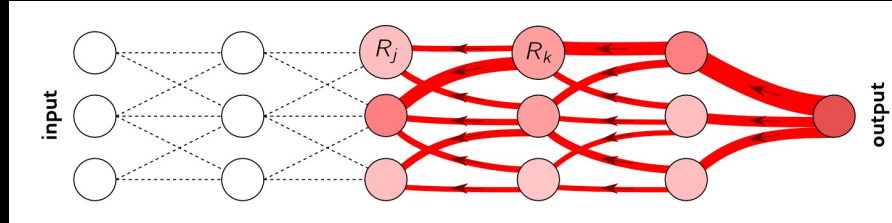
→ Right answer for the right reason

A few examples:

1. Layerwise Relevance Propagation (LRP)
2. Adjoint/gradient – Saliency maps

1. Layerwise Relevance Propagation (LRP)

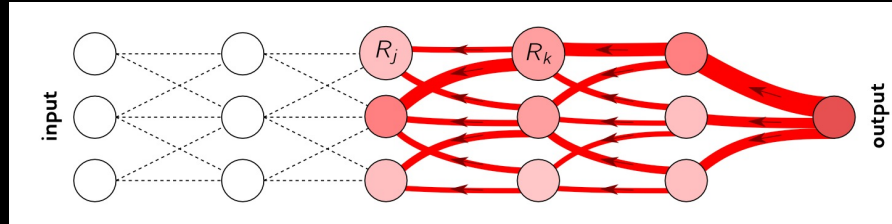
Trying to assess where the information is coming from
Backward – from output to input that most explain the output



Pass information/relevance backward – a la Kirchoff law (conservation of current):

1. Layerwise Relevance Propagation (LRP)

Trying to assess where the information is coming from
Backward – from output to input that most explain the output



Pass information/relevance backward – a la Kirchoff law (conservation of current):

$$R_j = \sum_k \frac{a_j w_{jk}}{\sum_{0,j} a_j w_{jk}} R_k$$

Layer j before layer k

a_j – output from layer j (after activation function)

w_{jk} - weights

1. Layerwise Relevance Propagation (LRP)

Different LRP rules

1. LRP-0

$$R_j = \sum_k \frac{a_j w_{jk}}{\sum_{0,j} a_j w_{jk}} R_k$$

Equivalent to gradient x input

2. LRP-epsilon

Limit noisy information – filter by small noise epsilon

$$R_j = \sum_k \frac{a_j w_{jk}}{\epsilon + \sum_{0,j} a_j w_{jk}} R_k$$

3. LRP-gamma

Favor positive contributions compared to negative contributions

$$R_j = \sum_k \frac{a_j \cdot (w_{jk} + \gamma w_{jk}^+)}{\sum_{0,j} a_j \cdot (w_{jk} + \gamma w_{jk}^+)} R_k$$

1. Layerwise Relevance Propagation (LRP)

Different LRP rules \rightarrow generic rule

$$R_j = \sum_k \frac{a_j \cdot \rho(w_{jk})}{\epsilon + \sum_{0,j} a_j \cdot \rho(w_{jk})} R_k$$

Can split computation into 4 substeps

$$\forall_k : z_k = \epsilon + \sum_{0,j} a_j \cdot \rho(w_{jk}) \quad (\text{forward pass})$$

$$\forall_k : s_k = R_k / z_k \quad (\text{element-wise division})$$

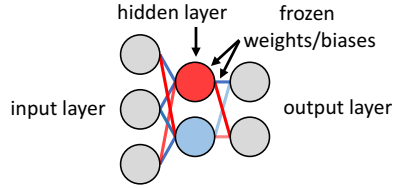
$$\forall_j : c_j = \sum_k \rho(w_{jk}) \cdot s_k \quad (\text{backward pass})$$

$$\forall_j : R_j = a_j c_j \quad (\text{element-wise product})$$

What we will do in the notebook – stay tuned

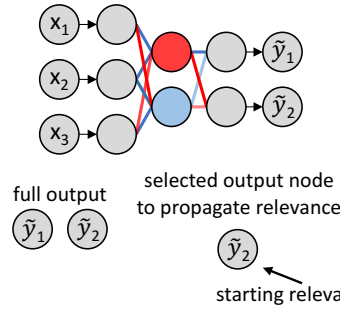
1. Layerwise Relevance Propagation (LRP)

- 1) Train network & freeze weights/biases

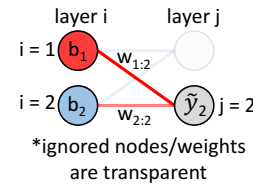


Information learned during training:
positive weights/biases
negative weights/biases

- 2) Input sample into frozen network and retain output



- 3) Propagate relevance from output node to previous layer



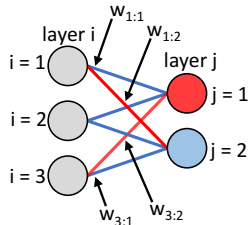
$$R_i = \sum_j \frac{\text{propagation rule}}{\sum_i a_i w_{ij}^+ + \max(0, b_j)} R_j$$

example relevance calculations

$$R_{i=1} = \left(\frac{a_1 w_{1:2}}{a_1 w_{1:2} + a_2 w_{2:2}} \right) \tilde{y}_2$$

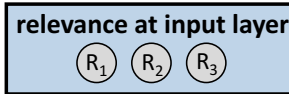
$$R_{i=2} = \left(\frac{a_2 w_{2:1}}{a_1 w_{1:2} + a_2 w_{2:2}} \right) \tilde{y}_2$$

- 4) Propagate relevance from hidden layer to input layer



$$R_i = \sum_j \left(\frac{w_{ij}^2}{\sum_i w_{ij}^2} \right) R_j$$

*all biases are ignored for this rule



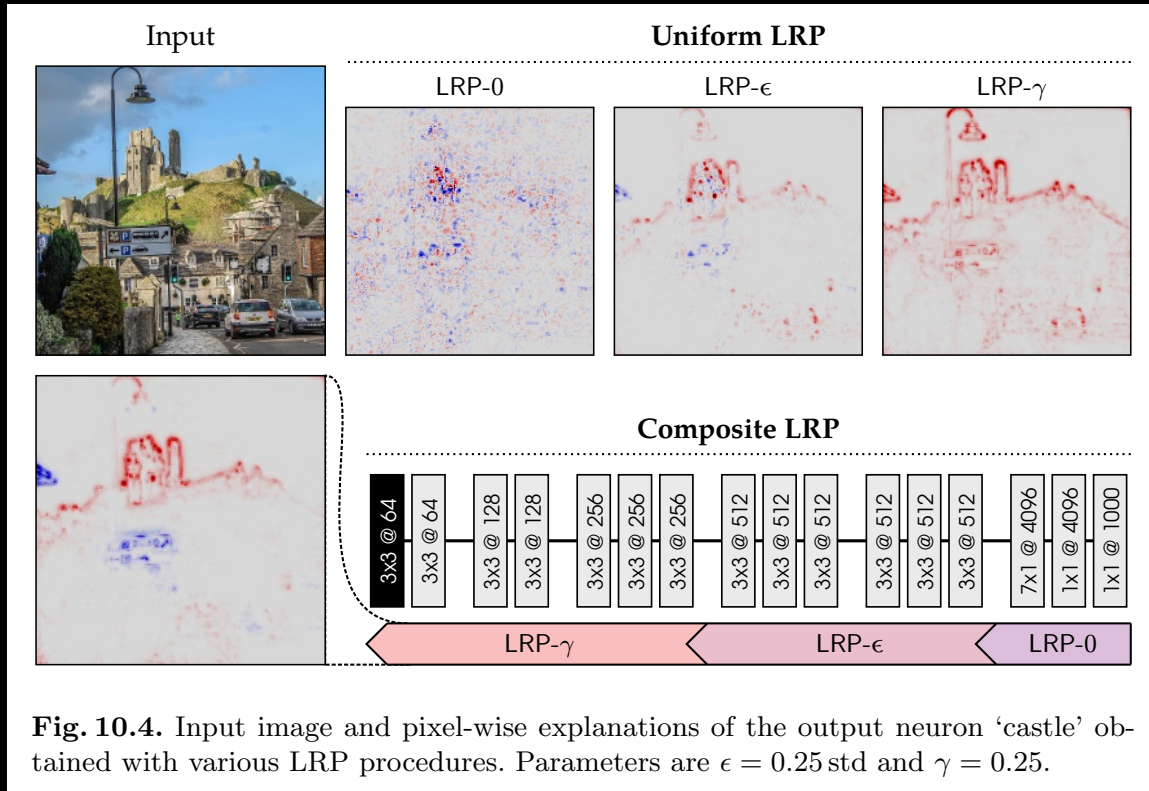
$$\text{example relevance calculation}$$

$$R_{i=1} = \left(\frac{w_{1:1}^2}{w_{1:1}^2 + w_{2:1}^2 + w_{3:1}^2} \right) R_{j=1} + \left(\frac{w_{1:2}^2}{w_{1:2}^2 + w_{2:2}^2 + w_{3:2}^2} \right) R_{j=2}$$

*relevance calculations are similar for other input nodes

- 5) Repeat for each sample of interest...

1. Layerwise Relevance Propagation (LRP)



2. Gradient/adjoint

Gradient/adjoint of output \mathbf{y} to inputs \mathbf{x}

$$\nabla_{\mathbf{x}} \mathbf{y}$$

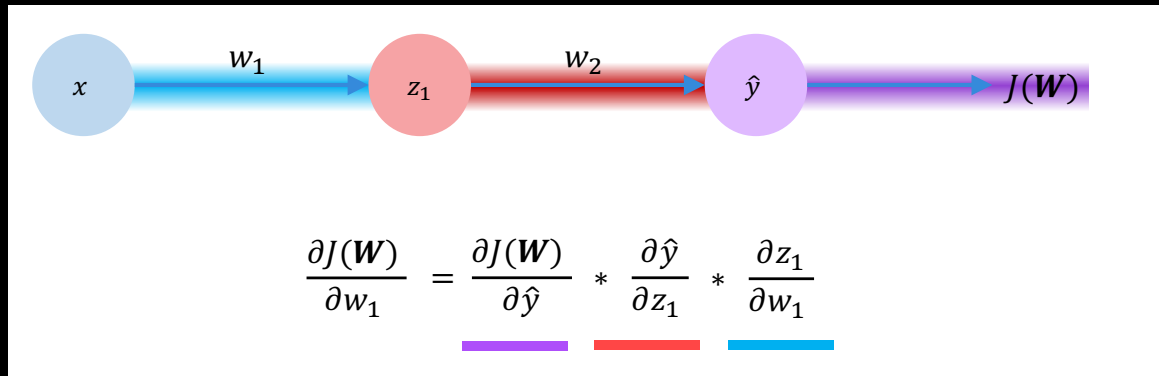
$$\mathbf{J} = \begin{pmatrix} \frac{\partial \mathbf{y}}{\partial x_1} & \dots & \frac{\partial \mathbf{y}}{\partial x_n} \end{pmatrix} = \begin{pmatrix} \frac{\partial y_1}{\partial x_1} & \dots & \frac{\partial y_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_m}{\partial x_1} & \dots & \frac{\partial y_m}{\partial x_n} \end{pmatrix}$$

2. Gradient/adjoint

Gradient/adjoint of output \mathbf{y} to inputs \mathbf{x}

$$J = \begin{pmatrix} \frac{\partial \mathbf{y}}{\partial x_1} & \dots & \frac{\partial \mathbf{y}}{\partial x_n} \end{pmatrix} = \begin{pmatrix} \frac{\partial y_1}{\partial x_1} & \dots & \frac{\partial y_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_m}{\partial x_1} & \dots & \frac{\partial y_m}{\partial x_n} \end{pmatrix}$$

Once upon a time: **backpropagation**
Chain's rule



Same thing!

2. Gradient/adjoint

Personal experience:

Non-linear so average over different samples

Use non-dimensional/normalized inputs (and ideally outputs)

Tends to work well for first cut – but can be noisy