FRM Review Notes

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The One And Only
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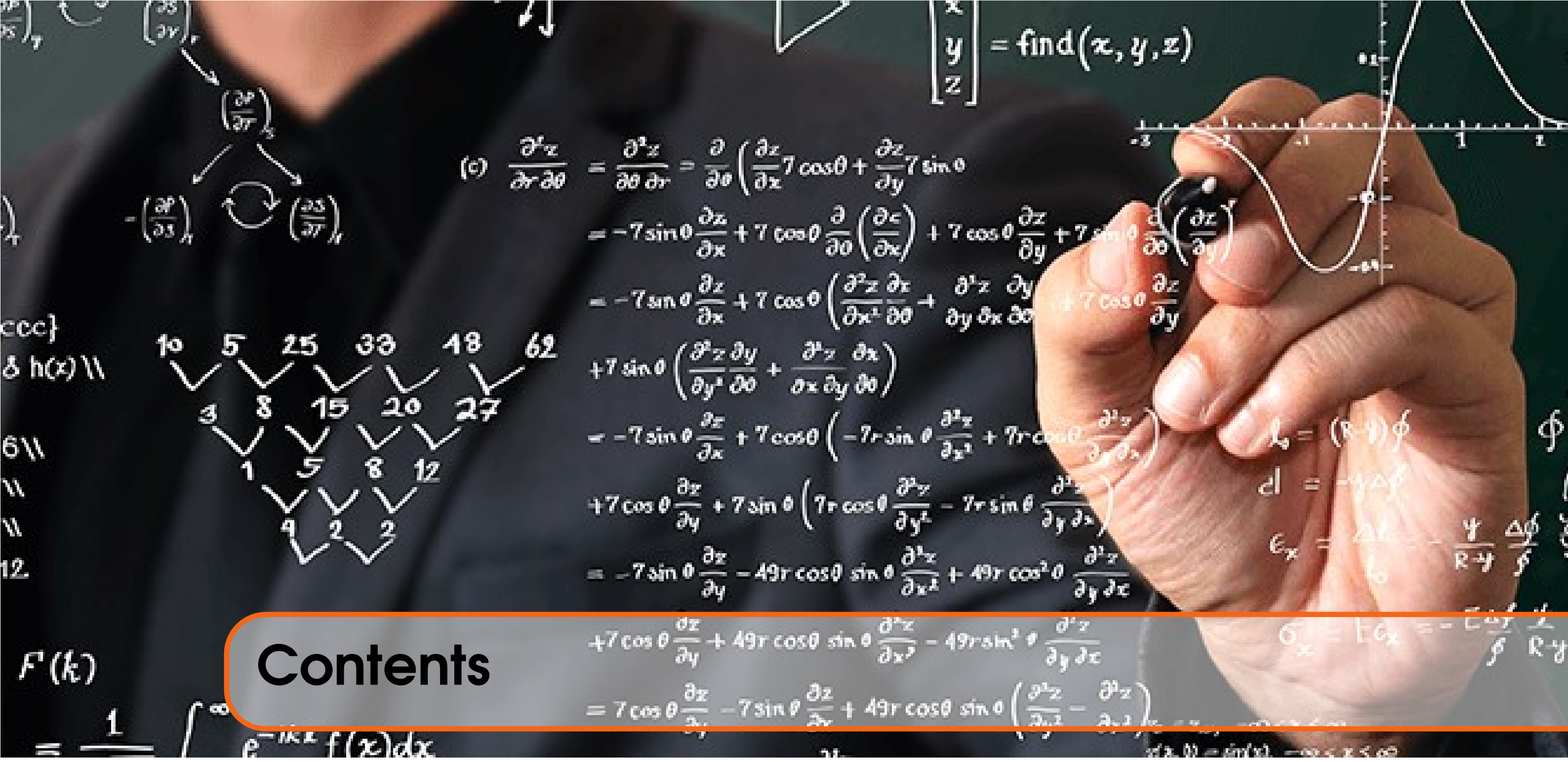

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1. Banks

Definition 1.0.1 Types of Banks

1. **Commercial Banks** are those that take deposits and make loans
   (a) retail banks-individuals and small firms
   (b) wholesale banks-corporate
2. **Investment Banks** are those that assist in raising capital for their customers and advising them on corporate finance matters such as M&A.

Definition 1.0.2 Major Risks Faced by Banks

1. **Credit Risk** refers to the risk that borrowers may default on loans or other counterparties contracts
2. **Market Risk** refers to the risk of losses from a bank's trading activities
3. **Operational Risk** refers to the possibility of losses arising from external events or failures of a bank's internal control

Definition 1.0.3 Types of Capitals

1. **Regulatory Capital** refers to the amount determined by bank regulators (must maintain this level!)
2. **Economic Capital** refers to the amount of capital that a bank believes is adequate based on its own risk models

Definition 1.0.4 IB Financing Arrangements

1. **Private Placement:** securities are sold directly to qualified investors with substantial wealth and investment knowledge
2. **Public Offering:** the securities are sold to the investing public at large
   (a) Firm Commitment: the IB agrees to purchase the entire issue and sell
   (b) Best Effort: sell as much as possible with commission
3. **Dutch Auction:** reduce price until all bidders have accepted all the shares.

R To avoid conflict of interest, large integrated banks must implement **Chinese walls** internal control.

Definition 1.0.5
1. **Banking Book** refers to loans made, which are the primary assets of a commercial bank.

$$\textbf{BV Value of a loan} = \textbf{Principal} + \textbf{Accrued Interest}$$

   nonperforming: payments overdue for more than 90 days
2. **Trading Book** refers to assets and liabilities related to a bank's trading activities

Types of Books

Definition 1.0.6 The Originate-To-Distribute Model
The model involves making loans and selling them to other parties. Government agencies using this model:
1. Ginnie Mae (GNMA)
2. Fannie Mae (FNMA)
3. Freddie Mac (FHLMC)

The benefit is increased liquidity but the drawback is the loose lending standards

2. Insurance Companies and Pension Plans

Definition 2.0.1 Categories of Insurance Companies

1. **Life Insurance**
2. **P&C Insurance**
   (a) Property insurance covers property losses such as fire and theft
   (b) Casualty (liability) insurance covers third-party liability
3. **Health insurance**
4. **Risks:**
   (a) Insufficient funds to satisfy poicyholders' claims
   (b) Poor return on investments
   (c) Liquidity risk of investments
   (d) Credit risk
   (e) Operational risk

Exercise 2.1 Breakeven Premium Payments Using Mortality Table
The relevant interest rate for insurance contracts is 3% per annum (semiannual compounding applies), and all premiums are paid annually at the beginning of the year. A $500,000 term insurance contract is being proposed for a 60-year-old male in average health. Assuming that payouts occur halfway throughout the year, calculate the insurance company's breakeven premium for a one-year term and a two-year term.

Solution:

1. One-year term:

$$P_{death,60,1} \times 500,000 = (1 - P_{survival,60,1}) \times 500,000 = 5,598.50$$

$$Premium_{breakeven,1} = \frac{5598.50}{1.015} = 5515.76$$

2. Two-year term:

$$P_{death,60,2} = (1 - P_{death,60,1})P_{death,60,1} = 0.011874$$

$$P_{death,60,2} \times 500,000 = 5937.27$$

the payout is 18 months after

$$Premium_{breakeven,2} = \frac{5937.27}{1.015} = 5677.91$$

3. Take total:

$$Y + \frac{P_{survival,60,1}Y}{1.015^2} = Premium_{breakeven,1} + Premium_{breakeven,2} = 11193.67$$

Solve for $Y = 5711.66$.

Definition 2.0.2 P&C Insurance Ratios

1. **Loss ratio** for a given year is he percentage of payouts versus premiums generated, usually between 60-80% and increasing over time
2. **Expense ratio** for a given year is the percentage of expense versus premiums generated, suually between 25-30% and decreasing over time
3. **Combined ratio** sum of loss and expense ratio
4. **Operating Ratio** for a given year is the combined ratio after dividends less investment income

Definition 2.0.3 Adverse Selection is where an insurer is unable to differentiate between a good risk and a bad risk

Definition 2.0.4

1. **Mortality Risk** refers to the risk of policyholder dying earlier than expected
2. **Longevity Risk** refers to the risk of policyholder living longer than expected

R If an insurance company has both life annuities and life insurance, then there is a natural hedge of these two risks. Otherwise, can consider reinsurance contracts.

Definition 2.0.5 Types of Pension Plans

1. **Defined Benefits Plans**
2. **Defined Contribution Plans**

R

1. Liability insurance is subject to long-tail risk, the risk of legitimate claims being submitted years after the insurance coverage has ended
2. Property and Casualty insurance companies typically have a greater amount of equity than a life insurance company

3. Mutual Funds and Hedge Funds

Definition 3.0.1 Types of Mutual Funds

$$\mathbf{NAV} = \frac{\textbf{fund assets} - \textbf{fund liabilities}}{\textbf{total shares outstanding}}$$

1. **Open-ended Mutual Funds**: most common, trades at the fund's **net asset value**, which is essentially the sum of all assets owned minus any liabilities of the fund then divided by shares outstanding.
   (a) Poor price visibility, market order only
   (b) Taxes passed onto investors
   (c) Fees required
      i. Management fee: as high as 2.5-3.0%
      ii. Advertising fee: 0.0-1.0%
      iii. Sales charge:
         A. Front-end load: charge when the asset is sold
         B. Back-end load: charge when the investor leaves a fund
2. **Closed-End Mutual Funds**
   (a) Niche investment
   (b) Number of funds remains static and can be purchased from other investors
   (c) Cannot sell the fund back to the company but need to find next investor
   (d) Transact at a price other than NAV with discount or premium
3. **Exchange-Traded Funds**
   (a) Like a daily traded closed-end funds with options available
   (b) Tremendous visibility
   (c) Low management fee

Definition 3.0.2 Hedge Funds
More complex compensation struture centered around incentive ees, **2 plus 20%**, 2% of all assets plues an additional 20% of all profits above a specified benchmark. Safeguards for

investors:

1. Hurdle Rate: benchmark that must be beaten
2. High-water mark clause: previous losses must first be recouped and hurdle rates surpassed before incentive fees once again apply
3. Clawback clause: enables investors to remain a portion of previously paid incentive fees to offset investment losses

Exercise 3.1 Calculate a Hedge Fund Manager's Expected Return
What is the expected return to a hedge fund if the fund uses a standard 2 and 20 incentive fee structure with an investment that has a 35% probability of making 55% and a 65% probability of losing 45%?
Solution

$$P(W)(2\% + 0.2 \times (W - 2\%)) + P(L) \times 2\%$$

$$= 0.35(2\% + 0.2(55\% - 2\%)) + 0.65 \times 2\% = 5.71\%$$

Definition 3.0.3 Hedge Fund Strategies

1. **Long/Short Equity**
2. **Dedicated Short**
3. **Distressed Securities**: high return if they can turn things around
4. **Merger Arbitrage**: cash deals and stock deals
5. **Convertible Arbitrage**: utilize convertible bond
6. **Fixed Income Arbitrage**
7. **Emerging Market**: developing country securities or American Depository Receipts (ADRs)
8. **Global Macro:** global macro trend that is in disequilibrium
9. **Managed Futures:** future of commodity prices

Definition 3.0.4 Hedge Fund Performance and Measurement Bias

1. **Meansurement Bias:** report good, avoid bad
2. **Backfill Bias:** use previous return to boost return rate
3. **Protection during period of stock market volatility**

R

1. Mutual funds must offer immediate access to withdrawals from their fund as an SEC requirement. Whereas, the hedge funds have advance notification and lock-up periods.

4. Option, Futures, and Other Derivatives

Definition 4.0.1 Derivative Markets

1. **Open outcry system/electronic trading system:** like NASDAQ
2. **OTC:** customized
3. **Traditional Exchange**

Definition 4.0.2 Basics of Derivative Securities

1. **Option Contract** is a contract that, in exchange for the otpion price, gives the option buyer the right, but not obligation, to buy/sell an asset at the exercise price from/to the option seller within a specified time period
2. **Forward Contract** is a contract that specifies the price and quantity of an asset to be delivered sometime in the future—foreign currency risk hedge
3. **Futures contract** is a more formalized, legally binding agreement to buy/sell a commodity/financial instrument in a pre-designated month in the future, at a price agreed upon today—exchange traded

Theorem 4.0.1 Call Option Payoff/Profit
For call option buyer

$$C_T = \max(0, S_T - X)$$

$$\textbf{Profit} = C_T - C_0$$

where S_T is the stock price at maturity and X is the strike price and C_0 is the call premium. **The put version is analogous**

Theorem 4.0.2 Forward Contract Payoff
For a forward contract long position

$$\textbf{Payoff} = S_T - K$$

where S_T is the spot price at maturity and K is the delivery price. **The case for a future is similar**

Exercise 4.1 How to use forward contracts to hedge?
Suppose that a company based in the United States will receive a payment o f EUR10M in three months. The company is worried that the euro will depreciate and is contemplating using a forward contract to hedge this risk. Compute the following:

1. The value of the EUR10M in U.S. dollars at maturity given that the company hedges the exchange rate risk with a forward contract at 1.25 $/**EUR**.
2. The value o f the EUR10M in U .S. dollars at maturity given that the company did not hedge the exchange rate risk and the spot rate at maturity is 1.2 $/**EUR**.

R **Speculative Strategies** Derivatives create significant leverage for the speculators.

5. Mechanics of Futures Markets

$$\textbf{Open Interest} = \textbf{Total \# Long Position} = \textbf{Total \# Short Position}$$

Definition 5.0.1 Futures Contract Characteristics

1. Quality of the underlying asset
2. Contract size
3. Delivery location
4. Delivery time
5. Price quotation and tick size: tick size is the minimum price fluctuation for the contract
6. Daily price limits:limit down (cannot go down further), limit up (cannot go up further)
7. Position limits: maximum number of contracts that a speculator may hold

Theorem 5.0.1 Future/Spot Convergence

$$\textbf{Basis} = \textbf{Spot Price} - \textbf{Futures Price}$$

when T approaches maturity, basis will converge to 0. Otherwise, arbitrage exists.

Definition 5.0.2 Operation of Margins

1. **Margin** is cash or highly liquid collateral placed in an account to ensure that any trading losses will be met.
2. **Marking to market** is the daily procedure of adjusting the margin account balance for daily movements in the future price.
3. **Initial Margin**
4. **Maintenance Margin:** lower than this, there will be a margin call
5. **Variation Margin:** the amount necessary to bring the margin account back to the initial margin amount

Exercise 5.1 Margin Trading
Let's return to our investor with the long gold contract. The investor entered the position at \$993.60. Each contract controls 100 troy ounces for a current market value of \$99,360. Assume that the initial margin is \$2,500, the maintenance margin is \$2,000, and the futures price drops to \$991.00 at the end of the first day and \$985.00 on the end of the second day. Compute the amount in the margin account at the end of each day for the long position and any variation margin needed.

Solution

1. First day: amount of loss
$$2.6 \times 100 = 260 < 2500 - 2000 = 500$$
No margin call, the margin balance is 2240.
2. Second day: amont of loss
$$6 \times 100 = 600 > 2240 - 2000 = 240$$
There is a margin call and the current balance is 1640, the variation margin is $2500 - 1640 = 860$.

■

Definition 5.0.3 OTC Markets

1. **Collateralization** is basically a marked to market feature for the OTC market where any loss is settled in cash at the end of the trading day.
2. In practice, the current OTC market is a mix of both bilateral agreements and transactions dealing with one or more clearinghouses. **Why clearinghouse?**
   (a) Automatic posting of collateral
   (b) Reduction of financial system credit risk
   (c) Increase transparaency of OTC trades

Definition 5.0.4

1. **Settlement price** is the average of the prices of the trades during the last period of trading
2. **Normal Market:** increasing settlement prices
3. **Inverted Market:** decreasing settlement prices

Definition 5.0.5 The Delivery Process of A Future Contract

1. Delivering the goods to the clearing house
2. Cash-settlement contract
3. Reverse/offsetting
4. Exchange for physicals: between traders, not the clearinghouse

Definition 5.0.6 Types of Orders

1. **Market order**
2. **Discretionary order**: delayed market order by the broker
3. **Limit order**
4. **Stop orders**
5. **Time-of-day order**
6. **Good-till-canceled order**
7. **Fill-or-kill order**: must execute immediately or the trade will not take place.

R

1. Commodity Futures Trading Commission (CFTC) is responsible for regulating futures markets
2. Hedging accounting specifies that gains/losses from a hedging instrument be recognized in the same period as gains/losses from the asset being hedged

6. Hedging Strategies Using Futures

Definition 6.0.1 Hedging With Futures
1. **Short Hedge** short a future contract to hedge against a price decrease in the existing long position
2. **Long Hedge** long a future contract to hedge against an increase in price in the existing short position

Definition 6.0.2 Basic Risk exists if one of the following is true
1. the asset in the existing position is often not the same as that underlying the futures
2. the hedging horizon may not match perfectly with the maturity of the futures contract

R To minimize basis risk, hedgers should select asset that is highly correlated to the spot position and contract maturity that is closet to the hedging horizon. Liquidity must be considered as well.

Definition 6.0.3 Sources of Basis Risks
1. Interruption in the convergence of the futures and spot prices
2. Changes in the cost of carry
3. Imperfect matching between the cash asset and the hedge asset: maturity/duration mismatch, liquidity mismatch, credit risk mismatch

Theorem 6.0.1 Optimal Hedge Ratio
A hedge ratio is the ratio of the size of the futures position relative to the spot position. The **optimal hedge ratio**, which minimizes the variance of the combined hedge position, is defined as follows:

$$HR = \rho_{S,F}\frac{\sigma_S}{\sigma_F} = \beta_{S,F} = \frac{\mathbf{Cov}_{S,F}}{\sigma_F^2} = \frac{\mathbf{Cov}_{S,F}}{\sigma_S\sigma_F}\frac{\sigma_S}{\sigma_F}$$

where S is the spot position and the F is the future position.

R This is optimal since it can minimize the variance. The **effectiveness of the hedge** measures the variance that is reduced by implementing the optimal hedge. We can measure it by $\rho^2_{S,F}$

Theorem 6.0.2 Hedging With Stock Index Futures

$$\textbf{\# of Contracts} = \beta_{port} \times \frac{\textbf{portfolio value}}{\textbf{value of future contract}} = \beta_{port} \times \frac{\textbf{portfolio value}}{\textbf{future price} \times \textbf{contract multiplier}}$$

Exercise 6.1 Tailing the Hedge
Suppose that you would like to make a tailing the hedge adjustment to the number of contracts needed in the previous example. Assume that when evaluating the next daily settlement period you find that the S&P 300 spot price is 1,095 and the futures price is now 1,160. Determine the number o f S&P 500 contracts needed after making a tailing the hedge adjustment.

Solution

We need to adjust the formula to spot-to-future ratio

$$\textbf{\# of Contracts} = \beta_{port} \times \frac{\textbf{portfolio value}}{\textbf{future price} \times \textbf{contract multiplier}} \times \frac{\textbf{spot price}}{\textbf{future price}}$$

$$= 1.4 \times \frac{20000000}{1150 \times 250} \times \frac{1095}{1160} = 92$$

Theorem 6.0.3 Adjusting Portfolio Beta

$$\textbf{\# of Contracts} = (\beta^* - \beta)\frac{P}{A}$$

where P is the portfolio value, A is the value of the underlying asset, β^* is the target beta and β is current portfolio beta

R Due to different maturities of the spot and future, the hedger needs to introduce a new future to roll the hedge forward and this will get rid of the old basis risk but introduce a new basis risk

7. Interest Rates

Definition 7.0.1 Types of Rates
1. Treasury rates: risk-free rates
2. LIBOR
3. Repo rates: implied rate on a repurchase agreement

R **We are ignoring a bunch of ACTSC231 stuff here. BORING.**

Theorem 7.0.1 Boostrap Spot Rate Curves
Given a set of treasury price $P_1, \dots, P_n$ and related periods $t_1, \dots, t_n$, then we calculate $z_{t_1}, \dots, z_{t_n}$ one-by-one.

Definition 7.0.2 Forward Rate Agreements is a forward contract obligating two parties to agree that a certain interest rate will apply to the principal amount during a specified future time.

Theorem 7.0.2 FRA Valuation

$$PV_{rec,R_K} = L(R_K - R_F)(T_2 - T_1)e^{-R_2 T_2}$$
$$PV_{pay,R_K} = L(R_F - R_K)(T_2 - T_1)e^{-R_2 T_2}$$

where L is the principal, R_K is the annualized rate on L, R_F is the forward rate between T_1 and T_2 continuous compounding period.

Exercise 7.1 Computing the Value of an FRA
Suppose the 3-month and 6-month LIBOR spot rates are 4% and 5%, respectively (continuously compounded rates). An investor enters into an FRA in which she will receive 8% (assuming quarterly compounding) on a principal of \$5,000,000 between months 3 and 6. Calculate the

value of the FRA.

Solution

$$e^{0.25R_{F,c}} = \frac{e^{0.5\cdot 0.05}}{e^{0.25\cdot 0.04}} \to R_{F,c} = 0.06 = 6\%$$

$$R_{F,quarter} = 4\left(e^{\frac{0.06}{4}} - 1\right) = 0.060452 = 6.05\%$$

$$PV_{rec,R_K} = 5000000(8\% - 6.05\%)(0.5 - 0.25)e^{-0.05\times 0.5} = 23773$$

Definition 7.0.3 Duration

The duration of a bond is the cash flow weighted average time until the cash flows ont the bond are received. Under continuous compounding:

$$D = \sum_{i=1}^{n} t_i \left(\frac{c_i e^{-yt_i}}{P_{Bond}}\right)$$

where t_i is the time until cash flow c_i is to be received and y is the continuously compound yield.

Theorem 7.0.3 Approximation of Bond Price Change

Under a parallel shift in the yield curve of Δy, we have

$$\frac{\Delta P_{Bond}}{P_{Bond}} = -D \times \Delta y$$

Definition 7.0.4

1. **Modified Duration:**

$$D^* = \frac{D}{1+\frac{y}{m}}$$

where m is the number of compounding periods per year

2. **Dollar Duration**:

$$\$D = D^* \times P_{Bond}$$

Theorem 7.0.4 Improved Approximation Using Convexity

$$\Delta P_{Bond} = P_{Bond}\left(-D\Delta y + \frac{1}{2}C(\Delta y)^2\right)$$

Proposition 7.0.5 Theories of The Term Structure

1. **Expectations Theory**: $R_F = \mathbb{E}(Z_t)$
2. **Market Segmentation Theory**: different maturity sectors yield different supply and demand
3. **Liquidity Preference Theory**: most depositors prefer short-term liquid deposits

8. Determination of Forward and Future Prices

Definition 8.0.1
1. **Investment Asset** is an asset that is held for the purpose of investing
2. **Consumption Asset** is an asset that is held for the purpose of consumption

Definition 8.0.2 Rules of Short Selling
1. The short seller must pay all dividends due to the lender of the security
2. The short seller must deposit collateral to guarantee the eventual repurchase of the security

Exercise 8.1 Net profit of a short sale
Assume that trader Alex Rodgers sold short XYZ stock in March by borrowing 200 shares and selling them for \$50/share. In April, XYZ stock paid a dividend of \$2/share. Calculate the net profit from the short sale assuming Rodgers bought back the shares in June for \$40/share in order to replace the borrowed shares and close out his short position.

Solution
Initial revenue=$200 \times 50 = 10000$
Get rid of dividend=$2 \times 200 = 400$
Return the stock=$40 \times 200 = 8000$
Total Profit=$10000 - 400 - 8000 = 1600$ ■

Definition 8.0.3 Future vs. Forwards
1. Similarities:
   (a) Deliverable or cash-settlement
   (b) Priced to have zero value at entrance
2. Differences:
   (a) Future:
      i. Trade on organized exchanges
      ii. highly standardized
      iii. A single clearinghouse is the counterparty to all futures contracts
      iv. Government regulated market

(b) Forward:
   i. Private contracts not on an exchange
   ii. customized contracts
   iii. Forwards are contracts with the originating counterparty
   iv. Forward contracts are usually not regulated

Theorem 8.0.1 Forward Pricing
Under the following assumptions:
1. No transaction costs or short-sale restrictions
2. Same tax rates on all net profits
3. Borrowing and lending at the risk-free rate
4. Arbitrage opportunities are exploited as they arise

No interim cash flows or carrying costs

$$F_0 = S_0 e^{rT}$$

If I is the PV of all cash flows over the T years, we have

$$F_0 = (S_0 - I)e^{rT}$$

If there is a dividend on the stock with δ continuous return

$$F_0 = S_0 e^{(r-\delta)T}$$

Theorem 8.0.2 Value of A Forward Contract
For the buyer of the contract

$$S_0 e^{-\delta T} - I - Ke^{rT}$$

where K is the obligated delivery price after inception

Theorem 8.0.3 Currency Futures

$$F_0 = S_0 e^{(r-r_f)T}$$

Theorem 8.0.4 Commodity Futures

$$F_0 = S_0 e^{(r+u-y)T}$$

where u is the storage cost and y is the convenience yield.

Definition 8.0.4
1. **Backwardation** refers to a situation where the futures price is below th spot price. Thre must be a significant benefit to holding the asset
2. **Contango** refers to a situation where the futures price is above the spot price. If there are no benefits to holding the asset

9. Interest Rate Futures

Definition 9.0.1 Day Count Convention

$$\textbf{Accrued Interest} = \textbf{Coupon} \times \frac{\textbf{\# of days from last coupon to the settlement date}}{\textbf{\# of days in coupon period}}$$

1. US Treasury bonds use actual/actual
2. US corporate and municipal bonds use 30/360
3. US money market (T-Bill) use actual/360

Definition 9.0.2 Quotation of T-Bonds
Quoted relative to \$100 par amount in dollars and 32nds.

$$95-05 \qquad 95\frac{5}{32} \qquad 95.15625$$

$$\textbf{Cash Price} = \textbf{Quoted Price} + \textbf{Accrued Interest}$$

The cash price is also known as the dirty price while the quoted price is the clean price.

Definition 9.0.3 Quotation for T-Bills
We use actual/360 and Y as the cash price and n days to maturity, then

$$\textbf{T-Bill discount rate} = \frac{360}{n}(100-Y)$$

Definition 9.0.4 Treasury Bond Futures For the short position to deliver, the cash received is

$$\textbf{cash received} = (QFP \times CF) + AI$$

where QFP is the quoted futures price, CF is the conversion factor, and AI is the accrued interest

$$CF = \frac{\textbf{discounted price of a bond} - AI}{\textbf{face value}}$$

Theorem 9.0.1 Cheapest-to-Deliver Bond

$$\textbf{CTD Bond} = \textbf{Quoted Bond Price} - (QFP \times CF)$$

we take the minimum!

R CTD Bonds Behaviours:

1. When yield $> 6\%$, CTD bonds tend to be low-coupon, long-maturity bonds
2. When yield $< 6\%$, CTD bonds tend to be high-coupon, short-maturity bonds
3. When the yield curve is upward sloping, CTD bonds tend to have longer maturities
4. When the yield curve is downward sloping, CTD bonds tend to have shorter maturities

Exercise 9.1 Theoretical Futures Price

Suppose that the CTD bond for a Treasury bond futures contract pays 10% semiannual coupons. This CTD bond has a conversion factor of 1.1 and a quoted bond price of 100. Assume that there are 180 days between coupons and the last coupon was paid 90 days ago. Also assume that Treasury bond futures contract is to be delivered 180 days from today, and the risk-free rate of interest is 3%. Calculate the theoretical price for this T-bond futures contract.

Solution

1. Step 1: calculate the dirty price of the bond

$$\textbf{Dirty Price} = 100 + 5 \times \frac{90}{180} = 102.5$$

2. Step 2: calculate the cash future price, since this is a treasury bond, we use actual/actual

$$F_0 = (102.5 - 5e^{-0.03\times(90/365)})e^{0.03(180/365)} = 98.99$$

3. Step 3: calculate the quoted futures price at delivery

$$98.99 - 5 \times \frac{90}{180} = 96.49$$

4. Step 4: calculate theoretical price using conversion factor

$$QFP = \frac{96.49}{1.1} = 87.72$$

■

Definition 9.0.5 Eurodollar Futures

The 3-month eurodollar futures contract trades on Chicago Mercantile Exchange (CME). One **tick** is \$25 per \$1 million contract. if Z is the quoted price, the contract price is

$$\textbf{Eurodollar future price} = 10000[100 - (0.25)(100 - Z)]$$

Theorem 9.0.2 Convexity Adjustment

$$\textbf{Forward Rate Implied By Futures} = (100 - Z)\%$$

$$\textbf{Actual Forward Rate} = \textbf{Forward Rate Implied By Futures} - \frac{1}{2}\sigma^2 T_1 T_2$$

where σ is the annual sd of the 90-LIBOR, T_1 is the maturity of the future contract and T_2 is 90 days of the underlying contract.

Theorem 9.0.3 Duration-Based Hedging
To create a combined position taht does not change in value when yields change by small amount.

$$N = -\frac{P \times D_P}{F \times D_F}$$

Exercise 9.2 Assume there is a 6-month hedging horizon and a portfolio value of \$100 million. Further assume that the 6-month T-bond contract is quoted at 105—09, with a contract size of \$100,000. The duration of the portfolio is 10, and the duration o f the futures contract is 12. Outline the appropriate hedge for small changes in yield.
Solution

$$N = -\frac{100000000 \times 10}{105\frac{9}{32} \times 0.01 \times 100000 \times 12} = -791.53$$

Thus, the manager should short 792 contracts. ■

10. Swaps

Definition 10.0.1 Financial Intermediaries in Swap Markets

1. Swaps typically require no payment by either party at initiation
2. Swaps are custom instruments
3. Swaps are not traded in any organized secondary market
4. Swaps are largely unregulated
5. Default risk is an important aspect of the contract
6. Most participants in the swaps market are large institutions
7. Individuals are rarely swap markets participants

Theorem 10.0.1 Discount Rate for Swaps
Implied forward rate is used to produce LIBOR, so under continuous compounding

$$R_{forward} = R_2 + (R_2 - R_1)\frac{T_1}{T_2 - T_1}$$

Theorem 10.0.2 IRS Value

$$V_{fltrec} = PV_{flt} - PV_{fix}$$

$$V_{fixrec} = PV_{fix} - PV_{flt}$$

R

1. Bond methodology use the fixed rate to calculate floating rate payment right away
2. FRA methodology use Theorem 10.0.1 to calculate each floating rate payment and discount accordingly

Theorem 10.0.3 Fixed-for-fixed Currency Swap

$$V_{swap}(USD) = B_{USD} - (S_0 \times B_{GBP})$$

Theorem 10.0.4 FRA Currency Swap We first calculate the forward rates using

$$F_t = S_0 e^{(r-r_f)t}$$

then, discount cash flows like before and calculate difference.

Definition 10.0.2 Other Types of Swaps

1. **Equity Swap** the return on a stock, a portfolio, or a stock index is paid each period by one party in return for a fixed-rate or floating-rate payment
2. **Swaption** is an onption which gives the holder the right to enter into an interest rate swap
3. **Volatility swap** involves the exchanging of volatility based on notional principal

11. Mechanics of Option Markets

Definition 11.0.1 Types of Options

1. **LEAP:** long-term equity anticipation securities, January expiration
2. **FLEX options** exchange-traded options on equity (indices) that allow alterations on specifications
3. **ETF options:** American-style options and utilize delivery rather than cash settlement
4. **Weekly options:** created on Thursday and matures next Friday
5. **Binary options:** pays $\$x$ when the strike price is reached
6. **CEBOs:** specific form of credit default swap
7. **DOOM options:** put option that has really low strike price in case of large downward price movement in the underlying asset.

R

1. Option is not adjusted for dividends but for stock-split. A 25% dividend is treated as a 5−for−4 stock split
2. Options with maturities nine months or fewer cannot be purchased on margin; otherwise, a maximum of 25% of the option value can be borrowed
3. **Naked options** refers to options in which the writer does not also own a position in the underlying asset.
4. OCC is the clearinghouse for exchange-traded options
5. Other option-like instruments are **warrants**, **employee stock options**, and **convertible bonds**.

12. Properties of Stock Options

Figure 1: Summary of Effects of Increasing a Factor on the Price of an Option

Factor	*European Call*	*European Put*	*American Call*	*American Put*
S	+	–	+	–
X	–	+	–	+
T	?	?	+	+
σ	+	+	+	+
r	+	–	+	–
D	–	+	–	+

Figure 12.0.1: *Stock Option Properties*

Theorem 12.0.1 Upper Bounds for European and American Option Prices

$$c_{Ame} \leq S_0 \qquad c_{Eur} \leq S_0$$

$$p_{Ame} \leq X \qquad p_{Eur} \leq Ke^{-rT}$$

Theorem 12.0.2 Upper Bounds for European and American Option Prices If the stock does not have dividend payments

$$\max(S_0 - Ke^{-rT}, 0) \leq c_{Eur} \qquad \max(Ke^{-rT} - S_0, 0) \leq p_{Eur}$$

$$\max(S_0 - Ke^{-rT}, 0) \le c_{Ame} \qquad \max(K - S_0, 0) \le p_{Ame}$$

Note that $S_0 - Ke^{-rT} \ge S_0 - K$, thus, if there is no dividend, American call option should never be exercised early.

Figure 2: Lower and Upper Bounds for Options

Option	*Minimum Value*	*Maximum Value*
European call	$c \ge \max(0, S_0 - Xe^{-rT})$	S_0
American call	$C \ge \max(0, S_0 - Xe^{-rT})$	S_0
European put	$p \ge \max(0, Xe^{-rT} - S_0)$	Xe^{-rT}
American put	$P \ge \max(0, X - S_0)$	X

Figure 12.0.2: *Stock Option Lower and Upper Bounds*

Theorem 12.0.3 Put-Call Parity Theorem Only holds for European Options!

$$c_{Eur} + Ke^{-rT} = p_{Eur} + Se^{-\delta T}$$

Theorem 12.0.4 American Option: Call and Put Relationship

$$S_0 - K \le c_{Ame} - p_{Ame} \le S_0 - Ke^{rT}$$

Theorem 12.0.5 Impact of Large Dividend on European Options Let D be the PV of the large dividend, then the put-call parity becomes

$$p_{Eur} + S_0 - D = c_{Eur} + Ke^{-rT}$$

Theorem 12.0.6 Impact of Large Dividend on American Options Let D be the PV of the large dividend, then the impact on American option is

$$S_0 - K - D \le c_{Ame} - p_{Ame} \le S_0 - Ke^{rT}$$

13. Trading Strategies Involving Options

Definition 13.0.1 Types of Strategies

1. **Protective Put:** long stock, long put
2. **Covered Call:** long stock, short call
3. **Bull Call Spread:** long lower call, short higher call
   The profit$= \max(0, S_T - K_L) - \max(0, S_T - K_H) - c_L + c_H$
4. **Bear Call Spread:** short lower call, long higher call
   The profit$= \max(0, S_T - K_H) - \max(0, S_T - K_L) + c_L - c_H$
5. **Butterfly Spreads:** if the strike prices of three call options are K_L, K_M, K_H, then to construct a butterfly spread
   (a) buy $K_H - K_M$ call with C_L
   (b) sell $K_H - K_L$ call with C_M
   (c) buy $K_M - K_L$ call with C_H
6. **Calendar Spread:** created by transacting in two options that have the same strike price but different expiration dates. This is not symmetric and not linear.
7. **Diagonal Spread:** based on calendar spread with different strike price
8. **Box Spread:** a bull call spread and a bear put spread, this provides a constant payoff and profit. Such arbitrage only exists for European options.
9. **Straddle:** long a put and a call at the same strike price to capture volatility
10. **Strangle:** long a put at K_L and a call at K_H to capture volatility with cheaper cost than Straddle
11. **Strips:** long more call than put at the same strike price
12. **Straps:** long more put than call at the same strike price

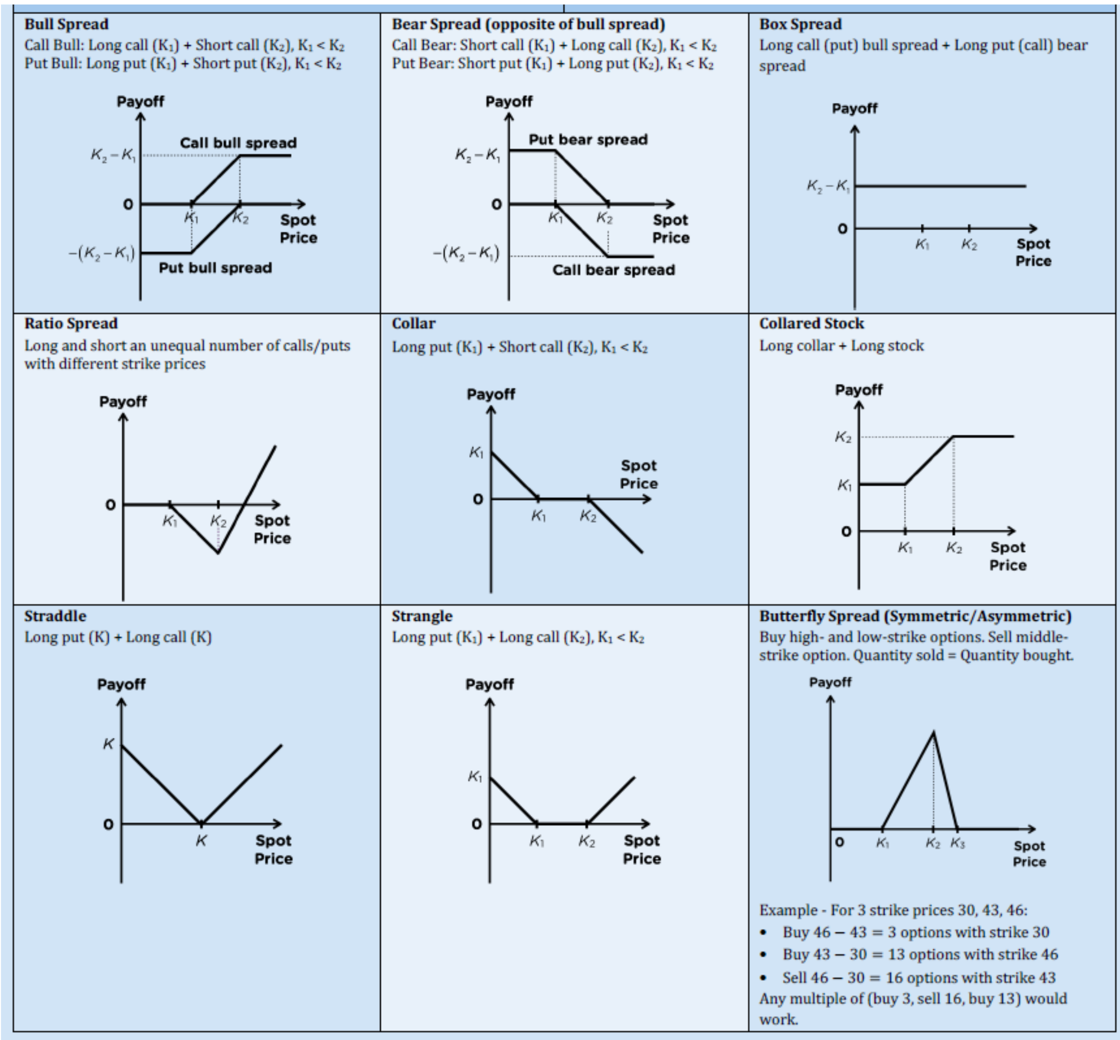


Figure 13.0.1: *Stock Option Strategies*

14. Exotic Options

Definition 14.0.1 Why Exotic Option?
The exotic options are developed to provide a unique hedge for a firm's underlying assets, tax and regulatory purposes, and speculation on future market. 4 factors need to be considered

1. Will the hedge be effective?
2. Cost of strategy?
3. Is pricing model needed?
4. How is the position reversed?

Definition 14.0.2 American Option Transformation

1. Restrict early exercise time—**Bermudan option**
2. lock out period
3. Changing strike price

Definition 14.0.3 Types of Exotic Options

1. **Gap Options:** two strike prices, X_2 is referred as the trigger price
   For a **gap call Options:** $X_2 > X_1$, when X_2 is reached, the payoff is $S_T - X_1$
   For a **gap put Options:** $X_2 < X_1$, when X_2 is reached, the payoff is $X_1 - S_T$
2. **Forward Start Options** are options that begin their existence at some time in the future
3. **Compound Options:** coc,cop,poc,pop
4. **Chooser Options:** choose to be a put or a call
5. **Barrier options:** down-and-in, down-and-out, up-and-in, up-and-out
   (a) Usually cheaper
   (b) Vega usually always positive for standard options, but maybe negative for barrier options
   (c) (down-and-in)+(down-and-out)=standard option=(up-and-in)+(up-and-out)
6. **Binary Options:**
   cash-or-nothing call pays fixed amount Q

$$c_{cash} = Qe^{-rT}N(d_2)$$

asset-or-nothing call pays the value of the stok when the contract is initiated if the stock price ends up above the strike price at expiration

$$c_{asset} = S_0 e^{-\delta T} N(d_1)$$

7. **Shout Options:** allows the owner to pick a date when he shouts to the option seller, the owner receives the $\max(shout, K)$
8. **Asian Options:** average price or average strike
9. **Exchange Options:** one asset for another asset
10. **Basket options:** underlying asset is a basket of assets (rainbow options)
11. **Volatility Swap/Variance Swap**: exchange volatility/variance based on a notional value

Definition 14.0.4 Option Replication

1. Dynamic Options Replication: requires frequent trading, which is really costly
2. Statis options replication: a short portfolio of actively traded options that approximates the option position to be hedged is constructed

15. Commodity Forwards and Futures

Theorem 15.0.1 Forward Price

$$F_{0,T} = S_0 e^{(r-\delta+\lambda)T}$$

where δ is the leasing rate and λ is the storage cost.
A synthetic commodity forward price can be derived by combining a long position on a commodity forward, $F_{0,T}$ and a long zero-coupon bond that pays $F_{0,T}$ at time T.

Exercise 15.1 Cash-and-carry arbitrage

EXAMPLE: Futures cash-and-carry arbitrage

Assume the spot price of gold is $900/oz., that the 1-year futures price is $975/oz., and that an investor can borrow or lend funds at 5%. Storage costs are 2% annually. **Calculate** the arbitrage profit.

Answer:

The futures price, according to the no-arbitrage principle, should be:

$$F_{0,T} = \$900e^{(0.05 + 0.02)1} = \$965$$

Instead, it's trading at $975. That means the futures contract is overpriced, so we should conduct cash and carry arbitrage by going short in the futures contract, buying gold in the spot market, and borrowing money to pay for the purchase. If we borrow $900 to fund the purchase of gold, we must repay $965 after 1 year (at maturity of the futures contract).

Today		1 Year From Today	
Transaction	**Cash Flow**	**Transaction**	**Cash Flow**
Short futures	$0	Settle short position by delivering gold	+$975
Buy gold in spot market	−$900		
Borrow at 5%	+$900	Repay loan	−$965
Total cash flow	$0	Total cash flow = arbitrage profit	+$10

The riskless profit is equal to the difference between the futures contract proceeds and the loan payoff, or $975 − $965 = $10. Notice that this profit is equal to the difference between the actual futures price of $975 and the no-arbitrage price of $965.

Definition 15.0.1
1. **Contango:** upward sloping forward curve with $r > \delta$
2. **Backwardation:** downward sloping forward curve with $r < \delta$

Definition 15.0.2 Convenience Yield: The convenience yield is only relevant when a commodity is stored (i.e., in a carry market).A convenience yield cannot be earned by the average investor who does not have a business reason for holding the commodity. Thus, we have a range for the $F_{0,T}$

$$S_0 e^{(r+\lambda-c)T} \leq F_{0,T} \leq S_0 e^{(r+\lambda)T}$$

R Arbitrage-free conditions dictate that continuous lease rates should be equal to either
1. Risk-adjusted required ror on commodity investment minus the expected price appreciation of the commodity
$$\delta = \alpha - \frac{1}{T}[E(S_T)/S_0]$$
2. The risk-free rate minus the forward premium on the commodity
$$\delta = r - \frac{1}{T}[F_{0,T}/S_0]$$

Definition 15.0.3 Types of commodity forward prices
1. **Gold Forward Prices:** positive leasing rate
2. **Corn Forward Prices:** the corn forward curve increases until harvest time, drops sharply and then slopes upward again after harvest time is over.
3. **Electricity Forward Prices:** not storable commodity
4. **Natural Gas Forward Prices:** constant production and seasonal demand, expensive to store. Peaks in the fall usually.
5. **Oil Forward Prices:** better to transport than natural gas, more stable long-run forward price

Definition 15.0.4 Commodity Spread results from a commodity that is an input in the production process of the other commodities.
1. crush spread: soybean
2. crack spread: crude oil, we need to know the notation

EXAMPLE: Pricing a crack (commodity) spread

Suppose we plan on buying crude oil in one month to produce gasoline and kerosene for sale in two months. The 1-month futures price for crude oil is currently \$30/barrel. The 2-month futures prices for gasoline and heating oil are \$41/barrel and \$31.50/barrel, respectively. **Calculate** the 5-3-2 crack (commodity) spread.

Answer:

The 5-3-2 spread tells us the amount of gross margin that can be locked in by buying five barrels of oil and producing three barrels of gasoline and two barrels of heating oil.

gross margin for a 5-3-2 spread = (3 × \$41) + (2 × \$31.50) − (5 × \$30) = \$123 + \$63 − \$150 = \$36 for five barrels, or \$7.20/barrel

Definition 15.0.5
1. **Strip Hedge:** Entering multiple future contracts as a long party, matching the maturities and quantities with their obligations under fixed price agreement
2. **Stack Hedge:** oil producer would enter into a one-month futures contract equaling the total value of the year's promised deliveries and redo this every month to reduce

transaction costs. **stack and roll**

3. **Cross Hedge:** a futures contract that is highly correlated with the underlying exposure is selected, this will introduce a basis risk, and usually evaluated based on
   (a) The liquidity of the futures contract
   (b) The correlation between the underlying for the futures contract and the assets being hedged
   (c) The maturity of the ftures contract

   —Used for weather derivatives for agriculture

16. Exchanges, OTC, Derivatives, DPCs and SPVs

Definition 16.0.1 Forms of Clearing

1. **Clearing** is the process of reconciling and amtching contracts between counterparties from the time the commitments are made until settlement
2. **Direct Clearing** is a mechanism for bilaterally reconciling commitments between two counterparties, a **clearing ring** is used to reduce counterparty exposure among more members.
3. **Complete Clearing** refers to clearing through a CCP

	Exchange-Traded Derivatives	**OTC Derivatives**
Terms	Standardized	Custom, negotiable
Maturity	Standardized	Negotiable, non-standard
Liquidity	Strong	Weak
Credit risk	Little (CCP guarantee)	High (bilateral)

Figure 16.0.1: *Differences Between Exchange and OTC*

R **Classes of OTC Derivatives**

1. Interest rate
2. Foreign exchange
3. Equity
4. Commodity
5. Credit derivatives

Definition 16.0.2 **Special Purpose Vehicles (SPVs)** are bankruptcy remote legal entities set up by a parent firm to shield the SPV from any financial distress of the firm.

1. SPV rating is stronger than the firm's credit rating
2. for issuing financial securities reasons
3. transfering counterparty risk into legal risk

Definition 16.0.3 **Derivatives Product Companies (DPCs)** are set up by firms as bankruptcy remote subsidiaries to originate derivatives products and sell them to investors. A DPC's AAA rating depends on 3 criteria

1. market risk minimization through participating on both sides of the market
2. parent support, with the bankruptcy remote status shielding against the parent's potential distress
3. credit risk and operational risk management through restrictions like limits, margin, and daily mark to market

Definition 16.0.4 **Monoline and Credit Derivative Product Companies (CDPCs)**

1. Monolines are highly-rated insurance companies that provide financial guarantees
2. CDPCs are similar to DPC

R

1. CCPs give priority to OTC derivatives counterparties to the detriment of other parties, including bondholders. This increases the risk in other markets.
2. Relying on a solid legal framework exposes CCPs and exchange members to legal risk. For example, as seen in the case of SPVs and DPCs, courts may change the priority of claims in a bankruptcy scenario, or courts in different jurisdictions may rule in contradictory ways.
3. Although CCPs share similarities with monolines and CDPCs in that they are highlyrated entities set up to manage counterparty risk, CCPs do not take residual risk in the market given that they maintain a matched book of trades. This is in contrast to monolines and CDPCs, which typically have one-way market exposures.
4. In contrast to monolines and CDPCs, which post no variation margin and often no initial margin, CCPs require members to post both initial and variation margin.

17. Basic Principles of Central Clearing

Definition 17.0.1 Loss mutualization is a form of insurance and refers to members' contributions to a default fund to cover future losses from member defaults.

Definition 17.0.2 Conditions to be centrally cleared

1. Standardization: legal and economic terms should be standard
2. Complexity: transactions need to be easily valued
3. Liquidity: cleared products are typically more liquid than OTC products

Definition 17.0.3 Conditions to be a clearing member

1. Admission criteria: credit quality and size
2. Financial commitment: contribute to the CCP's default fund
3. Operational criteria: posting margin, simulating default

R It is desirable to have only one CCP but not that feasible due to the following reasons

1. Regional Differences
2. Product Types
3. Regulatory reasons

	OTC Derivatives	*CCP/Exchanges*
Trading	Bilateral	Bilateral / Centralized
Counterparty	Original trade counterparty	CCP (replaces counterparty)
Participants	All	Clearing members (dealers)
Products	All (including non-standard, exotic)	Standard, vanilla
Margining	Bilateral, custom	Full margining set by CCP (initial, variation)
Loss buffers	Margin, regulatory capital	Initial margin, default fund, CCP capital

Definition 17.0.4

1. **Advantages of CCP**
   (a) transparaency
   (b) offsetting
   (c) loss mutualization
   (d) legal and operational efficiency
   (e) liquidity
   (f) default management
2. **Disavantages of CCP**
   (a) Moral hazard
   (b) Adverse selection
   (c) Procyclicality
   (d) Bifurcation: the separation of trading into cleared and non-cleared products can increase the volatility of cash flow even for hedged products

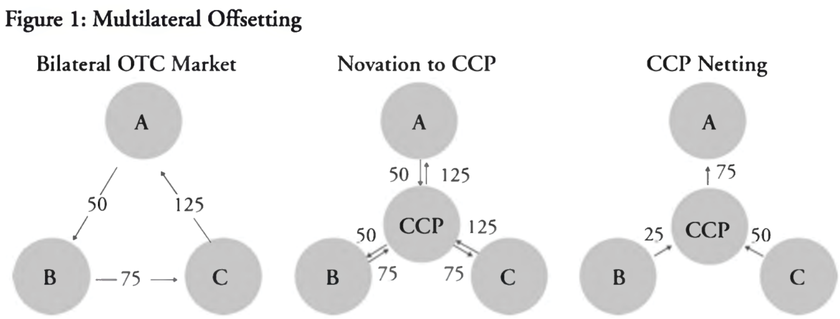


Figure 17.0.1: *How Novation and Netting Can Help Increasing Efficiency*

18. Risks Caused by CCPs

Definition 18.0.1 Risks Caused by CCPs

1. Default Risk
2. Model Risk
3. Liquidity Risk
4. Operational Risk
5. Legal Risk
6. Other Risks
   (a) Investment risk refers to the risk of losses of margin funds resulting from investment actions performed within or outside of the stated investment policy.
   (b) Settlement and payment risk
   (c) Foreign exchange risk
   (d) Custody risk
   (e) Concentration risk refers to the risk of clearing members, margins, or both that are located in a single geographic area. Essentially, it is a lack of diversification.
   (f) Sovereign risk: foreign government default on its debt obligation
   (g) Wrong-way risk refers to the risk that exposure to a counterparty is negatively correlated the credit quality of the counterparty

Definition 18.0.2 Lessons learned from CCP Failures

1. Operational risk must be controlled to the maximum extent possible.
2. Variation margins should be recalculated often and collected quickly
3. Initial margins and default funds should be sufficiently large in order to withstand significant negative asset value declines as well as increased return correlations during a crisis
4. CCPs must actively monitor positions, penalize overly concentrated positions, and promptly liquidate hedge extremely large positions.
5. CCPs must have one or more external sources of liquidity to avoid default due to illiquidity

19. Foreign Exchange Risk

Definition 19.0.1 Net Position Exposure on a Currency Let **CUR** be a specific foreign currency.

$$\textbf{net CUR exposure} = (\textbf{CUR assets-CUR liabilities}) + (\textbf{CUR bought-CUR sold})$$

R

1. Match foreign currency assets with its liabilities on the balance sheet
2. Be long the currency in one component and short the currency in the other component

Definition 19.0.2 The potential gain/loss exposure to a foreign currency (FC) is a function of the size of the position and the potential change in the value of the foreign currency

$$\textbf{\$ Gain/Loss in CUR} = \textbf{net CUR exposure (in \$)} \times \%\Delta\$/\textbf{CUR}$$

Definition 19.0.3 Types of FX Trading Activities

1. Enabling customers to participate in international commercial business transaction
2. Enabling customers to take positions in real or financial foreign investments
3. Offsetting exposure in a given currency for hedging purposes
4. Speculating on foreign currencies in search of profit

Theorem 19.0.1 Interest Rate Parity

$$\textbf{Forward} = \textbf{Spot}\left(\frac{1+r_{DC}}{1+r_{FC}}\right)^T$$

Forward and spot are denoted in DC/FC as well!

For a continuous compounded rate version, we have

$$\textbf{Forward} = \textbf{Spot} \times e^{(r_{DC} - r_{FC})T}$$

Theorem 19.0.2 Purchasing Power Parity (PPP)

$$\%\Delta S(\textbf{DC/FC}) = \textbf{inflation}(DC) - \textbf{inflation}(FC)$$

20. Corporate Bonds

Definition 20.0.1 Methods for Retiring Bonds

1. **Call Provision:**
   (a) Fixed-price call: call back the bonds a specific price
   (b) Make-whole call: market rates determine the call price
2. **Sinking fund provision:** issuing firm retires a specified portion of the debt each year as outlined in the indenture—similar to a maintenance and replacement fund
3. **Tender Offers:** the firm openly indicates an interest in buying back a certain dollar amount of bonds or, more oftern, all of the bonds at a set price

Definition 20.0.2 Types of Risks Related to Bonds

1. Credit sperad risk focuses on the difference between a corporate bond's yield and the yield on a comparable-maturity benchmark Treasury security. The difference is known as the credit spread
2. Event risk addresses the adverse consequences from possible events such as mergers, recapitalizations, restructurings, acquisitions, leveraged buyouts...
3. Businessman's risk refers to bonds with a rating at the bottom rung of the investment -grade category, short term it is volatile
4. Fallen angels: downgraded investment-grade bonds
5. restructurings and leveraged buyouts may increase the credit risk of a company today the point where the bonds cecome non-investment grade
6. **High-Yield Bond Structure**
   - Reset bonds: reset coupon payment based on market
   - Deferred-interest bonds: deep discount, so no interest in early years
   - Step-up bonds: low interest in early years but higher interest in later years
   - Payment-in-kind bonds: pay interest with additional bonds

Definition 20.0.3 Default Rates

1. **Issuer default rate** number of issuers that defaulted over a year divided by the total total

number of issuer at the beginning of the year
2. **Dollar default rate** the par value of all bonds that defualted in a diven calendar year divided by the total par value of all bonds outstanding during the year
3. **Recovery rate** is the amount received as a proportion of the toital obligation after a bond defaults

R

1. Bond rating agencies only handles default risk but not credit risk since it would be accessed by the the credit spread duration
2. Zero coupon bond has low reinvestment risk

21. Mortgages and MBS

Definition 21.0.1 Credit Guarantees
1. **Government Loans** GNMA Ginnie Mae
2. **Converntional Loans** government-sponsored enterprises, FHLMC, FNMA

Definition 21.0.2 Other Factors That Influence Prepayments
1. Seasonality: summertime—greatest risk
2. Age of mortgage pool: refinancing in later years
3. Personal: marital breakdown, family issues...
4. Housing prices: rising housing price will result in higher prepayment risk
5. Refinancing burnout: people who cannot refinance...

Definition 21.0.3 Prepayment Speed Let *CPR* be the conditional prepayment rate, which is the annula rate at which a mortgage pool balance is assumed to be prepaid during the life of the pool. A mortgage pool's CPR issue a function of past prepayment rates and expected future economic conditions.

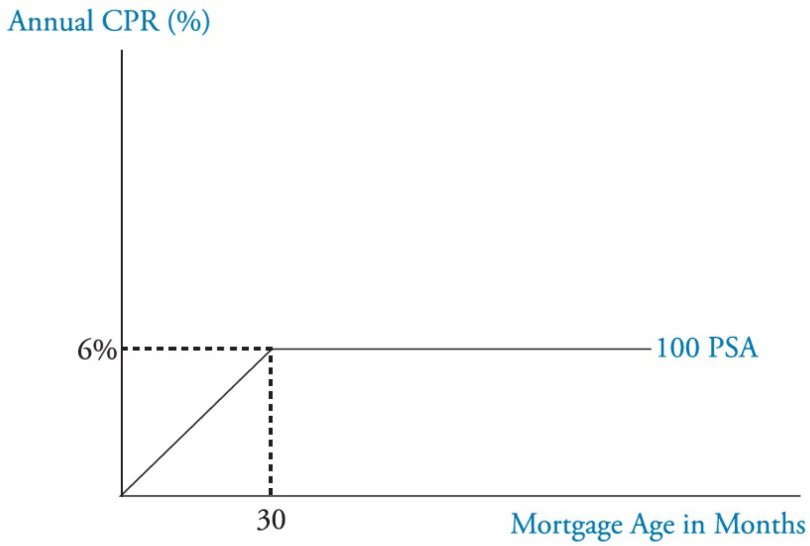
$$CPR = 1 - (1 - SMM)^{1}2$$

where *SMM* is the single monthly mortality rate.

The PSA standard benchmark is referred to as 100% PSA (or just 100 PSA). 100 PSA (see Figure 51.4) assumes the following graduated CPRs for 30-year mortgages:

- CPR = 0.2% for the first month after origination, increasing by 0.2% per month up to 30 months. For example, the CPR in month 14 is 14(0.2%) = 2.8%.
- CPR = 6% for months 30 to 360.

Figure 51.4: 100 PSA



Remember that the CPRs are expressed as annual rates.

II Valuation and Risk Models

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22. Quantifying Volatility in VaR Models

Definition 22.0.1 VaR Assuming normal distribution, then dollar VaR is

$$VaR(X\%)_{dollar} = VaR(X\%)_{deci} \times \textbf{Asset Value} = (z_{X\%}\sigma) \times \textbf{Asset Value}$$

where $z_{X\%}$ is the critical z-value based on the normal distribution and the selected $X\%$ probability and σ is the standard deviation of daily returns on a percentage basis.

R **Given Expected Return...** If expected return is given then

$$VaR = [E(R) - z \cdot \sigma]$$

at $X\%$ significance.

Theorem 22.0.1 VaR Time Conversion

$$VaR(X\%)_{J\textbf{-days}} = VaR(X\%)_{\textbf{1-day}}\sqrt{J}$$

Theorem 22.0.2 VaR Confidence Level Conversion Given original VaR confidence level α and the future confidence level is β, then

$$VaR(1-\beta) = VaR(1-\alpha) \times \frac{z_{(1-\beta)}}{z_{(1-\alpha)}}$$

Definition 22.0.2 The VaR Methods

1. **Linear Methods** replace portfolio positions with linear exposures on the appropriate risk factor. Cheaper but less accurate, usually use Delta-normal method
2. **Full Valuation Methods** fully reprice the portfolio for each scenario encountered over a historical period, or over a great number of hypothetical scenarios developed through

historical simulation or Monte Carlo simulation. More complicated and costly but more accurate

Definition 22.0.3 Delta-Normal Approach Let S be a measured risk factor and V as the portfolio value, we can model the relationship as

$$dV = \Delta_0 \times dS$$

the VaR at a given level of significance z can be written as

$$VaR = |\Delta_0| \times z \times \sigma \times S_0$$

where $z \times \sigma \times S_0 = VaR_S$

Definition 22.0.4
1. **Monte Carlo Simulation** approach revalues a portfolio for a large number of risk factor values, randomly selected from a normal distribution
2. **Historical simulation** revalues a protfolio using actual values for risk factors taken from historical data

Exercise 22.1 Delta-Normal VaR
The expected 1-day return for a \$100,000,000 portfolio is 0.00085 and the historical standard deviation of daily returns is 0.0011. Calculate daily value at risk (VaR) at 5% significance.
Solution
$z_{5\%} = qnorm(0.95) = 1.65$ then

$$VaR = [E(R) - z_{5\%}\sigma] \times V_p = [0.00085 - 1.65 \times 0.0011] \times 100000000 = -96500$$

R **Volatility Conversion**

$$\sigma_{daily} \cong \frac{\sigma_{annual}}{\sqrt{250}} \qquad \sigma_{monthly} \cong \frac{\sigma_{annual}}{\sqrt{12}}$$

R

1. **Advantages of Delta-Normal VaR**
   (a) Easy to implement
   (b) Calculations can be performed quickly
   (c) Conducive to analysis because risk factors, correlations, and volatilities are identified
2. **Disadvantages of Delta-Normal VaR**
   (a) The need to assume a normal distribution
   (b) The method is unable to properly account for distributions with fat tails, either because of unidentified time variation in risk or unidentified risk factors and/or correlations
   (c) Nonlinear relationships of option-like positions are note adequately described by the delta-normal method

R

1. Historical method just count the percentile of the historical data to calculate VaR assuming Normal distribution
2. **Advantages of Historical VaR**

   (a) The model is easy to implement when historical data is readily available
   (b) Calcuateions are simple
   (c) Horizon is positive choice based on the intervals of historical data
   (d) Based on actual prices not exposed to model risk
   (e) Includes all correlations
3. **Disadvantages of Historical VaR**
   (a) It may not be enough data
   (b) Time variation of risk in the past may not represent variation in the future
   (c) The model may not recognize changes in volatility and correlations from structural changes
   (d) The weighting might be different for new and old data EWMA can be used to solve this
   (e) A small number of actual observations may lead to insufficently defined distribution tails

R

1. **Advantages of the Monte Carlo method**
   (a) It is the most powerful model, account for both linear and nonlinear risks
   (b) It can include time variation in risk and correlations
   (c) Flexible and extensible
   (d) Nearly unlimited numbers of scenarios
2. **Disadvantages of the Monte Carlo method**
   (a) Lengthy computation and expensive
   (b) Model risk of the stochastic process chosen
   (c) Sampling variation at lower numbers of simulation

Definition 22.0.5 3 Common Deviation from Normality in Modeling Risk
1. **Fat-tailed**: most likely the result of the volatility and/or the mean of the distribution changing over time in a unconditional distribution, can be solved by regime-switching model
2. **Skewed**
3. **Unstable parameters**

Definition 22.0.6 Regime-Switching Volatility Model assumes different market regimes exist with high or low volatility. The conditional distributions of returns are always normal with a constant mean but either have a high or low volatility

R Asset return distributions tend to exhibit fat tails. As a result, VaR may underestimate the actual loss amount.

Definition 22.0.7 Estimating VaR
1. **Historical-Based**
   (a) Parametric approach: requires specific assumtions regarding the asset returns distribution, on K most recent data, we can estimate the future volatility to be

$$\sigma_t^2 = \left(r_{t-K,t-K+1}^2 + \dots r_{t-1,t}^2\right)/K$$

   delta-normal method is an example parametric approach
   (b) Nonparametric approach: historical data model
   (c) Hybrid
2. **Implied-volatility-based approach** uses derivative pricing models such as BSM option pricing model to estimate an implied volatility based on current market data rather than

historical data
(a) Advantage: forward-looking and reacts immediately to changing market conditions
(b) Disadvantage: model dependent

Definition 22.0.8 Exponential Smoothing Weighting Methods
1. Are parametric
2. Attempt to estimate conditional volatility
3. Use recent historical data
4. Apply a set of weights to past squared returns

Including EWMA and GARCH

Theorem 22.0.3 EWMA Model

$$\sigma_t^2 = (1-\lambda)\left(\lambda^0 r_{t-1,t}^2 + \cdots + \lambda^N r_{t-N-1,t-N}^2\right)$$

for $0 < \lambda < 1$ and N is the number of observations used to estimate volatility. We can reduce this to be

$$\sigma_t^2 = (1-\lambda) r_{t-1,t}^2 + \lambda \sigma_{t-1}^2$$

	Weight of Volatility Parameter		
	(1 − λ)λ^t	**1/k**	**(1 − λ)λ^t**
t	λ = 0.97	k = 75	λ = 0.92
0	0.0300	0.0133	0.0800
−1	0.0291	0.0133	0.0736
−2	0.0282	0.0133	0.0677
−3	0.0274	0.0133	0.0623
−4	0.0266	0.0133	0.0573

Figure 22.0.1: *Volatility Weighting Under Exponential Smoothing and Historical SD Approach*

R

1. When K is small as a shorter estimation window, the historical SD method results in forecasts that are more volatile
2. When λ is smaller, will result in more volatile forecast as well

Theorem 22.0.4 GARCH Model
To estimate volatility, the GARCH(p,q) model using the following formula

$$\sigma_t^2 = a + b_1 r_{t-1,t}^2 + \cdots + b_p r_{t-p,t-p+1}^2 + c_1 \sigma_{t-1}^2 + \cdots + c_q \sigma_{t-q}^2$$

p, q are lagging terms for return and volatility. The standard GARCH(1,1) model is

$$\sigma_t^2 = a + b r_{t-1,t}^2 + c \sigma_{t-1}^2$$

when $a = 0, b + c = 1$, this is identical to EWMA model

Definition 22.0.9 Nonparametric VaR Methods: historical simulation, multivariate density estiamtion, hybrid

1. **Advantages:**
   (a) No distribution assumption
   (b) MDE allows for weights to vary based on market environment instead of timing
2. **Disadvantages:**
   (a) Data is more efficient with parametric methods
   (b) Separating the full sample of data into different market regimes reduces the usable data
   (c) MDE may lead to overfitting

Definition 22.0.10 Historical Approach Same weighting for all return data, care for random distribution by taking average

Definition 22.0.11 Hybrid Approach Combine percentiles of return and exponential smoothing

Step 1: Assign weights for historical realized returns to the most recent K returns using an exponential smoothing process as follows:

$$\left[(1-\lambda)/(1-\lambda^{K})\right], \left[(1-\lambda)/(1-\lambda^{K})\right]\lambda^{1}, \ldots,$$
$$\left[(1-\lambda)/(1-\lambda^{K})\right]\lambda^{K-1}$$

Step 2: Order the returns.

Step 3: Determine the VaR for the portfolio by starting with the lowest return and accumulating the weights until x percentage is reached. Linear interpolation may be necessary to achieve an exact x percentage.

Definition 22.0.12 Multivariate Density Estimation (MDE)
For each market regime, the conditional volatility is calculated as

$$\sigma_t^2 = \sum_{i=1}^{K} \omega(X_{t-i}) r_{t-i}^2$$

$\omega(X_{t-i})$ is used to measure the realtive weiht in terms of near or distant from the current state

Definition 22.0.13 Mean Reversion
Recall AR model say

$$r_i = a + b r_{i-1} \rightarrow \lim_{i\to\infty} r_i = \frac{a}{1-b}$$

when $b < 1$, we say this is mean reverting.

23. Putting VaR to Work

Definition 23.0.1 VaR for Linear Derivatives
Assuming $F_t = \Delta S_t$ where Δ is the sensitivity of the derivative price wrt to the underlying asset price. Then

$$VaR(F_t) = \Delta VaR(S_t)$$

Definition 23.0.2 Structured Monte Carlo (SMC) Approach simulates thousands of valuation outcomes for the underlying assets based on the assumption of normality. The VaR for the portfolio of derivatives is then calculated from the simulated outcomes.

1. Advantage is that SMC can simulate multiple risk factors by assuming an underlying distribution
2. Disadvantage is that the estimation may not be accurate and cannot be improved by more simulations

R

1. A contagion effect often occurs where volatility and correlations both increase, thus mitigating any diversification benefits

24. Measures of Financial Risk

Definition 24.0.1 M-V Framework Limitations

1. Not appropriate for non-normal distributions
2. Especially positively skewed

Definition 24.0.2 Limitation of VaR

1. VaR increases when when the confidence level increases, VaR will increase at an increasing rate as the confidence level increases
2. VaR will increase with increases in the holding period

Definition 24.0.3 Coherent Risk Measures Let R be a set of random events and $\rho(R)$ to be the risk measure for the random events

1. **Monotonicity:** a portfolio with greater future returns will likely have less risk:

$$R_1 \leq R_2 \rightarrow \rho(R_1) \leq \rho(R_2)$$

2. **Subadditivity:**

$$\rho(R_1 + R_2) \leq \rho(R_1) + \rho(R_2)$$

3. **Positive Homogeneity:** for $\beta > 0$

$$\rho(\beta R) = \beta \rho(R)$$

4. **Translation Invariance:** for constant c

$$\rho(c + R) = \rho(R) + c$$

Definition 24.0.4 Expected Shortfall

1. ES and VaR both are coherent risk measures but ES can do better with non-elliptical distributions

2. The portfolio risk surface for ES is convex because the property of subbadditivity is met. ES is more appropriate for solving portfolio optimization problems than the VaR method
3. ES gives an estimate of the magnitude of a loss for unfavorable events
4. ES has less restrictive assumptions regarding risk/return decision rules

25. Binomial Trees

Theorem 25.0.1 Perfect Hedge Ratio

$$\Delta = \frac{C_U - C_D}{S_U - S_D}$$

one option contract is needed for each Δ stock

Theorem 25.0.2 Synthetic Call Replication
A combination of the delta, the stock price, and the PV of the borrowings can be used to price the call option

$$C = \Delta \times (S - PV(B))$$

Theorem 25.0.3 Risk-Neutral With No Dividend

$$U = e^{\sigma\sqrt{t}} \qquad D = \frac{1}{U} = e^{-\sigma\sqrt{t}}$$

$$\pi_u = \frac{e^{rt} - D}{U - D} \qquad \pi_d = 1 - \pi_u$$

R The above result is for **call** option pricing. You need to account for the expected cashflow at the end of the stage and discount it back. To get the put option price, use the put-call parity

$$p = c - S + Ke^{-rt}$$

Theorem 25.0.4 **Risk Neutral with Dividend** δ For stocks, δ is the dividend yield,

$$\pi_u = \frac{e^{(r-\delta)t} - D}{U - D} \qquad \pi_d = 1 - \pi_u$$

For currencies r_{FC} is the foreign currency risk-free rate

$$\pi_u = \frac{e^{(r_{DC}-r_{FC})t} - D}{U - D} \qquad \pi_d = 1 - \pi_u$$

For futures, since they are costless to enter into, we have

$$\pi_u = \frac{1 - D}{U - D} \qquad \pi_d = 1 - \pi_u$$

R For American option with dividends, we know that there is a possibility of early exercise. Thus, when dealing with multi-stage binomial tree. The actual payoff and the risk-neutral payoff at each intermediate node need to be compared to decide whether early exercise is available and proceed further. This is cumbersome and tricky for most of the time.

Theorem 25.0.5 If we shorten the length of the intervals to be arbitrarily small in the binomial model, then the option valuation will converge to its continuous time model called Black-Scholes-Merton Model.

26. The Black-Scholes-Merton Model

Definition 26.0.1 Lognormal Stock Price & Normal Stock Return
The BSM model assumes stock prices are lognomally distributed

$$\ln S_T \sim G\left(\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right)T, \sigma\sqrt{T}\right)$$

However, the stock returns are normally distributed when considering continuously compounded annual rate of return α (geometric return),

$$\alpha \sim G\left(\mu - \frac{\sigma^2}{2}, \frac{\sigma}{\sqrt{T}}\right)$$

Theorem 26.0.1 Expection of Stock Price

$$E(S_T) = S_0 e^{\mu T}$$

where μ is the arithmetic expected rate of return.

R When calculating **realized rate of return**, we need to use the geometric return

Definition 26.0.2 Black-Scholes-Merton Model BSM model values options in continuous time and is derived from the same no-arbitrage assumption used to value options with the binomial model. The assumptions are as follow:

1. The price of the underlying asset follows a lognormal distribution
2. The risk-free rate is continuous, constant, and known
3. The volatility of the underlying asset is constant and known
4. No transaction costs, no taxes, and no restrictions on short sales
5. The underlying assets has no cash flow

6. The options valued are European options

Theorem 26.0.2 BSM for non-dividend stock option Since we do not have dividend, we have $S = S_0$ and the following equations

$$c = SN(d_1) - Ke^{-rT}N(d_2) \qquad p = Ke^{-rT}(1 - N(d_2)) - S(1 - N(d_1))$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \qquad d_2 = d_1 - (\sigma\sqrt{T})$$

Theorem 26.0.3 BSM with dividend rate or payment Suppose we do have a continuous dividend yield of δ, then we need to adjust

$$S = S_0 e^{-\delta T}$$

and resubstitute. If we have a dividend payment, then adjust as follow

$$S = S_0 - PV(dividends)$$

Then, resubstitute and get the result.

R Even though we do not have a BSM for American options, we still have a result that the closer the option is to expiration and the larget the dividend, the more optimal early exercise will become. The early exercise of American **call option** will be not optimal if

$$D_n \leq K(1 - e^{-r(T-t_n)}), t_n < T$$

Here, if it is optimal to exercise early, then we need to apply BSM approximation by adjust S to only subtract PV of the dividends before the n-th dividend payment and the time should stop at t_n for evaluation. For, American **put option**, early exercise becomes less likely with larger dividends, it not optimal if

$$D_n \geq K(1 - e^{-r(T-t_n)}), t_n < T$$

Theorem 26.0.4 Warrant Valuation
Warrants are attachments to a bond issue that give the holder the right to purchase shares of a security at a stated price. Assuming no benefit to the company from issuing warrants, the value is

$$\frac{N}{N+M} \times \textbf{Value of regular call option}$$

where N is the number of shares outstanding and M is the number of new warrants issued

Definition 26.0.3 Historical Volatility Estimation

1. Convert a times series of N prices to returns

$$R_i = \frac{P_i - P_{i-1}}{P_{i-1}}, 1 \leq i \leq N$$

2. Convert the returns to continuously compounded returns

$$R_i = \ln(1 + R_i), 1 \leq i \leq N$$

3. Calculate the variance and SD of the continuously compounded returns

$$\sigma^2 = \frac{\sum_{i=1}^{N} (R_i - \bar{R})^2}{N - 1}$$

Theorem 26.0.5 **Implied Volatility by BSM** Given market option prices, we can use BSM to backtrack the volatility.

27. The Greek Letters

Definition 27.0.1 Positions

1. **Naked position** occurs when one party sells a call option without owning the underlying asset
2. **Covered position** occurs when the party selling a call option owns the underlying asset

Definition 27.0.2 Delta & Delta Hedging

$$\Delta = \frac{\partial C}{\partial S}$$

The option delta is just $N(d_1)$.
The future delta is $e^{(r-\delta)T}$.

Definition 27.0.3 Delta-neutral portfolio

$$\textbf{Number of options needed to delta hedge} = \frac{\textbf{number of shares hedged}}{\textbf{delta of call option}}$$

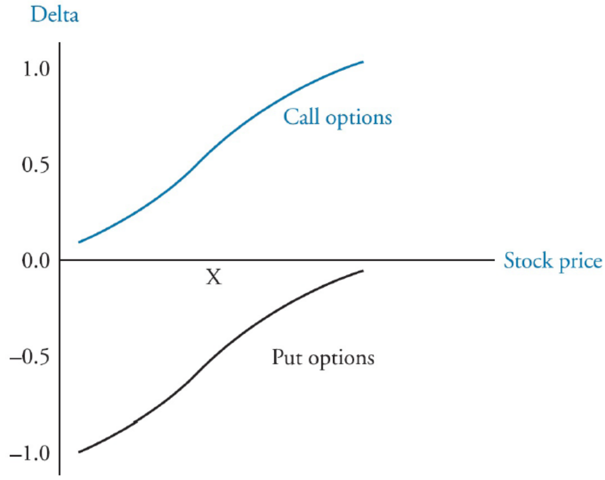
Definition 27.0.4 Portfolio Delta

$$\textbf{Portfolio Delta} = \Delta_p = \sum_{i=1}^{n} w_i \Delta_i$$

Definition 27.0.5 Theta

$$\theta = \frac{\partial C}{\partial t}$$

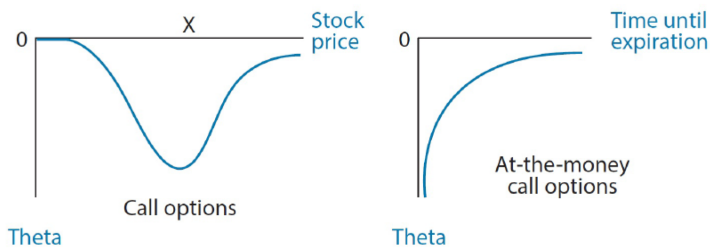
For European call options on non-dividend-paying stocks, θ can be calculated using the BSM



formula

$$\theta(call) = -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} - rKe^{-rT}N(d_2)$$
$$\theta(put) = -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} + rKe^{-rT}N(-d_2)$$

where $N'(x) = \frac{1}{\sqrt{2\pi}}e^{-(x^2/2)}$



Definition 27.0.6 Gamma

$$\Gamma = \frac{\partial^2 C}{\partial t^2} = \frac{N'(d_1)}{S_0 \sigma \sqrt{T}}$$

To get a Gamma-neutral portfolio, we need

$$-\frac{\Gamma_p}{\Gamma_T} = \textbf{number of options to long}$$

Theorem 27.0.1 Delta, Theta, Gamma

$$r\Pi = \theta + rS\Delta + \frac{1}{2}\sigma^2 S^2 \Gamma$$

Definition 27.0.7 Vega

$$\textbf{Vega} = \frac{\partial C}{\partial \sigma} = S_0 N'(d_1)\sqrt{T}$$

most sensitive at the money like Theta.

Definition 27.0.8 ρ

$$\rho = \frac{\partial C}{\partial r}$$

for calls, the in-the-money calls and puts are more sensitive to change of risk-free rates than out-of-the-money calls and puts.

$$\rho(C) = KTe^{-rT}N(d_2) \qquad \rho(P) = -KTe^{-rT}N(-d_2)$$

28. Prices, Discount Factors, and Arbitrage

29. Spot, Forward, and Par Rates

Definition 29.0.1 — Holding Period Return.

$$r = m\left[\left(\frac{FV_n}{PV_0}\right)^{\frac{1}{m\times n}} - 1\right]$$

Definition 29.0.2 — Swap Rate.
Given swap rate table

Maturity	Swap Rates
0.5	s_1
1.0	s_2

we have

$$\left(1+\frac{s_1}{2}\right)d(0.5) = 1$$

and

$$\left(\frac{s_2}{2}\right)d(0.5) + \left(1+\frac{s_2}{2}\right)d(1.0) = 1$$

Definition 29.0.3 — Par Rate.
The par rate at maturity is the rate at which the present value of a bond equals its par value (in fact they are swap rates)

$$\frac{P_T}{2}\times\sum_{t=1}^{2T} d\left(\frac{t}{2}\right) + d(T) = 1$$

30. Returns, Spreads, and Yields

31. One-Factor Risk Metrics and Hedges

Definition 31.0.1 — DV01.

$$DV01 = -\frac{\Delta BV}{10000 \times \Delta y}$$

Theorem 31.0.1 — Application to Hedging.

$$HR = \frac{DV01(\textbf{per 100 of initial position})}{DV01(\textbf{ per 100 of hedging instrument})}$$

Definition 31.0.2 — Duration.

1. Macualy Duration and Modfied Duration

$$D^* = \frac{D}{1+y} \qquad\qquad D^* = \frac{1}{BV}\frac{\Delta BV}{\Delta y}$$

2. Effective Duration for callable and putable bonds

$$D_e = \frac{BV_{-\Delta y} - BV_{+\Delta y}}{2 \times BV_0 \times \Delta y}$$

R DV01 is more useful when conducting quantitative hedging but duration is more convenient when just alerting investors.

Definition 31.0.3 — Convexity.

$$C = \frac{1}{BV}\frac{d^2BV}{dy^2}$$

too much work, we can use the approximated version

$$C_{approx} = \frac{BV_{-\Delta y} + BV_{+\Delta y} - 2 \times BV_0}{2 \times BV_0 \times \Delta y^2}$$

Theorem 31.0.2 — Portfolio Duration.

$$D_p = \sum_{j=1}^{K} w_j \times D_j$$

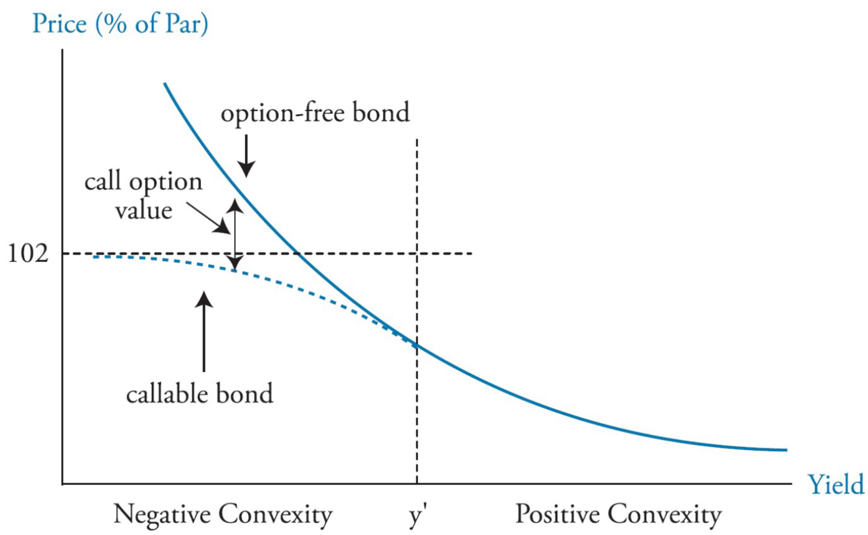


Figure 31.0.1: *Callable Bond's Yield Curve*

Definition 31.0.4 — Barbell Portfolio.
A barbell strategy is typically used when an investment manager uses bonds with short and long maturities, thus forgoing any bonds concentrated in the intermediate maturity range. This is preferred over bullet portfolio when the portfolio manager thinks the interest rate is volatile. **This requires the sum of PV to be the same and the duration of the portfolio matches the bullet portfolio**

32. Multi-Factor Risk Metrics and Hedges

33. Country Risk

34. External and Internal Ratings

35. Capital Structure in Banks

Definition 35.0.1 Expected Loss

$$EL = EA \times PD \times LR = EA \times PD \times (1 - RR)$$

Definition 35.0.2 Unexpected Loss For the horizon H, the asset with value V_H has $UL_H = \sqrt{var(V_H)}$. And

$$UL = EA \times \sqrt{PD \times \sigma_{LR}^2 + LR^2 \times \sigma_{PD}^2}$$

where $\sigma_{PD}^2 = PD(1 - PD)$ due to binomial model

Definition 35.0.3

1. Portfolio EL is linear

$$EL_p = \sum_i EL_i$$

2.

$$UL_p = \sqrt{\sum_i \sum_j \rho_{ij} UL_i UL_j}$$

Definition 35.0.4 Two-Asset Risk Contribution

$$RC_1 = UL_1 \times \frac{UL_1 + (\rho_{12} UL_2)}{UL_p} \qquad RC_2 = UL_2 \times \frac{UL_2 + (\rho_{12} UL_1)}{UL_p}$$

Definition 35.0.5 Economic Capital

$$EC_p = UL_p \times CM$$

R **Bottom-up Risk Measurement Limitations**

1. Credits are presumed to be illiquid assets
2. Credit risk models used in practice only use a one-year estimation horizon

3. Other risk components are separated from credit risk and

36. Operational Risk

Definition 36.0.1 Operational Risk

- Financial risk that is not caused by market risk
- Any risk developing from a breakdown in normal operations
- Any risk from internal sources
- Direct or indirect losses that result from ineffective or insufficient systems
- Legal risk, but not reputational risk or strategic risk since hard to quantify

The risk of direct and indirect loss resulting from inadequate or failed internal processes, people, and systems or from external events.

Definition 36.0.2 — Methods to Calculate Regulatory Capital.

1. **Basic Indicator Approach**: operational risk capital is based on 15% of the bank's annual gross income over a 3-year period
2. **Standardized Approach**: 8 Business line weighted total of operational risk using gross income over a 3-year period

Investment Banking (corporate finance)	18%
Investment Banking (trading and sales)	18%
Retail banking	12%
Commercial banking	15%
Settlement and payment services	18%
Agency and custody services	15%
Retail brokerage	12%
Asset Management	12%

Basel Committee recommended in order to use the standardized approach

- have an operational risk management function that is able to identify, assess, monitor, and control this type of risk

- Document losses for each business line
- report opertional risk losses on a regular basis
- have a system taht has the appropriate level of documentation
- conduct independent audits with both internal and external audits

3. **Advanced Measurement Approach**: also need to approximate unexpected loss, confidence level of 99.9% over a 1 year-time horizon for operational VaR

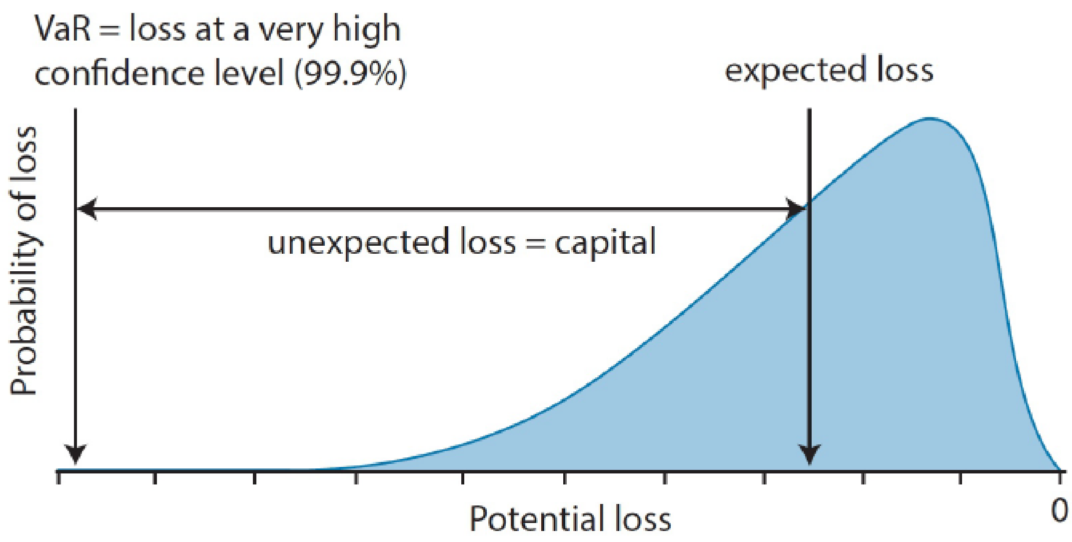


Figure 36.0.1: *Operational VaR and Loss Distribution*

Definition 36.0.3 — Operational Risk Categories.

1. Clients, products, and business prctices.
2. Internal fraud
3. External fraud
4. Damage to physical assets
5. Execution, delivery, and process management
6. Business disruption and system failures
7. Employment practives and workplace safety

R

1. It is very common to use **Poisson Distribution** to estimate loss frequency.
2. **RCSA** risk and control self assessment program is to survey those mangers directly responsible for the operations of the various business lines
3. **KRI** key risk indicators is helpful when attempting to identify operational risks when factor has a predictive relationship to losses and be accessible and measurable in a timely fashion

Theorem 36.0.1 — Firm Size Scale Adjustment. How to use other firms as benchmark

$$\textbf{Estimated Loss}_Y = \textbf{External Loss}_Z \times \left(\frac{\textbf{revenue}_Y}{\textbf{revenue}_Z}\right)^{0.23}$$

Definition 36.0.4 — Scorecard Data.
Scorecard Data approach is to survey each type of risk

Theorem 36.0.2 — The Power Law.
Since operational risk losses are likely to occur in thee tails

$$P(V > X) = K \times X^{-\alpha}$$

where V is the loss variable, X is some large value of V in the tail and K, α are constants.

37. Governance Over Stress Testing

Definition 37.0.1 — Roles of Management in Stress Testing.

1. **The board of director**
    - accountable for the entire organization and must be sufficiently knowledgeable about the organization's stress testing activities
    - should actively challenge the results of stress testing and supplement them with other tests as well as both quantitative and qualitative information
2. **Senior Management**
    - with oversight of the board, is responsible for establishin robust policies for stress tests, which could supplement other risk, capital, and adequacy measures
    - reviewing and coordinating stress test activities, assigning competent staff, challenging results and assumptions, and incorporating remedies to potential problems
    - ensure that there is sufficient range of stress testing
    - regularly report tot he board of directors on stress testing results

38. Stress Testing

39. Principles for Sound Stress Testing

III Additional Information

40 Question Findings 107

40. Question Findings

1. Both AR and ARMA are good at forecasting with seasonal patterns because they both both involve lagged observable variables, which are best for capturing a relationships in motion. It is the moving average representation that is best at capturing only random movements
2. Penalty factors for MSE:

$$\begin{cases} s^2 & \frac{T}{T-k} \\ AIC & e^{\frac{2k}{T}} \\ SIC & T^{(k/T)} \end{cases}$$

   SIC has the largest penalty factor.
3. For dummy variables of forecasting, we only need $n-1$
4. Including the full set of dummy variables and an intercepts term wouldproduce a forecasting model that exhibits perfect multicollinearity.
5. Geometric Brownian Motion:

$$\Delta S_t = S_t(\mu \Delta t + \sigma \varepsilon \sqrt{\Delta t})$$

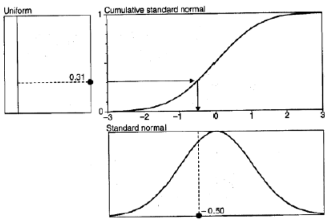
   or say

$$S_{t+1} = S_t(1 + \mu \Delta t + \sigma \varepsilon \sqrt{\Delta t})$$

6. The standard error estimate of a Monte Carlo simulation can be reduced by a factor of 10 by increasing N by a factor of 100
7. Monte Carlo simulation is suitable for pricing options in each case except when early exercise of the option isw possible.
8. Short option positions have long left tails, which makes it more difficult to estimate a left-tailed quantile precisely.
9. Boostrap: $S_{t+1} = S_t(1 + R_{m(1)})$
   - Advantages: can include fat tails, jumps, or any departure from the normal distribution Account for correlations across series because one draw consists of the simultaneous returns for N series, such as stock, bonds, and currency prices.

Variance Reduction Technique	
Antithetic Variates	▫ Reduces sampling error by rerunning the simulation using a complement set of the original set of random variables.
Control Variates	▫ Replaces a variable x that has unknown properties in a Monte Carlo simulation with a similar variable y that has known properties. The new x* variable estimate will have a smaller sampling error than the original x variable if the control statistic and statistic of interest are highly correlated.
Random Number Re-Usage	▫ Reusing sets of random number draws across Monte Carlo experiments reduces the estimate variability across experiments.

Inverse Transform Method:



Cholesky Factorization: $\sum = XX^T$

- $\varepsilon_1 = \eta_1$
- $\varepsilon_2 = \rho\eta_1 + (1-\rho^2)^{1/2}\eta_2$

- Limitations: for small sample sizes, it may be a poor approximation of the actual one; replies heavily on the assumption that returns are independent

10. GARCH model allows for time-varying volatility by describing the conditional variance as a function the previous period's volatility and the most recent variance estimate; it is good for simulating leptokertic return distributions with fat tails; the EWMA is a Special case of the GARCH model
11. Correlation Estimateion

$$\hat{\rho_{XY}} = \frac{cov_n}{\sigma_{x,n}\sigma_{y,n}}$$

$$cov_n = \lambda cov_{n-1} + (1-\lambda)x_{n-1}y_{n-1} = \omega + \alpha x_{n-1}y_{n-1} + \beta cov_{n-1}$$

12. The GARCH model will take the shortest time to revert to its mean is the model withdrawals the lowest persistence $\alpha + \beta$
13. When calculating rate of return using EWMA, make sure use continuous compounding
14. GARCH model is stable when the persistence $\alpha + \beta < 1$
15. EWMA only cares about most recent variance, the weighting for long-run average variance is zero.
16. The EWMA estimate of varaince is a weighted average of the average of the prior day's variance and prior day squared return