STAT 341: Tutorial 5 – Contour Plots and `optim`

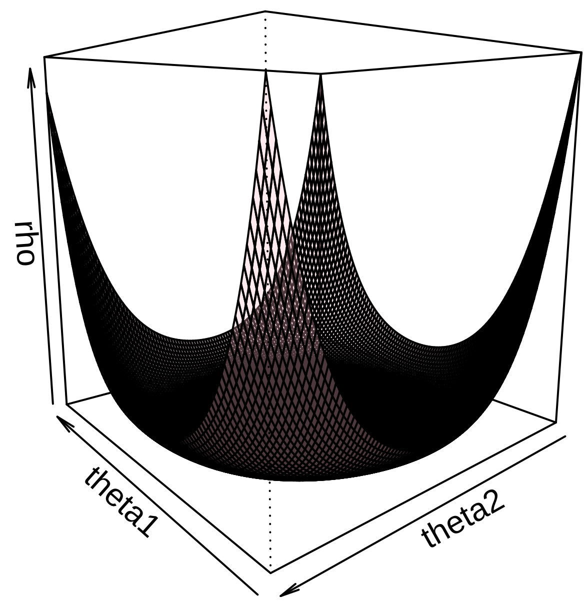
Friday February 14, 2020

Part I: Plot the Mystery Function

$$\rho(\theta_1, \theta_2) = (\theta_1^2 + \theta_2^2 - 1)^3 - \theta_1^2\theta_2^3$$

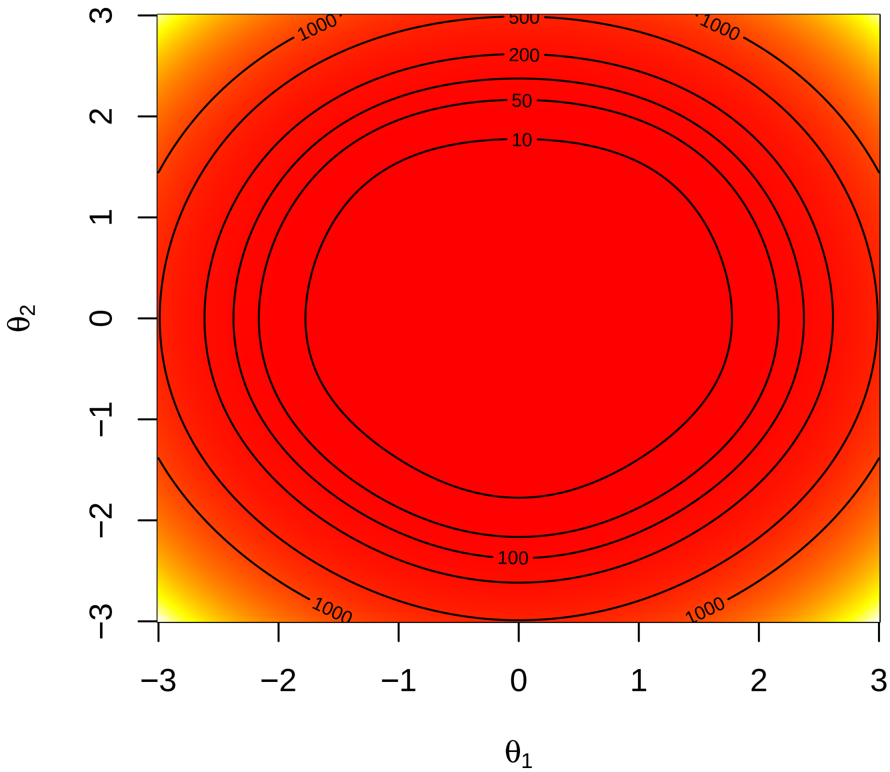
1. Plot the function as a 3D over the range $\theta_1, \theta_2 \in [-3, 3]$.

```
rho <- function(theta1,theta2){
  (theta1^2 + theta2^2 - 1)^3 - (theta1^2)*(theta2^3)
}
theta1 <- seq(-3,3,length=100)
theta2 <- seq(-3,3,length=100)
Rho <- outer(theta1,theta2,"rho")
persp(theta1, theta2, Rho, theta = 230, phi = 10, col=adjustcolor("pink", alpha = 0.3),
      xlab = "theta1", ylab = "theta2", zlab = "rho")
```



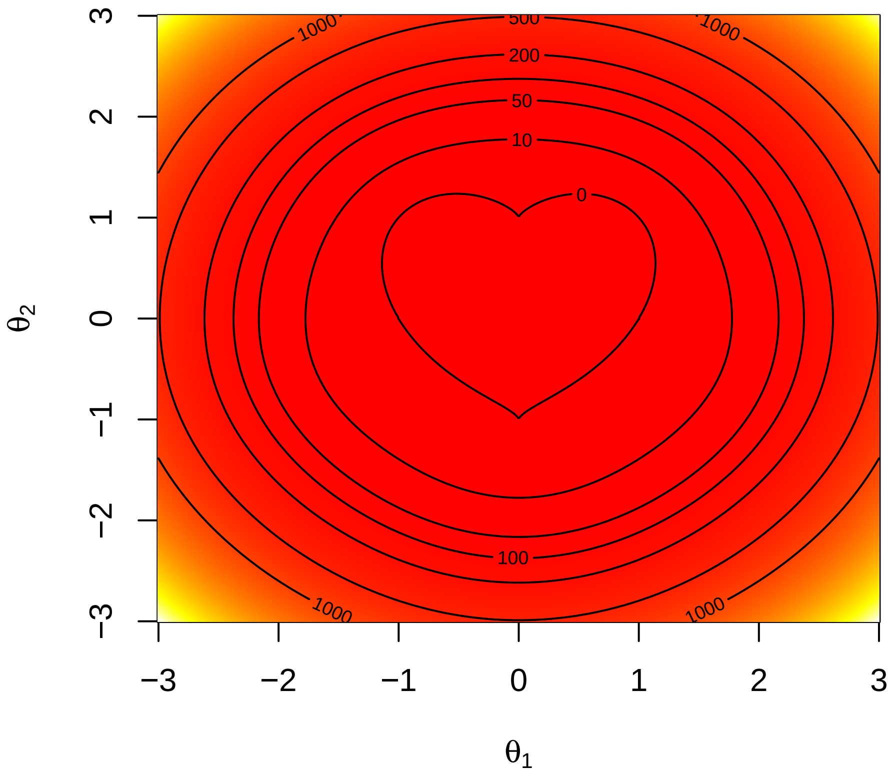
2. Create a 2D contour plot of the function over the range $\theta_1, \theta_2 \in [-3, 3]$.

```
theta1 <- seq(-3,3,length=500)
theta2 <- seq(-3,3,length=500)
Rho <- outer(theta1,theta2,"rho")
image(theta1, theta2, Rho, col = heat.colors(1000) , xlab = bquote(theta[1]), ylab = bquote(theta[2]), m
contour(theta1,theta2,Rho,add=T, levels = c(10, 50, 100, 200, 500, 1000))
```



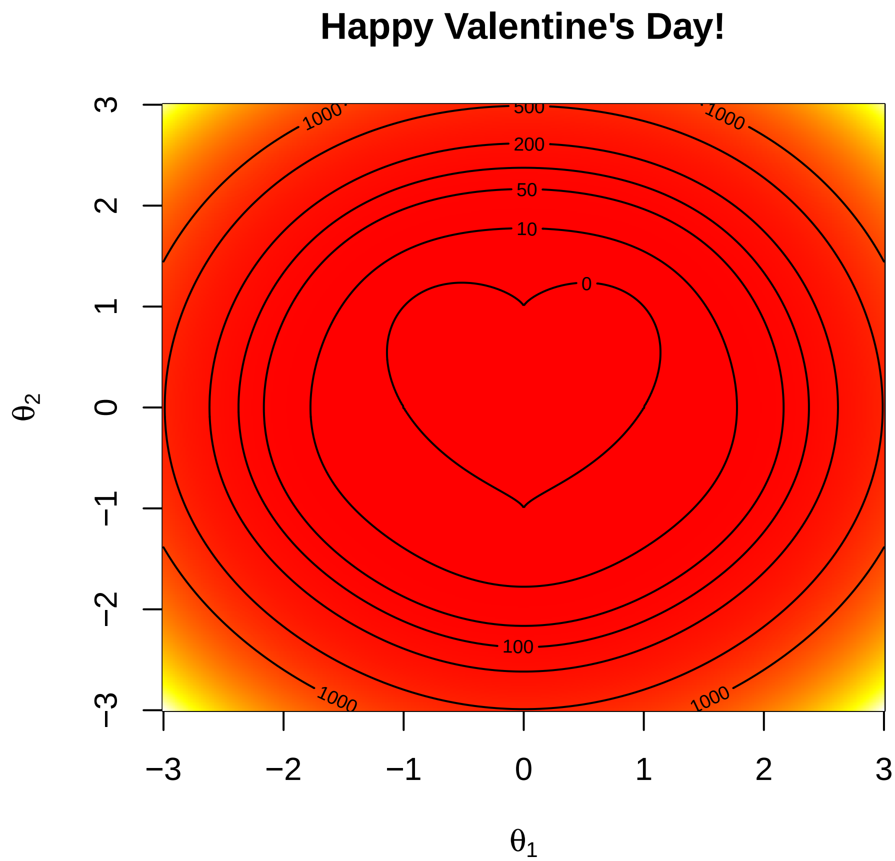
3. Add the 0 contour

```
image(theta1, theta2, Rho, col = heat.colors(1000) , xlab = bquote(theta[1]), ylab = bquote(theta[2]), m
contour(theta1,theta2,Rho,add=T, levels = c(0,10, 50, 100, 200, 500, 1000))
```



4. Add an appropriate plot title

```r
image(theta1, theta2, Rho, col = heat.colors(1000) , xlab = bquote(theta[1]), ylab = bquote(theta[2]),
      main = "")
contour(theta1,theta2,Rho,add=T, levels = c(0,10, 50, 100, 200, 500, 1000))
title(main = "Happy Valentine's Day!")
```



Part II: Find the Minimum of the Mystery Function

1. Determine the gradient of $\rho(\theta_1, \theta_2)$ and write the corresponding `rho` and `gradient` functions for use with `optim`.

```r
rho <- function(theta){
  theta1 <- theta[1]
  theta2 <- theta[2]
  (theta1^2 + theta2^2 - 1)^3 - (theta1^2)*(theta2^3)
}

grad <- function(theta){
  theta1 <- theta[1]
  theta2 <- theta[2]
  c(6*theta1*(theta1^2 + theta2^2 - 1)^2 - 2*theta1*theta2^3,
    6*theta2*(theta1^2 + theta2^2 - 1)^2 - 3*(theta1^2)*(theta2^2))
}
```

2. Explore the different optimization `methods` available to you in by checking out `? optim`

3. Using `Nelder-Mead` to find $\text{argmin}_{\theta \in \mathbb{R}}\, \rho(\theta)$, and start the algorithm from $\hat{\theta}_0 = (3, 3)$.

```
optim(par = c(3,3), fn = rho, gr = grad, method = "Nelder-Mead")
```

```
## $par
## [1] -0.0003073409 -0.0016774222
##
## $value
## [1] -0.9999913
##
## $counts
## function gradient
##       59       NA
##
## $convergence
## [1] 0
##
## $message
## NULL
```

4. Using `BFGS` to find $\text{argmin}_{\theta \in \mathbb{R}}\, \rho(\theta)$, and start the algorithm from $\hat{\theta}_0 = (3, 3)$.

```
optim(par = c(3,3), fn = rho, gr = grad, method = "BFGS")
```

```
## $par
## [1] 5.841424e-09 1.386031e-08
##
## $value
## [1] -1
##
## $counts
## function gradient
##       29       13
##
## $convergence
## [1] 0
##
## $message
## NULL
```

5. Using `CG` to find $\text{argmin}_{\theta \in \mathbb{R}}\, \rho(\theta)$, and start the algorithm from $\hat{\theta}_0 = (3, 3)$.

```
optim(par = c(3,3), fn = rho, gr = grad, method = "CG")
```

```
## $par
## [1] 7.216951e-08 8.968249e-08
##
## $value
## [1] -1
##
## $counts
```

```
## function gradient
##       40       15
##
## $convergence
## [1] 0
##
## $message
## NULL
```

6. Using L-BFGS-B to find $\text{argmin}_{\theta \in \mathbb{R}} \rho(\theta)$, and start the algorithm from $\hat{\theta}_0 = (3, 3)$.

```
optim(par = c(3,3), fn = rho, gr = grad, method = "L-BFGS-B")
```

```
## $par
## [1] 0.7862829 0.9629960
##
## $value
## [1] -0.3897013
##
## $counts
## function gradient
##       19       19
##
## $convergence
## [1] 0
##
## $message
## [1] "CONVERGENCE: REL_REDUCTION_OF_F <= FACTR*EPSMCH"
```

7. Using SANN to find $\text{argmin}_{\theta \in \mathbb{R}} \rho(\theta)$, and start the algorithm from $\hat{\theta}_0 = (3, 3)$.

```
optim(par = c(3,3), fn = rho, gr = grad, method = "SANN")
```

```
## $par
## [1] 3 3
##
## $value
## [1] 4670
##
## $counts
## function gradient
##    10000       NA
##
## $convergence
## [1] 0
##
## $message
## NULL
```