Quantum Computing (量子计算)

Get ready to grasp the leading edge technologies

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Periodic behavior

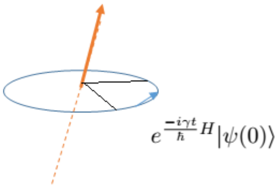
- If J is a matrix such that $J^2 = -I$ then
$$e^{tJ} = \cos(t)I + \sin(t)J.$$
Example: $J \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ J behaves as the imaginary unit i.
- If H is a Hermitian matrix such that $H^2 = I$ then
$$e^{-itH} = \cos(t)I - i\sin(t)H.$$
Examples: Pauli matrices $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ and $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. We can prove that in general we have $H = \eta_1\sigma_1 + \eta_2\sigma_2 + \eta_3\sigma_3$ with $\eta_1^2 + \eta_2^2 + \eta_3^2 = 1$ and $e^{\frac{-i\gamma t}{\hbar}H} = \cos(\frac{\gamma t}{\hbar})I - i\sin(\frac{\gamma t}{\hbar})H$ is a rotation around the vector $\eta^T = \begin{pmatrix} \eta_1, \eta_2, \eta_3 \end{pmatrix}$

Spin Schrodinger equation

The Schrodinger equation of a particle within a magnetic field with Hamiltonian γH is given by:

$$i\hbar \frac{\partial}{\partial t}|\psi(t)\rangle = \gamma H|\psi(t)\rangle.$$

Its solution $e^{\frac{-i\gamma t}{\hbar}H}|\psi(0)\rangle$ is a rotation (up to overall phase) of the initial state $|\psi(0)\rangle$ around the vector $\eta^T = \begin{pmatrix} \eta_1, \eta_2, \eta_3 \end{pmatrix}$ by an amount of $\frac{2\gamma t}{\hbar}$.



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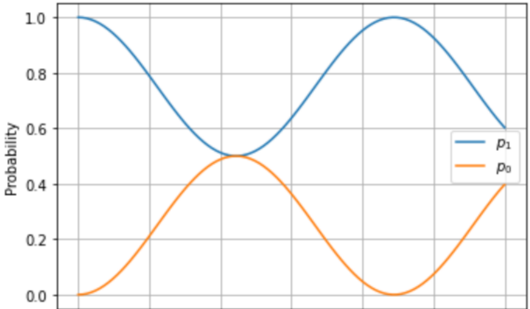
Its solution $e^{\frac{-i\gamma t}{\hbar}H}|\psi(0)\rangle$. Simulation for $H = J(\sigma_1 + \sigma_3)$. Create the trotterized circuit $e^{\frac{-i\gamma t}{\hbar}H} = \lim_{n\to+\infty}[e^{\frac{-i\gamma t}{n\hbar}\sigma_1}e^{\frac{-i\gamma t}{n\hbar}\sigma_3}]^n$



Create the trotterized circuit

quantum circuit

simulation

Assume we how to simulate $R_z(t) = e^{-itZ}$, and we want to simulate $R_{zz} = e^{-itZ\otimes Z}$.

- $R_{zz} = Cx \times \left(R_{zz} = e^{-itI\otimes Z}\right) \times Cx$
- $R_{zz} = \cos(t) I \otimes I - i\sin(t) Z \otimes Z$
- where we can write the Cnot Cx operator as follows: $Cx = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X$
- $R_{Iz} = \cos(t) I \otimes I - i\sin(t) I \otimes Z$
- $Cx \times R_{Iz} = \cos(t) Cx - i\sin(t)\, |1\rangle\langle 1| \otimes XZ$
- $Cx \times R_{Iz} \times Cx = \cos(t) I \otimes I - i\sin(t)\left(|0\rangle\langle 0| \otimes Z + |1\rangle\langle 1| \otimes XZX\right)$
- $Cx \times R_{Iz} \times Cx = \cos(t) I \otimes I - i\sin(t)\left(|0\rangle\langle 0| \otimes Z - |1\rangle\langle 1| \otimes Z\right)$
- $Z = |0\rangle\langle 0| - |1\rangle\langle 1|$