Quantum Computing (量子计算)

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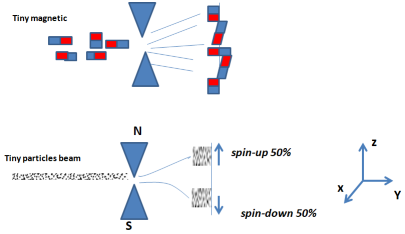
Spin operator[1]: Stern–Gerlach experiment



Stern-Gerlach experiment

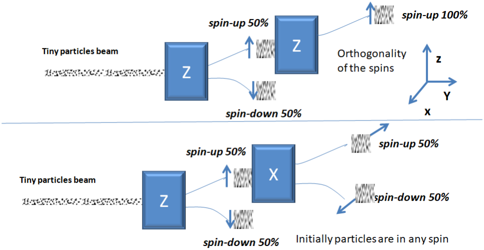
which corresponds to the following operator σ_z.

$\sigma_z = Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ has two eigenvalues 1 and -1 with eigenstates $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ spin-up state and $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ spin-down state.

[4]自旋算符

Spin operator: Stern–Gerlach experiment

If we act a second time using Z then nothing changes because we act on its eigenstate (first figure below):



Let now acts on the spin-up eigenstate of Z using the following operator (second figure above)

$$\sigma_x = X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

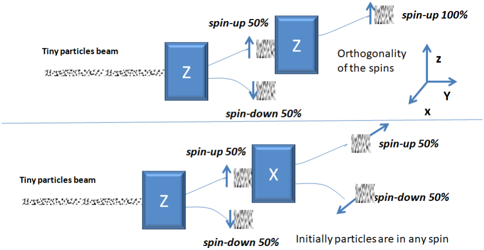
it splits this state on two new eigenstates of X

Spin operator: Stern–Gerlach experiment

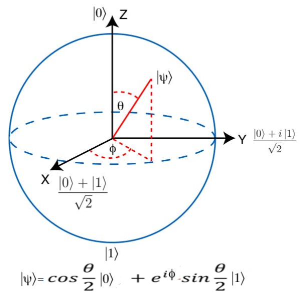
corresponds to eigenstates as linear combination of $|0\rangle$ and $|1\rangle$

$$\langle +| = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right), \quad \langle -| = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

with eigenvalues 1 and -1.



Spin states: Bloch sphere



Each state in Bloch sphere is a linear combination of $|0\rangle$ and $|1\rangle$

$$\langle\psi| = \cos(\frac{\theta}{2})\,|0\rangle + e^{i\phi}\sin(\frac{\theta}{2})\,|1\rangle$$

and its orthogonal state

$$\langle\psi| = \sin(\frac{\theta}{2})\,|0\rangle - e^{i\phi}\cos(\frac{\theta}{2})\,|1\rangle$$

A class of transformations

Each state in Bloch sphere is a linear combination of $|0\rangle$ and $|1\rangle$

$$\langle\psi| = \cos(\frac{\theta}{2})\,|0\rangle + e^{i\phi}\sin(\frac{\theta}{2})\,|1\rangle$$

and we have the general unitary transformation (酉变换)

$$U(\theta,\phi,\lambda) = \begin{pmatrix} \cos\frac{\theta}{2} & -e^{i\lambda}\sin\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} & e^{i(\phi+\lambda)}\cos\frac{\theta}{2} \end{pmatrix}$$

Transpose Conjugate (共轭转置)

$U^*(\theta,\phi,\lambda) = \begin{pmatrix} \cos\frac{\theta}{2} & e^{-i\phi}\sin\frac{\theta}{2} \\ -e^{-i\lambda}\sin\frac{\theta}{2} & e^{-i(\phi+\lambda)}\cos\frac{\theta}{2} \end{pmatrix}$ with $UU^* = I_d$

thus U is hermitian.

Pauli operators[2]

Three important matrices (矩阵) in quantum computation

$$\sigma_x = X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},$$
$$\sigma_z = Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

They fulfil the following relationships

$$\sigma_i^2 = I_d, \quad i = x, y, z$$
$$[\sigma_x, \sigma_y] = 2i\sigma_z, \quad [\sigma_y, \sigma_z] = 2i\sigma_x, \quad [\sigma_z, \sigma_x] = 2i\sigma_y.$$

[5]Pauli 算符

Rotation about axes : X, Y and Z

Let σ be an unitary hermitian operator : $\sigma^* = \sigma$ and $\sigma^2 = I_d$ then

$$e^{\frac{-i\theta\sigma}{2}} = \cos(\frac{\theta}{2}) - i\sin(\frac{\theta}{2})\sigma$$

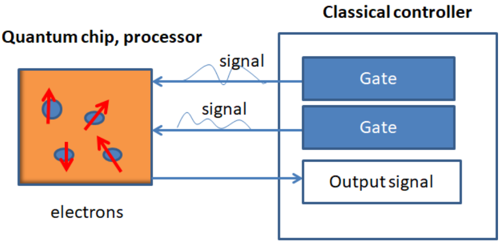
The rotation operator on the Bloch sphere axes are given

$$R_X = e^{\frac{-i\theta\sigma_x}{2}} = \begin{pmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}, \tag{1}$$

$$R_Y = e^{\frac{-i\theta\sigma_y}{2}} = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}, \tag{2}$$

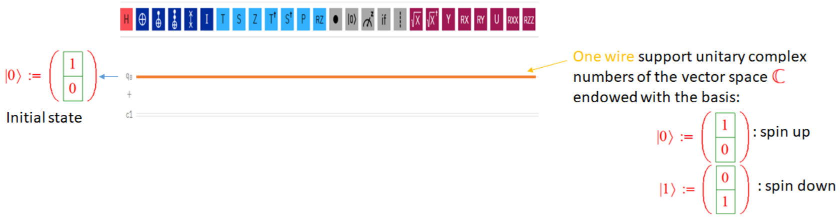
$$R_Z = e^{\frac{-i\theta\sigma_z}{2}} = \begin{pmatrix} e^{\frac{-i\theta}{2}} & 0 \\ 0 & e^{\frac{i\theta}{2}} \end{pmatrix}. \tag{3}$$

Simplified diagram of quantum computer



In the controlled part each wire that support a gate is as follows

Qubits[3]



The canonical basis is formed by $|0\rangle$ and $|1\rangle$. Thus the state of the qubit are linear combination of them, for example

$$|\psi\rangle = a\,|0\rangle + b\,|0\rangle\,,$$

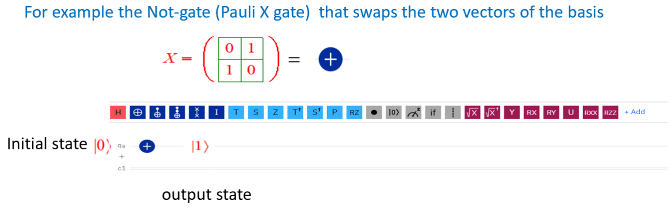
where a and b are complex numbers such that

$$|a|^2 + |b|^2 = 1$$

[6] 量子比特

Not X gate[4]

To change the initial state $|0\rangle$ and $|1\rangle$ into the state $|1\rangle$ and $|0\rangle$ respectively, we use the Not$X$



$$X\,|0\rangle = |1\rangle\,, \quad X\,|1\rangle = |0\rangle$$

[7] gate：量子门

Two wires

The two states on two wires (电路线) give rise to the tensor product (张量积).