Exercises of Quantum Computing

To Northeastern University

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1. If ψ is an eigenstate of $\hat{H}$ then what is its eigenvalue? If ψ is an eigenstate of $\hat{p}$ what is the corresponding eigenvalue?

2. Compute the commutator $[\hat{x}, \hat{p}]$ and explain what means the fact that is not equal to zero.

3. If we use two basis state $|0\rangle$ and $|1\rangle$, what is the general form of state presented on the Bloch sphere ?

4. X gate is one of the important gates in quantum computing, please show that we have the following two relations by calculation and simulation.

$$X|0\rangle = |1\rangle, \quad X|1\rangle = |0\rangle \tag{1}$$

5. Please explain what is superposition and entanglement, then provide an example for each one.

6. If we have a quantum state $|\psi\rangle = \frac{1}{\sqrt{5}}|1\rangle + \frac{2}{\sqrt{5}}|0\rangle)$, what is the probability of obtaining state $|0\rangle$?

7. What is the matrix form of controlled-NOT gate ? If the initial state are all $|0\rangle$, show the simulation circuit with controlled-NOT gate, what is the output state ?

8. Please calculate the output state of the following two circuits in Fig. 1, and point out if they are in the state of entanglement.

9. Find the unitary matrix of circuit in Fig. 2.

10. What is the form of first order quantum Fourier transform ?

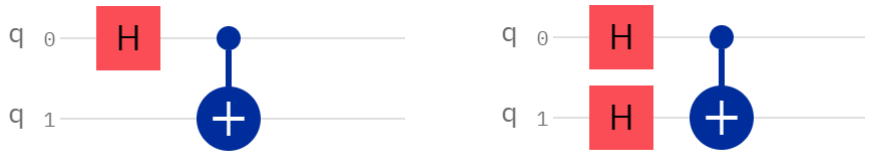

Figure 1: Circuit 1 and circuit 2

Figure 2: Circuit 3

11. Show that $Cx = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X$;
12. Build the Swap gate from three Cx note.
13. Show that $e^{-it(X+Z)} = cos(t)I - i \sin t(X + Z)$
14. Program Practice : Complete the following program

```
import numpy as np
from numpy import pi
# Importing standard Qiskit libraries
from qiskit import QuantumCircuit, transpile, assemble, Aer
from qiskit import IBMQ, execute
from qiskit.quantum_info import Statevector
from qiskit.visualization import plot_bloch_multivector
from qiskit.visualization import plot_histogram
from qiskit_textbook.problems import dj_problem_oracle
```

- The goal is to reach the state $|1\rangle$

```
def lab1_ex1():
    qc = QuantumCircuit(1)
    # FILL YOUR CODE IN HERE
    #
    return qc
state = Statevector.from_instruction(lab1_ex1())
plot_bloch_multivector(state)
```

- The goal is to reach the state $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

```
def lab1_ex2():
    qc = QuantumCircuit(1)
    # FILL YOUR CODE IN HERE
    return qc
state = Statevector.from_instruction(lab1_ex2())
plot_bloch_multivector(state)
```

- Construct the Bell state $|\Psi_+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

```
def lab1_ex5():
    qc = QuantumCircuit(2,2)
    # FILL YOUR CODE IN HERE
    return qc
qc = lab1_ex5()
qc.draw() # we draw the circuit
```