

# Computational Optimization

## Exam 2

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In this exam, you will implement a mixed integer linear program (MILP) to solve the  $k$ -traveling salesmen problem. In this problem, you are given a graph with nodes and edges. A single node, called '**start**' is designated as an initial location for  $k$  sales people. We have to find  $k$  disjoint cycles, one for each sales person, such that all nodes of the graph are visited exactly once.

One formulation of a mixed integer linear program for this problem is described as follows. You are given the following data:

- $N$  – a set of nodes
- $A$  – a set of directed arcs  $(u, v)$ , where both  $u$  and  $v$  are a node. In other words,  $u$  is the start node of the arc, and  $v$  is the end node of the arc.
- $S$  – a set of sales people
- $s$  – a start node, for us this is '**start**'
- $c_{u,v}$  – a cost for each edge  $(u, v)$

The MILP has the following variables:

- $y_{u,v}^k$  – a binary variable specifying if the  $k$ th sales person uses edge  $(u, v)$

The MILP is then:

$$\begin{aligned} \min \quad & \sum_{k \in S} \sum_{(u,v) \in A} c_{u,v} y_{u,v}^k \\ \text{s.t.} \quad & \sum_{(a,u) \in A} y_{a,u}^k - \sum_{(u,a) \in A} y_{u,a}^k = 0, \quad \forall u \in N, \forall k \in S \\ & \sum_{(a,s) \in A} y_{a,s}^k = 1, \quad \forall k \in S \\ & \sum_{(s,a) \in A} y_{s,a}^k = 1, \quad \forall k \in S \\ & \sum_{k \in S} \sum_{(a,u) \in A} y_{a,u}^k = 1, \quad \forall u \in N, u \neq s \\ & \sum_{k \in S} \sum_{(u,a) \in A} y_{u,a}^k = 1, \quad \forall u \in N, u \neq s \\ & y_{u,v}^k \in \{0, 1\}, \quad \forall (u, v) \in A, \forall k \in S. \end{aligned}$$

Download the file `exam2_data.zip`, it contains three data files `arcs.csv`, `nodes.csv`, and `salespeople.csv` that have all the relevant data for the above problem. Use these files to create the MILP and

solve it. In the end, read the solution variables, and print out the solution to the screen with statements similar to:

```
Boris's path is ['start', 'A', 'start']
Chang's path is ['start', 'B', 'start']
Amelie's path is ['start', 'D', 'C', 'start']
Objective function value is 11.0
```

To debug, add **'Deeter'** to the sales people CSV and re-run your code. You should get the answer:

```
Boris's path is ['start', 'B', 'start']
Deeter's path is ['start', 'A', 'start']
Chang's path is ['start', 'D', 'start']
Amelie's path is ['start', 'C', 'start']
Objective function value is 8.0
```

### What to turn in

- Your code in a file titled `exam2.py`. **Please add some brief comments, documenting your thought process in adding variables, constraints, etc.**
- Your code should run and print the solution to the screen, if I place CSVs in the same directory.