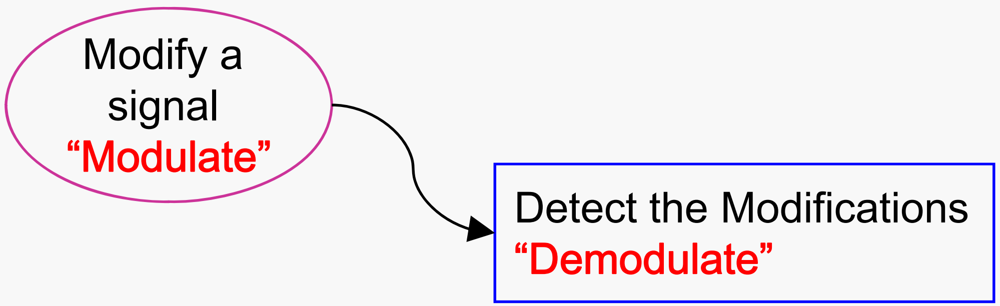
Principles of Communications

Chapter 3: Analog Modulation
Part I: Amplitude Modulation

Textbook: Ch3

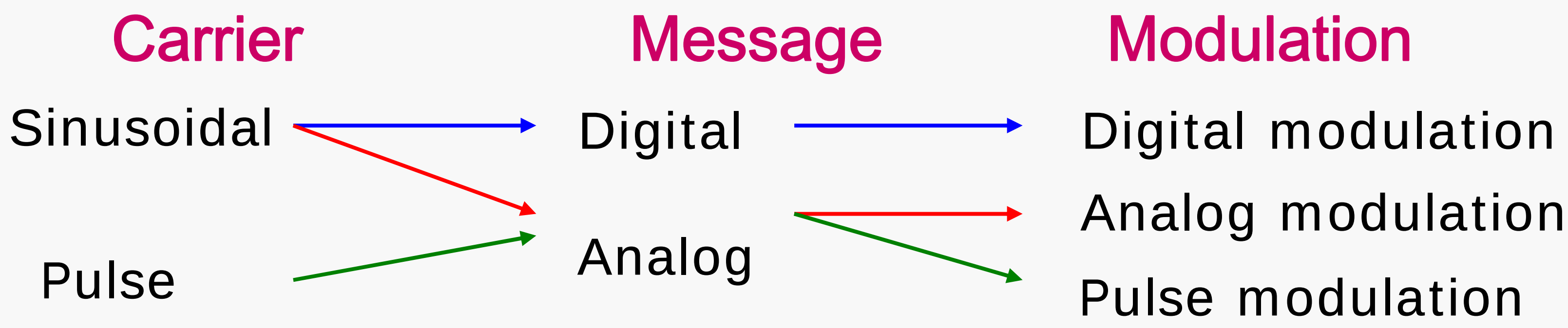
Modulation

- 3-step signal transmission over a band-pass channel
  - A pure **carrier** (usually sinusoidal) is generated at the transmitter
  - The carrier is modulated with **the information** to be transmitted. Any reliably detectable change in signal characteristics can carry information
  - At the receiver the signal modifications or changes are detected and demodulated



Modulation

- Modulation objectives
  - Frequency translation from lowpass to passband
  - Frequency-division multiplexing
  - Increasing noise and interference immunity
- Modulation types



Analog Modulation

- Characteristics that be modified in sin carrier
  - Amplitude $\rightarrow$ Amplitude modulation
  - Frequency
  - Phase

  $\left.\begin{array}{l}\text{Frequency}\\ \text{Phase}\end{array}\right\} \rightarrow$ Angle modulation

- In the following we
  - Consider the transmission and reception of analog signals by amplitude modulation
  - Compare their bandwidth requirement and implementation complexity
  - Discuss the performance in the presence of noise

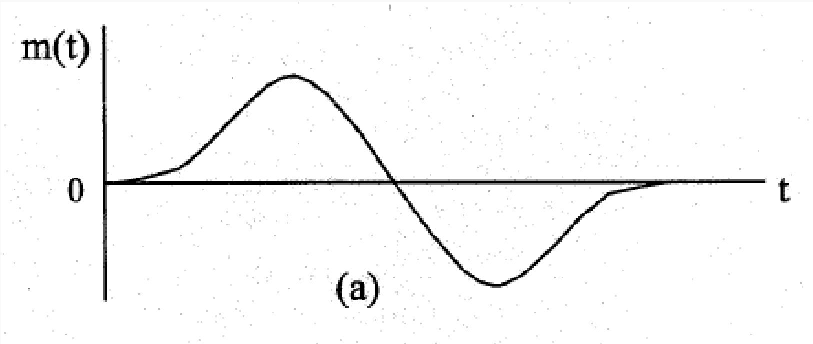
Analog Modulation

- 3.1.Amplitude modulation
- 3.2. Effect of noise on AM systems
- 3.3.Angle modulation
- 3.4.Effect of noise on angle modulation

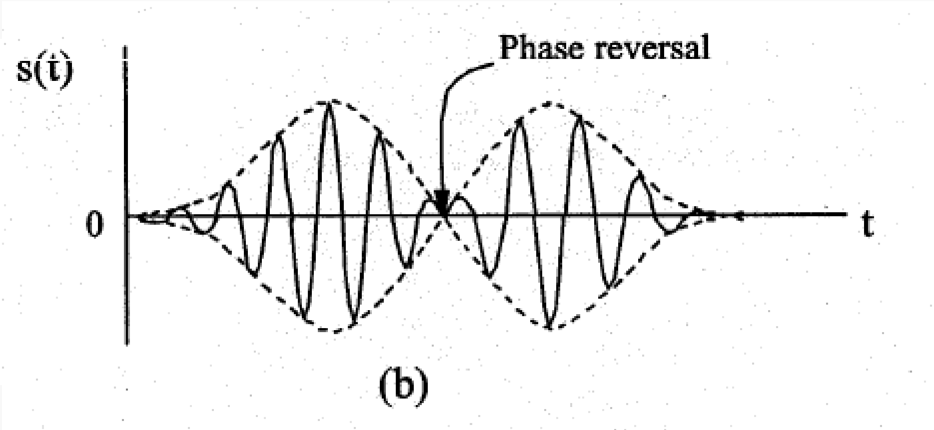
Double-Sideband Suppressed-Carrier AM (DSB-SC)

- Carrier wave : $c(t) = A_c \cos(\omega_c + \theta_0)$
- Baseband signal (modulating wavel) $m(t)$
- Modulated wave

$$s(t) = c(t)m(t) = A_c m(t)\cos\left(\omega_c t + \theta_0\right)$$



Modulating wave

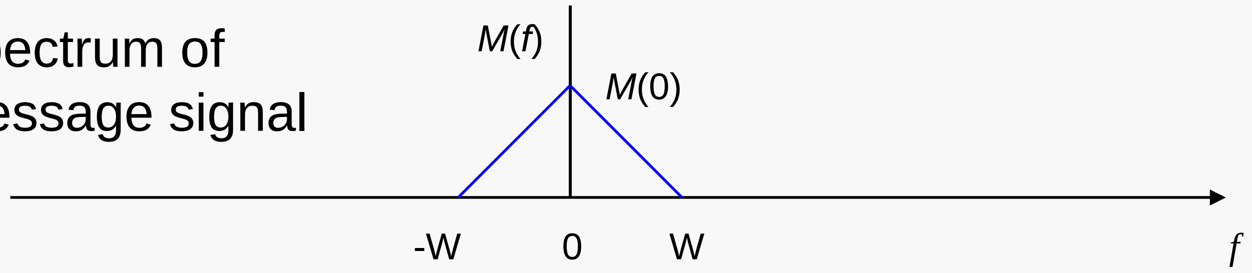


Modulated wave

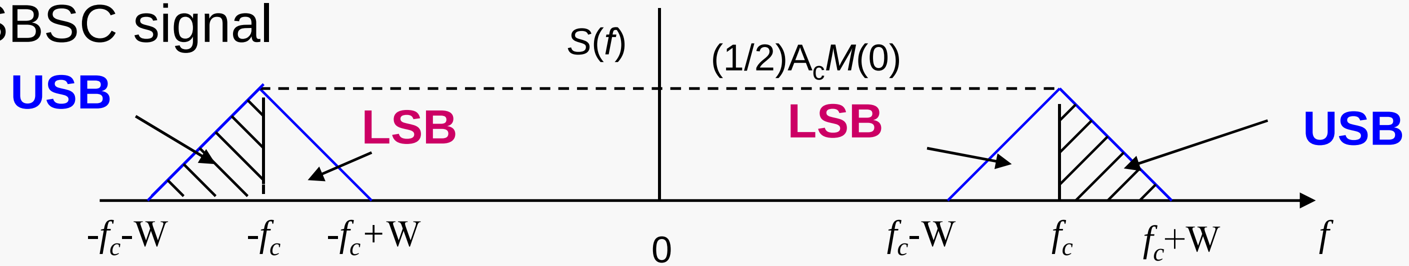
Spectrum of DSB-SC Signals

$$S(f) = \frac{1}{2} A_c \left[M(f - f_c) + M(f + f_c) \right]$$

Spectrum of message signal

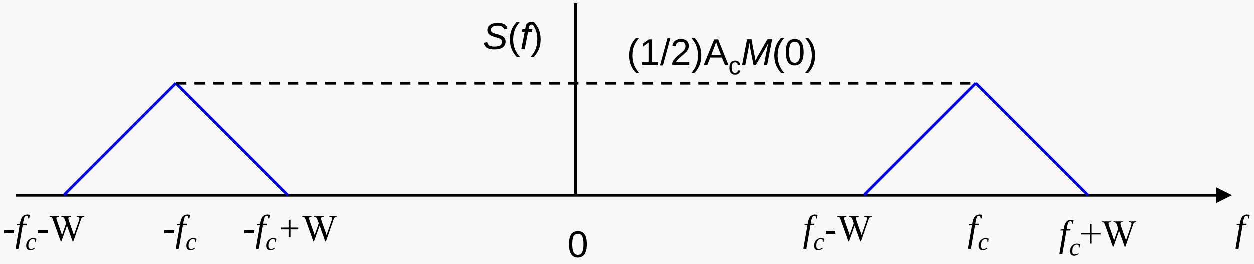


Spectrum of DSBSC signal



- Translation of the original message spectrum to $\pm f_c$
- Suppression of the carrier

Bandwidth and Power of DSB-SC



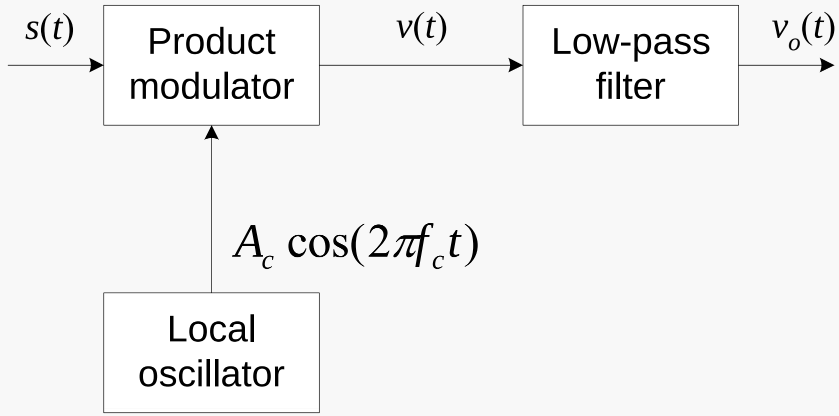
- Channel bandwidth required to transmit the modulated signal is $B_c = 2W$, 2 times of the message bandwidth
- Power content

$$P_s = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} s^2(t)dt = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} A_c^2 m^2(t)\cos^2(\omega_c t + \theta_0)dt$$

$$= \frac{A_c^2}{2}\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} m^2(t)\left[1+\cos(2\omega_c t + 2\theta_0)\right]dt = \frac{A_c^2}{2}P_m$$

Demodulation of DSB-SC Signals

- The local oscillator is assumed to be exactly coherent or synchronized to original c(t) => <u>coherent detection or synchronous detection</u>



- If there is a phase error ϕ, then

$$v(t) = \cos(2\pi f_c t + \phi)s(t) = A_c \cos(2\pi f_c t)\cos(2\pi f_c t + \phi)m(t)$$

$$= \underbrace{\frac{1}{2}A_c \cos(\phi)m(t)}_{\text{Scaled version of message signal}} + \underbrace{\frac{1}{2}A_c \cos(4\pi f_c t + \phi)m(t)}_{\text{Unwanted terms}}$$

Phase Error ϕ

$A_c \cos(\phi) m(t)$

- $\cos(\phi)$ = Attenuation factor
- If $\phi = 0 \Rightarrow$ amplitude of demodulated signal is maximized
- If $\phi = \pm\pi/2 \Rightarrow$ amplitude is zero, called quadrature null effect
- In practice, ϕ varies randomly with time, resulting in undesired effect
- Need additional circuitry to ensure synchronization
- The increased receiver complexity is the price that must be paid for suppressing the carrier wave to save transmit power

Example: Single-tone DSBSC Modulation

- Consider a single tone modulating wave $m(t) = A_m \cos(2\pi f_m t)$
- The DSBSC modulated wave is

$$s(t) = A_c A_m \cos(2\pi f_c t)\cos(2\pi f_m t)$$

$$= \frac{1}{2} A_c A_m \cos[2\pi(f_c + f_m)t] + \frac{1}{2} A_c A_m \cos[2\pi(f_c - f_m)t]$$

- With perfect synchronization, the output of product modulator is

$$v(t) = \cos(2\pi f_c t)s(t)$$

$$= \frac{1}{2} A_c A_m \cos(2\pi f_m t) + \frac{1}{4} A_c A_m \cos[2\pi(2f_c - f_m)t]$$

$$+ \frac{1}{4} A_c A_m \cos[2\pi(2f_c + f_m)t]$$

Removed by LPF

Conventional Amplitude Modulation

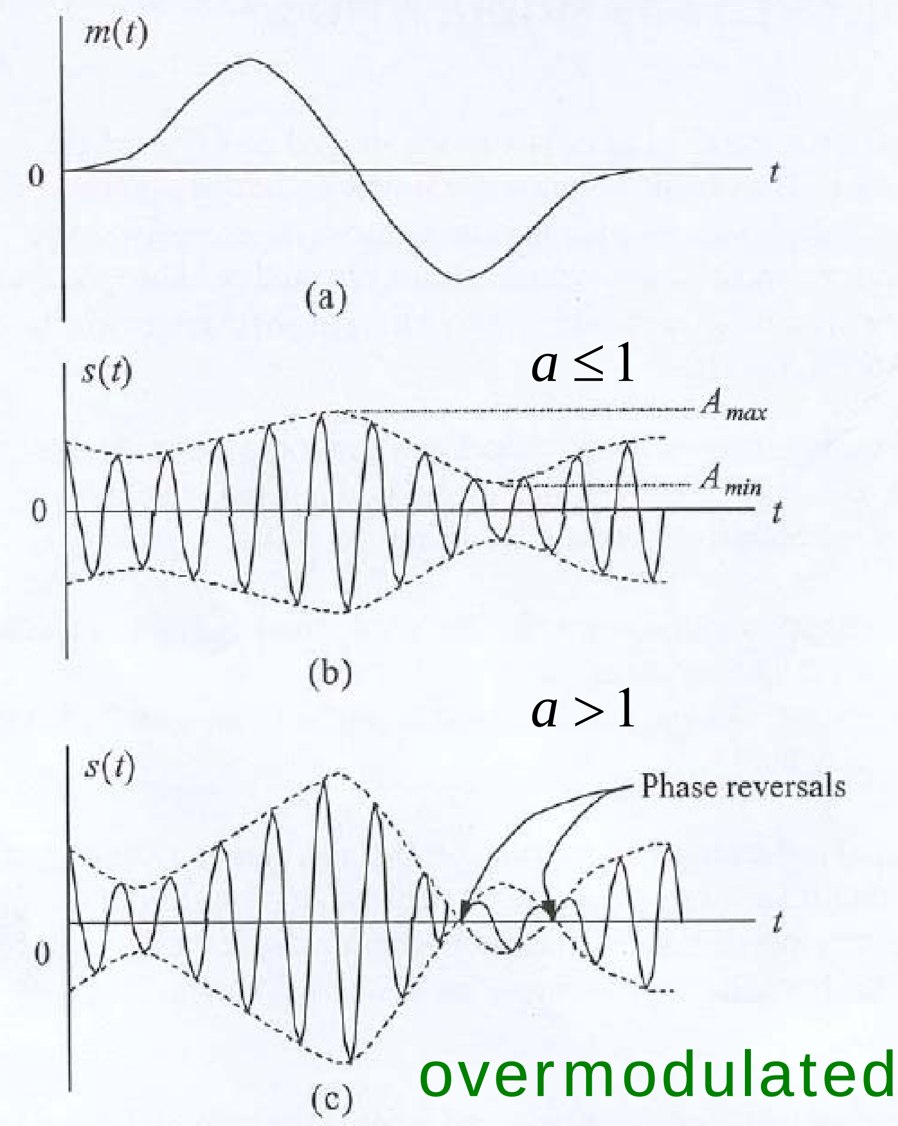
- Modulated signal :

$$s(t) = A_c\left[1 + am(t)\right]\cos\left(2\pi f_c t\right)$$

$$= A_c am(t)\cos\left(2\pi f_c t\right)$$
$$+ A_c \cos\left(2\pi f_c t\right)$$

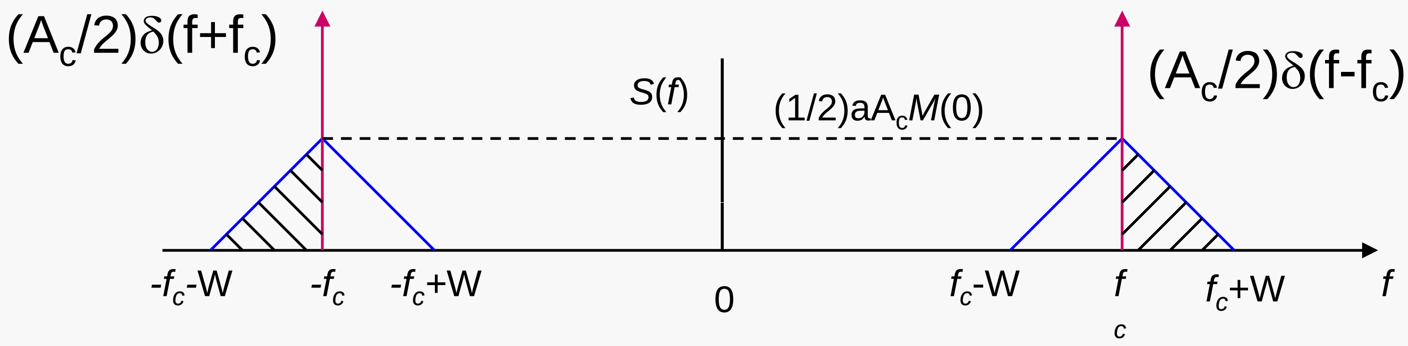
$m(t)$: normalized message

a : modulation index



Spectrum of Conventional AM

$$S(f)=\frac{A_c}{2}\left[\delta\left(f-f_c\right)+\delta\left(f+f_c\right)\right]+\frac{A_c a}{2}\left[M\left(f-f_c\right)+M\left(f+f_c\right)\right]$$



Power for the Conventional AM

- Power
$$S = E\left[s^2(t)\right] = E\left\{A_c^2\left[1+am(t)\right]^2\cos^2\omega_c t\right\}$$
$$= \frac{A_c^2}{2} + \frac{a^2A_c^2}{2}E\left[m^2(t)\right] = \underbrace{\frac{A_c^2}{2}}_{\text{Power in the carrier component}} + \underbrace{\frac{a^2A_c^2}{2}P_m}_{\text{Power in sidebands}}$$

- Modulation efficiency

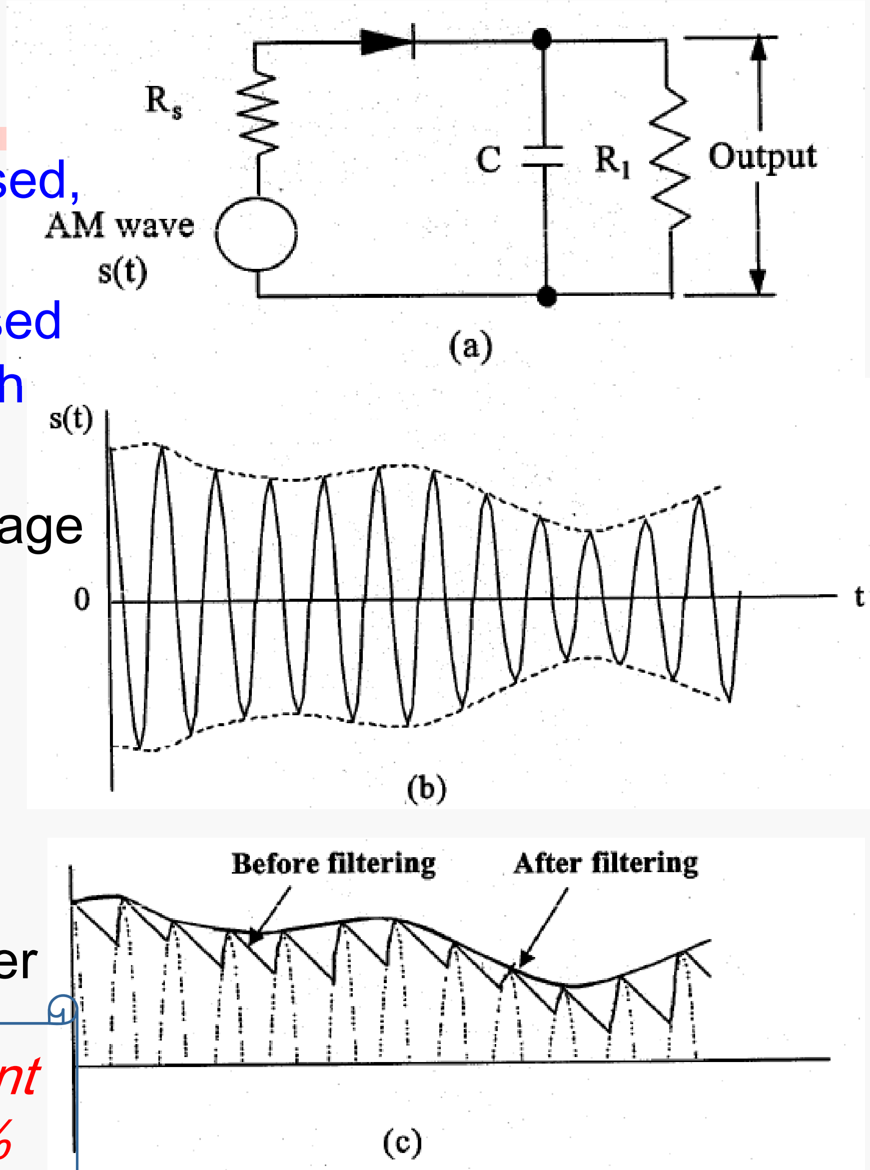
$$E = \frac{\text{power in sideband}}{\text{total power}} = \frac{\frac{a^2}{2}A_c^2P_m}{\frac{A_c^2}{2}+\frac{a^2}{2}A_c^2P_m} = \frac{a^2P_m}{1+a^2P_m}$$

Example

- The signal $m(t) = 3\cos(200\pi t) + \sin(600\pi t)$ is used to modulate the carrier $c(t) = \cos(2\times 10^{-5} t)$ The modulation index is a=0.85. Determine the power in the carrier component and in the sideband components of the modulated signal

Demodulation of AM signals

Envelop Detection



- On +ve half cycle, diode is forward-biased, capacitor is charged to peak value
- On –ve half cycle, diode is reverse-biased and capacitor discharges slowly through load resistor R_l.
- Assume AM wave was supplied by voltage source with internal impedance R_s.
- Also assume short charging time, i.e.

$$R_s C \ll 1/f_c,$$

and long discharging time, i.e.

$$1/f_c \ll R_l C \ll 1/W.$$

- Ripple can be removed by low-pass filter

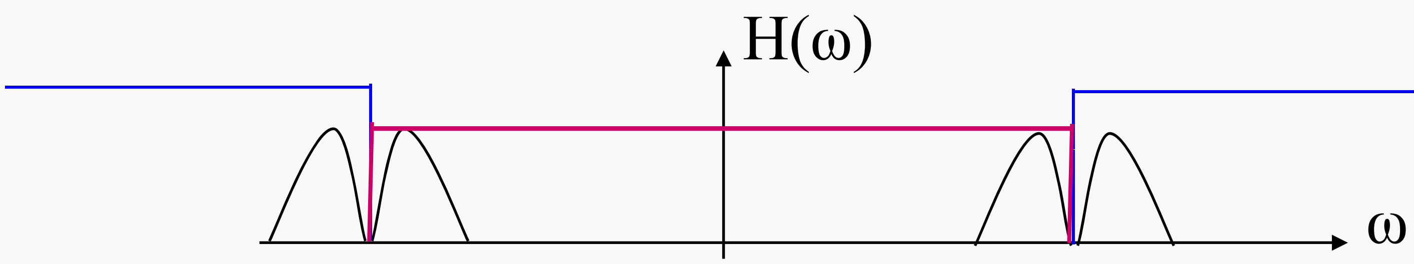
Envelop Detector is Simple and efficient when percentage modulation < 100%

Single Sideband (SSB) AM

- Common problem in AM and DSBSC:

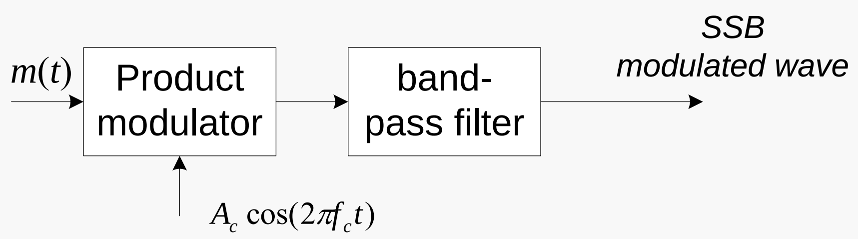
  bandwidth wastage because the transmission bandwidth equals to twice the message bandwidth

$\Rightarrow$ SSB is very bandwidth efficient



Generation of SSB Waves: Frequency discrimination method

- Requirements on message signal m(t)
  - Little or no low-frequency components, i.e. “holes” at 0Hz. E.g: audio signal (speech or music). In telephony, the useful frequency content of a speech signal is restricted to 0.3~3.4 kHz
  - The highest frequency component W << carrier frequency f_c
- Block diagram



- Two conditions for bandpass filter
  - Passband occupies the same frequency range as desired SSB wave
  - Guardband, separating the passband from the stopband where the unwanted sideband of the filter input lies, should be less than twice the lowest frequency, f_l, in $m(t)$, i.e. must be between f_c-f_l to f_c+f_l

Expression of SSB signals

- The baseband signal can be written as the sum of finite sinusoid signal

$$m(t)=\sum_{i=1}^{n} x_i \cos(2\pi f_i t+\theta_i)\,,\quad f_i \le f_c$$

- Then its USB component is

$$m_c(t)=\frac{A_c}{2}\sum_{i=1}^{n} x_i \cos\left[2\pi(f_c+f_i)t+\theta_i)\right]$$

- After maniputation

$$m_c(t)=\frac{A_c}{2}\left\{\left[\sum_{i=1}^{n} x_i \cos(2\pi f_i t+\theta_i)\right]\cos 2\pi f_c t-\left[\sum_{i=1}^{n} x_i \sin(2\pi f_i t+\theta_i)\right]\sin 2\pi f_c t\right\}$$

$$=\frac{A_c}{2}m(t)\cos 2\pi f_c t-\frac{A_c}{2}\widehat{m}(t)\sin 2\pi f_c t$$

Hilbert transform of m(t)

$$S_{SSB}(\omega)=\frac{1}{2}[M(\omega-\omega_c)+M(\omega+\omega_c)]H(\omega)$$

$$H(\omega)=\frac{1}{2}[Sgn(\omega+\omega_c)-Sgn(\omega-\omega_c)]$$

$$S_{SSB}(\omega)=\frac{1}{4}[M(\omega+\omega_c)+M(\omega-\omega_c)]$$

$$(\Leftrightarrow \frac{1}{2}m(t)\cos\omega_c t)$$

$$+\frac{1}{4}[M(\omega+\omega_c)Sgn(\omega+\omega_c)-M(\omega-\omega_c)Sgn(\omega-\omega_c)]$$

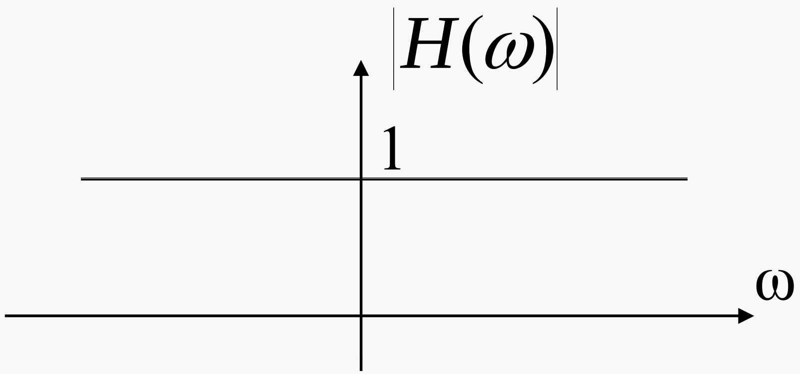
$$(\Leftrightarrow \frac{1}{2}\hat{m}(t)\sin\omega_c t)$$

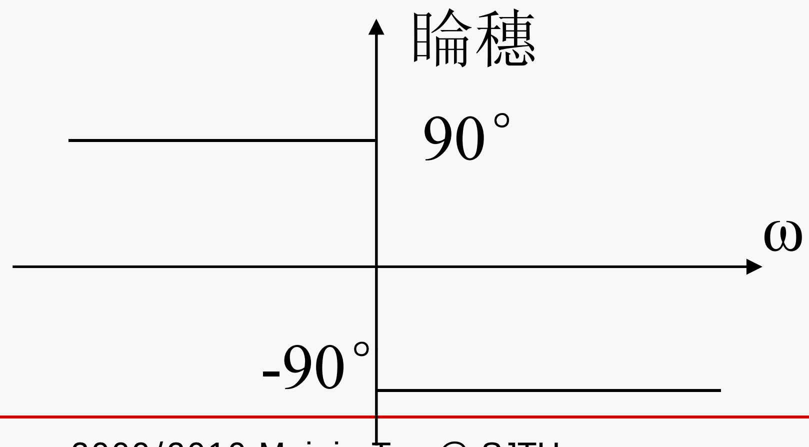
About Hilbert Transform

- $x(t) \Leftrightarrow \hat{x}(t) \Rightarrow \hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau = x(t) * \frac{1}{\pi t}$

  $X(w) \Leftrightarrow \hat{X}(w) \Rightarrow \hat{X}(w) = [-j\,\mathrm{sgn}(w)]\,X(w)$

$$H(f) = \begin{cases} -j, & f > 0 \\ j, & f < 0 \\ 0, & f = 0 \end{cases}$$





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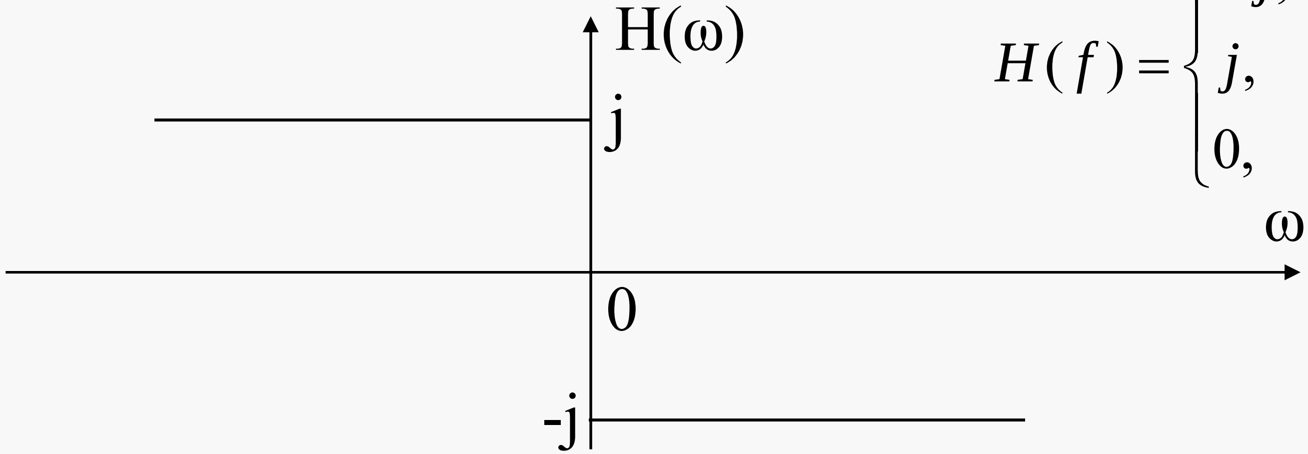
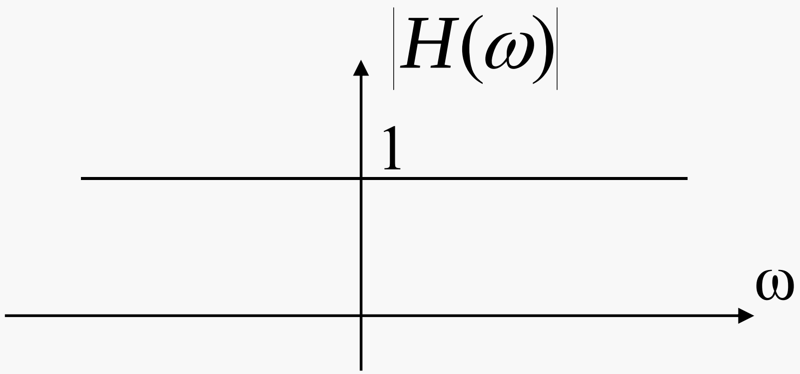
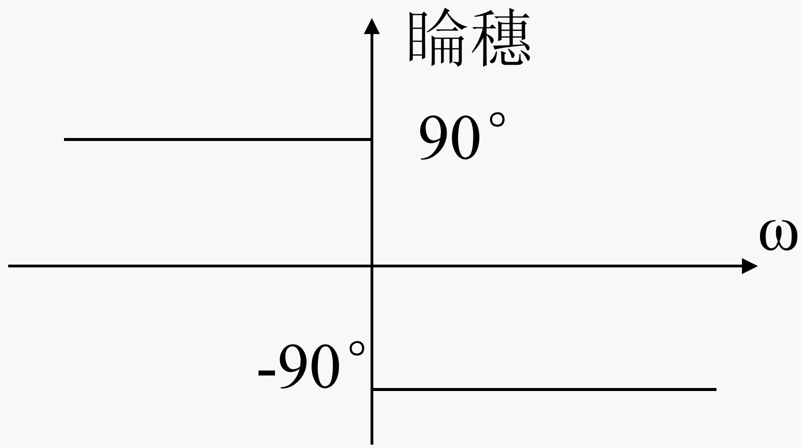
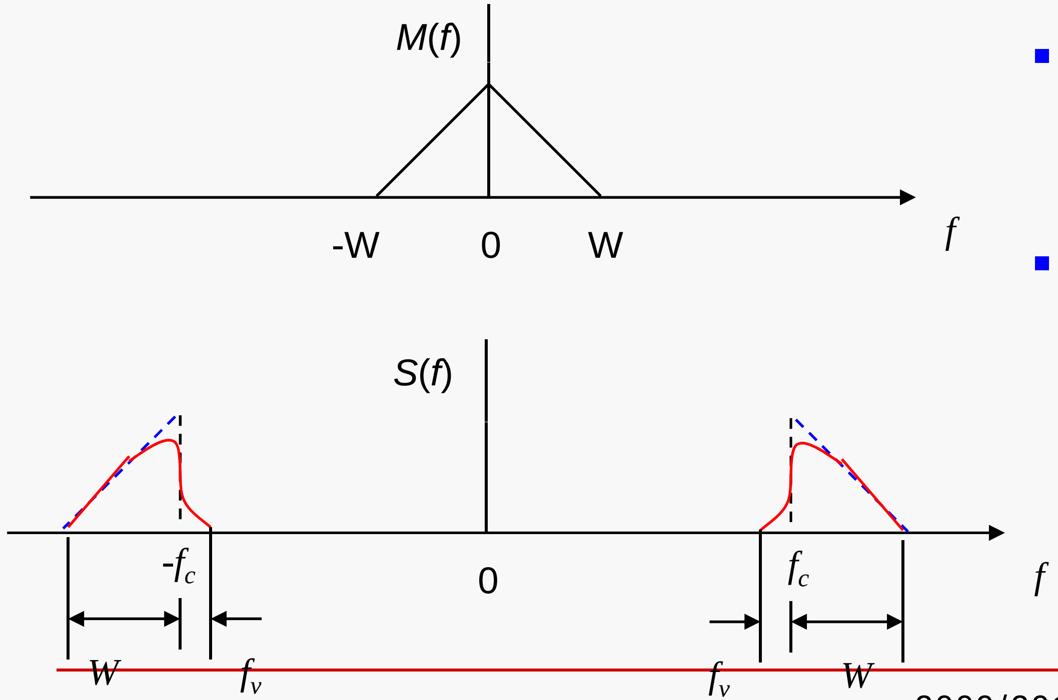






Vestigial Sideband: VSB

- SSB is not suitable when signals have significant low frequency components
- VSB is a compromise between SSB and DSBSC
- VSB frequency domain description



- VSB signal bandwidth is
  $B = W + f_v$
  f_v: width of the vestigial sideband
- VSB is used in TV broadcasting and similar signals where good phase characteristics are required and low frequency components are significant

Comparison of AM Techniques

- Conventional AM demodulation uses simple envelop detector or square-law detector. Avoids complexity of coherent detection. E.g. AM radio broadcast systems
- Suppressed-carrier systems are more power efficient, making transmitters less expensive. Suitable for point-to-point transmissions
- SSB modulation requires minimum transmitter power and bandwidth. Suitable for point-to-point and over long distances
- VSB bandwidth requirements are between SSB and DSBSC. Suitable for TV transmission
- In SSB and VSB, the role of the quadrature component is to interfere with the in-phase component so as to eliminate power in one of the sideband achieve bandwidth saving

Frequency-Division Multiplexing

- Multiplexing is a technique where a number of independent signals are combined and transmitted in a common channel
- These signal are de-multiplexed at the receiver
- Two common methods for signal multiplexing
  - TDM (time-division multiplexing): usually used to transmit digital information
  - FDM (frequency-division multiplexing: may be used for either analog or digital signal transmission

Block Diagram of FDM

LPF: ensure signal bandwidth limited to W

MOD (modulator): shift message frequency range to mutually exclusive high frequency bands

BPF: restrict the band of each modulated wave to its prescribed range

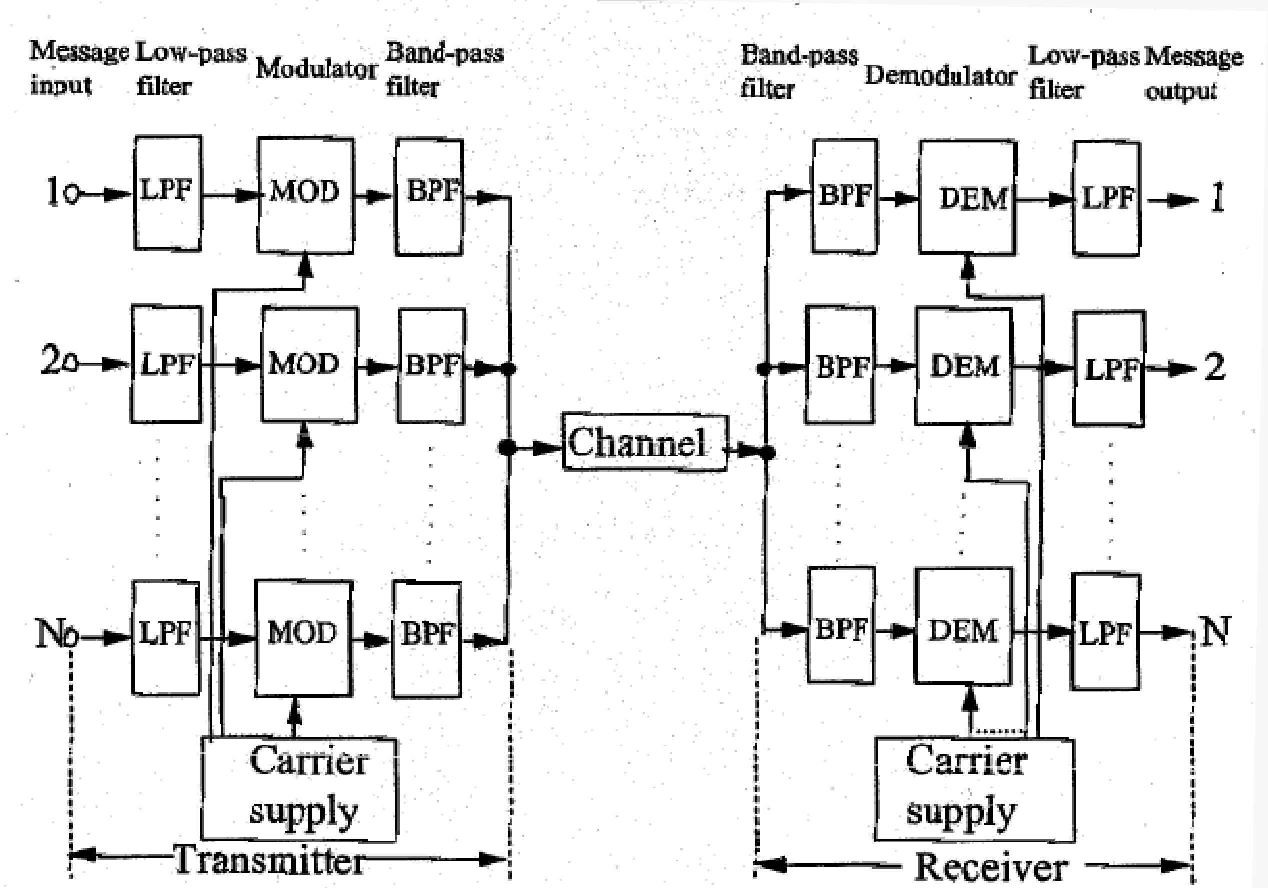
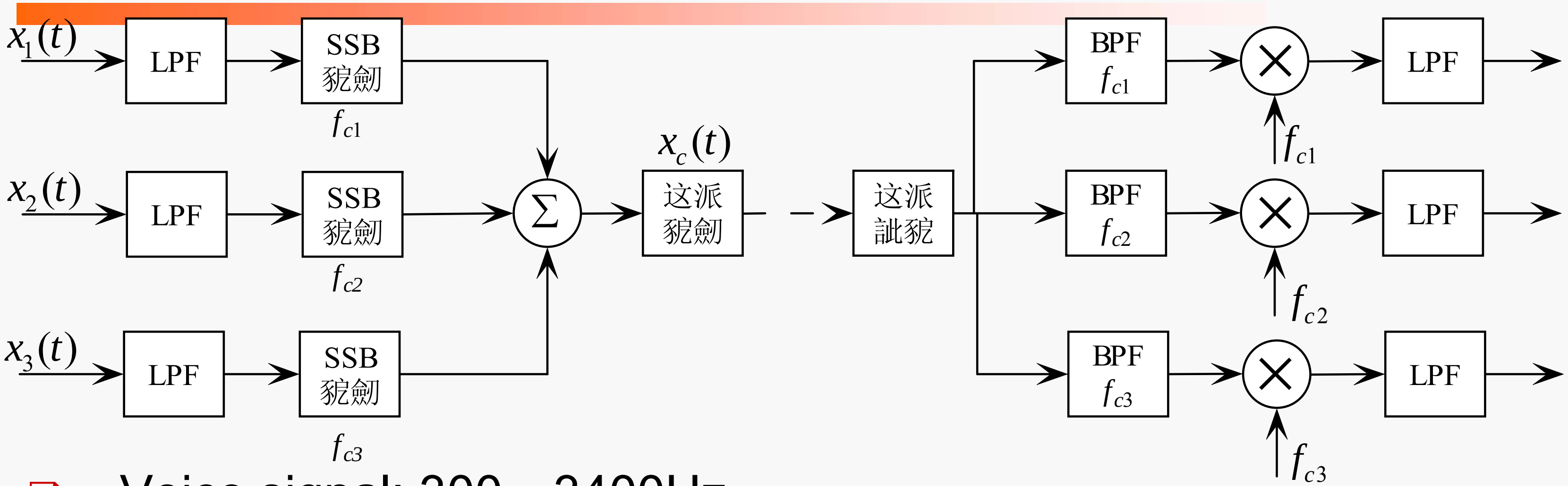


Figure 7.21 - Block diagram of FDM system.

FDM application in Telephone comm.



- Voice signal: 300 ~ 3400Hz
- Message is SSB modulated.
- In 1st-level multiplexing, 12 signal are stacked in frequency, with a freq. separation of 4 kHz between adjacent carriers
- A composite 48 kHz channel, called a group channel, transmits 12 voice-band signals
- Higher-order FDM is obtained by combining several group channels => FDM hierarchy in telephone comm. systems

Quadrature-Carrier Multiplexing

- Quadrature-carrier multiplexing: transmit two messages on the same carrier as

$$s(t) = A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t)$$

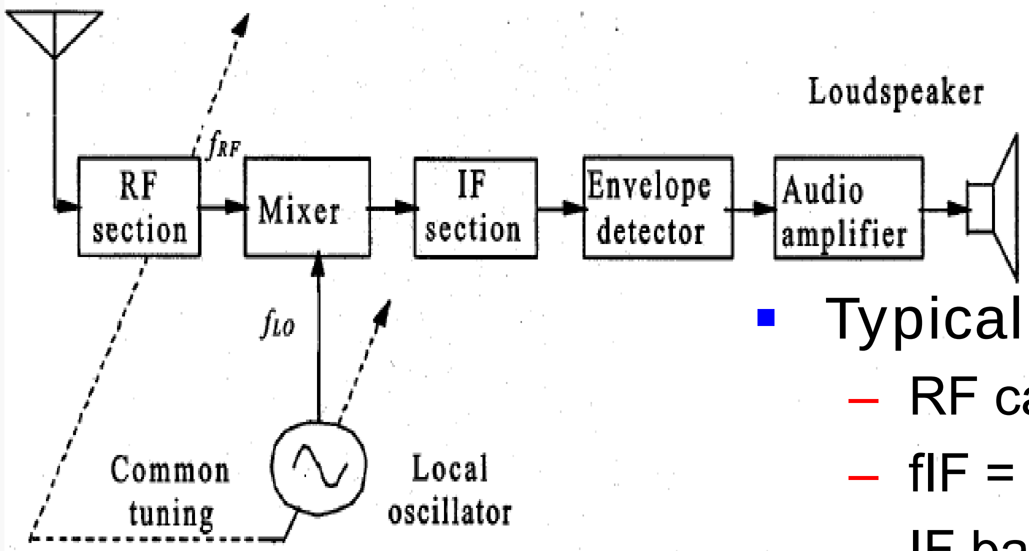
  - cos() and sin() are two quadrature carriers
  - Each message signal is modulated by DSB-SC
- Demodulation of m1(t):

$$s(t)\cos(2\pi f_c t) = A_c m_1(t)\cos^2(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t)\cos(2\pi f_c t)$$

$$= \underbrace{\frac{A_c}{2} m_1(t)}_{\text{LPF}} + \frac{A_c}{2} m_1(t)\cos(4\pi f_c t) + \frac{A_c}{2} m_2(t)\sin(4\pi f_c t)$$

Application: AM Radio Broadcasting

- Commercial AM radio uses conventional AM
- The radio receiver is of the superheterodyne type, i.e. involves the freq conversion or heterodyning from the variable carrier freq of the incoming RF (radio freq) signal to the fixed IF (intermediate freq signal)



- Typical freq parameters
  - RF carrier range = 0.535 ~ 1.605 MHz
  - fIF = 455kHz
  - IF bandwidth = 10kHz