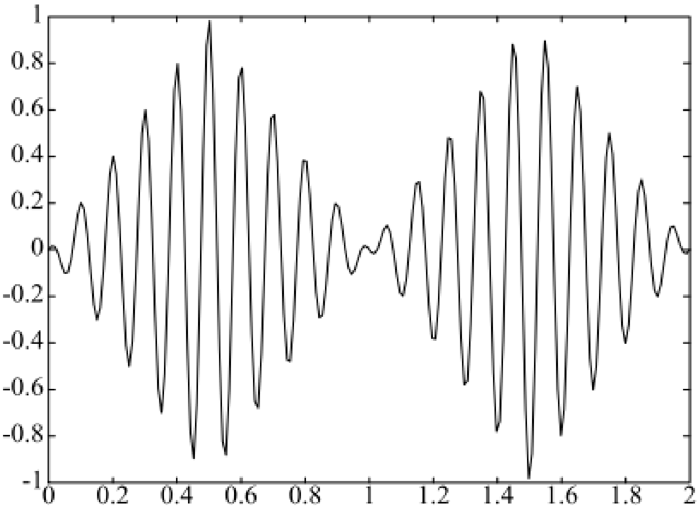
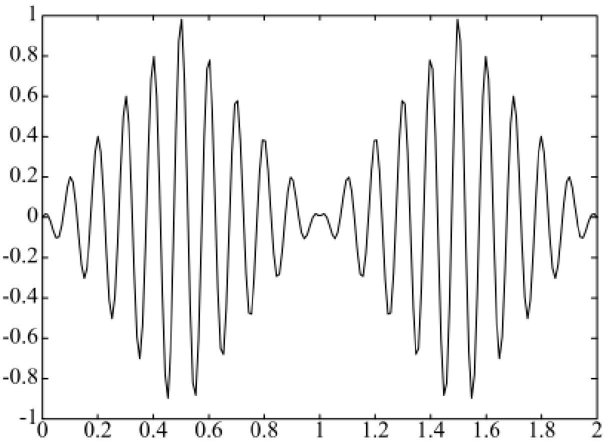
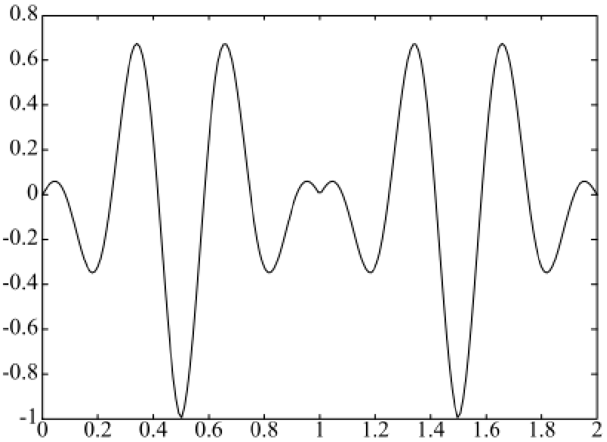
Problem 3.3

The following figure shows the modulated signals for $A = 1$ and $f_0 = 10$. As it is observed both signals have the same envelope but there is a phase reversal at $t = 1$ for the second signal $Am_2(t)\cos(2\pi f_0 t)$ (right plot). This discontinuity is shown clearly in the next figure where we plotted $Am_2(t)\cos(2\pi f_0 t)$ with $f_0 = 3$.





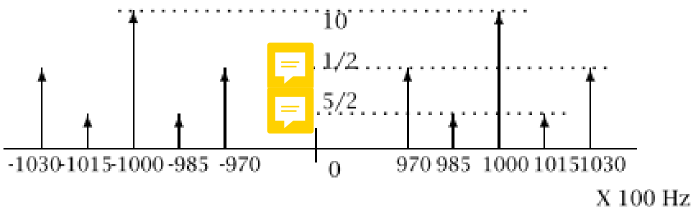


Problem 3.7

1) The spectrum of $u(t)$ is

$$\begin{aligned} U(f) &= \frac{20}{2}[\delta(f-f_c)+\delta(f+f_c)] \\ &+\frac{2}{4}[\delta(f-f_c-1500)+\delta(f-f_c+1500) \\ &+\delta(f+f_c-1500)+\delta(f+f_c+1500)] \\ &+\frac{10}{4}[\delta(f-f_c-3000)+\delta(f-f_c+3000) \\ &+\delta(f+f_c-3000)+\delta(f+f_c+3000)] \end{aligned}$$

The next figure depicts the spectrum of $u(t)$.



2) The square of the modulated signal is

$$\begin{aligned} u^2(t) &= 400\cos^2(2\pi f_c t)+\cos^2(2\pi(f_c-1500)t)+\cos^2(2\pi(f_c+1500)t) \\ &+25\cos^2(2\pi(f_c-3000)t)+25\cos^2(2\pi(f_c+3000)t) \\ &+\text{ terms that are multiples of cosines} \end{aligned}$$

If we integrate $u^2(t)$ from $-\frac{T}{2}$ to $\frac{T}{2}$, normalize the integral by $\frac{1}{T}$ and take the limit as $T\to\infty$, then all the terms involving cosines tend to zero, whereas the squares of the cosines give a value of $\frac{1}{2}$. Hence, the power content at the frequency $f_c = 10^5$ Hz is $P_{f_c} = \frac{400}{2} = 200$, the power content at the frequency P_{f_c+1500} is the same as the power content at the frequency P_{f_c-1500} and equal to $\frac{1}{2}$, whereas $P_{f_c+3000} = P_{f_c-3000} = \frac{25}{2}$.

3)

$$\begin{aligned} u(t) &= (20+2\cos(2\pi 1500t)+10\cos(2\pi 3000t))\cos(2\pi f_c t) \\ &= 20(1+\frac{1}{10}\cos(2\pi 1500t)+\frac{1}{2}\cos(2\pi 3000t))\cos(2\pi f_c t) \end{aligned}$$

This is the form of a conventional AM signal with message signal

$$\begin{aligned} m(t) &= \frac{1}{10}\cos(2\pi 1500t)+\frac{1}{2}\cos(2\pi 3000t) \\ &= \cos^2(2\pi 1500t)+\frac{1}{10}\cos(2\pi 1500t)-\frac{1}{2} \end{aligned}$$

The minimum of $g(z) = z^2+\frac{1}{10}z-\frac{1}{2}$ is achieved for $z = -\frac{1}{20}$ and it is $\min(g(z)) = -\frac{201}{400}$. Since $z = -\frac{1}{20}$ is in the range of $\cos(2\pi 1500t)$, we conclude that the minimum value of $m(t)$ is $-\frac{201}{400}$. Hence, the modulation index is

$$\alpha = -\frac{201}{400}$$

4)

$$\begin{aligned} u(t) &= 20\cos(2\pi f_c t)+\cos(2\pi(f_c-1500)t)+\cos(2\pi(f_c-1500)t) \\ &= 5\cos(2\pi(f_c-3000)t)+5\cos(2\pi(f_c+3000)t) \end{aligned}$$

The power in the sidebands is

$$P_{\text{sidebands}} = \frac{1}{2}+\frac{1}{2}+\frac{25}{2}+\frac{25}{2} = 26$$

The total power is $P_{\text{total}} = P_{\text{carrier}}+P_{\text{sidebands}} = 200+26 = 226$. The ratio of the sidebands power to the total power is

$$\frac{P_{\text{sidebands}}}{P_{\text{total}}} = \frac{26}{226}$$

Problem 3.18

The signal $x(t)$ is $m(t) + \cos(2\pi f_0 t)$. The spectrum of this signal is $X(f) = M(f) + \frac{1}{2}(\delta(f - f_0) + \delta(f + f_0))$ and its bandwidth equals to $W_x = f_0$. The signal $y_1(t)$ after the Square Law Device is

$$\begin{aligned} y_1(t) &= x^2(t) = (m(t) + \cos(2\pi f_0 t))^2 \\ &= m^2(t) + \cos^2(2\pi f_0 t) + 2m(t)\cos(2\pi f_0 t) \\ &= m^2(t) + \frac{1}{2} + \frac{1}{2}\cos(2\pi 2 f_0 t) + 2m(t)\cos(2\pi f_0 t) \end{aligned}$$

The spectrum of this signal is given by

$$Y_1(f) = M(f) \star M(f) + \frac{1}{2}\delta(f) + \frac{1}{4}(\delta(f - 2f_0) + \delta(f + 2f_0)) + M(f - f_0) + M(f + f_0)$$

and its bandwidth is $W_1 = 2f_0$. The bandpass filter will cut-off the low-frequency components $M(f) \star M(f) + \frac{1}{2}\delta(f)$ and the terms with the double frequency components $\frac{1}{4}(\delta(f - 2f_0) + \delta(f + 2f_0))$. Thus the spectrum $Y_2(f)$ is given by

$$Y_2(f) = M(f - f_0) + M(f + f_0)$$

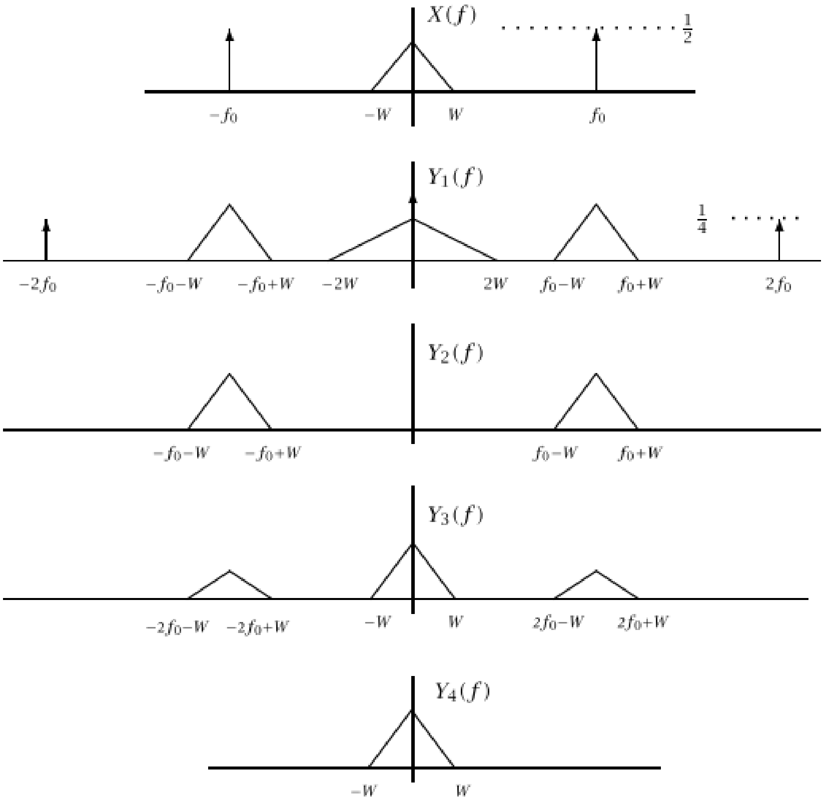
and the bandwidth of $y_2(t)$ is $W_2 = 2W$. The signal $y_3(t)$ is

$$y_3(t) = 2m(t)\cos^2(2\pi f_0 t) = m(t) + m(t)\cos(4\pi f_0 t)$$

with spectrum

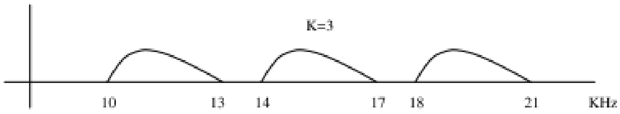
$$Y_3(t) = M(f) + \frac{1}{2}(M(f - f_0) + M(f + f_0))$$

and bandwidth $W_3 = f_0 + W$. The lowpass filter will eliminate the spectral components $\frac{1}{2}(M(f - f_0) + M(f + f_0))$, so that $y_4(t) = m(t)$ with spectrum $Y_4 = M(f)$ and bandwidth $W_4 = W$. The next figure depicts the spectra of the signals $x(t)$, $y_1(t)$, $y_2(t)$, $y_3(t)$ and $y_4(t)$.



Problem 3.24

1) The next figure illustrates the spectrum of the SSB signal assuming that USSB is employed and $K = 3$. Note, that only the spectrum for the positive frequencies has been plotted.



2) With $LK = 60$ the possible values of the pair (L, K) (or (K, L)) are $\{(1, 60), (2, 30), (3, 20), (4, 15), (6, 10)\}$. As it is seen the minimum value of $L + K$ is achieved for $L = 6$, $K = 10$ (or $L = 10$, $K = 6$).

3) Assuming that $L = 6$ and $K = 10$ we need 16 carriers with frequencies

$$\begin{array}{ll} f_{k_1} = 10 \text{ KHz} & f_{k_2} = 14 \text{ KHz} \\ f_{k_3} = 18 \text{ KHz} & f_{k_4} = 22 \text{ KHz} \\ f_{k_5} = 26 \text{ KHz} & f_{k_6} = 30 \text{ KHz} \\ f_{k_7} = 34 \text{ KHz} & f_{k_8} = 38 \text{ KHz} \\ f_{k_9} = 42 \text{ KHz} & f_{k_{10}} = 46 \text{ KHz} \end{array}$$

and

$$\begin{array}{ll} f_{l_1} = 290 \text{ KHz} & f_{l_2} = 330 \text{ KHz} \\ f_{l_3} = 370 \text{ KHz} & f_{l_4} = 410 \text{ KHz} \\ f_{l_5} = 450 \text{ KHz} & f_{l_6} = 490 \text{ KHz} \end{array}$$

Problem 4.4

1) Since an angle modulated signal is essentially a sinusoidal signal with constant amplitude, we have

$$P = \frac{A_c^2}{2} \Longrightarrow P = \frac{100^2}{2} = 5000$$

The same result is obtained if we use the expansion

$$u(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta) \cos(2\pi(f_c + nf_m)t)$$

along with the identity

$$J_0^2(\beta) + 2\sum_{n=1}^{\infty} J_n^2(\beta) = 1$$

2) The maximum phase deviation is

$$\Delta\phi_{\max} = \max|4\sin(2000\pi t)| = 4$$

3) The instantaneous frequency is

$$\begin{aligned} f_i &= f_c + \frac{1}{2\pi}\frac{d}{dt}\phi(t) \\ &= f_c + \frac{4}{2\pi}\cos(2000\pi t)2000\pi = f_c + 4000\cos(2000\pi t) \end{aligned}$$

Hence, the maximum frequency deviation is

$$\Delta f_{\max} = \max|f_i - f_c| = 4000$$

4) The angle modulated signal can be interpreted both as a PM and an FM signal. It is a PM signal with phase deviation constant $k_p = 4$ and message signal $m(t) = \sin(2000\pi t)$ and it is an FM signal with frequency deviation constant $k_f = 4000$ and message signal $m(t) = \cos(2000\pi t)$.

Problem 4.6

1) If the output of the narrowband FM modulator is,

$$u(t) = A\cos(2\pi f_0 t + \phi(t))$$

then the output of the upper frequency multiplier ($\times n_1$) is

$$u_1(t) = A\cos(2\pi n_1 f_0 t + n_1\phi(t))$$

After mixing with the output of the second frequency multiplier $u_2(t) = A\cos(2\pi n_2 f_0 t)$ we obtain the signal

$$\begin{aligned} y(t) &= A^2\cos(2\pi n_1 f_0 t + n_1\phi(t))\cos(2\pi n_2 f_0 t) \\ &= \frac{A^2}{2}(\cos(2\pi(n_1+n_2)f_0 + n_1\phi(t)) + \cos(2\pi(n_1-n_2)f_0 + n_1\phi(t))) \end{aligned}$$

The bandwidth of the signal is $W = 15$ KHz, so the maximum frequency deviation is $\Delta f = \beta_f W = 0.1 \times 15 = 1.5$ KHz. In order to achieve a frequency deviation of $f = 75$ KHz at the output of the wideband modulator, the frequency multiplier n_1 should be equal to

$$n_1 = \frac{f}{\Delta f} = \frac{75}{1.5} = 50$$

Using an up-converter the frequency modulated signal is given by

$$y(t) = \frac{A^2}{2}\cos(2\pi(n_1+n_2)f_0 + n_1\phi(t))$$

Since the carrier frequency $f_c = (n_1+n_2)f_0$ is 104 MHz, n_2 should be such that

$$(n_1+n_2)100 = 104 \times 10^3 \Longrightarrow n_1 + n_2 = 1040 \text{ or } n_2 = 990$$

2) The maximum allowable drift (d_f) of the 100 kHz oscillator should be such that

$$(n_1+n_2)d_f = 2 \Longrightarrow d_f = \frac{2}{1040} = .0019 \text{ Hz}$$

Problem 4.12

1) Assuming that $u(t)$ is an FM signal it can be written as

$$\begin{aligned} u(t) &= 100\cos(2\pi f_c t + 2\pi k_f \int_{-\infty}^{\infty} \alpha\cos(2\pi f_m \tau)d\tau) \\ &= 100\cos(2\pi f_c t + \frac{k_f \alpha}{f_m}\sin(2\pi f_m t)) \end{aligned}$$

Thus, the modulation index is $\beta_f = \frac{k_f\alpha}{f_m} = 4$ and the bandwidth of the transmitted signal

$$B_{\text{FM}} = 2(\beta_f + 1)f_m = 10 \text{ KHz}$$

2) If we double the frequency, then

$$u(t) = 100\cos(2\pi f_c t + 4\sin(2\pi 2 f_m t))$$

Using the same argument as before we find that $\beta_f = 4$ and

$$B_{\text{FM}} = 2(\beta_f + 1)2f_m = 20 \text{ KHz}$$

3) If the signal $u(t)$ is PM modulated, then

$$\beta_p = \Delta\phi_{\max} = \max[4\sin(2\pi f_m t)] = 4$$

The bandwidth of the modulated signal is

$$B_{\text{PM}} = 2(\beta_p + 1)f_m = 10 \text{ KHz}$$

4) If f_m is doubled, then $\beta_p = \Delta\phi_{\max}$ remains unchanged whereas

$$B_{\text{PM}} = 2(\beta_p + 1)2f_m = 20 \text{ KHz}$$

Problem 4.18

The modulation index is

$$\beta = \frac{k_f \max[|m(t)|]}{f_m} = \frac{10 \times 10}{8} = 12.5$$

The output of the FM modulator can be written as

$$\begin{aligned} u(t) &= 10\cos(2\pi 2000t + 2\pi k_f \int_{-\infty}^{t} 10\cos(2\pi 8\tau)d\tau) \\ &= \sum_{n=-\infty}^{\infty} 10 J_n(12.5)\cos(2\pi(2000 + n8)t + \phi_n) \end{aligned}$$

At the output of the BPF only the signal components with frequencies in the interval $[2000 - 32, 2000 + 32]$ will be present. These components are the terms of $u(t)$ for which $n = -4, \ldots, 4$. The power of the output signal is then

$$\frac{10^2}{2}J_0^2(12.5) + 2\sum_{n=1}^{4}\frac{10^2}{2}J_n^2(12.5) = 50 \times 0.2630 = 13.15$$

Since the total transmitted power is $P_{\text{tot}} = \frac{10^2}{2} = 50$, the power at the output of the bandpass filter is only 26.30% of the transmitted power.