Principles and Experiments of Communications

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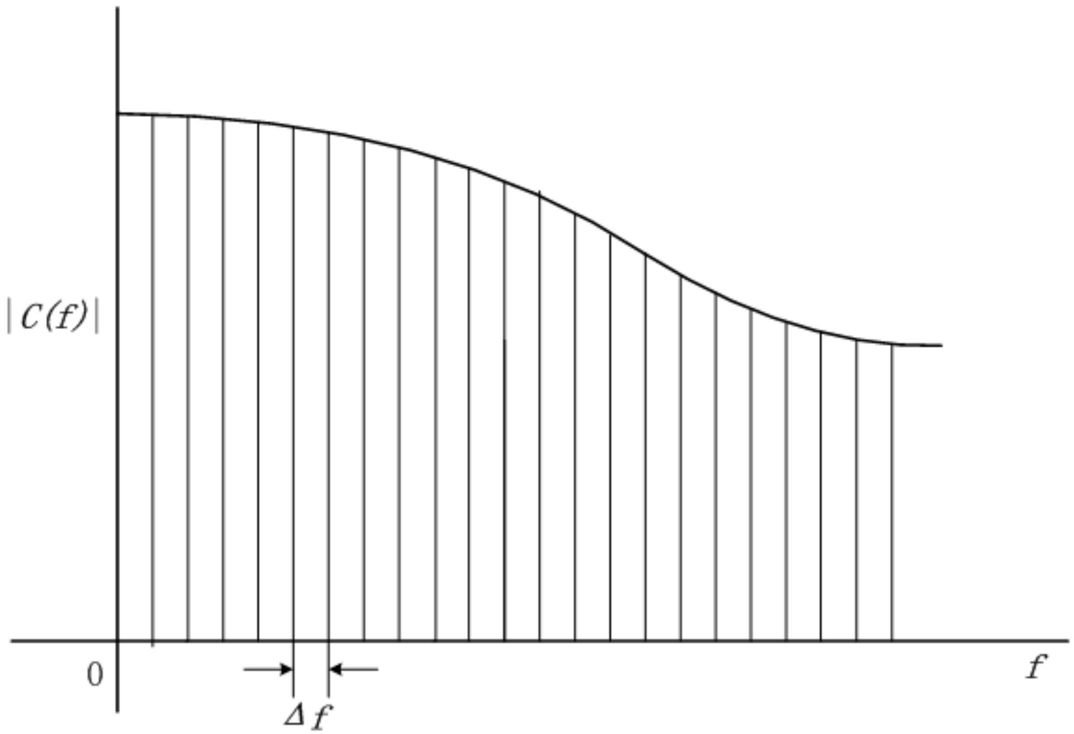
Textbook: Chapter 11

Lecture 06: Multicarrier modulation and OFDM

Multicarrier Modulation

- Multipath channels cause intersymbol interference (ISI) when the reciprocal of the symbol rate is smaller than the dispersion of the nonideal channel.
- A solution:
  - Subdivide the available channel bandwidth into a number of equal-bandwidth subchannels;
  - The frequency response characteristics of each subchannel are nearly ideal.

Subdivision of Channel Bandwidth



$$\Delta f = \frac{1}{T}$$

where T is the symbol interval

Orthogonal Frequency Division Multiplexing (1/2)

- Each subchannel associate a carrier

$$x_k(t) = \cos 2\pi f_k t, \quad k = 0, 1, \cdots, N-1$$

- Orthogonal Frequency Division Multiplexing (OFDM)
  - The subcarriers are orthogonal over the symbol interval;
  - Independently of the relative phase relationship between subcarriers.

$$\int_0^T \cos(2\pi f_i t + \phi_i) \cos(2\pi f_j t + \phi_j)\, dt = 0$$

Orthogonal Frequency Division Multiplexing (2/2)

- ISI can be made arbitrarily small through the selection of N:
  - Select sufficiently large N
  - Since N is large sufficiently, the symbol interval can be larger than the time duration of the channel-time dispersion

Modulation

- Each OFDM symbol is the sum of signals modulated by subcarriers.
- OFDM symbol can express as

$$s(t) = \mathrm{Re}\left\{ \sum_{i=-N/2}^{N/2-1} d_{i+N/2} \exp\left[j2\pi \left(f_c - \frac{i+0.5}{2} \right) (t - t_s) \right] \right\} \quad t_s \le t \le t_s + T$$

equivalent baseband signal

$$s(t) = \sum_{i=-N/2}^{N/2-1} d_{i+N/2} \exp\left[j2\pi \frac{i}{T} (t - t_s) \right] \quad t_s \le t \le t_s + T$$

Demodulation

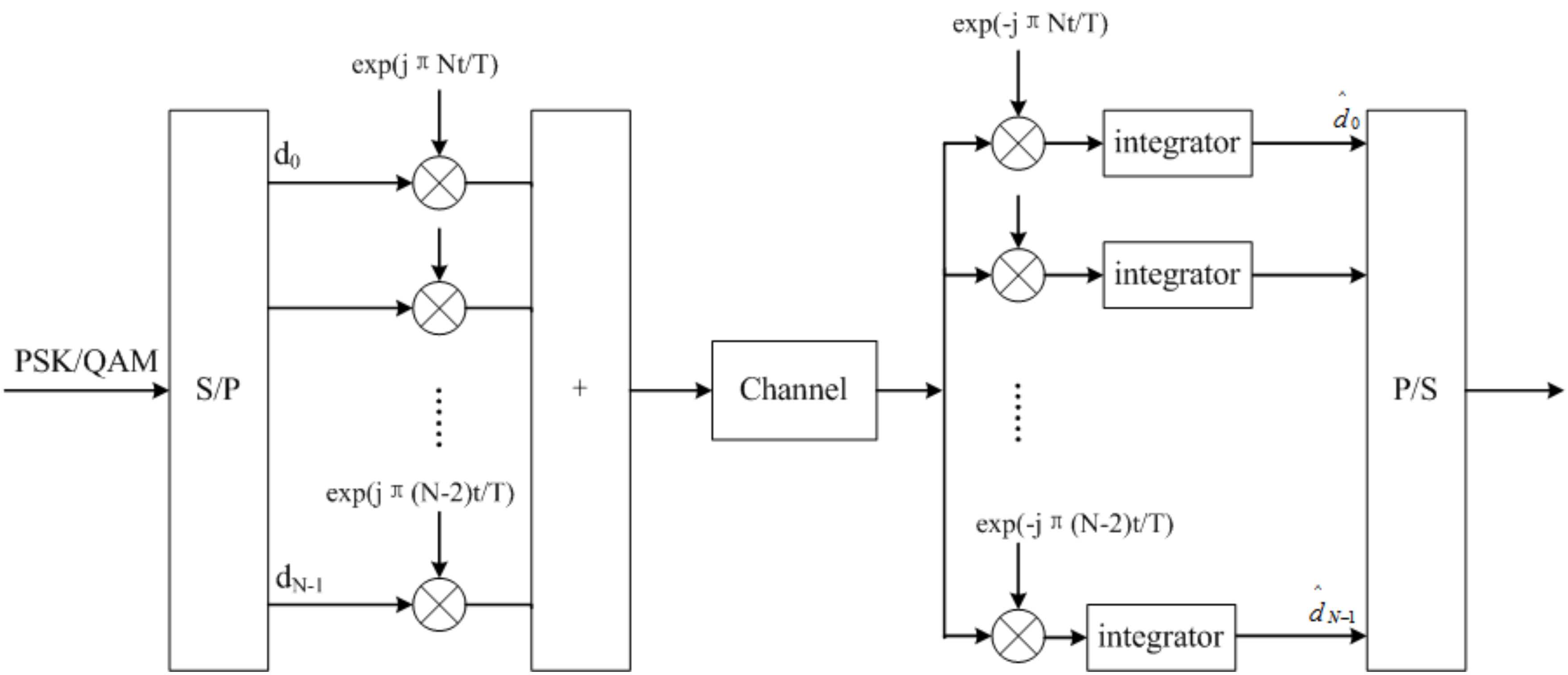
- For the *k* th signal

$$\hat{d}_k = \frac{1}{T}\int_{t_s}^{t_s+T} \exp\left[j2\pi \frac{k-N/2}{T}\left(t-t_s\right)\right] \sum_{i=-N/2}^{N/2-1} d_{i+N/2} \exp\left[-j2\pi \frac{i}{T}\left(t-t_s\right)\right] dt$$

$$= \frac{1}{T}\sum_{i=-N/2}^{N/2-1} d_{i+N/2} \int_{t_s}^{t_s+T} \exp\left[j2\pi \frac{i-k+N/2}{T}\left(t-t_s\right)\right] dt$$

$$= d_k$$

Modulation and Demodulation of OFDM System



FFT and IFFT

- Inverse DFT and IDFT are critical in the implementation of an OFDM system

$$\text{IDFT } \mathrm{x[n]} = \frac{1}{N}\sum_{n=0}^{N-1} x[k] e^{-j\frac{2\pi}{N}kn}$$

$$DFT \ \mathrm{x[k]} = \sum_{n=0}^{N-1} x[n] e^{j\frac{2\pi}{N}kn}$$

- IFFT and FFT algorithms are the fast implementation for the IDFT and DFT

Modulation Using IDFT

- OFDM equivalent baseband signal can implement with IDFT.

$$s(t)=\sum_{i=-N/2}^{N/2-1} d_{i+N/2}\exp\left[j2\pi\frac{i}{T}(t-t_s)\right]\quad t_s\le t\le t_s+T$$

let $t_s=0$, $t=kT/N, k=0,1,\cdots,N-1$

$$s(k)=s\left(\frac{kT}{N}\right)=\sum_{i=0}^{N-1} d_i\exp\left(j\frac{2\pi ki}{N}\right)\quad 0\le k\le N-1$$

$s(k)$ is IDFT of d_i

Demodulation Using DFT

- At the receiver

$$d_i = \sum_{k=0}^{N-1} s(k) \exp\left(-j\frac{2\pi ki}{N}\right) \quad 0 \le i \le N-1$$

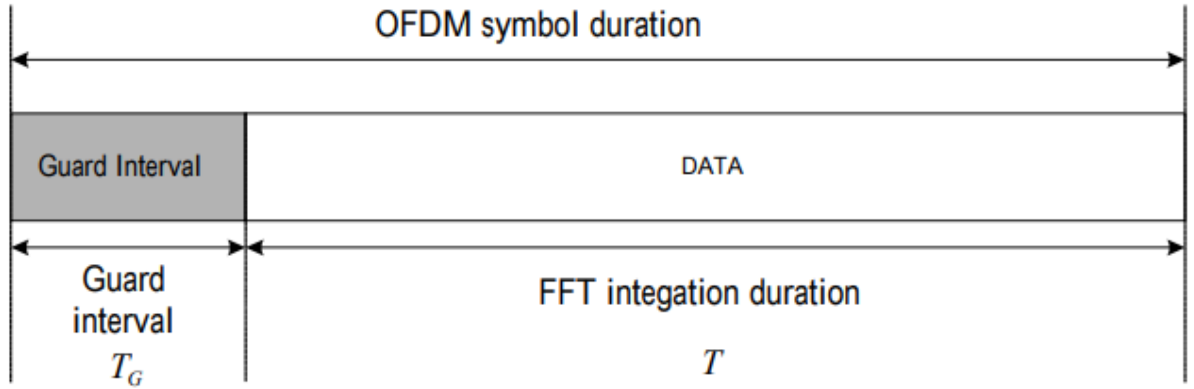
- Modulating the transmitted signal and demodulating the received signal is equivalent to the computation of the IDFT and its inverse.

OFDM via the FFT Algorithm

- Algorithm complexity
  - DFT/IDFT: $O\left(N^2\right)$
  - FFT/IFFT: $O\left(\frac{N}{2}\log_2(N)\right)$
- Implement the OFDM system via the FFT algorithm instead of DFT algorithm can be achieved more easily.

Eliminate ISI (1/2)

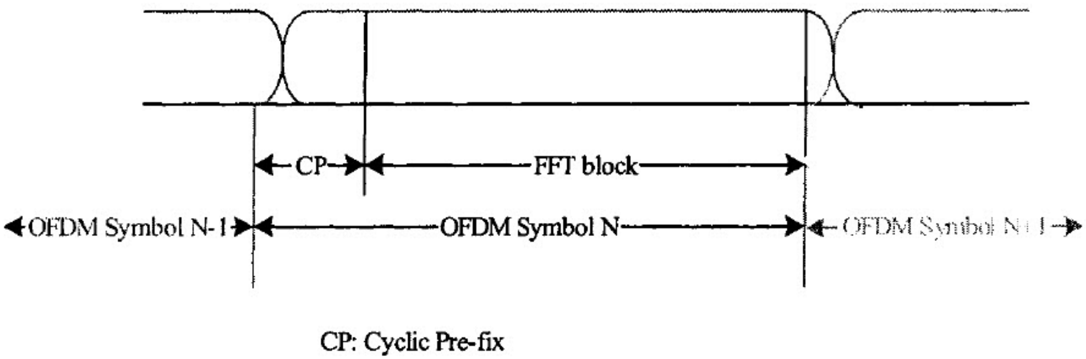
- To eliminate ISI, we need to insert guard interval between OFDM symbols.



- If the guard interval is larger than the delay spread, the multipath component of a symbol does not interfere with adjacent symbols.

Eliminate ISI (2/2)

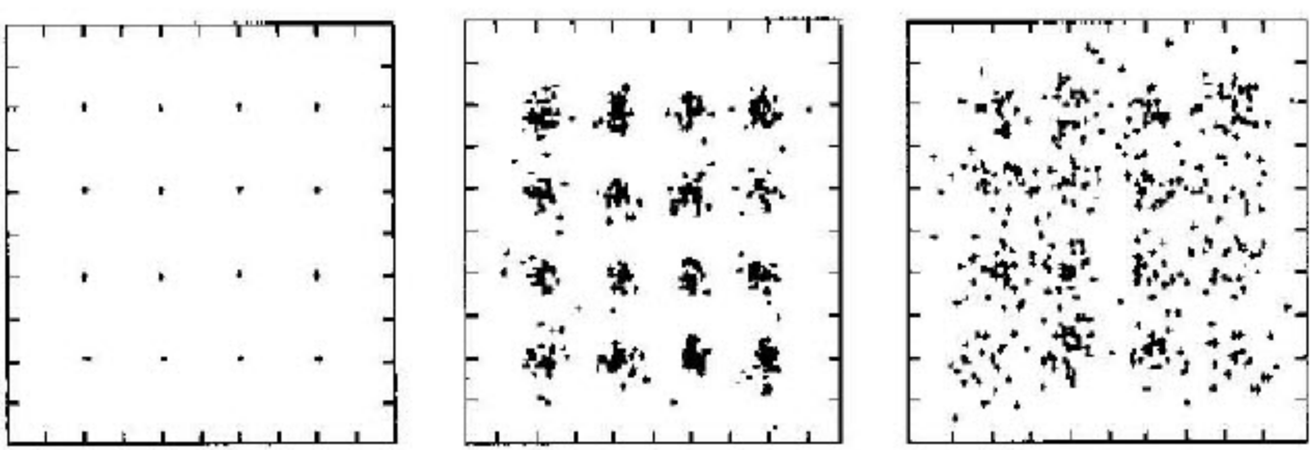
- An alternative approach is to append cyclic prefix to each OFDM symbol



- Since the ISI in any pair of successive signal transmission block only affects the cyclic prefix, we discard the affected samples of received signal to eliminate ISI

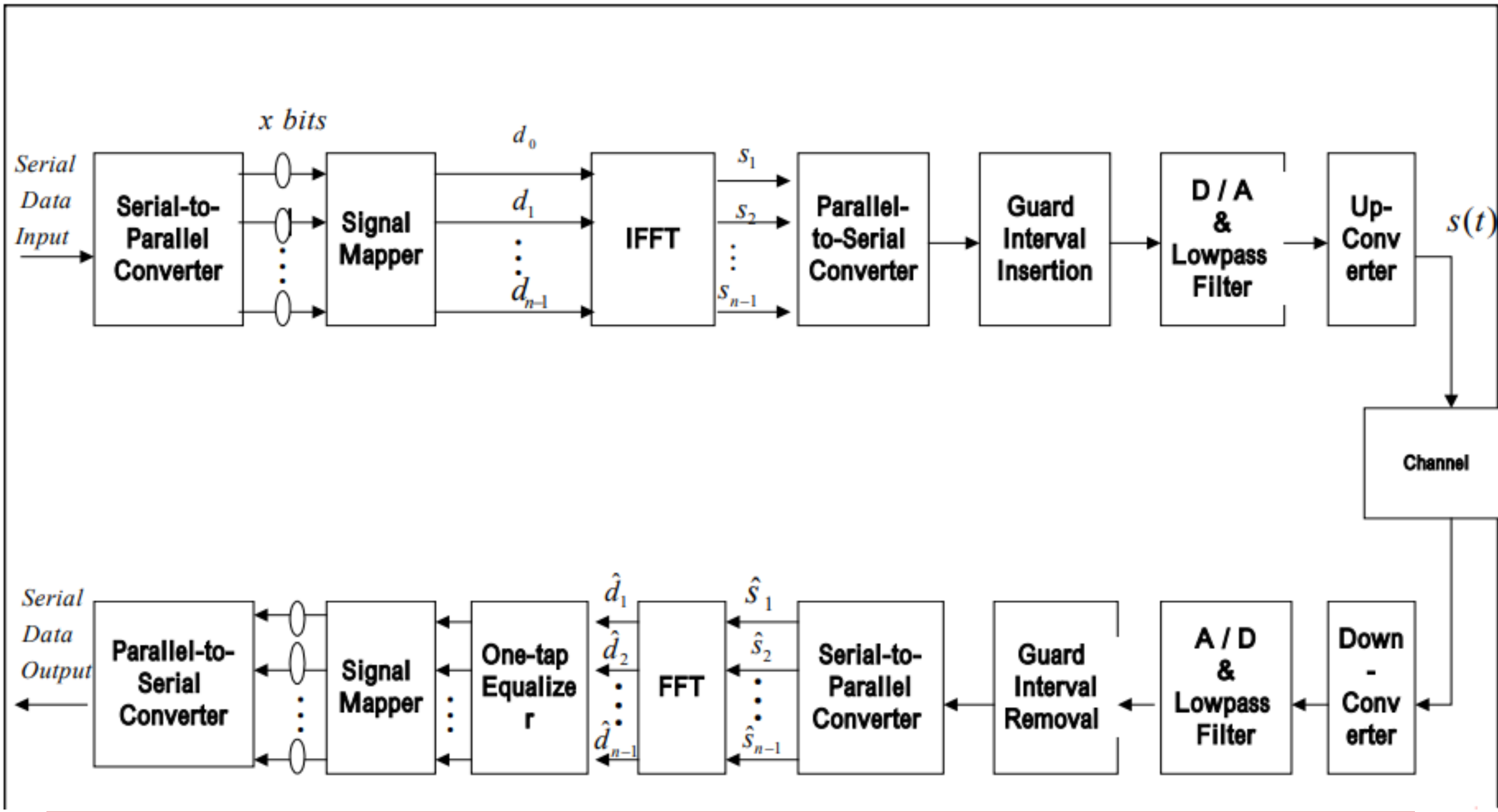
Cyclic Prefix and Delay Spread

- 3 cases:
  - (a) Delay Spread < Cyclic Prefix
  - (b) Delay Spread is 3% over Cyclic Prefix
  - (c) Delay Spread is 10% over Cyclic Prefix



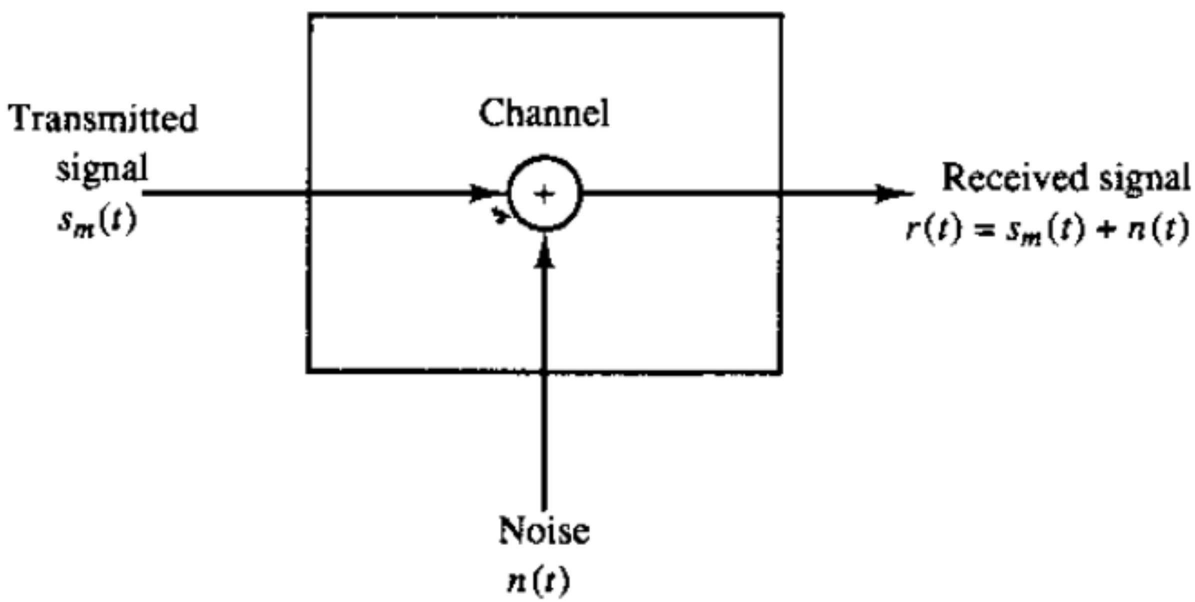
(a) (b) (c)

Multicarrier OFDM Comunication System

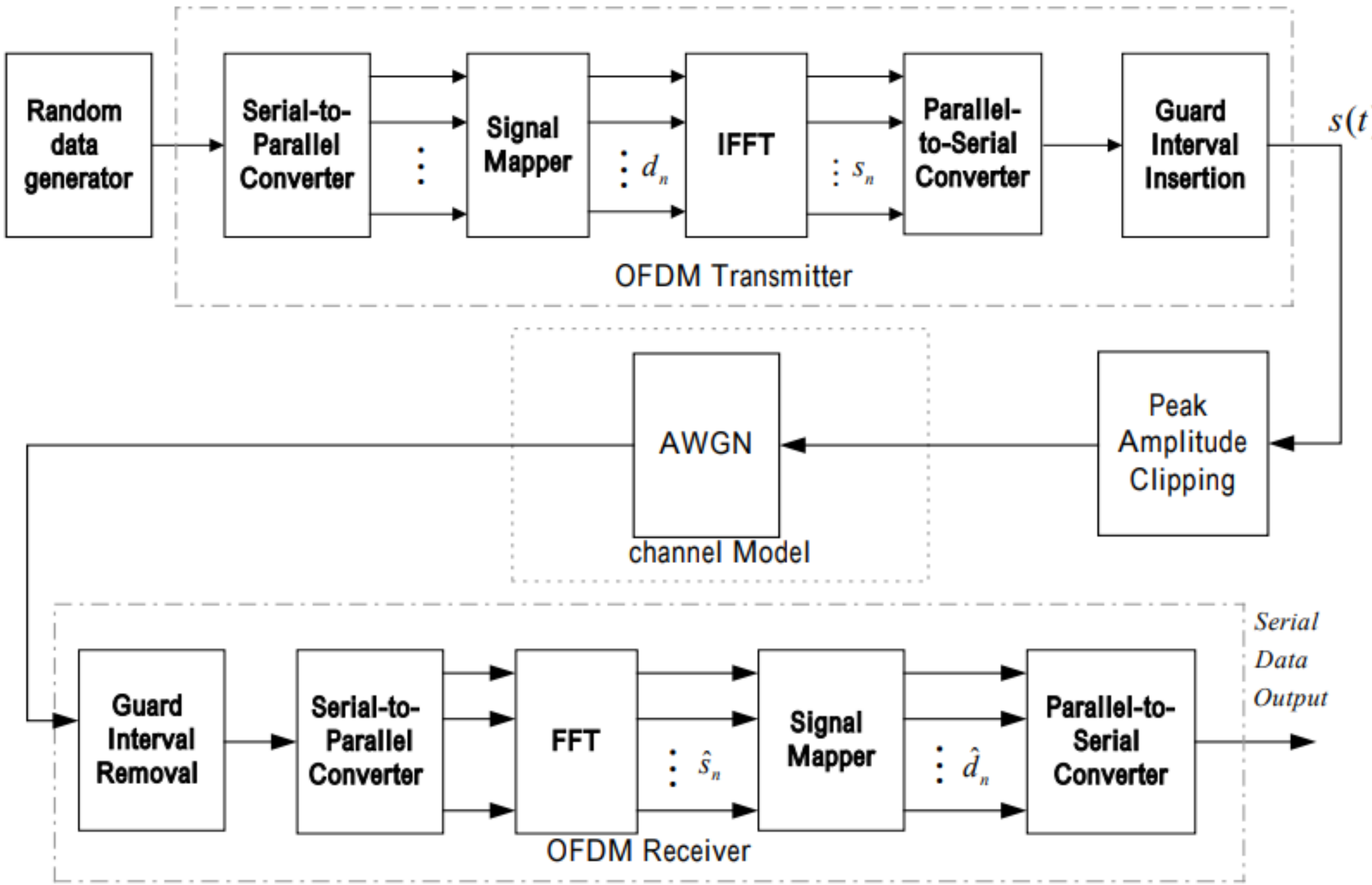


The AWGN Channel Model

- The channel is assumed to corrupt the signal by the addition of white Gaussian noise as shown in the following figure.



OFDM System Performance over AWGN Channel (1/3)



OFDM System Performance over AWGN Channel (2/3)

- QAM

$$\mathrm{p}_{e,M-QAM} = \frac{1}{\log_2 M}\left\{1-\frac{1}{M}\left[\left(\sqrt{M}-2\right)^2 \bullet p(c\,|\,I)+4\left(\sqrt{M}-2\right)p(c\,|\,II)+4p(c\,|\,III)\right]\right\}$$

$$a=\sqrt{\frac{3E_s}{2(M-1)}}$$

$$p(c\,|\,I)=\left[1-2Q\left(\sqrt{\frac{2a^2}{N_0}}\right)\right]^2$$

$$p(c\,|\,II)=\left[1-2Q\left(\sqrt{\frac{2a^2}{N_0}}\right)\right]\left[1-Q\left(\sqrt{\frac{2a^2}{N_0}}\right)\right]$$

$$p(c\,|\,III)=\left[1-Q\left(\sqrt{\frac{2a^2}{N_0}}\right)\right]^2$$

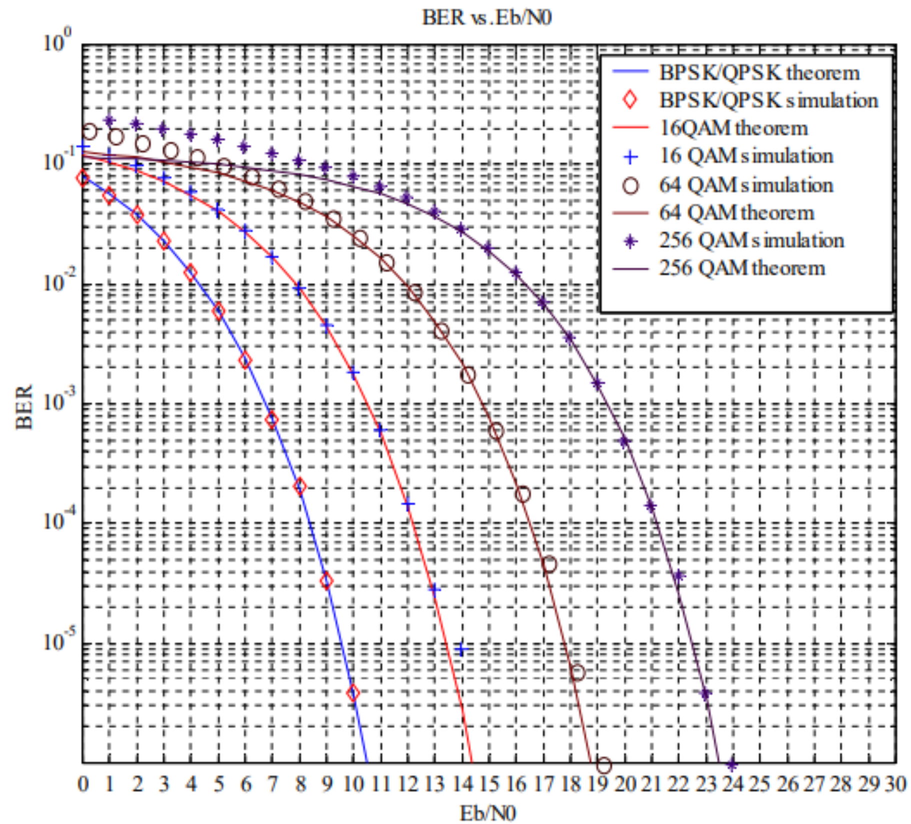
- M-ary PSK

$$\mathrm{p}_e \approx \mathrm{erfc}\left(\sqrt{\frac{E_s}{N_0}}\sin\left(\frac{\pi}{M}\right)\right)$$

OFDM System Performance over AWGN Channel (3/3)

- BER versus SNR curves for the OFDM system in AWGN channel using BPSK/QPSK, 16QAM, 64QAM, 256QAM

Simulation parameter	Value
Channel	AWGN
FFT size	1024
Subcarrier #	1024
Modulation	BPSK, QPSK, 16QAM, 64QAM, 256QAM
Guard Type	Cyclic Prefix
SNR	0 - 30 dB
Delay spread	No
Doppler frequency	No
Reflected power	No
Power Clipped	No



SNR per subchannel

- The SNR per subchannel can be defined as:

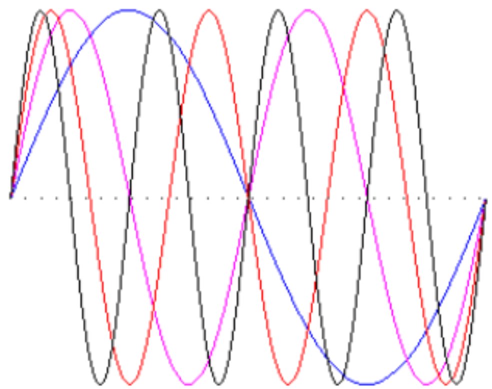
$$\mathrm{SNR}_k = \frac{TP_k \left|C_k\right|^2}{\sigma_{nk}^2}$$

- In subchannels with high SNR, we transmit more bits/symbol. Thus, the bit rate on each subchannel can be optimized

Spectra of OFDM Signal (1/2)

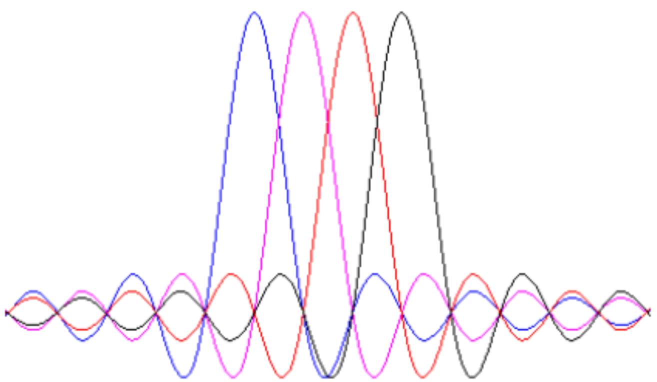
- OFDM signals are orthogonal in the time domain but have significant overlap in frequency domain.

Time domain



Example of four subcarriers within one OFDM symbol

Frequency domain



Spectra of individual subcarriers

Spectra of OFDM Signal (2/2)

- The large spectral overlap of the OFDM signals causes intersubchannel interference (**ICI**) when the communication channel is a fading channel
- OFDM may not be as robust as a single carrier system in radio communications where the receiving terminal is moving at high speed

Peak-to-Average Power Ratio

- Major problem: high PAPR
  - Large peaks occurs when the signals in the K subchannels add constructively in phase.
  - Such large signal peaks may saturate the power amplifier at the transmission, thus, causing intermodulation distortion in the transmitted signal.
- Solution:
  - Insert different phase shifts in each of the subcarriers;
  - Modulate a small subset of the subcarriers with dummy symbol.

Applications of OFDM

- Digital audio broadcasting (DAB)
- Digital video broadcasting (DVB)
- Digital subscriber lines (DSL)
- Wireless local area networks (LANs)

Advantages and disadvantages of OFDM

- Advantages:
  - High data rate
  - Immunity to delay spread
  - Efficient bandwidth
- Disadvantages:
  - The problem of Synchronization
  - Need FFT units at transmitter, reciver
  - The problem of high peak to average power ratio (PAPR)