Problem 7.4

1)

$$\begin{aligned} x_p(t) &= \sum_{n=-\infty}^{\infty} x(nT_s)p(t-nT_s) \\ &= p(t) \star \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t-nT_s) \\ &= p(t) \star x(t) \sum_{n=-\infty}^{\infty} \delta(t-nT_s) \end{aligned}$$

Thus

$$\begin{aligned} X_p(f) &= P(f)\cdot \mathcal{F}\left[x(t)\sum_{n=-\infty}^{\infty}\delta(t-nT_s)\right] \\ &= P(f)X(f) \star \mathcal{F}\left[\sum_{n=-\infty}^{\infty}\delta(t-nT_s)\right] \\ &= P(f)X(f) \star \frac{1}{T_s}\sum_{n=-\infty}^{\infty}\delta(f-\frac{n}{T_s}) \\ &= \frac{1}{T_s}P(f)\sum_{n=-\infty}^{\infty}X(f-\frac{n}{T_s}) \end{aligned}$$

2) In order to avoid aliasing $\frac{1}{T_s} > 2W$. Furthermore the spectrum $P(f)$ should be invertible for $|f| < W$.

3) $X(f)$ can be recovered using the reconstruction filter $\Pi(\frac{f}{2W_\Pi})$ with $W < W_\Pi < \frac{1}{T_s} - W$. In this case

$$X(f) = X_p(f)T_sP^{-1}(f)\Pi(\frac{f}{2W_\Pi})$$

Problem 7.9

1) From Table 7.1 we find that for a unit variance Gaussian process, the optimal level spacing for a 16-level uniform quantizer is .3352. This number has to be multiplied by σ to provide the optimal level spacing when the variance of the process is σ^2. In our case $\sigma^2 = 10$ and $\Delta = \sqrt{10}\cdot 0.3352 =$

1.060. The quantization levels are

$$
\begin{aligned}
\hat{x}_1 = -\hat{x}_{16} &= -7 \cdot 1.060 - \frac{1}{2} \cdot 1.060 = -7.950 \\
\hat{x}_2 = -\hat{x}_{15} &= -6 \cdot 1.060 - \frac{1}{2} \cdot 1.060 = -6.890 \\
\hat{x}_3 = -\hat{x}_{14} &= -5 \cdot 1.060 - \frac{1}{2} \cdot 1.060 = -5.830 \\
\hat{x}_4 = -\hat{x}_{13} &= -4 \cdot 1.060 - \frac{1}{2} \cdot 1.060 = -4.770 \\
\hat{x}_5 = -\hat{x}_{12} &= -3 \cdot 1.060 - \frac{1}{2} \cdot 1.060 = -3.710 \\
\hat{x}_6 = -\hat{x}_{11} &= -2 \cdot 1.060 - \frac{1}{2} \cdot 1.060 = -2.650 \\
\hat{x}_7 = -\hat{x}_{10} &= -1 \cdot 1.060 - \frac{1}{2} \cdot 1.060 = -1.590 \\
\hat{x}_8 = -\hat{x}_9 &= -\frac{1}{2} \cdot 1.060 = -0.530
\end{aligned}
$$

The boundaries of the quantization regions are given by

$$
\begin{aligned}
a_1 = a_{15} &= -7 \cdot 1.060 = -7.420 \\
a_2 = a_{14} &= -6 \cdot 1.060 = -6.360 \\
a_3 = a_{13} &= -5 \cdot 1.060 = -5.300 \\
a_4 = a_{12} &= -4 \cdot 1.060 = -4.240 \\
a_5 = a_{11} &= -3 \cdot 1.060 = -3.180 \\
a_6 = a_{10} &= -2 \cdot 1.060 = -2.120 \\
a_7 = a_9 &= -1 \cdot 1.060 = -1.060 \\
a_8 &= 0
\end{aligned}
$$

2) The resulting distortion is $D = \sigma^2 \cdot 0.01154 = 0.1154$.

3) Substituting $\sigma^2 = 10$ and $D = 0.1154$ in the rate-distortion bound, we obtain

$$R = \frac{1}{2} \log_2 \frac{\sigma^2}{D} = 3.2186$$

5) The distortion of the 16-level optimal quantizer is $D_{16} = \sigma^2 \cdot 0.01154$ whereas that of the 8-level optimal quantizer is $D_8 = \sigma^2 \cdot 0.03744$. Hence, the amount of increase in SQNR (db) is

$$10 \log_{10} \frac{\text{SQNR}_{16}}{\text{SQNR}_8} = 10 \cdot \log_{10} \frac{0.03744}{0.01154} = 5.111 \text{ db}$$

Problem 7.18

1)

$$\begin{aligned} R_X(t+\tau,t) &= E[X(t+\tau)X(t)] \\ &= E[Y^2\cos(2\pi f_0(t+\tau)+\Theta)\cos(2\pi f_0 t+\Theta)] \\ &= \frac{1}{2}E[Y^2]E[\cos(2\pi f_0\tau)+\cos(2\pi f_0(2t+\tau)+2\Theta)] \end{aligned}$$

and since

$$E[\cos(2\pi f_0(2t+\tau)+2\Theta)] = \frac{1}{2\pi}\int_0^{2\pi}\cos(2\pi f_0(2t+\tau)+2\theta)d\theta = 0$$

we conclude that

$$R_X(t+\tau,t) = \frac{1}{2}E[Y^2]\cos(2\pi f_0\tau) = \frac{3}{2}\cos(2\pi f_0\tau)$$

2)

$$10\log_{10}\text{SQNR} = 10\log_{10}\left(\frac{3\times 4^{\nu}\times R_X(0)}{x_{\max}^2}\right) = 40$$

Thus,

$$\log_{10}\left(\frac{4^{\nu}}{2}\right) = 4 \text{ or } \nu = 8$$

The bandwidth of the process is $W = f_0$, so that the minimum bandwidth requirement of the PCM system is $\text{BW} = 8f_0$.

3) If SQNR = 64 db, then

$$\nu' = \log_4(2\cdot 10^{6.4}) = 12$$

Thus, $\nu' - \nu = 4$ more bits are needed to increase SQNR by 24 db. The new minimum bandwidth requirement is $\text{BW}' = 12f_0$.

10. 3.

(1) The received signal:

$$v(t) = \sum_{k=-\infty}^{+\infty} A_k\, p(t-kT) + n_o(t) \qquad n_o(t) = n(t) * h_R(t)$$

$$t_m = (m \pm 0.1)T \qquad \text{Let } t_m = (m-0.1)T$$

$$\begin{aligned} v(t_m) &= \sum_{k=-\infty}^{+\infty} A_k p((m-0.1-k)T) + n_o((m-0.1)T) \\ &= A_m\, p(-0.1T) + A_{m-1}\, p(0.9T) + n_o((m-0.1)T) \\ &= 0.9 A_m A^2T + 0.1 A_{m-1} A^2 T + n_o((m-0.1)T) \end{aligned}$$

$$SNR = \frac{(0.9A^2T)^2}{6^2}$$

If there is no mistiming, $SNR_o = \frac{(A^2T)^2}{6^2}$

$\therefore$ loss $= 0.81 = -0.915\ dB$

(2) $ISI = 0.1 A_{m-1} A^2 T$.

If $A_m = 1$, $P_{e_1} = \frac{1}{2} P_e(A_m = 1, A_{m-1} = 1) + \frac{1}{2} P_e(A_m = 1, A_{m-1} = -1)$

$$= \frac{1}{2} Q\left(\sqrt{\frac{2A^2T}{N_0}}\right) + \frac{1}{2} Q\left(0.8\sqrt{\frac{2A^2T}{N_0}}\right)$$

If $A_m = 0$, $P_{e_0} = P_{e_1}$

$\therefore P_e = P_{e_1}$

If there is no mistiming. $P_e' = Q\left(\sqrt{\frac{2A^2T}{N_0}}\right)$

$\therefore \Delta P_e = \frac{1}{2} Q\left(0.8\sqrt{\frac{2A^2T}{N_0}}\right) - \frac{1}{2} Q\left(\sqrt{\frac{2A^2T}{N_0}}\right)$

10.5.

$$X_{rc}(f) = \begin{cases} T & 0 \le |f| \le (1-\alpha)/2T \\ \frac{T}{2}\left[1 + \cos\left(\frac{\pi T}{\alpha}\left(|f| - \frac{1-\alpha}{2T}\right)\right)\right] & \frac{1-\alpha}{2T} \le |f| \le \frac{1+\alpha}{2T} \\ 0 & |f| > \frac{1+\alpha}{2T} \end{cases}$$

$$p(t) = \operatorname{sinc}\left(\frac{t}{T}\right) \frac{\cos(\pi\alpha t/T)}{1 - 4\alpha^2 t^2/T^2}$$

when $t = 0$, $\operatorname{sinc}\left(\frac{t}{T}\right) = 1$. $\frac{\cos(\pi\alpha t/T)}{1-4\alpha^2t^2/T^2} = 1$.

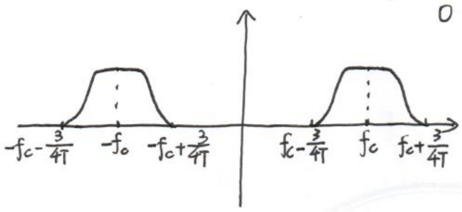
when $t = nT$, $\operatorname{sinc}\left(\frac{t}{T}\right) = 0$,

if $1 - 4\alpha^2t^2/T^2 = 0$, then $\alpha t = \frac{T}{2}$. $\lim\limits_{\alpha t \to \frac{T}{2}} \frac{\cos(\pi\alpha t/T)}{1-4\alpha^2t^2/T^2} = \lim\limits_{n\to 1} \frac{\cos(\frac{\pi}{2}n)}{1-n^2} = \frac{\pi}{4}$.

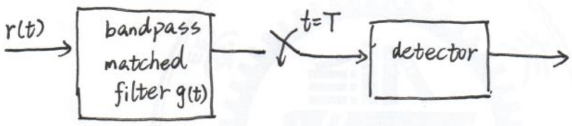
10.17.

(1) $S_V(f) = \frac{1}{T} |X_{rc}(f)|^2$

$T = \frac{1}{2400}$ $$X_{rc}(f) = \begin{cases} T & 0 \le |f| \le \frac{1}{4T} \\ \frac{T}{2}\left[1 + \cos\left(2\pi T\left(|f| - \frac{1}{4T}\right)\right)\right] & \frac{1}{4T} \le |f| \le \frac{3}{4T} \\ 0 & |f| > \frac{3}{4T}. \end{cases}$$



(2)



$g(t) = A X_{rc}(T-t) \cos(2\pi f_c(T-t))$

10.28.

(1) $c_{-1} = -0.4762$, $c_0 = 1.4286$, $c_1 = -0.4762$.

(2) $q_2 = -0.1429$, $q_{-2} = -0.1429$.

$q_3 = 0$. $q_{-3} = 0$