

Ans. to week 7:

(1). the key point is to show the convergence of the series. They are indeed uniformly convergent in $x \in \mathbb{T}^n$.

• the 1st one follows from

$$|f(x+m)| \leq C_N (1+|m|)^{-N} \quad \forall N.$$

• the second one follows from

$$|\hat{f}(m)| \leq C_N (1+|m|)^{-N} \quad \forall N.$$

Moreover

$$F_2(x) = \sum_{m \in \mathbb{Z}^n} \hat{f}(m) e^{2\pi i x \cdot m}$$

~~$$= \sum_{m \in \mathbb{Z}^n} \int_{\mathbb{R}^n} f(y) e^{-2\pi i y \cdot m} dy e^{2\pi i x \cdot m}$$~~

~~$$= \int_{\mathbb{R}^n} \sum_{m \in \mathbb{Z}^n} \int_{\mathbb{R}^n} f(y) e^{-2\pi i y \cdot m + 2\pi i x \cdot m} dy$$~~

\Downarrow Fubini theorem on \mathbb{T}^n

$$\hat{f}_2(m) = \hat{f}(m) := \int_{\mathbb{T}^n} F_2(x) e^{-2\pi i m x} dx$$

while the Fourier transform of F_1 on \mathbb{T}^n is

$$\widehat{F_1}(m) = \int_{\mathbb{T}^n} \sum_{k \in \mathbb{Z}^n} f(x+k) e^{-2\pi i m \cdot x} dx$$

$$= \sum_{k \in \mathbb{Z}^n} \int_{\mathbb{T}^n} f(x+k) e^{-2\pi i m \cdot x} dx$$

$$= \sum_{k \in \mathbb{Z}^n} \int_{k+\mathbb{T}^n} f(x) e^{-2\pi i m \cdot (x-k)} dx$$

$$= \int_{\mathbb{R}^n} f(x) e^{-2\pi i m \cdot x} dx$$

$$= \widehat{f}(m).$$

then: $F_1 = F_2$ follows from the uniqueness.

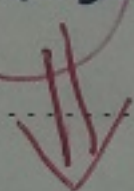
(2) the 1st assertion is indeed the restatement of absolute conv. of integration (at ∞)

while the 2nd one follows from the 1st one by exchanging the order of summation and the integration.

(3) As can be done in the same vein in (1), where the condition that guarantees the exchange of order of summation and integration is replaced by the almost sharp decay condition.

(5) If you do this, you can find some properties of Jacobi θ -function, which plays a role in the proof of ~~analytic~~ meromorphic extension of Riemann zeta-function.

(4)



See, thm 2.17, Chapter VII, in Stein Weiss, Introduction to Fourier Analysis on Euclidean spaces.