Week 4
Ex1 Haisenborg's uncortainty principle.
1) n=1.
a) Show that the commutator [8x, x]:= 2x(x.)-xdx:
Satisfies $[\partial_x, x]f = f$ for $f \in f(\mathbb{R})$ .
b) conclude from a) that
b) conclude from a) that    xf  2.   2xf  2 ≥ c  f  2, f∈S(IR)
for some $\epsilon > 0$ .
2) n>1, first show the commutar [Vx, x]. defin
by Vx. (x.) - 7. Vzf Satorfies
$[\nabla x, x] f = n f f \in J(\mathbb{R}^n)$
and conclude from shis that
(I)  xf  2.   xf  2 = c  f  2. f = Some
3) Now suppose 52 eRn is abounded domain. Let
f be a function on so such that there exists a function GEGIRA, with GIRC = 0 and GIR= f
function GE (IRM). With GIRC = 0 and GIR=
Conclude from (Z) that
(II)
for Some C>0.
Problem under what conditions can we replace the term
Problem: under what conditions can we replace the term

(4) If we apply Plancherel's theorem. We then have
1/2/ = 1/22iz/ 1/12.
And hence (I) reads
1/2 fle 1/8 \$ 1/2 = = 1/2 1/2.
This is Heiserby's hincortainly principle
(5). In the same spirit, one can show, with r=X
$\Gamma + \nabla_{x}(+)$ , $x - 1 + -2 + + \epsilon \int (\mathbb{R}^{n})$
Consequently, we have
$  x-1f  _{L^{2}}^{2}=2(2\pi/2f), \chi f > f \in S(\mathbb{R}^{n})$
5 how that
Show that $\nabla_{\mathbf{x}}(\frac{1}{ \mathbf{x} }f) = \frac{1}{ \mathbf{x} }\nabla_{\mathbf{x}}f + \frac{\mathbf{x}}{ \mathbf{x} ^3}f.$
from this, we conclude
(1) 1/1+1/2 = 2 ( xf, x, r-2f), fe g(1Rh
It then follows from Cauchy Schwarz mequations that
그리어 보고 있다. 그렇게 하는 속하면 보는 사람이 가장 주었다면 하면 하면 하면 가장 하는 것이 되었다면 하는 것이 되었습니다. 그 하게 되어 하다 하다 하다.
11x1+fl12 < 2= 18fl2
this is Hardy's linea.
The state of the s
<del>는 사람들이 되는 사람들이 되었다. 그는 사람들이 되었다면 하는 사람들이 되었다. 그는 사람들이 되었다면 하는 것이다. 그는 사람들이 되었다. 그는 사람들이 되었다. 그는 사람들이 되었다. 그는 사람들이 되었다. 그는 사람들이 되었다면 보다 되었다면 보다 되었다. 그는 사람들이 되었다면 보다 되었다면 보다 되었다면 보다 되었다면 보다. 그런 사람들이 되었다면 보다 되었다면 보다 되었다면 보다 되었다면 보다 되었다면 보다 되었다면 보다 되었다면 보다.</del>

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Ex5 Roverny lemmas
for B=Bixir) denote +B=Bixitr)
(1) Wiener's Vitali-type lemma.
let {BifieT be a collection of balls in Rn. Then
there exists an at mosa countable Subcollersion of
disjoint Dalls EBRY such that
UBj. Z () Bk
Use this lemma to show weak (1.1) boundedness of Hardy-Littlewood Maximal operator.
Hardy-Littlewood Maximal operator.
(2) Besi coviter-Morse covering lemma
let A be a bounded set in 121, and Suppose that 18/1xeA
is a collection of balls St. Bx=B(x, k), k>0. Then there
exists an at most countable Subcollection of balls ? Bj?
and a constant Cn, depending only on the dimension,
Such that
$A = \bigcup B_j$ and $\sum \chi_{B_j}(x) \leqslant C_n$
Finnely overlesp.
Ex6. by the same technique as in thmz. and assuring T* is weak (p.g.)
Show that the Set
is zlosed in P(x, µ).
15 = 105ed in l'(X, //).

Ex2 Lot (x, u), (Y, u) be two measure spaces. Let f: Xx/ >0
Ex? Lot (x, u), (Y, v) be two measure spaces, let f: xx/> a be a measurable function. Show that for 0 <p(9<00)< td=""></p(9<00)<>
11 + 11 Peder 119 (dr) (   H1 12 (dr) 1 P (du)
('a priority.)
(1) Let (X, M) = (X, V) = (IR", Leb.), Show Hore ourg's Ineq.
we have  \[ \left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}\left(\frac{1}\left(\frac{1}2\left(\frac{1}2\left(\frac{1}\left(\frac{1}2\left(\fr
where 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
where
$1+\frac{1}{r}=\frac{1}{p}+\frac{1}{q}$ , $r>0,p>0,q>0.$
(3) let 065 <n, &="" 1×1-5="" [5(1rm)].="" c="" can="" consent="" for="" have,="" nevertheless,="" one="" see="" some="" still="" then="" we="">0,</n,>
Newstholess, we still have, for some contain C>0,
11 1.1 × 9 11, x < c 118 11/2
where $1+\frac{1}{4} = \frac{5}{5} + \frac{7}{4}$
Ex3 prove Riesz-Thorin Interpolation theorem.
Ex3 prove Riesz. Thorin Interpolation theorem.  Ref. T. Ransford, Potential theory in the complex plane.
Pase 162.
Ext. Show, for fellocien, that
1   So     Fix-y) - fixedy = 0, a.e.
L-DO+ IBLI JBL 1 1 1