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ECE 310
PS J.

1. a) FT: $G(\vec{k}, w) = \int_{\vec{r} \in \mathbb{R}^2} \sum_{n=-\infty}^{+\infty} g(\vec{r}, n) e^{-j(\vec{k}\cdot\vec{r} + w\cdot n)} d\vec{r}$

IFT: $g(\vec{r}, n) = \frac{1}{(2\pi)^3} \int_{k \in \mathbb{R}^2} \int_{-\pi}^{\pi} G(\vec{k}, w) e^{j(\vec{k}\cdot\vec{r} + wn)} d\vec{k}\, dw.$

b). $\int_{r \in \mathbb{R}^2} \sum_{n=-\infty}^{+\infty} g(\vec{r}, n) h^*(\vec{r}, n)\, d\vec{r} = \frac{1}{(2\pi)^3} \int_{-\pi}^{\pi} \int_{\vec{k} \in \mathbb{R}^2} G(\vec{k}, w) H^*(\vec{k}, w)\, d\vec{k}\, dw$

c). $\frac{1}{(2\pi)^3} \int_{-\pi}^{\pi} \int_{\vec{k} \in \mathbb{R}^2} G(\vec{k}', w') H(\vec{k} - \vec{k}', w - w')\, d\vec{k}\, dw' = G(\vec{k}, w) * H(\vec{k}, w)$

d) $\mathcal{F}^{-1}\{U * V\} = \frac{1}{(2\pi)^3} \int_{-\pi}^{\pi} \int_{k \in \mathbb{R}^2} \left\{ \frac{1}{(2\pi)^3} \int_{-\pi}^{\pi} \int U(\vec{k}', w') V(\vec{k} - \vec{k}', w - w') d\vec{k}' d\right.$

$= \frac{1}{(2\pi)^3} \cdot \frac{1}{(2\pi)^3} \int_{-\pi}^{\pi} \int_{\vec{k} \in \mathbb{R}} \int_{-\pi}^{\pi} \int_{\vec{k}' \in \mathbb{R}^2} U(\vec{k}', w') e^{j(\vec{k}\cdot\vec{r} + w'n)} V(\vec{k} - \vec{k}', w - w') e^{j(\vec{k} - \vec{k}')\cdot\vec{r} + (w - w')n)} d\vec{k}' dw' d\vec{k}\, dw$

Let $\vec{k}'' = \vec{k} - \vec{k}'$ $w'' = w - w'$.

$\Rightarrow \mathcal{F}^{-1}\{U * V\}$

$= \left(\frac{1}{(2\pi)^3} \int_{-\pi}^{\pi} \int_{\vec{k}' \in \mathbb{R}^2} U(\vec{k}', w') e^{j(\vec{k}'\cdot\vec{r} + wn)} d\vec{k}' dw'\right)$

$\cdot \left(\frac{1}{(2\pi)^3} \int_{-\pi}^{\pi} \int_{\vec{k}'' \in \mathbb{R}^2} V(\vec{k}'', w'') e^{j(\vec{k}''\cdot\vec{r}) + w''n)} d\vec{k}'' dw''\right)$

$= U(\vec{r}, n) \cdot V(\vec{r}, n) = U \cdot V$

e). $\int_{\vec{r} \in \mathbb{R}^2} \sum_{-\infty}^{+\infty} |h(\vec{r}, n)|\, d\vec{r} < \infty$

2. $h = \frac{1}{6}(-20\delta[x,y] + 1\delta[x+1,y+1] + 4\delta[x,y+1] + 1\delta[x-1,y+1]$
$+ 4\delta[x+1,y] + 4\delta[x-1,y] + 1\delta[x+1,y-1] + 4\delta[x,y-1]$
$+ 1\delta[x-1,y-1])$.

$H(z_1,z_2) = \frac{1}{6}(-20 + 1z_1^{-1}z_2^{-1} + 4z_2^{-1} + 1z_1z_2^{-1} + 4z_1^{-1} + 4z_1 + 1z_1^{-1}z_2 + 4z_2$
$+ 1z_1z_2)$

$H(w_1,w_2) = \frac{1}{6}(-20 + 4(z_1+z_1^{-1}) + 4(z_2+z_2^{-1}) + (z_1z_2 + z_1^{-1}z_2^{-1}) + (z_1^{-1}z_2 + z_1z_2^{-1}$

$= \frac{1}{6}(-20 + 8\cos(w_1) + 8\cos(w_2) + 2\cos(w_1+w_2) + 2\cos(w_1-w_2))$

[redacted]