Weizhe Guo

1. a). $\pm 10kHz \pm n50kHz$.

$\therefore 40kHz, 60kHz, 90kHz, 110kHz, 140kHz, 160kHz, 190kHz$

b). ~~[illegible]~~

$|f| \le \frac{f_s}{2} = \frac{50kHz}{2} = 25kHz$

c). Analog radian: $w = 2\pi f = 2\pi \cdot 10k = 20\pi\, k\ rad/s$.

normalized d. radian: $w = \frac{2\pi f}{f_s} = \frac{20\pi k}{50k} = \frac{2}{5}\pi\ rad$

normalized to sampling rate: $f_s \Rightarrow 1 \quad f = \frac{2}{5}\pi / 2\pi = \frac{1}{5}$

nom. to Nyquist bandwidth: $f = \frac{1}{5} \times 2 = \frac{2}{5}$.

d). $\frac{2}{5}\pi\ rad = 2\pi f_0 / 100k$.

$f_0 = 20kHz$.

$\therefore f_0 \pm nf_s. \quad \pm 20kHz \pm n100kHz$

$\Rightarrow 20kHz, 80kHz, 120kHz, 180kHz,$

e). $|f| \le \frac{f_s}{2} = \frac{100kHz}{2} = 50kHz$.

3. $h = \{3, -1, 2, 1\} \quad x = \{2, -1, 2, 3\}$.

$y_2[2] = h_0x_2 + h_1x_1 + h_2x_0 = 3\times 2 + (-1)\cdot(-1) + 2\cdot 2 = 11$

$y_4[2] = h_0x_2 + h_1x_1 + h_2x_0 + h_3x_3 = 11 + 3 = 14$

$y_8[2] = y_2[2] = 11$.

4. $x = \{2, -1, 2, 3\}$.

$$X_8[3] = \sum_{n=0}^{8-1} x[n] e^{-j\frac{2\pi n \cdot 3}{8}} = 2 - e^{-j\frac{3\pi}{4}} + 2e^{-j\frac{3\pi}{2}} + 3e^{-j\frac{9\pi}{4}}$$

$$= 2 - (\cos(\tfrac{3\pi}{4}) + j\sin(\tfrac{3\pi}{4})) + 2(\cos(-\tfrac{3\pi}{2}) + j\sin(-\tfrac{3}{2}\pi)) + 3(\cos(-\tfrac{9}{4}\pi) + j\sin(-\tfrac{9}{4}\pi))$$

$$= (2 + 3\sqrt{2}) + j(2 - \sqrt{2}).$$

J. a). $W_N^{2p} = (e^{-j\frac{2\pi}{N}})^{2p} = e^{-j\frac{4\pi p}{N}}$.

$W_{N/2}^{p} = (e^{-j\frac{2\pi}{N/2}})^{p} = e^{-j\frac{4\pi p}{N}}$.

$\therefore W_N^{2p} = W_{N/2}^{p}$.

b). $$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{2nk}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x[2n] W_N^{2nk} + \sum_{n=0}^{\frac{N}{2}-1} x[2n+1] W_N^{(2k+1)n}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x_0[n] W_N^{2nk} + W_N^{k} \sum_{n=1}^{\frac{N}{2}-1} x_1[n] W_N^{2kn}.$$

$$= X_0[k] + W_N^{k} X_1[k].$$