

# Computer Graphics (COMP3271)

## Written Assignment 1

**Due Date: 23:59 Oct 20th, 2022**

**1. (10 marks)**

Given a unit quaternion  $q = w + x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$  and a 3D vector  $\mathbf{v} \in \mathbb{R}^3$ , find the rotated vector  $\mathbf{v}'$  operated by  $q$ .

(a)  $q = \frac{\sqrt{2}}{2} + 0\mathbf{i} + 0\mathbf{j} + \frac{\sqrt{2}}{2}\mathbf{k}$ ,  $\mathbf{v} = (1, 2, 1)^T$

(b)  $q = -\frac{1}{2} + \frac{1}{2}\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{1}{2}\mathbf{k}$ ,  $\mathbf{v} = (1, 0, 0)^T$

**2. (10 marks)**

Derive the rotation transformation of  $\theta$  degrees about the point  $(a, b)^T$  in the 2D plane.

(a) Write the transformation in the form of  $X' = MX + B$ .

(b) Write the transformation in the form of  $X' = NX$  in homogeneous coordinates.

**3. (20 marks)**

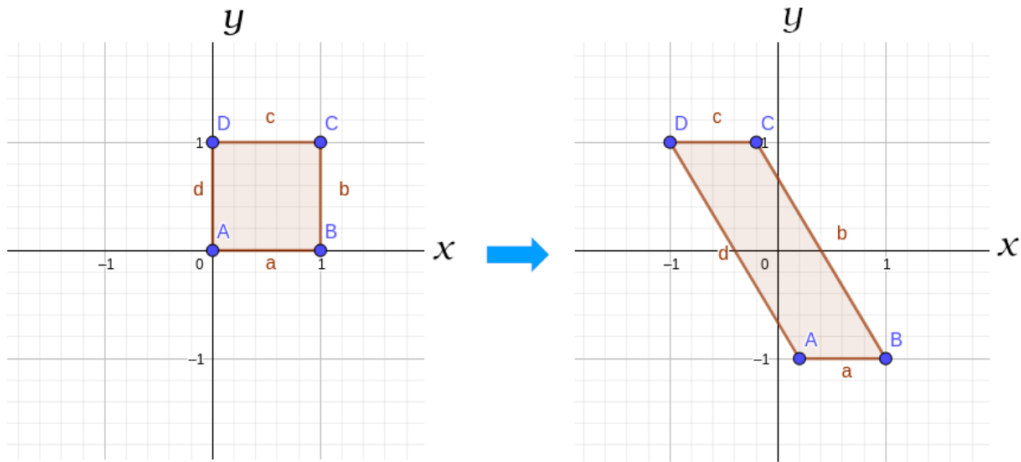
Derive the  $3 \times 3$  homogeneous transformation matrix for the reflection about the line  $y = cx + d$  in the 2D plane.

**4. (10 marks)**

Given two triangles  $\triangle ABC$  and  $\triangle A'B'C'$  in 2D plane with  $A = (0, 0)^T, B = (2, 0)^T, C = (0, 1)^T$ ,  $A' = (0, 1)^T$ ,  $B' = (0, 2)^T$ , and  $C' = (-1, -2)^T$ . Derive the affine transformation matrix for the transformation  $T$  that maps  $\triangle ABC$  to  $\triangle A'B'C'$  such that  $T(A) = A'$ ,  $T(B) = B'$ ,  $T(C) = C'$ .

**5. (10 marks)**

Give a sequence of 4 x 4 matrices that transforms the unit square in the left figure to the parallelogram in the right. Find a sequence of OpenGL function calls that implements these transformations.



6. (10 marks) Show that any sequence of rotations and translations can be replaced by a single rotation about the origin followed by a translation.
7. (10 marks) Consider the line in  $\mathbb{R}^3$  given by

$$L(t) = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}.$$

Let  $Q_1$  and  $Q_2$  be two points on this line that are at distance 1 apart from each other. We now transform the line by applying transformation  $T$  given by

$$\begin{pmatrix} 2 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

What is the distance between  $T(Q_1)$  and  $T(Q_2)$ ?

8. (20 marks)

Given an affine transformation  $X' = MX + B$ , where  $M$  is a 2 by 2 matrix and  $B$  is a 2D vector, find the equation of the image  $E$  of the circle  $x^2 + y^2 - 1 = 0$  under this transformation. Show that  $E$  is an ellipse.