1.

(a) $\boxed{q = \frac{\sqrt{2}}{2} + 0i + 0j + \frac{\sqrt{2}}{2}k \ , \quad v = (1,2,1)^T}$

Since, $q = \cos\theta + u\sin\theta$

$= \cos(45) + \sin(0)\cdot i + \sin(0)j + \sin(45)\cdot k$

and, $v = (1,2,1)^T$

$q_v = 0 + i + 2j + k$

$R(\text{rotated vector}) = [\cos(\frac{\theta}{2}) ; \sin(\frac{\theta}{2})\cdot x , \sin(\frac{\theta}{2}) y , \sin(\frac{\theta}{2})\cdot z]$

Hence, $R = [\cos(\frac{45}{2}) , \sin(0)\cdot 1 , \sin(0)\cdot 2 , \sin(\frac{45}{2})\cdot 1]$

$= \boxed{[-0.873 , 0 , 0 , -0.487]}$ ✗

(b) $\boxed{q = -\frac{1}{2} + \frac{1}{2}i + \frac{1}{2}j + \frac{1}{2}k \ , \quad v = (1,0,0)^T}$

-/0

Similar as (a),

$R = [\cos(\frac{\theta}{2}) , \sin(\frac{\theta}{2})\cdot x , \sin(\frac{\theta}{2})\cdot y , \sin(\frac{\theta}{2})\cdot z]$

$= [\cos(\frac{30}{2}) , \sin(\frac{30}{2})\cdot 1 , \sin(\frac{\theta}{2})\cdot 0 , \sin(\frac{\theta}{2})\cdot 0]$

$= \boxed{[-0.759 , 0.65 , 0 , 0]}$ ✗

2.

(a) We know that the rotation matrix is :

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

then :

$$X' = RX + B$$

$$\boxed{\underbrace{\begin{bmatrix} X'_x \\ X'_y \end{bmatrix}}_{X'} = \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} X_x \\ X_y \end{bmatrix}}_{X} + \underbrace{\begin{bmatrix} a \\ b \end{bmatrix} - \begin{bmatrix} a\cos\theta & -b\sin\theta \\ a\sin\theta & b\cos\theta \end{bmatrix}}_{B}}$$

(b) In homogeneous coordinate where $(a, b) = (0, 0)$

we can simply do :

$$\boxed{\begin{bmatrix} x'-a \\ y'-b \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x-a \\ y-b \end{bmatrix}}$$

-5

3. Since reflection is merely transform a line to one of the axes.

That is:

$$P' = P \cdot R \cdot R_x \cdot R^{-1}$$

where R is some transformation
R_x is reflection trans.
R^{-1} is the inverse of the first transformation

In this case:

we can do a ϕ degree rotation, where $\phi = \tan^{-1}(c) = -c$ of line $y = cx + d$ (clockwise) toward x-axis. Then do a reflection transformation. finally, do an inverse rotation of ϕ.

So. $R = R_r R_x R_r^{-1}$

$= R_{T1} R_{r1} R_x \cdot R_{r2} \cdot R_{T2}$

$$P' = \begin{bmatrix} 1 & 0 & -c/d \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos(\phi) & -\sin(\phi) & 0 \\ \sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot$$

Intersection point between $y = cx + d$ and x-axis.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos(\phi) & \sin(\phi) & 0 \\ -\sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\cdot \begin{bmatrix} 1 & 0 & c/d \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2(\phi) - \sin^2(\phi) & 2\sin(\phi)\cos(\phi) & \dfrac{c\left(\left(\cos^2(\phi) - \sin^2(\phi)\right)\right) - c}{d} \\ 2\cos(\phi)\sin(\phi) & \sin^2(\phi) - \cos^2(\phi) & \dfrac{2c \cdot \cos(\phi)\sin(\phi)}{d} \\ 0 & 0 & 1 \end{bmatrix}$$

where $\phi = \tan^{-1}(c) = -c$

(slope of $y = cx + d$)

so. the transformation matrix for the reflection about the line $y = cx + d$

$$R = \begin{bmatrix} \cos^2(-c) - \sin^2(-c) & 2\sin(-c)\cos(-c) & \frac{c(\cos^2(-c) - \sin^2(-c)) - c}{d} \\ 2\cos(-c)\sin(-c) & \sin^2(-c) - \cos^2(-c) & \frac{2c \cdot \cos(-c)\sin(-c)}{d} \\ 0 & 0 & 1 \end{bmatrix}$$

4.

$$T = \Delta A'B'C' \cdot \Delta ABC^{-1}$$

$$= \begin{bmatrix} 0 & 0 & -1 \\ 1 & 2 & -2 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 0 & 0 & -1 \\ 1 & 2 & -2 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} -\frac{1}{2} & -1 & 1 \\ \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\boxed{T = \begin{bmatrix} 0 & -1 & 0 \\ \frac{1}{2} & -3 & 1 \\ 0 & 0 & 1 \end{bmatrix}}$$

✓

$$T(A) = \begin{bmatrix} 0 & -1 & 0 \\ \frac{1}{2} & -3 & 1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$T(B) = \begin{bmatrix} 0 & -1 & 0 \\ \frac{1}{2} & -3 & 1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

$$T(C) = \begin{bmatrix} 0 & -1 & 0 \\ \frac{1}{2} & -3 & 1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

5、

```
glScalef(0.4, 2.0, 1.0);

GLfloat shearTrans[16] = { 1, -0.8, 0, 0,
                           0, 1, 0, 0,
                           0, 0, 1, 0,
                           0, 0, 0, 1});

glMultMatrixf( shearTrans );

glTranslatef(0.6, -1, 0);
```

⑥.

If we multiply a 4x4 rotation matrix by a 4x4 translation matrix. we can see the left three column of the product matrix are the same as the left three column of the rotation matrix. the right first column is the same as the right first colum of the translation matrix.

Moreover, if we do $R \cdot T \cdot R'$, where R、R' are two rotation matrix, and T is a translation matrix. We can notice ~~that~~ the first three colums of the product matrix of RTR' are the same as the product matrix RR''s first three colemns.

the right first colum of RTR', although not the same as the right first column of T, but maintains the form of a translation matrix column like, where the last element of right first column maintains as 1.

Hence, a sequence of rotations and translations can be replaced by a rotation and a translation.

7.

$$Q1 = \begin{bmatrix} t1 \\ 1+t1 \\ 1 \\ 1 \end{bmatrix} \quad Q2 = \begin{bmatrix} t2 \\ 1+t2 \\ 1 \\ 1 \end{bmatrix}$$

$$1 = \sqrt{(t1-t2)^2 + (1+t1-1-t2)^2 + (1-1)^2 + (1-1)^2}$$

$$1 = 2(t1-t2)^2$$

$$1 = \sqrt{2}(t1-t2)$$

$$\boxed{\frac{\sqrt{2}}{2}} = t1 - t2$$

Then $T(Q_1) = T\begin{pmatrix} t_1 \\ 1+t_1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2t_1+2 \\ t_1+2 \\ 5 \\ 1 \end{pmatrix}$

$$T(Q_2) = T\begin{pmatrix} t_2 \\ 1+t_2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2t_2+2 \\ t_2+2 \\ 5 \\ 1 \end{pmatrix}$$

$$\sqrt{(2t_1+2-2t_2-2)^2+(t_1+2-t_2-2)^2+(5-5)^2+(1-1)^2}$$

$$= \sqrt{4(t_1-t_2)^2+(t_1-t_2)^2}$$

$$= \sqrt{5}\,(t_1-t_2)$$

$$= \sqrt{5}\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{10}}{2}$$

The distance between $T(Q_1)$ and $T(Q_2)$ is $\boxed{\frac{\sqrt{10}}{2}}$

8.

let M be $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, B be $\begin{bmatrix} e \\ f \end{bmatrix}$

then

$$X' = \begin{bmatrix} a & b \\ c & d \end{bmatrix} X + \begin{bmatrix} e \\ f \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}$$

$$= \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}$$

$$= \begin{bmatrix} ax + by + e \\ cx + dy + f \end{bmatrix}$$

hence

$$x' = ax + by + e$$

$$y' = cx + dy + f$$

since $x'^2 + y'^2 = 1$

$$(ax + by + e)^2 + (cx + dy + f)^2 = 1$$

$$\left(a^2x^2 + axby + axe + axby + b^2y^2 + bye + axe + bye + e^2 \right.$$

$$+$$

$$\left. c^2x^2 + cxdy + cxf + cxdy + d^2y^2 + dyf + cxf + dyf + f^2 \right) = 1$$

$$\left[(a^2+c^2)x^2+(b^2+d^2)y^2+(ab+ab+cd+cd)xy+(ae+ae+cf+cf)x+(be+be+df+df)y+e^2+f^2-1\right]=0$$

since (a^2+c^2) and (b^2+d^2) must be positice

hence E is an eclipse