

Artificial Intelligence

Homework no. 2

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Find the L^AT_EX code of this masterpiece at my Github @ github.com/WellOfSorrows.

Question 1

1.1

A special case of "*Local Beam Search*" when $k = 1$ is called "*Hill-Climbing Search*".

1.2

This is also *hill-climbing search* since we always keep the best individual of the population, so the mutated individual only replace current individual when the fitness function improves. This is exactly *hill-climbing search*.

Question 2

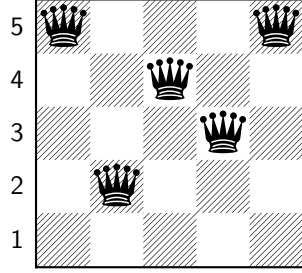
In each genetic algorithm problem, we must go through the following steps:

1. Chromosome encoding
2. Initial population
3. Fitness evaluation
4. Selection
5. Crossover
6. Mutation
7. Population update
8. Iteration

We shall go through each step one by one.

2.1 Encoding

The best way is to represent a chromosome as a string (or a list) of length N where N is the number of queens; in our case $N = 5$. The value of each index shows the row of the queen in a column. For example, for the following configuration, we would have:



Chromosome :

5	2	3	4	5
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2.2 Initial Population

Now, we initialize our population with random chromosomes. Let's take a population of 4 chromosomes as follows:

$$C_1 = \begin{bmatrix} 5 & 2 & 3 & 4 & 5 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 4 & 3 & 5 & 1 & 4 \end{bmatrix}$$

$$C_3 = \begin{bmatrix} 2 & 1 & 3 & 2 & 4 \end{bmatrix}$$

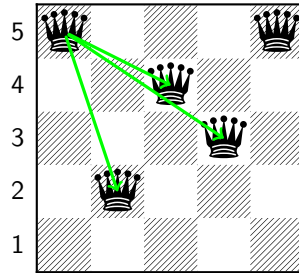
$$C_4 = \begin{bmatrix} 5 & 2 & 3 & 4 & 1 \end{bmatrix}$$

2.3 Fitness Evaluation

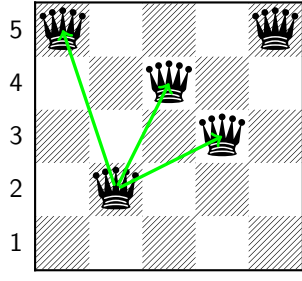
Since in genetic algorithms, higher scores are better, thus we define the fitness function $F_{total}(C)$ as number of the pairs of non-attacking queens in configuration C . We also assign each queen a unique name by using its index in the chromosome as a subscription and define $f(x_i)$ as the number of queens the queen x_i does NOT attack. So, we would attain:

$$F_{total}(C) = \frac{\sum_{i=1}^5 f(x_i)}{2}$$

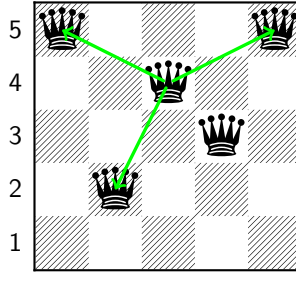
Note that we divided the sum by two because of *hand-shaking lemma*. For example, for the chromosome $C_1 = \begin{bmatrix} 5 & 2 & 3 & 4 & 5 \end{bmatrix}$, we would have



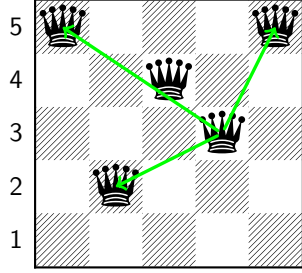
$$\Rightarrow x_1 = 3$$



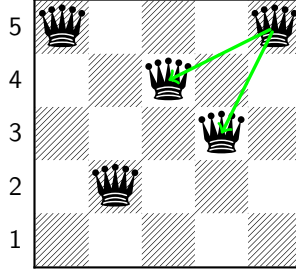
$$\Rightarrow x_2 = 3$$



$$\Rightarrow x_3 = 3$$



$$\Rightarrow x_4 = 3$$



$$\Rightarrow x_5 = 2$$

$$\Rightarrow F_{total}(C_1) = \frac{14}{2} = 7$$

Then, we need to compute the probability of being chosen from the fitness function. This will be needed for the selection step. First, we need to add all fitness functions of the chromosomes population. Thus, in our case, since the population is of size 4, we would have:

$$P(C_j) = \frac{F_{total}(C_j)}{\sum_{k=1}^4 F_{total}(C_k)}$$

Calculating the fitness function for other chromosomes, we would attain:

$$F_{total}(C_2) = 6,$$

$$F_{total}(C_3) = 6,$$

$$F_{total}(C_4) = 5$$

which means that:

$$P(C_1) = \frac{7}{24} = 29\%$$

$$P(C_2) = \frac{6}{24} = 25\%$$

$$P(C_3) = \frac{6}{24} = 25\%$$

$$P(C_4) = \frac{5}{24} = 21\%$$

2.4 Selection

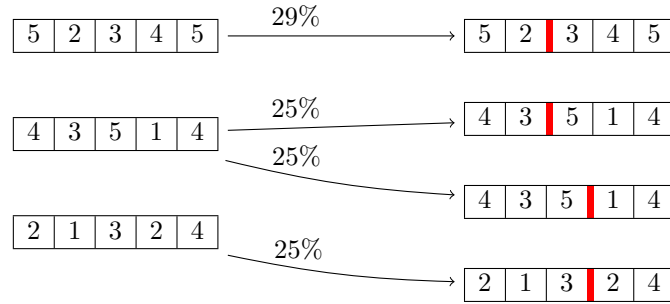
In this step, we randomly choose the two pairs to reproduce based on their probabilities. Then, selected chromosomes will act as parents and will be combined using a crossover operator to make children. For the crossover operation, we need to pick a crossover point per pair. Here we took the following chromosomes based on their probabilities:

$$C_1 = \begin{bmatrix} 5 & 2 & 3 & 4 & 5 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 4 & 3 & 5 & 1 & 4 \end{bmatrix}$$

$$C_3 = \begin{bmatrix} 2 & 1 & 3 & 2 & 4 \end{bmatrix}$$

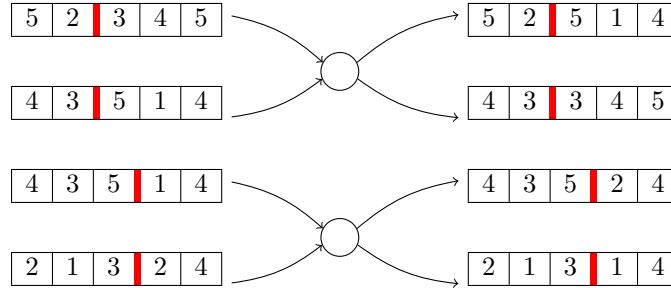
We notice that the chromosome $C_4 = \begin{bmatrix} 5 & 2 & 3 & 4 & 1 \end{bmatrix}$ is omitted because its probability was the least among chromosomes, making it unfit to go to the next step. Thus, the selection step would derive:



2.5 Crossover

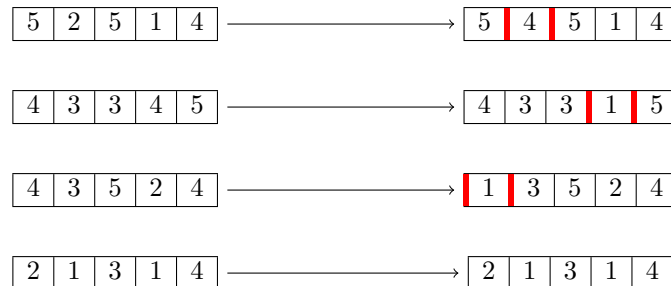
Now we use what we derived in the last step to make new children.

In order to do so, a child would be created in the following way: we concatenate the first part of the child's parent with the second part of its other parent. Thus:



2.6 Mutation

In the mutation process, we may randomly alter one or more gene values in chromosomes we attained after crossover. So in this example, a random mutation will look like the following:



2.7 Population Update

In this step, the chromosomes derived in the last step replaces the original population. So our population would become:

$$C_1 = \begin{array}{|c|c|c|c|c|} \hline 5 & 4 & 5 & 1 & 4 \\ \hline \end{array}$$

$$C_2 = \begin{array}{|c|c|c|c|c|} \hline 4 & 3 & 3 & 1 & 5 \\ \hline \end{array}$$

$$C_3 = \begin{array}{|c|c|c|c|c|} \hline 1 & 3 & 5 & 2 & 4 \\ \hline \end{array}$$

$$C_4 = \begin{array}{|c|c|c|c|c|} \hline 2 & 1 & 3 & 1 & 4 \\ \hline \end{array}$$

2.8 Iteration

We iterate the same procedure we illustrated (from *step 3* to *step 7*) until the F_{total} of one chromosome becomes:

$$\exists j \in \{1, 2, 3, 4\} \Rightarrow F_{total}(C_j) = \binom{N}{2} = \binom{5}{2} = 10$$

This is because each queen must not attack all other queens. So, we must have a complete graph of size N . The number of edges of a complete graph is $\binom{N}{2}$.