## The 1223 new periodic orbits of planar three-body problem with unequal mass and zero angular momentum

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Abstract We present 1349 families of Newtonian periodic planar three-body orbits with unequal mass and zero angular momentum and the initial conditions in case of isosceles collinear configurations. These 1349 families of the periodic collisionless orbits can be divided into seven classes according to their geometric and algebraic symmetries. Among these 1349 families, 1223 families are entirely new, to the best of our knowledge. Furthermore, some periodic orbits have the same free group element for different mass. We find that the scale-invariant average period linearly increases with the mass of one body when the masses of the other two are constant. It is suggested that the masses of bodies may play an important role in periodic three-body systems. The movies of these periodic orbits are given on the website http://numericaltank.sjtu.edu.cn/three-body/three-body-unequal-mass.htm.

The study of periodic three-body problem has received lots of attention in recent years. In 2013, Suyakov and Dmitrašinović [1] found 13 new distinct collisionless periodic orbits of Newtonian planar three-body problem with equal mass and zero angular momentum. In 2014, Dmitrašinović et al. [2] investigated gravitational waves from these periodic three-body systems. In the same year, Iasko and Orlov[3] found nine new close to periodic orbits of threebody problem with equal mass and Suvakov [4] found eleven solutions in the vicinity of the figure-eight orbits with equal mass. In 2015, Hudomal [5] reported 25 families of periodic orbits Newtonian planar three-body problem with equal mass, including the 11 families found in [1]. In 2016, Rose [6] gained 90 periodic planar collisionless orbits with equal mass in case of isosceles collinear configurations. Recently, Li and Liao [7] found more than six hundred new families of planar three-body problem with equal mass. These researchers focused on periodic three-body problem with equal mass. However, the three-body problem with unequal mass is more general. In 2002, Galán et al. [8] studied stability properties of figure-eight [9][10] as the masses are unequal. Doedel et al. [11] gained periodic orbits from the figure-eight as the mass of one body is varied. In 2015, Yan et al. [12] investigated the spatial isosceles three-body problem with unequal masses in case of one body moves up and down on a vertical line. However, little attention has been paid to search for the collisionless periodic orbits of the planar three-body problem with unequal mass and zero angular momentum. In this paper, we present results of collisionless periodic orbits in Newtonian planar three-body problem with unequal mass and zero angular momentum.

The motions of Newtonian planar three-body system can be described by Newtonian second law and gravitational law:  $\ddot{r}_i = \sum_{j=1, j\neq i}^3 \frac{Gm_j(r_j-r_i)}{|r_i-r_j|^3}$ , where  $r_i$  and  $m_i$  denote vector position and mass of the *i*th body (i=1,2,3), G denotes the Newtonian gravity coefficient, and the dot denotes the derivative with respect to the time t, respectively. We consider a planar three-body system with zero angular momentum and unequal mass  $(m_1 = m_2 \neq m_3)$  in the case of G=1 and the initial conditions in case of the isosceles collinear configurations:

$$\begin{cases}
\mathbf{r}_1(0) = (x_1, x_2) = -\mathbf{r}_2(0), & \mathbf{r}_3(0) = (0, 0), \\
\dot{\mathbf{r}}_1(0) = \dot{\mathbf{r}}_2(0) = (v_1, v_2), & \dot{\mathbf{r}}_3(0) = -\frac{m_1 + m_2}{m_2} \dot{\mathbf{r}}_1(0)
\end{cases}$$
(1)

With these configurations, if  $r_i(t)$  (i = 1, 2, 3) with  $m_i$  (i = 1, 2, 3) denotes a periodic orbit with the period T of a three-body system, then

$$\mathbf{r}_i'(t') = \mathbf{r}_i(t), \ \mathbf{v}_i'(t') = \alpha \ \mathbf{v}_i(t), \ t' = t/\sqrt{\alpha}, \ m_i' = \alpha^2 \ m_i,$$
 (2)

has a same periodic orbit with the period  $T' = T/\sqrt{\alpha}$  for arbitrary  $\alpha > 0$ . Therefore, without loss of generality, we consider  $m_1 = m_2 = 1$  and  $m_3$  is varied. Note that 11 families were found by Šuvakov and Dmitrašinović [1] and more than six hundred new families of periodic orbits were found by Li and Liao [7] in case of  $m_3 = 1$ . In this paper, we search for periodic orbits in case of  $m_3 \neq 1$ .

For any given  $m_3$ , the orbits are determined by the four parameters  $(x_1, x_2, v_1, v_2)$ . Write  $\mathbf{y}(t) = (\mathbf{r}_1(t), \dot{\mathbf{r}}_1(t))$ . A periodic solution with the period  $T_0$  is the root of the equation  $\mathbf{y}(T_0) - \mathbf{y}(0) = 0$ , where  $T_0$  is unknown.

Firstly, we use the grid search method to find approximated initial conditions to satisfy the equation  $\mathbf{y}(T_0) - \mathbf{y}(0) = 0$ . We set the initial positions  $x_1 = -1$  and  $x_2 = 0$  and we search for initial conditions in a square plane:  $v_1 \in [0, 1]$  and  $v_2 \in [0, 1]$ . We set 4000 points in each dimension and thus have 16 million grid points in the square search plane. With these different 16 million initial conditions, the motion equations are integrated up to the time  $T_0 = 200$  by

TABLE I. The initial velocities and periods T of some newly-found periodic orbits of the three-body system with unequal mass and zero angular momentum in the case of  $\mathbf{r}_1(0) = (-1,0) = -\mathbf{r}_2(0)$ ,  $\dot{\mathbf{r}}_1(0) = (v_1,v_2) = \dot{\mathbf{r}}_2(0)$  and  $\mathbf{r}_3(0) = (0,0)$ ,  $\dot{\mathbf{r}}_3(0) = (-2v_1/m_3, -2v_2/m_3)$  when G = 1 and  $m_1 = m_2 = 1$ , where  $T^* = T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group word (element). Here, the superscript *i.c.* indicates the case of the initial conditions with isosceles collinear configuration, due to the fact that there exist periodic orbits in many other cases.

Class, number and $m_3$	$v_1$	$v_2$	T	$T^*$	$L_f$
$I.A_4^{i.c.}(0.5)$	0.2009656237	0.2431076328	19.0134164290	19.086	16
$I.A_{68}^{i.c.}(0.5)$	0.2138410831	0.0542938396	83.8472907647	118.112	104
$I.B_{59}^{i.c.}(0.75)$	0.4101378717	0.1341894173	121.0976361440	183.067	102
$I.A_1^{i.c.}(2)$	0.6649107583	0.8324167864	12.6489061509	42.121	8
$II.D_2^{i.c.}(2)$	0.3057224330	0.5215124257	8.8237067653	64.567	12
$I.A_2^{i.c.}(4)$	0.9911981217	0.7119472124	17.6507807837	276.852	24

means of the ODE solver dop853 developed by Hairer et al. [13], which is based on an explicit Runge-Kutta method of order 8(5,3) in double precision with adaptive step size control. The corresponding initial conditions and the period  $T_0$  are chosen as the candidates when the return proximity function

$$|\mathbf{y}(t) - \mathbf{y}(0)| = \sqrt{\sum_{i=1}^{4} (y_i(t) - y_i(0))^2}$$
 (3)

is less than  $10^{-1}$ .

Secondly, we improve these candidates by means of the Newton-Raphson method [14–16]. At this stage, the motion equations are solved numerically by means of the same ODE solver dop853 [13]. The candidates are corrected until the level of the return proximity function (3) is less than  $10^{-3}$ . As mentioned by in Li and Liao [7], many periodic orbits might be lost by means of traditional algorithms in double precision. Thus, we further integrate the motion equations by means of "Clean Numerical Simulation" (CNS) [17–20] with negligible numerical noises in a long enough interval of time, which is based on the arbitrary order of Taylor expansion method [21–24] in arbitrary precision [25, 26], plus a convergence check by means of an additional computation with even smaller numerical noises. A periodic orbit is found when the level of the return proximity function (3) is less than  $10^{-6}$ . Note that the initial positions  $\mathbf{r}_1 = (x_1, x_2)$  will depart from (-1, 0) a little.

It is well-known that, if  $r_i(t)$  (i = 1, 2, 3) denotes a periodic orbit with the period T of a three-body system, then

$$\mathbf{r}_i'(t') = \alpha \, \mathbf{r}_i(t), \quad \mathbf{v}_i'(t') = \mathbf{v}_i(t)/\sqrt{\alpha}, \quad t' = \alpha^{3/2} t,$$
 (4)

is also a periodic orbit with the period  $T' = \alpha^{3/2} T$  for arbitrary  $\alpha > 0$ . Thus, through coordinate transformation and then the scaling of the spatial and temporal coordinates, we can always enforce (-1,0), (1,0) and (0,0) as the initial positions of the body-1, 2 and 3, respectively. In this way, the periodic orbits are only dependent upon two physical parameters  $(v_1, v_2)$ , the initial velocity of Body-1.

We identify these periodic orbits by means of Montgomery's topological identification and classification method [27]. The positions  $r_1, r_2$  and  $r_3$  of the three-body corresponds to a unit vector n in the so-called "shape sphere" with the Cartesian components

$$n_x = \frac{2\boldsymbol{\rho} \cdot \boldsymbol{\lambda}}{R^2}, \ n_y = \frac{\lambda^2 - \rho^2}{R^2}, \ n_z = \frac{2(\boldsymbol{\rho} \times \boldsymbol{\lambda}) \cdot \boldsymbol{e}_z}{R^2},$$

where  $\rho = \frac{1}{\sqrt{2}}(r_1 - r_2)$ ,  $\lambda = \frac{1}{\sqrt{6}}(r_1 + r_2 - 2r_3)$  and the hyper-radius  $R = \sqrt{\rho^2 + \lambda^2}$ . Then a periodic orbit is associated with a closed curve on the shape sphere, which can be characterized by its topology with three punctures (two-body collision points). With one of the punctures as the "north pole", the sphere can be mapped onto a plane by a stereographic projection. And a closed curve can be mapped onto a plane with two punctures and its topology can be described by the so-called "free group element" (word) with letters a (a clockwise around right-hand side puncture), b (a counter-clockwise around left-hand side puncture) and their inverses  $a^{-1} = A$  and  $b^{-1} = B$ . For details, please refer to [4].

The periodic orbits can be divided into different classes on basis of their geometric and algebraic symmetries [1]. There are two types of geometric symmetries in the shape space: (I) the reflection symmetries of two orthogonal axes — the equator and the zeroth meridian passing through the "far" collision point; and (II) a central reflection symmetry

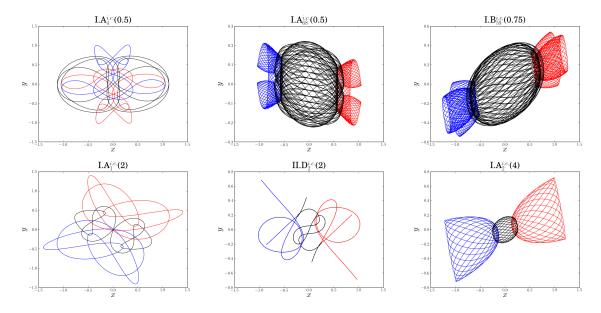


FIG. 1. (color online.) Brief overview of the six newly-found families of periodic three-body orbits.

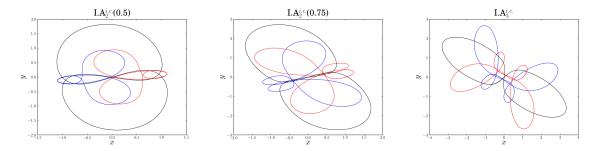


FIG. 2. (color online.) The periodic orbits with the same free group element for  $m_3 = 0.5, 0.75$  and 1.

about one point — the intersection of the equator and the aforementioned zeroth meridian. Besides, Suvakov and Dmitrašinović [1] mentioned three types of algebraic exchange symmetries for the free group elements: (A) the free group elements are symmetric with  $a \leftrightarrow A$  and  $b \leftrightarrow B$ , (B) free group elements are symmetric with  $a \leftrightarrow b$  and  $A \leftrightarrow B$ , and (C) free group elements are not symmetric under either (A) or (B). However, in this paper, we find a new algebraic symmetry class (D) with free group words symmetric under  $a \leftrightarrow B$  and  $b \leftrightarrow A$ . And then (C) can be regarded as free group words are not symmetric under one of (A), (B) and (D). We observe that the algebraic symmetry class (D) always corresponds to the geometric class (II) for present orbits.

Within the period  $T \in [0, 200]$ , we find 565, 401, 237, 85, 35, 17 and 9 families of periodic orbits in case of  $m_3 = 0.5, 0.75, 2, 4, 5, 8$  and 10, respectively. Among these 1349 families of periodic orbits, 1223 families of periodic orbits are entirely new, to the best of our knowledge. These 1349 families of the periodic collisionless orbits can be divided into seven classes: I.A, I.B, I.C, II.A, II.B, II.C amd II.D, as listed in Table S I-LXXXI in Supplementary material [28]. The initial velocities of the periodic orbits are listed in Tables S I-XXIX in [28]. The free group elements of the 1349 families are listed in Table S XXIX-LXXXI in [28]. Due to the limited length, only six new families are listed in Table I, and their orbits in real space are shown in FIG. 1. Note that Doedel et al. [11] found periodic orbits which have the same topology with the figure-eight as the mass of one body is varied. Here we also find that some periodic orbits have the same free group element for different  $m_3$ . For instance, as shown in FIG. 2, the periodic orbits have the same free group element (BaBabAbA) for  $m_3 = 0.5, 0.75$  and 1. The movies of these periodic orbits are given on the website: http://numericaltank.sjtu.edu.cn/three-body/three-body-unequal-mass.htm.

As mentioned by Li and Liao [7], the scale-invariant average period  $\bar{T}^* = \bar{T}|E|^{3/2}$  is approximately equal to a universal constant in case of  $m_3 = 1$ , i.e.  $\bar{T}^* \approx 2.433 \pm 0.075$ , with the definition of the average period  $\bar{T} = T/L_f$ , where  $L_f$  is the length of free group element of periodic orbit of a three-body system. Here the scale-invariant average period is equal to 1.14, 1.77, 2.43, 5.39, 11.53, 14.64, 23.99 and 30.25 for  $m_3 = 0.5, 0.75, 2, 4, 5, 8$  and 10, respectively. It is found that the scale-invariant average period agree well with the formula  $\bar{T}^* = 3.074m_3 - 0.617$  as shown in

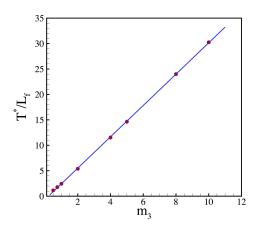


FIG. 3. (color online). The scale-invariant average period  $T^*/L_f = T|E|^{3/2}/L_f$  versus  $m_3$ . Symbols: computed results; line:  $T^*/L_f = 3.074m_3 - 0.617$ .

FIG. 3. The standard deviation is  $\sigma = 0.135$ . It is suggested that the scale-invariant average period linearly increases with  $m_3$ .

In this paper, we totally find 1349 families of Newtonian periodic planar three-body orbits with unequal mass and zero angular momentum and the initial conditions in case of isosceles collinear configurations. These 1349 families of the periodic collisionless orbits can be divided into seven classes according to their geometric and algebraic symmetries. Among these 1349 families, 1223 families are entirely new, to the best of our knowledge. Furthermore, some periodic orbits have the same free group element for different  $m_3$ . It is found that the scale-invariant average period linearly increases with  $m_3$ . It is suggested that the masses of bodies may play an important role in periodic three-body systems.

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## Supplementary information for "The 1223 new periodic orbits of planar three-body problem with unequal mass and zero angular momentum"

Table S. I. Initial conditions and periods T of the periodic three-body orbits for class I.A in the case of  $\mathbf{r}_1(0) = (-1,0) = -\mathbf{r}_2(0)$ ,  $\dot{\mathbf{r}}_1(0) = (v_1,v_2) = \dot{\mathbf{r}}_2(0)$  and  $\mathbf{r}_3(0) = (0,0)$ ,  $\dot{\mathbf{r}}_3(0) = (-2v_1/m_3, -2v_2/m_3)$  when G = 1 and  $m_1 = m_2 = 1$  and  $m_3 = 0.5$  by means of the search grid  $4000 \times 4000$  in the interval  $T_0 \in [0,200]$ , where  $T^* = T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

Class and number	$v_1$	$v_2$	T	$T^*$	$L_f$
$I.A_1^{i.c.}(0.5)$	0.2869236336	0.0791847624	4.1761292190	4.538	4
$I.A_2^{i.c.}(0.5)$	0.3420307307	0.1809369236	13.9153339459	9.063	8
$I.A_3^{i.c.}(0.5)$	0.3697718457	0.1910065395	25.9441952945	13.095	12
$I.A_4^{i.c.}(0.5)$	0.2009656237	0.2431076328	19.0134164290	19.086	16
$I.A_5^{i.c.}(0.5)$	0.2613236072	0.2356235235	28.4358575383	23.513	20
$I.A_6^{i.c.}(0.5)$	0.1908428490	0.1150772110	15.9682350284	22.361	20
$I.A_7^{i.c.}(0.5)$	0.1579313682	0.0949852732	14.5766076405	22.363	20
$I.A_8^{i.c.}(0.5)$	0.0979965852	0.0369408875	15.6059191780	27.112	24
$I.A_9^{i.c.}(0.5)$	0.3589116510	0.0578397225	35.2777168591	27.120	24
$I.A_{10}^{i.c.}(0.5)$	0.2066204352	0.1123859298	22.8770013381	30.956	28
$I.A_{11}^{i.c.}(0.5)$	0.3095805649	0.1012188182	37.8353981553	36.122	32
$I.A_{12}^{i.c.}(0.5)$	0.2935606362	0.2168613674	54.5846159117	41.571	36
$I.A_{13}^{i.c.}(0.5)$	0.2614113685	0.1097599351	39.2849176561	45.204	40
$I.A_{14}^{i.c.}(0.5)$	0.3049866810	0.0979042378	46.1065937257	45.210	40
$I.A_{15}^{i.c.}(0.5)$	0.1644199050	0.0637816144	29.0215071279	45.244	40
$I.A_{16}^{i.c.}(0.5)$	0.2698142826	0.0360688014	37.8687787781	45.458	40
$I.A_{17}^{i.c.}(0.5)$	0.1451647294	0.0318334148	30.5079373557	49.973	44
$I.A_{18}^{i.c.}(0.5)$	0.3467747647	0.0474429378	59.7722919460	49.973	44
$I.A_{19}^{i.c.}(0.5)$	0.3025694869	0.0951546278	54.5401904272	54.294	48
$I.A_{20}^{i.c.}(0.5)$	0.2726720005	0.0478754379	46.2148464304	54.543	48
$I.A_{21}^{\tilde{i.c.}}(0.5)$	0.2997637007	0.0934329270	62.7105115603	63.376	56
$I.A_{22}^{\tilde{i.c.}}(0.5)$	0.2747511246	0.0544869553	54.5744279508	63.626	56
$I.A_{23}^{\tilde{i.c.}}(0.5)$	0.2867479329	0.0521752523	57.0633556930	63.626	56
$I.A_{24}^{\overline{i.c.}}(0.5)$	0.2172290935	0.0383448898	45.1666009592	63.631	56
$I.A_{25}^{\tilde{i.c.}}(0.5)$	0.3108794721	0.1023369865	71.4799545005	67.698	60
$I.A_{26}^{\tilde{i.c.}}(0.5)$	0.2979925625	0.0918951185	70.9842467059	72.456	64
$I.A_{27}^{\tilde{i.c.}}(0.5)$	0.2366779591	0.0914177522	56.6833946453	72.485	64
$I.A_{28}^{\tilde{i.c.}}(0.5)$	0.1628551705	0.0589464762	46.2097799724	72.484	64
$I.A_{29}^{\tilde{i.c.}}(0.5)$	0.2763361520	0.0588302447	62.9447137981	72.706	64
$I.A_{30}^{\tilde{i.c.}}(0.5)$	0.1936757357	0.0730232621	49.7181917085	72.488	64
$I.A_{31}^{\tilde{i.c.}}(0.5)$	0.3017504100	0.1030778699	77.7653686390	76.789	68
$I.A_{32}^{i.c.}(0.5)$	0.1671144104	0.0438815944	49.1675240282	77.185	68
$I.A_{33}^{i.c.}(0.5)$	0.3274705985	0.0612651208	84.0143824473	77.185	68
$I.A_{34}^{i.c.}(0.5)$	0.2668455153	0.0138391891	63.2419174415	77.282	68
$I.A_{35}^{i.c.}(0.5)$	0.3220251063	0.0754954232	87.6821712800	81.574	72
$I.A_{36}^{i.c.}(0.5)$	0.2965579937	0.0906370328	79.2439520137	81.536	72
$I.A_{37}^{i.c.}(0.5)$	0.2775882955	0.0619333069	71.3240509215	81.786	72
$I.A_{38}^{i.c.}(0.5)$	0.3558062278	0.0405108521	108.4971611336	86.349	76
$I.A_{39}^{i.c.}(0.5)$	0.3060017590	0.0986219478	88.0890690057	85.876	76
$I.A_{40}^{i.c.}(0.5)$	0.2689229383	0.0312527426	71.5697332821	86.372	76
$I.A_{41}^{i.c.}(0.5)$	0.1317126561	0.0254909293	51.5736578935	86.350	76
$I.A_{42}^{i.c.}(0.5)$	0.1428972736	0.0445901978	55.4783856796	90.717	80
$I.A_{43}^{i.c.}(0.5)$	0.3132151994	0.1046181562	96.6632360131	90.178	80
$I.A_{44}^{i.c.}(0.5)$	0.2954964679	0.0895434067	87.5354654697	90.615	80
$I.A_{45}^{i.c.}(0.5)$	0.2749526022	0.0648656500	78.6571356819	90.866	80

Table S. II. Initial conditions and periods T of the periodic three-body orbits for class I.A in the case of  $\mathbf{r}_1(0) = (-1,0) = -\mathbf{r}_2(0)$ ,  $\dot{\mathbf{r}}_1(0) = (v_1,v_2) = \dot{\mathbf{r}}_2(0)$  and  $\mathbf{r}_3(0) = (0,0)$ ,  $\dot{\mathbf{r}}_3(0) = (-2v_1/m_3, -2v_2/m_3)$  when G=1 and  $m_1=m_2=1$  and  $m_3=0.5$  by means of the search grid  $4000 \times 4000$  in the interval  $T_0 \in [0,200]$ , where  $T^* = T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

Class and number	$v_1$	$v_2$	T	$T^*$	$L_f$
$I.A_{46}^{i.c.}(0.5)$	0.2153858894	0.0499108529	64.5903822323	90.874	80
$I.A_{47}^{i.c.}(0.5)$	0.2770512888	0.1061164594	87.0243497655	94.961	84
$I.A_{48}^{i.c.}(0.5)$	0.3213655238	0.0897415420	103.6411578855	94.961	84
$I.A_{49}^{i.c.}(0.5)$	0.2706252379	0.0398693716	79.9076627886	95.459	84
$I.A_{50}^{i.c.}(0.5)$	0.2786174957	0.2292777524	131.7357122012	103.058	88
$I.A_{51}^{i.c.}(0.5)$	0.2469591661	0.0904031712	80.4191164038	99.719	88
$I.A_{52}^{i.c.}(0.5)$	0.3228250487	0.0968450484	110.3291503458	99.273	88
$I.A_{53}^{i.c.}(0.5)$	0.2954791318	0.1096218279	98.7725671166	99.273	88
$I.A_{54}^{i.c.}(0.5)$	0.2945920946	0.0886163420	95.8143486677	99.694	88
$I.A_{55}^{i.c.}(0.5)$	0.1945402732	0.0706830008	68.4003313978	99.727	88
$I.A_{56}^{i.c.}(0.5)$	0.1889789414	0.0744407920	70.7522226916	104.113	92
$I.A_{57}^{i.c.}(0.5)$	0.3011607127	0.0965243843	104.0576058514	104.046	92
$I.A_{58}^{i.c.}(0.5)$	0.2938398485	0.0878089007	104.0941488385	108.772	96
$I.A_{59}^{i.c.}(0.5)$	0.3323949017	0.0892628147	125.2963737946	108.361	96
$I.A_{60}^{i.c.}(0.5)$	0.1303739201	0.0110880478	64.8930053804	109.158	96
$I.A_{61}^{i.c.}(0.5)$	0.3388203286	0.0534486578	130.4329576227	113.548	100
$I.A_{62}^{i.c.}(0.5)$	0.1528319560	0.0357128745	70.2840899540	113.548	100
$I.A_{63}^{i.c.}(0.5)$	0.2931864677	0.0871031920	112.3665360659	117.850	104
$I.A_{64}^{i.c.}(0.5)$	0.2712558612	0.0702892333	101.3437101345	118.101	104
$I.A_{65}^{i.c.}(0.5)$	0.3227971415	0.0926025930	129.6770893175	117.455	104
$I.A_{66}^{i.c.}(0.5)$	0.2862860474	0.1079517198	112.0449969617	117.455	104
$I.A_{67}^{i.c.}(0.5)$	0.3078743981	0.0999722100	121.7962271737	117.455	104
$I.A_{68}^{i.c.}(0.5)$	0.2138410831	0.0542938396	83.8472907647	118.112	104
$I.A_{69}^{i.c.}(0.5)$	0.2672229781	0.0183673134	96.9436256746	118.196	104
$I.A_{70}^{i.c.}(0.5)$	0.2762141023	0.1007012568	110.7531694029	122.210	108
$I.A_{71}^{i.c.}(0.5)$	0.3151460645	0.0881775189	129.0111010994	122.210	108
$I.A_{72}^{i.c.}(0.5)$	0.2816347029	0.1165460881	115.5464074541 82.3194421170	$121.753 \\ 122.548$	108
$I.A_{73}^{i.c.}(0.5)$ $I.A_{74}^{i.c.}(0.5)$	0.1897313629	0.0569798644 $0.0783479856$			108 108
$I.A_{74} (0.5)$ $I.A_{75}^{i.c.} (0.5)$	$\begin{array}{c} 0.2365156031 \\ 0.1892308321 \end{array}$	0.0763479630	94.4496491279 85.7557681263	$122.548 \\ 126.964$	112
$I.A_{75}^{i.c.}(0.5)$ $I.A_{76}^{i.c.}(0.5)$	0.3167473355	0.0564409230	130.6376396627	120.904	112
$I.A_{76}^{i.c.}(0.5)$ $I.A_{77}^{i.c.}(0.5)$	0.3063566174	0.0988758502	130.0686378697	126.543	112
$I.A_{78}^{i.c.}(0.5)$ $I.A_{78}^{i.c.}(0.5)$	0.2686058325	0.0293186861	105.2715049896	127.286	112
$I.A_{79}^{i.c.}(0.5)$ $I.A_{79}^{i.c.}(0.5)$	0.3115828668	0.1029439863	138.7657196429	130.849	116
$I.A_{80}^{i.c.}(0.5)$	0.1926049152	0.0732402826	89.8940701833	131.356	116
$I.A_{81}^{i.c.}(0.5)$	0.2678790398	0.0910268984	117.9225138007	136.004	120
$I.A_{82}^{i.c.}(0.5)$	0.2769745628	0.1073466261	124.4856938122	135.627	120
$I.A_{83}^{i.c.}(0.5)$	0.2817808026	0.0707431739	121.7428951384	136.256	120
$I.A_{84}^{i.c.}(0.5)$	0.2887331369	0.1103223170	135.3850884074	139.941	124
$I.A_{85}^{i.c.}(0.5)$	0.2820080025	0.0568015273	124.4362856628	140.873	124
$I.A_{86}^{i.c.}(0.5)$	0.1635241804	0.0417506200	88.9883269271	140.761	124
$I.A_{87}^{i.c.}(0.5)$	0.2167821354	0.0426860123	100.0973395584	140.885	124
$I.A_{88}^{i.c.}(0.5)$	0.3111417615	0.0939330679	151.0501281052	144.714	128
$I.A_{89}^{\stackrel{\circ}{i.c.}}(0.5)$	0.2821974598	0.0715109410	130.1650998679	145.334	128
$I.A_{90}^{i.c.}(0.5)$	0.2277884543	0.0646616649	107.9084014726	145.349	128
$I.A_{91}^{i.c.}(0.5)$	0.2708795826	0.0409737605	121.9472637204	145.460	128
$I.A_{92}^{i.c.}(0.5)$	0.2744503104	0.0976059036	133.9471506576	149.452	132
$I.A_{93}^{i.c.}(0.5)$	0.3114904704	0.0864935535	154.6502149192	149.452	132
$I.A_{94}^{i.c.}(0.5)$	0.1794922768	0.0605219047	101.6064275160	154.197	136
$I.A_{95}^{i.c.}(0.5)$	0.3128476865	0.1042021457	163.9473666361	153.329	136
$I.A_{96}^{i.c.}(0.5)$	0.2718250586	0.0447887187	130.2931785836	154.545	136
$I.A_{97}^{i.c.}(0.5)$	0.2773004224	0.0612425070	138.4568988662	159.033	140
$I.A_{98}^{i.c.}(0.5)$	0.1882207909	0.0737375490	110.5070750985	162.981	144
$I.A_{99}^{i.c.}(0.5)$	0.2829061443	0.0727791246	147.0223286431	163.488	144
$I.A_{100}^{i.c.}(0.5)$	0.2176616157	0.0285927952	115.8358005700	163.643	144

Table S. III. Initial conditions and periods T of the periodic three-body orbits for class I.A in the case of  $\mathbf{r}_1(0) = (-1,0) = -\mathbf{r}_2(0)$ ,  $\dot{\mathbf{r}}_1(0) = (v_1, v_2) = \dot{\mathbf{r}}_2(0)$  and  $\mathbf{r}_3(0) = (0,0)$ ,  $\dot{\mathbf{r}}_3(0) = (-2v_1/m_3, -2v_2/m_3)$  when G = 1 and  $m_1 = m_2 = 1$  and  $m_3 = 0.5$  by means of the search grid  $4000 \times 4000$  in the interval  $T_0 \in [0, 200]$ , where  $T^* = T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

Class and number	$v_1$	$v_2$	T	$T^*$	$L_f$
$I.A_{101}^{i.c.}(0.5)$	0.3242506160	0.0631949983	180.1794835948	167.981	148
$I.A_{102}^{i.c.}(0.5)$	0.2684432435	0.0282692084	138.9734917593	168.200	148
$I.A_{103}^{\overline{i.c.}}(0.5)$	0.2778616536	0.0625774643	146.8402992438	168.113	148
$I.A_{104}^{i.c.}(0.5)$	0.3051413550	0.0931938899	174.3071401892	171.964	152
$I.A_{105}^{i.c.}(0.5)$ $I.A_{106}^{i.c.}(0.5)$	0.1629885654	0.0522640671	109.4669718191	172.377	152
$I.A_{106}^{i.c.}(0.5)$	0.1462535243	0.0217299404	105.3228173310	172.730	152
$I.A_{107}^{i.c.}(0.5)$	0.2906493900	0.0841512530	161.9536048399	172.315	152
$I.A_{108}^{i.c.}(0.5)$	0.3307845107	0.0282510145	186.8989238269	172.762	152
$I.A_{109}^{i.c.}(0.5)$	0.2734339548	0.0504407955	147.0001395753	172.712	152
$I.A_{110}^{i.c.}(0.5)$	0.1864663107	0.0314460085	116.6973116737	177.222	156
$I.A_{111}^{i.c.}(0.5)$	0.3054857593	0.0982549763	180.3047709490	176.296	156
$I.A_{112}^{i.c.}(0.5)$	0.1556984762	0.0375406067	110.2475386504	177.124	156
$I.A_{113}^{i.c.}(0.5)$	0.2693793982	0.0338185849	147.3066947737	177.288	156
$I.A_{113}^{i.c.}(0.5)$ $I.A_{114}^{i.c.}(0.5)$	0.1759573432	0.0582869204	118.5242718315	181.429	160
$I.A_{115}^{i.c.}(0.5)$	0.3063956054	0.0665241524	179.3796751444	181.667	160
$I.A_{115}^{i.16}(0.5)$ $I.A_{116}^{i.c.}(0.5)$ $I.A_{117}^{i.c.}(0.5)$	0.1844792914	0.0664186417	124.2585706817	185.835	164
$I.A_{117}^{i.c.}(0.5)$	0.2788305752	0.0647737865	163.6184241261	186.271	164
$I.A_{118}^{i.c.}(0.5)$	0.3181202953	0.0262108895	189.0552591547	186.385	164
$I.A_{119}^{i.c.}(0.5)$	0.2760366688	0.1001106086	172.0404786725	190.126	168
$I.A_{120}^{\overline{i.c.}}(0.5)$	0.2901045319	0.0834748902	178.4639514916	190.469	168
$I.A_{121}^{\overline{i.c.}}(0.5)$	0.2710039691	0.0415005602	163.9870417061	195.461	172
$I.A_{122}^{\overline{i.c.}}(0.5)$	0.2792522152	0.0656898524	172.0127626343	195.350	172
$I.A_{123}^{\overline{i.c.}}(0.5)$	0.2797022972	0.0856462692	179.7442254623	199.546	176
$I.A_{124}^{i.c.}(0.5)$	0.1897491024	0.0464972761	133.3930483814	199.814	176
$I.A_{125}^{i.c.}(0.5)$	0.2955617289	0.0815450298	190.9002964594	199.546	176
$I.A_{126}^{i.c.}(0.5)$	0.2991146914	0.0928880688	196.3997389850	199.208	176
$I.A_{127}^{i.c.}(0.5)$	0.2753246510	0.0561181124	172.0902983279	199.958	176
$I.A_{127}^{i.c.}(0.5)$ $I.A_{128}^{i.c.}(0.5)$	0.1608928391	0.0423566492	128.6102391540	204.338	180
$I.A_{129}^{i.c.}(0.5)$	0.2858473085	0.1123019709	194.7380952367	203.092	180
$I.A_{130}^{i.c.}(0.5)$	0.1632254337	0.0372831480	128.8601185998	204.338	180
$I.A_{131}^{i.c.}(0.5)$	0.2203920025	0.0720677540	149.5147366218	204.249	180
$I.A_{132}^{i.c.}(0.5)$	0.1747313962	0.0572750530	135.8934495248	208.659	184
$I.A_{133}^{i.c.}(0.5)$	0.2841883501	0.0749607778	189.2225836059	208.874	184
$I.A_{134}^{i.c.}(0.5)$	0.2799959773	0.0672515431	188.8112079689	213.506	188
$I.A_{135}^{i.c.}(0.5)$	0.1477960023	0.0331559448	130.9407612438	213.493	188
$I.A_{136}^{i.c.}(0.5)$	0.2843861676	0.0752851658	197.6692863336	217.951	192
$I.A_{137}^{i.c.}(0.5)$	0.2166248717	0.0438265532	155.0134625948	218.138	192
$I.A_{138}^{i.c.}(0.5)$	0.2141903816	0.0534864829	158.0856536626	222.606	196
$I.A_{139}^{i.c.}(0.5)$	0.2550856649	0.0780342905	185.5037127808	227.045	200
$I.A_{140}^{i.c.}(0.5)$	0.2027586331	0.0547976409	156.8933680291	227.054	200
$I.A_{141}^{i.c.}(0.5)$	0.2735238924	0.0507318572	197.3931765300	231.797	204
$I.A_{142}^{i.c.}(0.5)$	0.2174237597	0.0354281968	170.8625404977	240.899	212
$I.A_{143}^{i.c.}(0.5)$	0.1702863136	0.0542463019	163.7282370536	254.054	224
$I.A_{144}^{i.c.}(0.5)$	0.2114296338	0.0633519827	183.7049132401	258.716	228
$I.A_{145}^{i.c.}(0.5)$	0.2040531625	0.0308354022	183.6432714881	268.146	236
$I.A_{146}^{i.c.}(0.5)$	0.2128981089	0.0560440442	196.4519555242	277.080	244
$I.A_{147}^{i.c.}(0.5)$	0.1914804176	0.0167542245	198.8845878317	300.002	264

Table S. IV. Initial conditions and periods T of the periodic three-body orbits for class I.B in the case of  $\mathbf{r}_1(0) = (-1,0) = -\mathbf{r}_2(0)$ ,  $\dot{\mathbf{r}}_1(0) = (v_1, v_2) = \dot{\mathbf{r}}_2(0)$  and  $\mathbf{r}_3(0) = (0,0)$ ,  $\dot{\mathbf{r}}_3(0) = (-2v_1/m_3, -2v_2/m_3)$  when G = 1 and  $m_1 = m_2 = 1$  and  $m_3 = 0.5$  by means of the search grid  $4000 \times 4000$  in the interval  $T_0 \in [0, 200]$ , where  $T^* = T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$egin{array}{c} L_f \ 6 \ 10 \ 14 \ 14 \ 14 \ \end{array}$
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$\begin{array}{cccccccccccccccccccccccccccccccccccc$	14
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	
	1.4
	14
10.TO	18
$I.B_7^{i.c.}(0.5)$ 0.2674226718 0.2139289499 23.9372167355 20.903	18
$I.B_8^{i.c.}(0.5)$ 0.2878430093 0.3151477978 42.2778567687 19.118	18
$I.B_9^{i.c.}(0.5)$ 0.3030963188 0.0966599779 25.1036320618 24.876	22
$I.B_{10}^{i.c.}(0.5)$ 0.1099852485 0.0308448543 14.5241562996 24.961	22
$I.B_{11}^{i.c.}(0.5)$ 0.2291294485 0.2119828182 24.7437423714 25.221	22
$I.B_{12}^{i.c.}(0.5)$ 0.3692649167 0.0417694147 34.2771859316 24.965	22
$I.B_{13}^{i.c.}(0.5)$ 0.2737871583 0.0515706441 25.1965811355 29.542	26
$I.B_{14}^{i.c.}(0.5)$ 0.2988140236 0.0926281921 33.4227154398 33.958	30
$1.8_{15}^{i.c.}(0.5)$ $0.2788426894$ $0.0601828236$ $33.7944313342$ $38.623$	34
$I.B_{16}^{i.c.}(0.5)$ 0.2790832581 0.2276230751 50.6192127242 39.773	34
$I.B_{17}^{i.c.}(0.5)$ $0.2163072511$ $0.0457824324$ $27.4550707427$ $38.627$	34
$I.B_{18}^{i.c.}(0.5)$ 0.2631918196 0.0971022505 36.9742814503 43.036	38
$I.B_{19}^{i.c.}(0.5)$ 0.1940565085 0.0716923953 29.5220753291 43.054	38
$I.B_{20}^{i.c.}(0.5)$ 0.2790460219 0.0652447476 41.9537927832 47.703	42
$I.B_{21}^{\overline{5.c.}}(0.5)$ 0.1166234512 0.0062473070 27.8908904718 47.784	42
$I.B_{22}^{\vec{i.c.}}(0.5)$ 0.2941922385 0.0882015308 49.9756827423 52.116	46
$I.B_{23}^{\overline{i.c.}}(0.5)$ 0.1482620649 0.0469161409 32.2082149709 52.162	46
$I.B_{24}^{\overline{5.c.}}(0.5)$ 0.3383350929 0.0633820096 60.3349978637 52.165	46
$1.8_{25}^{\tilde{i.c.}}(0.5)$ 0.3086986007 0.1005760508 58.8260422524 56.455	50
$I.B_{26}^{\overline{5.c.}}(0.5)$ 0.2804820502 0.0682367913 50.3545241044 56.781	50
$I.B_{27}^{\bar{i.c.}}(0.5)$ 0.2928957198 0.0867806393 58.2512450352 61.194	54
$I.B_{28}^{\bar{i.c.}}(0.5)$ 0.3121801357 0.0954940881 68.9135784414 65.543	58
$1.\overline{B}_{29}^{\overline{i.c.}}(0.5)$ 0.2815530004 0.0703169410 58.7667019161 65.859	58
$I.B_{30}^{i.c.}(0.5)$ 0.2702462061 0.0720377706 56.4003264775 65.859	58
$I.B_{31}^{i.c.}(0.5)$ 0.2131987197 0.0555522173 46.7195262030 65.865	58
$I.B_{32}^{i.c.}(0.5)$ 0.1832565230 0.0627595131 46.7521066510 70.290	62
$I.B_{33}^{i.c.}(0.5)$ $0.2919082649$ $0.0856563914$ $66.5195343482$ $70.272$	62
$I.B_{34}^{i.c.}(0.5)$ $0.2703631586$ $0.0386904791$ $58.8880393273$ $70.458$	62
$I.B_{35}^{i.c.}(0.5)$ $0.1859380756$ $0.0731374234$ $50.3601763743$ $74.679$	66
$1.B_{36}^{i.c.}(0.5)$ $0.2489094150$ $0.0992895950$ $61.2653969718$ $74.663$	66
$I.B_{37}^{i.c.}(0.5)$ $0.2823888686$ $0.0718580778$ $67.1889348357$ $74.936$	66
$I.B_{38}^{i.c.}(0.5)$ $0.1615485916$ $0.0277748182$ $46.9823741559$ $74.980$	66
$I.B_{39}^{i.c.}(0.5)$ 0.2911309221 0.0847384936 74.7822861358 79.349	70
$I.B_{40}^{i.c.}(0.5)$ $0.1771098445$ $0.0784887228$ $52.5734020100$ $79.040$	70
$I.B_{41}^{i.c.}(0.5)$ 0.1838565013 0.0752899792 53.1629113586 79.040	70
$I.B_{42}^{i.c.}(0.5)$ 0.2143486812 0.0918266864 58.0738450139 79.038	70
$I.B_{43}^{i.c.}(0.5)$ 0.3247862862 0.0695897171 85.9588123017 79.387	70
$I.B_{44}^{i.c.}(0.5)$ 0.1610459823 0.0516375322 50.2179896145 79.382	70
$I.B_{45}^{i.c.}(0.5)$ 0.2722601340 0.0464077813 67.2341769532 79.543	70
$I.B_{46}^{i.c.}(0.5)$ 0.3192498609 0.0879575434 90.1594809343 83.709	74
$I.B_{47}^{i.c.}(0.5)$ 0.2830620767 0.0730518241 75.6195474605 84.014	74
$I.B_{48}^{i.c.}(0.5)$ 0.2905035826 0.0839717300 83.0409561905 88.427	78
$I.B_{49}^{i.c.}(0.5)$ 0.3002585565 0.1059795568 88.9553085721 88.032	78
$I.B_{50}^{i.c.}(0.5)$ 0.2174816082 0.0342966208 62.8453538957 88.634	78

Table S. V. Initial conditions and periods T of the periodic three-body orbits for class I.B in the case of  $\mathbf{r}_1(0) = (-1,0) = -\mathbf{r}_2(0)$ ,  $\dot{\mathbf{r}}_1(0) = (v_1,v_2) = \dot{\mathbf{r}}_2(0)$  and  $\mathbf{r}_3(0) = (0,0)$ ,  $\dot{\mathbf{r}}_3(0) = (-2v_1/m_3, -2v_2/m_3)$  when G=1 and  $m_1=m_2=1$  and  $m_3=0.5$  by means of the search grid  $4000 \times 4000$  in the interval  $T_0 \in [0,200]$ , where  $T^* = T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

Class and number	$v_1$	$v_2$	T	$T^*$	$L_f$
$\overline{\text{I.B}_{51}^{i.c.}(0.5)}$	0.2913964797	0.0479668990	80.7592905815	88.627	78
$I.B_{52}^{i.c.}(0.5)$	0.2960647431	0.0951352334	90.5465691204	92.793	82
$I.B_{53}^{i.c.}(0.5)$	0.2836170353	0.0740064147	84.0569632087	93.091	82
$I.B_{54}^{i.c.}(0.5)$	0.2679928311	0.0763276805	79.4113786013	93.090	82
$I.B_{55}^{i.c.}(0.5)$	0.2899831050	0.0833218568	91.2952846897	97.504	86
$I.B_{56}^{i.c.}(0.5)$	0.3080355719	0.1000897653	100.8065558913	97.122	86
$I.B_{57}^{i.c.}(0.5)$	0.1751508803	0.0576966004	63.5831421300	97.522	86
$I.B_{58}^{i.c.}(0.5)$	0.2341908442	0.0837260648	75.0584041949	97.525	86
$I.B_{59}^{i.c.}(0.5)$	0.2750441739	0.0553294312	83.9531808683	97.709	86
$I.B_{60}^{i.c.}(0.5)$	0.3442855217	0.0202360484	113.5049294912	97.768	86
$I.B_{61}^{i.c.}(0.5)$	0.3138785714	0.1054199562	109.2112794427	101.417	90
$I.B_{62}^{i.c.}(0.5)$	0.2840833689	0.0747873466	92.4999175257	102.168	90
$I.B_{63}^{\tilde{i.c.}}(0.5)$	0.1918308588	0.0726163450	69.5933264452	101.922	90
$I.B_{64}^{i.c.}(0.5)$	0.2967400478	0.0807114885	102.3821088719	106.581	94
$I.B_{65}^{i.c.}(0.5)$	0.1671740666	0.0535233977	68.2728344880	106.607	94
$I.B_{66}^{i.c.}(0.5)$	0.3062135700	0.0987733794	109.0789742536	106.210	94
$I.B_{67}^{i.c.}(0.5)$	0.2852343987	0.1063718482	100.6294915837	106.209	94
$I.B_{68}^{i.c.}(0.5)$	0.3169586307	0.0723491117	111.3358326383	106.613	94
$I.B_{69}^{i.c.}(0.5)$	0.2760984711	0.0582105683	92.3235074759	106.790	94
$I.B_{70}^{i.c.}(0.5)$	0.2977432358	0.0916649424	108.5470086473	110.955	98
$I.B_{71}^{i.c.}(0.5)$	0.2844794178	0.0754369919	100.9465701326	111.245	98
$I.B_{72}^{i.c.}(0.5)$	0.3249845265	0.0202864165	116.6138500488	111.383	98
$I.B_{73}^{i.c.}(0.5)$	0.2694680508	0.0342913622	92.5876783108	111.373	98
$I.B_{74}^{i.c.}(0.5)$	0.2891747738	0.0822826126	107.7974854583	115.658	102
$I.B_{75}^{i.c.}(0.5)$	0.2885302862	0.0933767620	113.1508640370	120.034	106
$I.B_{76}^{i.c.}(0.5)$	0.2666426613	0.0786440441	102.3898928785	120.321	106
$I.B_{77}^{i.c.}(0.5)$	0.2848187359	0.0759836961	109.3953593575	120.322	106
$I.B_{78}^{i.c.}(0.5)$	0.1977132984	0.0525053406	82.0586018240	120.336	106
$I.B_{79}^{i.c.}(0.5)$	0.2707787504	0.0405404036	100.9274275269	120.459	106
$I.B_{80}^{i.c.}(0.5)$	0.2449409545	0.0850729560	99.3632704793	124.756	110
$I.B_{81}^{i.c.}(0.5)$	0.1773510110	0.0248360734	80.6083002912	124.987	110
$I.B_{82}^{i.c.}(0.5)$	0.2888572586	0.0818643534	116.0480068229	124.734	110
$I.B_{83}^{\tilde{i.c.}}(0.5)$	0.2181512383	0.0833244460	94.9920652210	129.161	114
$I.B_{84}^{i.c.}(0.5)$	0.2904092593	0.1083815356	125.0092177942	128.699	114
$I.B_{85}^{i.c.}(0.5)$	0.2851109699	0.0764474074	117.8446993799	129.398	114
$I.B_{86}^{i.c.}(0.5)$	0.2703553256	0.0851792459	116.2605246289	133.811	118
$I.B_{87}^{i.c.}(0.5)$	0.2985222840	0.0787345637	129.2429556733	133.811	118
$I.B_{88}^{i.c.}(0.5)$	0.3138600006	0.0908763661	140.5350980635	133.464	118
$I.B_{89}^{i.c.}(0.5)$	0.3135219744	0.1049833885	142.8584537333	132.992	118
$I.B_{90}^{i.c.}(0.5)$	0.3098615763	0.0844309289	141.5123187727	138.190	122
$I.B_{91}^{i.c.}(0.5)$	0.2853633746	0.0768427234	126.2931104761	138.475	122
$I.B_{92}^{i.c.}(0.5)$	0.2176335170	0.0299228620	98.1757759947	138.640	122
$I.B_{93}^{i.c.}(0.5)$	0.2883498729	0.0811835015	132.5546327228	142.888	126
$I.B_{94}^{i.c.}(0.5)$	0.1749022486	0.0766378437	94.0590542640	142.271	126
$I.B_{95}^{i.c.}(0.5)$	0.1636510704	0.0474589703	90.6558775234	142.963	126
$I.B_{96}^{i.c.}(0.5)$	0.2150995281	0.0509041474	101.7118032345	143.121	126
$I.B_{97}^{i.c.}(0.5)$	0.2822851529	0.0922191464	135.1262821798	147.271	130
$I.B_{98}^{i.c.}(0.5)$	0.2855816329	0.0771807489	134.7392792763	147.552	130
$I.B_{99}^{i.c.}(0.5)$	0.2947320378	0.0887615187	141.6521915390	147.271	130
$I.B_{100}^{i.c.}(0.5)$	0.3344339309	0.0263022352	162.8324323639	147.763	130
	0.001100000	0.0200022002	102.002 1020000	111100	100

Table S. VI. Initial conditions and periods T of the periodic three-body orbits for class I.B in the case of  $\mathbf{r}_1(0) = (-1,0) = -\mathbf{r}_2(0)$ ,  $\dot{\mathbf{r}}_1(0) = (v_1, v_2) = \dot{\mathbf{r}}_2(0)$  and  $\mathbf{r}_3(0) = (0,0)$ ,  $\dot{\mathbf{r}}_3(0) = (-2v_1/m_3, -2v_2/m_3)$  when G = 1 and  $m_1 = m_2 = 1$  and  $m_3 = 0.5$  by means of the search grid  $4000 \times 4000$  in the interval  $T_0 \in [0, 200]$ , where  $T^* = T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

Class and number	$v_1$	$v_2$	T	$T^*$	$L_f$
$I.B_{101}^{i.c.}(0.5)$	0.3258205442	0.0948987232	170.1648469173	151.183	134
$I.B_{102}^{i.c.}(0.5)$	0.2887071810	0.1116133434	146.5524883273	151.183	134
$I.B_{103}^{i.c.}(0.5)$	0.3136039168	0.0546949113	153.7578002255	152.216	134
$I.B_{104}^{i.c.}(0.5)$	0.2881475675	0.0809077918	140.8129334279	151.965	134
$I.B_{105}^{i.c.}(0.5)$	0.2690556508	0.0320231034	126.2887308493	152.287	134
$I.B_{106}^{i.c.}(0.5)$	0.2857703636	0.0774702399	143.1821020744	156.629	138
$I.B_{107}^{i.c.}(0.5)$	0.2845169706	0.0979582675	149.9521586362	160.710	142
$I.B_{108}^{i.c.}(0.5)$	0.3360387142	0.0408766303	180.7765714891	161.357	142
$I.B_{109}^{i.c.}(0.5)$	0.2667815506	0.0845599684	138.0065476191	161.041	142
$I.B_{110}^{i.c.}(0.5)$	0.2700536880	0.0372395731	134.6253858410	161.374	142
$I.B_{111}^{i.c.}(0.5)$	0.1620074707	0.0567511584	105.1701997563	165.454	146
$I.B_{112}^{i.c.}(0.5)$	0.2846001734	0.0542198926	147.7254345023	165.875	146
$I.B_{113}^{i.c.}(0.5)$	0.2937225183	0.0876854814	158.2087324044	165.427	146
$I.B_{114}^{i.c.}(0.5)$	0.2752695212	0.0559644171	142.7112029555	165.875	146
$I.B_{115}^{i.c.}(0.5)$	0.2170568347	0.0402768809	117.8057908660	165.889	146
$I.B_{116}^{i.c.}(0.5)$	0.3256765948	0.0919189282	189.6279855460	169.365	150
$I.B_{117}^{i.c.}(0.5)$	0.2878232371	0.0804604016	157.3437190277	170.118	150
$I.B_{118}^{i.c.}(0.5)$	0.2709508831	0.0412767654	142.9671399385	170.460	150
$I.B_{119}^{i.c.}(0.5)$	0.2325404803	0.0634100827	128.0692487359	170.359	150
$I.B_{120}^{i.c.}(0.5)$	0.2777928735	0.0911581231	157.0896063985	174.505	154
$I.B_{121}^{i.c.}(0.5)$	0.1884011033	0.0613519263	120.4032477307	179.229	158
$I.B_{122}^{i.c.}(0.5)$	0.2876940968	0.0802805105	165.6171419591	179.195	158
$I.B_{123}^{i.c.}(0.5)$	0.2981478673	0.0920364541	175.3905472876	178.871	158
$I.B_{124}^{i.c.}(0.5)$	0.2717607209	0.0445435233	151.3128521285	179.545	158
$I.B_{125}^{i.c.}(0.5)$	0.2694901377	0.0990271302	157.6461325706	178.868	158
$I.B_{126}^{i.c.}(0.5)$	0.2780311897	0.0798240174	163.4425288092	183.859	162
$I.B_{127}^{i.c.}(0.5)$	0.1535774570	0.0490036967	114.5530130654	183.703	162
$I.B_{128}^{i.c.}(0.5)$	0.3084553159	0.0430030307	191.9788446279	187.951	166
$I.B_{129}^{i.c.}(0.5)$	0.2819693119	0.0959873338	173.1019417858	187.951	166
$I.B_{130}^{i.c.}(0.5)$	0.2724981752	0.0472634419	159.6637122720	188.629	166
$I.B_{131}^{i.c.}(0.5)$	0.2812272902	0.0472054419	167.8845519750	188.499	166
$I.B_{132}^{i.c.}(0.5)$	0.2851403139	0.1114556037	183.1661242112	191.850	170
$I.B_{132}^{i.c.}(0.5)$ $I.B_{133}^{i.c.}(0.5)$	0.2925384369	0.0863782875	183.0243366435	192.661	170
$I.B_{134}^{i.c.}(0.5)$	0.1940041780	0.0442273745	132.9965797155	197.594	170
$I.B_{135}^{i.c.}(0.5)$	0.1940041780	0.0799909840	182.1812064952	197.349	174
	0.2743698810	0.0799909840	179.0226701835	201.738	174
$I.B_{136}^{i.c.}(0.5)$ $I.B_{137}^{i.c.}(0.5)$	0.2691006601	0.0969556708	180.8849024635	201.738	182
$1.D_{137}(0.5)$ $1.D_{i.c.}(0.5)$	0.2091000001 $0.1872185341$			206.678	182
$I.B_{138}^{i.c.}(0.5)$		0.0408400425	136.8385909602 182.9605968884		
$I.B_{139}^{i.c.}(0.5)$	0.2708877517	0.0813214333		211.089	186
$I.B_{140}^{i.c.}(0.5)$	0.2632104741	0.0837121394	182.1621606443	215.501	190
$I.B_{141}^{i.c.}(0.5)$	0.2821312739	0.0713901022	193.1416258963	215.731	190
$I.B_{142}^{i.c.}(0.5)$	0.2394605930	0.0614826122	168.8258165112	220.374	194
$I.B_{143}^{i.c.}(0.5)$	0.1924591719	0.0733547400	150.2890500922	219.658	194
$I.B_{144}^{i.c.}(0.5)$	0.2124129905	0.0567064578	159.2830391611	224.832	198
$I.B_{145}^{i.c.}(0.5)$	0.2716732997	0.0442108364	193.3516822884	229.546	202
$I.B_{146}^{i.c.}(0.5)$	0.1784558095	0.0599818427	156.5079579799	238.104	210
$I.B_{147}^{i.c.}(0.5)$	0.2168487625	0.0421574292	172.7342330299	243.143	214
$I.B_{148}^{i.c.}(0.5)$	0.2116222333	0.0574328794	178.3098811885	252.068	222
$I.B_{149}^{i.c.}(0.5)$	0.1734501113	0.0584514203	172.5037487239	265.336	234
$I.B_{150}^{i.c.}(0.5)$	0.1773937613	0.0586759020	173.8818026019	265.336	234
$I.B_{151}^{i.c.}(0.5)$	0.1589938384	0.0147834329	168.2376674277	270.519	238
$I.B_{152}^{i.c.}(0.5)$	0.1732220167	0.0304907626	193.6476848963	302.170	266

Table S. VII. Initial conditions and periods T of the periodic three-body orbits for class II.A and II.B in the case of  $\mathbf{r}_1(0) = (-1,0) = -\mathbf{r}_2(0)$ ,  $\dot{\mathbf{r}}_1(0) = (v_1,v_2) = \dot{\mathbf{r}}_2(0)$  and  $\mathbf{r}_3(0) = (0,0)$ ,  $\dot{\mathbf{r}}_3(0) = (-2v_1/m_3, -2v_2/m_3)$  when G=1 and  $m_1=m_2=1$  and  $m_3=0.5$  by means of the search grid  $4000\times 4000$  in the interval  $T_0\in[0,200]$ , where  $T^*=T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

Class and number	$v_1$	$v_2$	T	$T^*$	$L_f$
$II.A_1^{i.c.}(0.5)$	0.3225427028	0.1048724883	30.3858430513	27.025	24
$II.A_2^{i.c.}(0.5)$	0.2131394477	0.2536523446	41.0020774214	38.035	32
$II.A_3^{i.c.}(0.5)$	0.2831992418	0.2242675073	71.8227958466	56.038	48
$II.A_4^{i.c.}(0.5)$	0.2890737487	0.1189680270	57.6132727750	58.603	52
$II.A_5^{i.c.}(0.5)$	0.1160519606	0.0374080642	37.3113944214	63.514	56
$II.A_6^{i.c.}(0.5)$	0.2814144757	0.0894379180	115.6439696541	126.928	112
$II.A_7^{i.c.}(0.5)$	0.2647972881	0.0999834687	180.6781521304	208.284	184
$II.B_1^{i.c.}(0.5)$	0.2522118365	0.2382777564	63.7808362694	54.281	46
$II.B_2^{i.c.}(0.5)$	0.2921124451	0.1099360087	117.3258510268	119.607	106
$II.B_3^{i.c.}(0.5)$	0.1843047650	0.0735504184	111.0333400751	165.168	146
$II.B_4^{i.c.}(0.5)$	0.3051194885	0.0996833215	191.8933811547	187.542	166

Table S. VIII. Initial conditions and periods T of the periodic three-body orbits for class II.C in the case of  $\mathbf{r}_1(0) = (-1,0) = -\mathbf{r}_2(0)$ ,  $\dot{\mathbf{r}}_1(0) = (v_1, v_2) = \dot{\mathbf{r}}_2(0)$  and  $\mathbf{r}_3(0) = (0,0)$ ,  $\dot{\mathbf{r}}_3(0) = (-2v_1/m_3, -2v_2/m_3)$  when G = 1 and  $m_1 = m_2 = 1$  and  $m_3 = 0.5$  by means of the search grid  $4000 \times 4000$  in the interval  $T_0 \in [0, 200]$ , where  $T^* = T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

Class and number	$v_1$	$v_2$	T	$T^*$	$L_f$
$II.C_1^{i.c.}(0.5)$	0.2057599772	0.2910772545	16.4482452694	13.225	10
$II.C_2^{i.c.}(0.5)$	0.0621756721	0.0261903906	6.2854133740	11.285	10
$II.C_3^{i.c.}(0.5)$	0.0657658390	0.1124034346	8.0617096205	13.572	12
$II.C_4^{i.c.}(0.5)$	0.0169747300	0.0752136850	8.8877784855	15.845	14
$II.C_5^{i.c.}(0.5)$	0.2251711660	0.3443496457	32.8998590701	17.385	14
$II.C_6^{i.c.}(0.5)$	0.1194732446	0.0207612630	10.6764792286	18.190	16
$II.C_7^{i.c.}(0.5)$	0.0551050358	0.3697557117	28.5843693576	20.500	16
$II.C_8^{i.c.}(0.5)$	0.0108740090	0.3192205525	25.7212160836	25.333	18
$II.C_9^{i.c.}(0.5)$	0.3448435595	0.0697188543	24.6216211722	20.364	18
$II.C_{10}^{i.c.}(0.5)$	0.0117243540	0.3525284661	28.5310959155	23.470	18
$II.C_{11}^{i.c.}(0.5)$	0.1780039242	0.2004919509	16.5294086907	20.135	18
$II.C_{12}^{i.c.}(0.5)$	0.1282291061	0.3649372977	40.6015240363	26.471	20
$II.C_{13}^{i.c.}(0.5)$	0.3234173131	0.2471156877	44.4546188773	24.471	22
$II.C_{14}^{i.c.}(0.5)$	0.2104677107	0.1057158539	18.2820987595	24.716	22
$II.C_{15}^{i.c.}(0.5)$	0.1983865989	0.1226004003	19.7881493182	26.930	24
$II.C_{16}^{i.c.}(0.5)$	0.2287935035	0.0923297799	22.5126057181	29.433	26
$II.C_{17}^{i.c.}(0.5)$	0.0913811473	0.0093493963	16.8086808175	29.586	26
$II.C_{18}^{i.c.}(0.5)$	0.2383747181	0.2447155547	35.0733137252	30.771	26
$II.C_{19}^{i.c.}(0.5)$	0.0992256924	0.0269686482	18.2257172127	31.728	28
$II.C_{20}^{i.c.}(0.5)$	0.3345991412	0.0565118924	35.7753676955	31.788	28
$II.C_{21}^{i.c.}(0.5)$	0.1490452956	0.0531097638	21.0786048660	33.979	30
$II.C_{22}^{i.c.}(0.5)$	0.2263132666	0.0992959988	25.8750904330	33.785	30
$II.C_{23}^{i.c.}(0.5)$	0.1822531374	0.1005779979	22.9574138909	33.376	30
$II.C_{24}^{i.c.}(0.5)$	0.3130939094	0.0872917851	35.4487192804	33.958	30
$II.C_{25}^{i.c.}(0.5)$	0.2379407604	0.2522685220	44.6369551897	38.031	32
$II.C_{26}^{i.c.}(0.5)$	0.1230951336	0.0496740170	21.5462119008	36.147	32
$II.C_{27}^{i.c.}(0.5)$	0.1326230917	0.0945397322	24.0011090101	38.376	34
$II.C_{28}^{i.c.}(0.5)$	0.1440416968	0.1072211552	24.7755961375	38.378	34
$II.C_{29}^{i.c.}(0.5)$	0.1444470268	0.0693298831	25.2083739245	40.495	36
$II.C_{30}^{i.c.}(0.5)$	0.3196418099	0.1040424691	47.3539072837	42.814	38

Table S. IX. Initial conditions and periods T of the periodic three-body orbits for class II.C in the case of  $\mathbf{r}_1(0) = (-1,0) = -\mathbf{r}_2(0)$ ,  $\dot{\mathbf{r}}_1(0) = (v_1, v_2) = \dot{\mathbf{r}}_2(0)$  and  $\mathbf{r}_3(0) = (0,0)$ ,  $\dot{\mathbf{r}}_3(0) = (-2v_1/m_3, -2v_2/m_3)$  when G = 1 and  $m_1 = m_2 = 1$  and  $m_3 = 0.5$  by means of the search grid  $4000 \times 4000$  in the interval  $T_0 \in [0, 200]$ , where  $T^* = T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

Class and number	$v_1$	$v_2$	T	$T^*$	$L_f$
$\overline{\text{II.C}_{31}^{i.c.}(0.5)}$	0.0987975586	0.0968340577	25.8114857482	42.954	38
$II.C_{32}^{i.c.}(0.5)$	0.2432511280	0.0916195182	34.3617337272	43.051	38
$II.C_{33}^{i.c.}(0.5)$	0.3382801410	0.0351009613	48.8084195841	43.188	38
$II.C_{34}^{i.c.}(0.5)$	0.0852972761	0.0255811336	24.1174290003	42.561	38
$II.C_{35}^{i.c.}(0.5)$	0.3220538773	0.0646621470	48.2270219401	45.398	40
$II.C_{36}^{i.c.}(0.5)$	0.2718709077	0.0946174473	42.0581440899	47.577	42
$II.C_{37}^{i.c.}(0.5)$	0.3183080806	0.0764243838	50.2696384479	47.594	42
$II.C_{38}^{i.c.}(0.5)$	0.1710053276	0.0753437523	31.0798039439	47.424	42
$II.C_{39}^{i.c.}(0.5)$	0.2934877947	0.2212457298	65.1670851943	48.795	42
$II.C_{40}^{i.c.}(0.5)$	0.1428288108	0.0553224707	30.6121545358	49.772	44
$II.C_{41}^{i.c.}(0.5)$	0.2635213411	0.1069239583	43.3809654410	49.749	44
$II.C_{42}^{i.c.}(0.5)$	0.1937551146	0.0588647172	36.9605508290	54.466	48
$II.C_{43}^{i.c.}(0.5)$	0.2501765655	0.0899074157	46.1520234471	56.667	50
$II.C_{44}^{i.c.}(0.5)$	0.0819979788	0.3405135958	79.1849364880	66.109	50
$II.C_{45}^{i.c.}(0.5)$	0.1421031951	0.1000963218	36.0751418819	56.519	50
$II.C_{46}^{i.c.}(0.5)$	0.1769229292	0.0317210521	36.6796720789	56.798	50
$II.C_{47}^{i.c.}(0.5)$	0.2231965871	0.0917189066	44.2893524922	58.865	52
$II.C_{48}^{i.c.}(0.5)$	0.3149842289	0.0687327481	60.7978969392	59.011	52
$II.C_{49}^{i.c.}(0.5)$	0.1615942955	0.1179674433	40.9841942386	60.738	54
$II.C_{50}^{i.c.}(0.5)$	0.2810932036	0.2296505498	82.0206638443	63.285	54
$II.C_{51}^{i.c.}(0.5)$	0.1735416113	0.0688826379	39.9994858929	61.055	54
$II.C_{52}^{i.c.}(0.5)$	0.1035547024	0.0422685017	35.0762402950	60.451	54
$II.C_{53}^{i.c.}(0.5)$	0.2690033537	0.0919983273	53.3422504976	61.194	54
$II.C_{54}^{i.c.}(0.5)$	0.1173397763	0.0109376998	38.5590034737	65.976	58
$II.C_{55}^{i.c.}(0.5)$	0.1315056462	0.0526807760	39.5934146422	65.562	58
$II.C_{56}^{i.c.}(0.5)$	0.2613378486	0.1103086095	56.9874074323	65.537	58
$II.C_{57}^{i.c.}(0.5)$	0.3281658348	0.0363891724	73.0735492094	68.188	60
II. $C_{58}^{i.c.}(0.5)$ II. $C_{59}^{i.c.}(0.5)$	0.1395436149	0.0265973488	41.1870639981	68.161	60
$II.C_{59}^{i.c.}(0.5)$	0.1878900821	0.0561739494	45.5265171501	68.082	60
$II.C_{60}^{i.c.}(0.5)$	0.2635734120	0.1081725286	61.2354053056	70.083	62
$II.C_{61}^{i.c.}(0.5)$	0.2540232703	0.0883181524	57.8669161621	70.283	62
$II.C_{62}^{i.c.}(0.5)$	0.1821497093	0.0381106799	46.0695631799	70.412	62
$II.C_{63}^{i.c.}(0.5)$	0.2811013873	0.1118997053	67.8894384900	72.243	64
$II.C_{64}^{i.c.}(0.5)$	0.2669232153	0.0902025793	64.5730278183	74.810	66
$II.C_{65}^{i.c.}(0.5)$	0.1792102190	0.1389127769	53.8787781562	74.660	66
$II.C_{66}^{i.c.}(0.5)$	0.3085501674	0.0768386796	75.4497521560	74.824	66
$II.C_{67}^{i.c.}(0.5)$	0.1564466743	0.0570450740	48.4839903391	77.011	68
II. $C_{68}^{i.c.}(0.5)$	0.2374442052	0.0698194855	60.9419117383	79.483	70
$II.C_{69}^{i.c.}(0.5)$	0.1495364869	0.0339284734	50.3005990140	81.760	72
$II.C_{70}^{i.c.}(0.5)$	0.3151482492	0.0494781713	82.8987639982	81.803	72
$II.C_{71}^{i.c.}(0.5)$	0.3414149024	0.0554960419	95.4752935994	81.760	72
$II.C_{72}^{i.c.}(0.5)$	0.1863427441	0.0555295217	54.4276379358	81.697	72
$II.C_{73}^{i.c.}(0.5)$	0.3285842514	0.0936843712	95.1841177804	83.484	74
$II.C_{74}^{i.c.}(0.5)$	0.2562981114	0.0869856262	69.5011518434	83.898	74
$II.C_{75}^{i.c.}(0.5)$	0.1322093913	0.0211571451	51.5542382142	86.350	76
$II.C_{76}^{i.c.}(0.5)$	0.3185759283	0.0928737708	92.8294620014	85.876	76
$II.C_{77}^{i.c.}(0.5)$	0.3226279066	0.1036036069	96.1122549182	85.628	76
$II.C_{78}^{i.c.}(0.5)$	0.1206929092	0.0421948480	50.9428983671	86.044	76 <b>7</b> 6
$II.C_{79}^{i.c.}(0.5)$	0.2763819973	0.1013126179	80.1001367274	88.252	78 <b>7</b> 8
$II.C_{80}^{i.c.}(0.5)$	0.2654060732	0.0888677277	75.7855698562	88.426	78 70
$II.C_{81}^{i.c.}(0.5)$	0.3057951517	0.0766709356	88.0351418174	88.439	78
$II.C_{82}^{i.c.}(0.5)$	0.2823465042	0.1166804112	85.8497303391	90.177	80
$II.C_{83}^{i.c.}(0.5)$	0.2638802619	0.0961802884	77.9476788146	90.613	80
$II.C_{84}^{i.c.}(0.5)$	0.1581811787	0.1114689424	61.7013854081	92.806	82
$II.C_{85}^{i.c.}(0.5)$	0.2475778287	0.0746419321	73.9745774361	93.099	82

Table S. X. Initial conditions and periods T of the periodic three-body orbits for class II.C in the case of  $\mathbf{r}_1(0) = (-1,0) = -\mathbf{r}_2(0)$ ,  $\dot{\mathbf{r}}_1(0) = (v_1, v_2) = \dot{\mathbf{r}}_2(0)$  and  $\mathbf{r}_3(0) = (0,0)$ ,  $\dot{\mathbf{r}}_3(0) = (-2v_1/m_3, -2v_2/m_3)$  when G = 1 and  $m_1 = m_2 = 1$  and  $m_3 = 0.5$  by means of the search grid  $4000 \times 4000$  in the interval  $T_0 \in [0, 200]$ , where  $T^* = T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

Class and number	$v_1$	$v_2$	T	$T^*$	$L_f$
$\overline{\text{II.C}_{86}^{i.c.}(0.5)}$	0.1516949119	0.0244769729	57.3481255424	93.166	82
$II.C_{87}^{i.c.}(0.5)$	0.3220495093	0.0374666542	96.8850696521	93.189	82
$II.C_{88}^{i.c.}(0.5)$	0.1858022899	0.0553034082	63.4160528904	95.312	84
$II.C_{89}^{i.c.}(0.5)$	0.2655776786	0.1010849772	84.7984660370	97.331	86
$II.C_{90}^{i.c.}(0.5)$	0.3207800454	0.0938599983	106.3208891400	97.122	86
$II.C_{91}^{i.c.}(0.5)$	0.1834880615	0.0537582335	66.0505896692	99.853	88
$II.C_{92}^{i.c.}(0.5)$	0.3018070276	0.0795462027	100.0943865475	102.041	90
$II.C_{93}^{i.c.}(0.5)$	0.3038401134	0.0764563615	100.6645786318	102.054	90
$II.C_{94}^{i.c.}(0.5)$	0.3052322511	0.1102837421	105.6095849909	101.417	90
$II.C_{95}^{i.c.}(0.5)$	0.3566421417	0.0401336571	132.0028545187	104.539	92
$II.C_{96}^{i.c.}(0.5)$	0.2657655740	0.1056700425	91.2995867266	104.043	92
$II.C_{97}^{i.c.}(0.5)$	0.1446403514	0.0463024934	64.0268715528	104.324	92
$II.C_{98}^{i.c.}(0.5)$	0.1257825213	0.0270171780	61.9602861500	104.540	92
$II.C_{99}^{i.c.}(0.5)$	0.2636263542	0.0943925785	89.3740530822	104.231	92
$II.C_{100}^{i.c.}(0.5)$	0.3189489999	0.0684963314	109.6451976549	104.408	92
$II.C_{101}^{i.c.}(0.5)$	0.2425203938	0.0657640364	81.0766276092	104.493	92
$II.C_{102}^{i.c.}(0.5)$	0.2865282765	0.0260626161	94.3794078874	106.829	94
$II.C_{103}^{i.c.}(0.5)$	0.3294573946	0.0567093697	119.3411186852	108.973	96
$II.C_{104}^{i.c.}(0.5)$	0.3300293300	0.0595323594	120.0149738801	108.973	96
$II.C_{105}^{i.c.}(0.5)$	0.1887576280	0.0377729761	72.3298978206	109.035	96
$II.C_{106}^{i.c.}(0.5)$	0.1665864480	0.0397866710	69.2172422426	108.973	96
$II.C_{107}^{i.c.}(0.5)$	0.3289575551	0.0668810663	122.6577661872	111.174	98
$II.C_{108}^{i.c.}(0.5)$	0.2749671985	0.1116521323	103.5125111065	112.909	100
$II.C_{109}^{i.c.}(0.5)$	0.2252171434	0.0831314124	84.9641254583	113.344	100
$II.C_{110}^{i.c.}(0.5)$	0.3035132687	0.0742550960	111.4832387743	113.467	100
$II.C_{111}^{i.c.}(0.5)$	0.2995384163	0.0855174656	110.8383172246	113.311	100
$II.C_{112}^{i.c.}(0.5)$	0.2634205915	0.0870091576	98.2072237371	115.656	102
$II.C_{113}^{i.c.}(0.5)$	0.3024538187	0.0762577398	113.3596459196	115.668	102
$II.C_{114}^{i.c.}(0.5)$	0.3110478575	0.0862561185	119.2212947649	115.494	102
$II.C_{115}^{i.c.}(0.5)$	0.3526389241	0.0428144462	145.8125822163	118.133	104
$II.C_{116}^{i.c.}(0.5)$	0.1918597167	0.0736315430	80.4647582647	117.735	104
$II.C_{117}^{i.c.}(0.5)$	0.2698687648	0.1023315358	104.3314555538	117.667	104
$II.C_{118}^{i.c.}(0.5)$	0.1414881922	0.0354161821	71.8047330418	118.134	104
$II.C_{119}^{i.c.}(0.5)$	0.1619102285	0.0244352124	74.0209337785	118.169	104
II. $C_{120}^{i.c.}(0.5)$ II. $C_{121}^{i.c.}(0.5)$	0.2502762644	0.0696607780	94.2261347512	118.110	104
$II.C_{121}^{i.c.}(0.5)$	0.1870717535	0.0147811565	77.5861543862	118.190	104
$II.C_{122}^{i.c.}(0.5)$	0.1905368730	0.0303149606	79.9609883712	120.424	106
$II.C_{123}^{i.c.}(0.5)$	0.1675018862	0.0354981141	76.4546465382	120.379	106
$II.C_{124}^{i.c.}(0.5)$	0.2382138646	0.0760779564	94.7217192960	122.548	108
$II.C_{125}^{i.c.}(0.5)$	0.3251008379	0.0627976669	132.0053020644	122.583	108
$II.C_{126}^{i.c.}(0.5)$	0.3268797519	0.0654741271	133.5845577449	122.583	108
$II.C_{127}^{i.c.}(0.5)$	0.2824206151	0.1120611754	117.3283797375	124.152	110
$II.C_{128}^{i.c.}(0.5)$	0.3355862027	0.0688344773	142.8830057530	124.693	110
$II.C_{129}^{i.c.}(0.5)$	0.2650997364	0.0972324666	107.7796238869	124.572	110
$II.C_{130}^{i.c.}(0.5)$	0.3155083362	0.0780524043	130.0655193752	124.605	110
$II.C_{131}^{i.c.}(0.5)$	0.2911251331	0.0747106544	116.3271632708	124.860	110
II. $C_{132}^{i.c.}(0.5)$	0.3023181843	0.0747198638	124.2563024241	127.082	112
II. $C_{133}^{i.c.}(0.5)$	0.2877451269	0.0973212810	119.7416237253	126.751	112
II. $C_{134}^{i.c.}(0.5)$	0.2730233157	0.0958101195	114.8135152620	129.113	114
II. $C_{135}^{i.c.}(0.5)$	0.2522438389	0.2397521752	158.1926187312	133.822	114
II. $C_{136}^{i.c.}(0.5)$	0.1972498965	0.0367328629	87.4975613393	129.499	114
$II.C_{137}^{i.c.}(0.5)$	0.2551006961	0.0719840482	106.9936965308	131.727	116
$II.C_{138}^{i.c.}(0.5)$	0.1466907115	0.0325820647	80.6365220274	131.733	116
$II.C_{139}^{i.c.}(0.5)$	0.3114989143	0.0680246052	133.2896024822	131.635	116
$II.C_{140}^{i.c.}(0.5)$	0.3295340331	0.0484649621	143.3806235446	131.777	116
$II.C_{141}^{i.c.}(0.5)$	0.1909835958	0.0219691369	87.3877632060	131.809	116
$II.C_{142}^{i.c.}(0.5)$	0.2413919468 $0.1733103989$	0.0306502812	99.7934250857	131.831	116
$II.C_{143}^{i.c.}(0.5)$	0.1755103989	0.0446120548	87.8145595559	136.194	120

Table S. XI. Initial conditions and periods T of the periodic three-body orbits for class II.C in the case of  $\mathbf{r}_1(0) = (-1,0) = -\mathbf{r}_2(0)$ ,  $\dot{\mathbf{r}}_1(0) = (v_1, v_2) = \dot{\mathbf{r}}_2(0)$  and  $\mathbf{r}_3(0) = (0,0)$ ,  $\dot{\mathbf{r}}_3(0) = (-2v_1/m_3, -2v_2/m_3)$  when G = 1 and  $m_1 = m_2 = 1$  and  $m_3 = 0.5$  by means of the search grid  $4000 \times 4000$  in the interval  $T_0 \in [0, 200]$ , where  $T^* = T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

Class and number	$v_1$	$v_2$	T	$T^*$	$L_f$
$\overline{\text{II.C}}_{144}^{i.c.}(0.5)$	0.2930149393	0.0749995913	130.0780110428	138.475	122
$II.C_{1.45}^{i.c.}(0.5)$	0.3010872261	0.0827870509	137.9566276722	140.544	124
$II.C_{146}^{i.c.}(0.5)$	0.2837900515	0.1048066565	131.7075390111	140.171	124
$II.C_{147}^{i.c.}(0.5)$	0.1654894951	0.0372403932	89.1316363022	140.762	124
$II.C_{148}^{i.c.}(0.5)$	0.2005754999	0.0307687587	95.6979960354	140.884	124
$II.C_{149}^{i.c.}(0.5)$	0.1950195608	0.0382427268	96.2751827640	143.118	126
$II.C_{150}^{i.c.}(0.5)$	0.3069226729	0.1114459071	151.7018490077	144.231	128
$II.C_{151}^{i.c.}(0.5)$	0.2583666188	0.0736044941	119.5432346546	145.343	128
$II.C_{152}^{i.c.}(0.5)$	0.2136148238	0.0824671746	105.2690821399	144.976	128
$II.C_{153}^{i.c.}(0.5)$	0.3026649056	0.0527959420	139.5203274022	145.428	128
$II.C_{154}^{i.c.}(0.5)$	0.1869669478	0.0727253609	99.4296116997	147.169	130
$II.C_{155}^{i.c.}(0.5)$	0.3253795688	0.0922726657	166.6893561658	149.032	132
$II.C_{1.56}^{i.c.}(0.5)$	0.3510221862	0.0469440817	183.8293814238	149.920	132
II. $C_{156}^{i.c.}(0.5)$ II. $C_{157}^{i.c.}(0.5)$	0.2976026689	0.0679239870	142.5318450479	149.883	132
$II.C_{158}^{i.c.}(0.5)$	0.2476930572	0.0788426863	119.5507824703	149.779	132
$II.C_{159}^{i.c.}(0.5)$	0.1744790844	0.0483169316	96.9984259826	149.806	132
$II.C_{160}^{i.c.}(0.5)$	0.2329477926	0.0962157533	116.2632004744	149.480	132
$II.C_{161}^{i.c.}(0.5)$	0.2049799809	0.0695040711	105.1805254163	149.783	132
$II.C_{162}^{i.c.}(0.5)$	0.2000398001	0.0570752651	102.9916394933	149.783	132
$II.C_{163}^{i.c.}(0.5)$	0.3208285254	0.0707218166	161.5242818190	152.011	134
$II.C_{164}^{i.c.}(0.5)$	0.2323718985	0.0470731914	113.1431229338	152.265	134
$II.C_{165}^{i.c.}(0.5)$	0.2732770465	0.1112359067	139.7843087122	153.574	136
$II.C_{166}^{i.c.}(0.5)$	0.1973164730	0.0325297824	104.2330202025	154.504	136
$II.C_{167}^{i.c.}(0.5)$ $II.C_{168}^{i.c.}(0.5)$	0.3555507477	0.0410620694	193.9086747432	154.510	136
$II.C_{168}^{i.c.}(0.5)$	0.3011590042	0.0819088183	151.2083294223	154.160	136
$II.C_{169}^{i.c.}(0.5)$	0.3069577168	0.0877702567	156.2447738532	153.992	136
$II.C_{170}^{i.c.}(0.5)$	0.3243977377	0.0286054600	161.8647903378	154.573	136
$II.C_{171}^{i.c.}(0.5)$	0.2708273208	0.0936225609	137.4958739955	156.348	138
$II.C_{172}^{i.c.}(0.5)$	0.3032598179	0.0211927097	148.3161503321	156.832	138
$II.C_{173}^{i.c.}(0.5)$	0.2499690317	0.0880524793	128.9029384970	158.722	140
$II.C_{174}^{i.c.}(0.5)$	0.2439808630	0.0927858470	128.9093834720	160.913	142
$II.C_{175}^{i.c.}(0.5)$	0.2921786881	0.1158238005	158.5701925931	160.019	142
$II.C_{176}^{i.c.}(0.5)$	0.3187109289	0.0697453074	171.6434882025	163.419	144
$II.C_{177}^{i.c.}(0.5)$	0.2345393298	0.0733775891	124.6080587586	163.399	144
II. $C_{178}^{i.c.}(0.5)$ II. $C_{179}^{i.c.}(0.5)$	0.2817443085	0.0820058547	149.7102508372	165.580	146
$II.C_{179}^{i.c.}(0.5)$	0.2683324131	0.0800302736	142.0849053423	165.705	146
$II.C_{180}^{i.c.}(0.5)$	0.2911513179	0.1023124996	159.3225451639	165.048	146
$II.C_{181}^{i.c.}(0.5)$	0.2450691759	0.0560195533	128.7590915568	165.887	146
$II.C_{182}^{i.c.}(0.5)$	0.1637533270	0.0120392511	104.0378908319	165.953	146
$II.C_{183}^{i.c.}(0.5)$	0.3189840056	0.0986343953	182.4517653543	166.971	148
$II.C_{184}^{i.c.}(0.5)$	0.3277428450	0.0422660249	180.4850020783	168.174	148
$II.C_{185}^{i.c.}(0.5)$	0.2822423292	0.0943666677	154.2006195402	167.612	148
$II.C_{186}^{i.c.}(0.5)$	0.1529038054	0.0482036149	105.9017717709	170.094	150
$II.C_{187}^{i.c.}(0.5)$	0.2502629663	0.0420236439	133.3212700219	170.461	150
$II.C_{188}^{i.c.}(0.5)$	0.2595662993	0.0702980306	140.2477320721	170.352	150
$II.C_{189}^{i.c.}(0.5)$	0.2719916833	0.1024099935	153.7109398488	171.961	152
$II.C_{190}^{i.c.}(0.5)$	0.3318880756	0.0584426073	191.7202695166	172.549	152
$II.C_{191}^{i.c.}(0.5)$	0.2642631613	0.0846766978	148.3494808332	174.656	154
$II.C_{192}^{i.c.}(0.5)$	0.2950993654	0.1054674522	171.5975520159	173.910	154
$II.C_{193}^{i.c.}(0.5)$	0.2952577971	0.0652451441	164.2296591527	174.893	154
$II.C_{194}^{i.c.}(0.5)$	0.1919407079	0.0544880286	119.2943656794	177.015	156
$II.C_{195}^{i.c.}(0.5)$	0.2341210185	0.0775543587	135.3588931652	177.015	156
$II.C_{196}^{i.c.}(0.5)$	0.2740046014	0.0751646038	154.2765378448	177.104	156
$II.C_{197}^{i.c.}(0.5)$	0.2741492762	0.1053054597	161.7072926859	178.670	158
$II.C_{198}^{i.c.}(0.5)$	0.3114471237	0.0925603589	186.4022437357	178.674	158
$II.C_{199}^{i.c.}(0.5)$	0.2517718983	0.0599951023	142.7458386836	179.508	158
$II.C_{200}^{i.c.}(0.5)$	0.2780070267	0.0823491926	159.7348584169	179.195	158

Table S. XII. Initial conditions and periods T of the periodic three-body orbits for class II.C in the case of  $\mathbf{r}_1(0) = (-1,0) = -\mathbf{r}_2(0)$ ,  $\dot{\mathbf{r}}_1(0) = (v_1, v_2) = \dot{\mathbf{r}}_2(0)$  and  $\mathbf{r}_3(0) = (0,0)$ ,  $\dot{\mathbf{r}}_3(0) = (-2v_1/m_3, -2v_2/m_3)$  when G = 1 and  $m_1 = m_2 = 1$  and  $m_3 = 0.5$  by means of the search grid  $4000 \times 4000$  in the interval  $T_0 \in [0, 200]$ , where  $T^* = T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

Class and number	$v_1$	$v_2$	T	$T^*$	$L_f$
$\overline{\text{II.C}_{201}^{i.c.}(0.5)}$	0.3208251493	0.0312083220	184.9613703373	179.574	158
$II.C_{202}^{{i.c.}}(0.5)$	0.2823197687	0.1061568833	169.2512195359	180.837	160
$II.C_{203}^{i.c.}(0.5)$	0.3058525824	0.0859666113	182.5097652558	181.230	160
$II.C_{204}^{i.c.}(0.5)$	0.2711017290	0.0880228048	158.5702051413	181.391	160
$II.C_{205}^{\tilde{i}.c.}(0.5)$	0.2832710026	0.0993650437	168.4082593848	181.046	160
$II.C_{205}^{\overline{i.c.}}(0.5)$ $II.C_{206}^{i.c.}(0.5)$	0.2580794098	0.0662802067	148.3342287109	181.742	160
$II.C_{207}^{i.c.}(0.5)$	0.2168915835	0.0791081052	134.0444642533	183.636	162
$II.C_{208}^{i.c.}(0.5)$	0.3182346683	0.0550212092	190.1060999565	184.007	162
$II.C_{209}^{\stackrel{200}{i.c.}}(0.5)$	0.2602548035	0.0468178359	149.1982024781	184.087	162
$II.C_{210}^{\stackrel{200}{i.c.}}(0.5)$	0.2957290452	0.1115815297	184.7739565799	184.904	164
$II.C_{211}^{\overline{i.c.}}(0.5)$	0.2713556472	0.1013159584	165.2338308800	185.584	164
$II.C_{211}^{i.c.}(0.5)$ $II.C_{212}^{i.c.}(0.5)$	0.1400364517	0.0295516193	112.7541853074	186.294	164
$II.C_{213}^{\stackrel{?}{i.c.}}(0.5)$	0.2538303347	0.0857213734	152.5491452121	185.952	164
$II.C_{214}^{\stackrel{i.c.}{i.c.}}(0.5)$	0.1661354659	0.0434379136	118.3442264866	186.158	164
$II.C_{215}^{\vec{i.c.}}(0.5)$	0.2947852597	0.0614773180	173.9781290999	186.283	164
$II.C_{216}^{i.c.}(0.5)$	0.2959154222	0.1062666806	183.5801829920	185.154	164
II. $C_{216}^{\overline{i.c.}}(0.5)$ II. $C_{217}^{i.c.}(0.5)$	0.1872594057	0.0518107976	125.4822943491	188.412	166
$II.C_{218}^{\vec{i.c.}}(0.5)$	0.1946620874	0.0643435760	130.1369103426	190.634	168
$II.C_{219}^{\overline{i.c.}}(0.5)$	0.1960305611	0.0313147606	129.8525440051	193.128	170
$II.C_{220}^{\overline{i.c.}}(0.5)$	0.2756620575	0.0823970177	170.3420494464	192.810	170
$II.C_{221}^{i.c.}(0.5)$	0.3006677095	0.0801050248	190.4434520843	195.007	172
$II.C_{222}^{i.c.}(0.5)$	0.2138125453	0.0765286284	140.8953101446	195.052	172
$II.C_{223}^{i.c.}(0.5)$	0.2817233384	0.1045112371	181.1173423050	194.465	172
$II.C_{224}^{i.c.}(0.5)$	0.2826464621	0.0982734891	180.3452656908	194.667	172
$II.C_{225}^{i.c.}(0.5)$	0.2632094008	0.0725802686	165.0883588774	197.586	174
$II.C_{226}^{i.c.}(0.5)$	0.2731816009	0.0806117521	172.4907007388	197.474	174
$II.C_{227}^{i.c.}(0.5)$	0.2960515524	0.1072263897	195.1428066451	196.397	174
$\begin{array}{c} \text{II.C}_{227}^{i.c.}(0.5) \\ \text{II.C}_{228}^{i.c.}(0.5) \\ \text{II.C}_{229}^{i.c.}(0.5) \\ \end{array}$	0.2951231894	0.0577880334	184.2949392209	197.670	174
$II.C_{229}^{i.c.}(0.5)$	0.2700922029	0.0499192216	166.1828634798	197.713	174
$II.C_{230}^{i.c.}(0.5)$	0.2677573881	0.0945684253	173.5769232952	199.385	176
$II.C_{231}^{i.c.}(0.5)$	0.2434451873	0.0747898054	156.7395420237	199.814	176
$II.C_{232}^{i.c.}(0.5)$	0.2936385507	0.0678957262	188.9849393321	202.125	178
$II.C_{233}^{i.c.}(0.5)$	0.2624985469	0.0847688537	170.4292589966	201.886	178
$II.C_{234}^{i.c.}(0.5)$	0.1968157177	0.0738072529	139.3969959798	201.651	178
$II.C_{235}^{i.c.}(0.5)$	0.2823384174	0.1064870414	188.3906435625	201.170	178
$II.C_{236}^{i.c.}(0.5)$	0.2136874355	0.0627089113	145.7957129717	204.251	180
$II.C_{237}^{i.c.}(0.5)$	0.2861638798	0.0874577519	191.0899162207	206.277	182
$II.C_{238}^{i.c.}(0.5)$	0.2926689949	0.0405343684	190.6416032233	209.088	184
$II.C_{239}^{i.c.}(0.5)$	0.1910600294	0.0654138331	142.9115531752	210.871	186
$II.C_{240}^{i.c.}(0.5)$	0.1794185340	0.0647107348	142.2515470251	215.270	190
$II.C_{241}^{i.c.}(0.5)$	0.2749406093	0.0726374464	188.1053317808	215.731	190
$II.C_{242}^{i.c.}(0.5)$	0.2498197638	0.0791829559	175.1478065921	217.861	192
$II.C_{243}^{i.c.}(0.5)$	0.2465045273	0.0440978667	170.5920826149	220.463	194
$II.C_{244}^{i.c.}(0.5)$	0.2632419381	0.0497931425	182.7043137437	222.714	196
II. $C_{245}^{i.c.}(0.5)$	0.2549695079	0.0579934165	184.1495051302	229.515	202
$II.C_{246}^{i.c.}(0.5)$	0.2632833046	0.0670452882	194.5495654808	233.984	206
II. $C_{247}^{i.c.}(0.5)$	0.2515183994	0.0784843668	198.0785646371	245.200	216
$II.C_{248}^{i.c.}(0.5)$	0.2462955611	0.0455335432	189.9687042853	245.464	$\frac{216}{216}$
II. $C_{249}^{i.c.}(0.5)$ II. $C_{250}^{i.c.}(0.5)$	0.2069465170 $0.2182720723$	0.0690747434 $0.0536172219$	$172.8812870460 \\ 177.7299779781$	$245.099 \\ 247.614$	216 218
II. $C_{250}^{i.c.}(0.5)$ II. $C_{251}^{i.c.}(0.5)$	0.2182720723	0.0330172219 $0.0311726007$	154.5748576016	247.014 $254.455$	$\frac{218}{224}$
II. $C_{251}^{i.c.}(0.5)$ II. $C_{252}^{i.c.}(0.5)$	0.1421093213 $0.1540640475$	0.0494956780	164.2313972673	263.085	$\frac{224}{232}$
$II.C_{252}^{i.c.}(0.5)$ $II.C_{253}^{i.c.}(0.5)$	0.154040473	0.0494930780	170.1977938681	272.489	240
II. $C_{253}^{i.c.}(0.5)$ II. $C_{254}^{i.c.}(0.5)$	0.1495219644	0.0367502784	170.6390914389	277.069	240
II. $C_{254}^{i.c.}(0.5)$ II. $C_{255}^{i.c.}(0.5)$	0.1495219044 $0.1964997354$	0.0307502764	196.7183026566	293.141	$\frac{244}{258}$
11.O <sub>255</sub> (0.0)	0.1304337304	0.0214040301	100.1100020000	200.141	200

Table S. XIII. Initial conditions and periods T of the periodic three-body orbits for class I.A in the case of  $\mathbf{r}_1(0) = (-1,0) = -\mathbf{r}_2(0)$ ,  $\dot{\mathbf{r}}_1(0) = (v_1,v_2) = \dot{\mathbf{r}}_2(0)$  and  $\mathbf{r}_3(0) = (0,0)$ ,  $\dot{\mathbf{r}}_3(0) = (-2v_1/m_3, -2v_2/m_3)$  when G=1 and  $m_1=m_2=1$  and  $m_3=0.75$  by means of the search grid  $4000\times4000$  in the interval  $T_0\in[0,200]$ , where  $T^*=T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

Class and number	$v_1$	$v_2$	T	$T^*$	$L_f$
$I.A_1^{i.c.}(0.75)$	0.4227625247	0.2533646387	5.9858187252	6.993	4
$I.A_2^{i.c.}(0.75)$	0.5337490177	0.3041674607	25.0460382386	12.114	8
$I.A_3^{i.c.}(0.75)$	0.3662988976	0.1741689247	17.3598689063	28.658	16
$I.A_4^{i.c.}(0.75)$	0.4329138577	0.3286773639	30.2157279993	26.520	16
$I.A_5^{i.c.}(0.75)$	0.2658200014	0.3876978560	20.4071584640	26.484	16
$I.A_6^{i.c.}(0.75)$	0.0848079590	0.0632351933	10.5323245447	28.878	16
$I.A_7^{i.c.}(0.75)$	0.1632908635	0.4709992142	36.5167219251	41.488	24
$I.A_8^{i.c.}(0.75)$	0.3986630485	0.2098556495	30.3699479925	42.738	24
$I.A_9^{i.c.}(0.75)$	0.3972439494	0.1439208014	32.1901761672	50.237	28
$I.A_{10}^{i.c.}(0.75)$	0.1902461690	0.1147396347	20.4870964105	50.262	28
$I.A_{11}^{i.c.}(0.75)$	0.4062910529	0.2262182980	42.8004488374	56.762	32
$I.A_{12}^{i.c.}(0.75)$	0.3444998604	0.0249623855	29.4488166879	57.520	32
$I.A_{13}^{i.c.}(0.75)$	0.3588572795	0.1923161209	39.1877154517	64.373	36
$I.A_{14}^{i.c.}(0.75)$	0.3585145568	0.1395242890	40.8139997342	71.804	40
$I.A_{15}^{i.c.}(0.75)$	0.4042841150	0.2375692455	54.2573733044	70.767	40
$I.A_{16}^{i.c.}(0.75)$	0.4356001994	0.3176227502	82.1366960817	74.183	44
$I.A_{17}^{i.c.}(0.75)$	0.3992933028	0.2021813618	55.1053036718	78.451	44
$I.A_{18}^{i.c.}(0.75)$	0.3465301653	0.0611178943	41.1425735253	79.087	44
$I.A_{19}^{i.c.}(0.75)$	0.4747944843	0.0277185939	62.4983813817	79.156	44
$I.A_{20}^{i.c.}(0.75)$	0.1788553784	0.1250024592	34.8754522497	86.012	48
$I.A_{21}^{i.c.}(0.75)$	0.4123273935	0.2418549526	67.6579019767	84.763	48
$I.A_{22}^{i.c.}(0.75)$	0.4002811362	0.2152589233	66.7743434315	92.493	52
$I.A_{23}^{i.c.}(0.75)$	0.3570732994	0.1330553625	52.5165797298	93.368	52
$I.A_{23}^{i.c.}(0.75)$ $I.A_{24}^{i.c.}(0.75)$	0.3204248773	0.1988777895	55.6667301422	100.077	56
$I.A_{24}^{i.c.}(0.75)$ $I.A_{25}^{i.c.}(0.75)$	0.4200665164	0.2432598967	81.5599720155	98.754	56
$I.A_{25}^{i.c.}(0.75)$ $I.A_{26}^{i.c.}(0.75)$	0.3477173243	0.2432393907	52.8403128515	100.651	56
$I.A_{26}^{i.c.}(0.75)$ $I.A_{27}^{i.c.}(0.75)$	0.4043929787	0.2232119633	79.2711602148	106.517	60
$I.A_{27} (0.75)$ $I.A_{28}^{i.c.} (0.75)$					60
	0.3642842051	0.1636761967	63.8886666871	107.559	
$I.A_{29}^{i.c.}(0.75)$	0.4181714913	0.2478074981	93.4047379916	112.743	64 64
$I.A_{30}^{i.c.}(0.75)$	0.4353986367	0.1115724837	81.3432979489	114.946	
$I.A_{31}^{i.c.}(0.75)$	0.3561647035	0.1287849051	64.2195094868	114.931	64
$I.A_{32}^{i.c.}(0.75)$	0.4012766820	0.1981267054	80.1768711298	114.162	64
$I.A_{33}^{i.c.}(0.75)$	0.1901620868	0.0896947011	46.1543229949	115.000	64
$I.A_{34}^{i.c.}(0.75)$	0.4064136540	0.2296155572	91.5695991198	120.529	68
$I.A_{35}^{i.c.}(0.75)$	0.3662485364	0.1831531016	74.6480334841	121.694	68
$I.A_{36}^{i.c.}(0.75)$	0.3484936438	0.0810521814	64.5396545591	122.214	68
$I.A_{37}^{i.c.}(0.75)$	0.3625081838	0.1561981586	75.6201745758	129.136	72
$I.A_{38}^{i.c.}(0.75)$	0.3555412053	0.1257508069	75.9229791803	136.494	76
$I.A_{39}^{i.c.}(0.75)$	0.3490401568	0.0856323568	76.2397282094	143.776	80
$I.A_{40}^{i.c.}(0.75)$	0.4023237219	0.1953477132	105.1410526930	149.871	84
$I.A_{41}^{i.c.}(0.75)$	0.3368899556	0.1858462998	89.4831471991	157.407	88
$I.A_{42}^{i.c.}(0.75)$	0.4037216808	0.2221234572	115.7611144325	156.271	88
$I.A_{43}^{i.c.}(0.75)$	0.3637279697	0.1884044215	96.5977490387	157.408	88
$I.A_{44}^{i.c.}(0.75)$	0.3550873014	0.1234828055	87.6269195614	158.055	88
$I.A_{45}^{i.c.}(0.75)$	0.3651069701	0.1674068653	98.6376062555	164.877	92
$I.A_{46}^{i.c.}(0.75)$	0.3984203610	0.2064633948	115.6749862554	163.928	92
$I.A_{47}^{i.c.}(0.75)$	0.4262489553	0.0672054586	109.3699499056	165.348	92
$I.A_{48}^{i.c.}(0.75)$	0.3494454129	0.0888406710	87.9401405108	165.338	92
$I.A_{49}^{i.c.}(0.75)$	0.4164619789	0.2419173608	143.2372283584	176.522	100
$I.A_{50}^{i.c.}(0.75)$	0.3547423337	0.1217229642	99.3312692044	179.617	100

Table S. XIV. Initial conditions and periods T of the periodic three-body orbits for class I.A in the case of  $\boldsymbol{r}_1(0)=(-1,0)=-\boldsymbol{r}_2(0),\ \dot{\boldsymbol{r}}_1(0)=(v_1,v_2)=\dot{\boldsymbol{r}}_2(0)$  and  $\boldsymbol{r}_3(0)=(0,0),\ \dot{\boldsymbol{r}}_3(0)=(-2v_1/m_3,-2v_2/m_3)$  when G=1 and  $m_1=m_2=1$  and  $m_3=0.75$  by means of the search grid  $4000\times 4000$  in the interval  $T_0\in[0,200]$ , where  $T^*=T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

$\overline{\text{I.A}_{51}^{i.c.}(0.75)}$ $\overline{\text{I.A}_{52}^{i.c.}(0.75)}$	0.3666229117				$L_f$
$I.A_{52}^{i.c.}(0.75)$	0.5000229117	0.1802557053	109.4686390383	179.012	100
	0.4034422709	0.2320847945	139.2068145953	184.296	104
$I.A_{53}^{i.c.}(0.75)$	0.4017850173	0.1937496080	129.5985275936	185.581	104
$I.A_{54}^{i.c.}(0.75)$	0.1878826213	0.1228511711	76.2757360731	186.545	104
$I.A_{55}^{i.c.}(0.75)$	0.3638942950	0.1619913559	110.3963190576	186.458	104
$I.A_{56}^{i.c.}(0.75)$	0.3497577254	0.0912169635	99.6406881029	186.899	104
$I.A_{57}^{i.c.}(0.75)$	0.3477310897	0.1903660735	113.6756165257	193.118	108
$I.A_{58}^{i.c.}(0.75)$	0.4012296139	0.2174859716	139.6773663749	192.002	108
$I.A_{59}^{i.c.}(0.75)$	0.3592003644	0.1424958332	110.7390745623	193.846	108
$I.A_{60}^{i.c.}(0.75)$	0.3453188013	0.0436341225	100.0379934281	194.129	108
$I.A_{61}^{i.c.}(0.75)$	0.3987295714	0.2040518543	140.4203692852	199.641	112
$I.A_{62}^{i.c.}(0.75)$	0.3544714361	0.1203177331	111.0359806099	201.178	112
$I.A_{63}^{i.c.}(0.75)$	0.4027598089	0.1099065756	126.7174189742	201.180	112
$I.A_{64}^{i.c.}(0.75)$	0.3085492973	0.1235638992	99.8775281408	201.180	112
$I.A_{65}^{i.c.}(0.75)$	0.3500056587	0.0930489404	111.3412553208	208.461	116
$I.A_{66}^{i.c.}(0.75)$	0.3659182597	0.1843854011	131.8265841519	214.729	120
$I.A_{67}^{i.c.}(0.75)$	0.3459823155	0.0539982359	111.7320070042	215.696	120
$I.A_{68}^{i.c.}(0.75)$	0.1906186812	0.1029059018	87.2101531089	215.531	120
$I.A_{69}^{i.c.}(0.75)$ $I.A_{69}^{i.c.}(0.75)$	0.3894810083	0.0456452590	125.3277220883	215.697	120
$I.A_{69}^{i.c.}(0.75)$ $I.A_{70}^{i.c.}(0.75)$	0.3654656585	0.1691775243	133.3741145779	222.194	124
$I.A_{70}^{i.c.}(0.75)$ $I.A_{71}^{i.c.}(0.75)$	0.3714351327	0.1091773243	145.6730252269	228.826	124
$I.A_{71} (0.75)$ $I.A_{72}^{i.c.} (0.75)$	0.3714331327 $0.2497702662$	0.1973098003	100.3713839877	230.057	128
$I.A_{72} (0.75)$ $I.A_{73}^{i.c.} (0.75)$			133.8460648608		
	0.3619204754	0.1537575193		229.609	128
$I.A_{74}^{i.c.}(0.75)$	0.3504692196	0.1957352300	136.7945571599	228.826	128
$I.A_{75}^{i.c.}(0.75)$	0.3502071722	0.0945047358	123.0417715598	230.022	128
$I.A_{76}^{i.c.}(0.75)$	0.3579495351	0.1370259459	134.1447275594	236.977	132
$I.A_{77}^{i.c.}(0.75)$	0.3540736229	0.1182148529	134.4463621214	244.301	136
$I.A_{78}^{i.c.}(0.75)$	0.3644051317	0.1875040842	153.7854226046	250.442	140
$I.A_{79}^{i.c.}(0.75)$	0.3503741196	0.0956894365	134.7421912222	251.583	140
$I.A_{80}^{i.c.}(0.75)$	0.2850469461	0.1237066414	125.9073062746	265.878	148
$I.A_{81}^{i.c.}(0.75)$	0.3604411206	0.1477540307	157.2539569259	272.751	152
$I.A_{82}^{i.c.}(0.75)$	0.2623247173	0.0630697393	122.9004467283	280.406	156
$I.A_{83}^{i.c.}(0.75)$	0.3537959008	0.1167175132	157.8578914671	287.422	160
$I.A_{84}^{i.c.}(0.75)$	0.3404668705	0.1788234152	166.8446546886	293.645	164
$I.A_{85}^{i.c.}(0.75)$	0.3506344970	0.0975004375	158.1426267292	294.705	164
$I.A_{86}^{i.c.}(0.75)$	0.3649683941	0.1667479259	179.9043199632	301.096	168
$I.A_{87}^{i.c.}(0.75)$	0.1936102478	0.0742911296	123.6422146647	309.182	172
$I.A_{88}^{i.c.}(0.75)$	0.2440200098	0.1100378530	135.0238328296	309.180	172
$I.A_{89}^{i.c.}(0.75)$	0.3450126559	0.0378080014	158.9350378593	309.170	172
$I.A_{90}^{i.c.}(0.75)$	0.1906314309	0.1067262314	128.1965967535	316.058	176
$I.A_{91}^{i.c.}(0.75)$	0.3507379026	0.0982077679	169.8426052773	316.265	176
$I.A_{92}^{i.c.}(0.75)$	0.3564275786	0.1300379619	180.9554087549	323.231	180
$I.A_{93}^{i.c.}(0.75)$	0.3480099279	0.0767141118	170.2199214581	323.516	180
$I.A_{94}^{i.c.}(0.75)$	0.2806075517	0.1252726560	155.4447267514	330.581	184
$I.A_{95}^{i.c.}(0.75)$	0.3616931000	0.1527867827	192.0738064899	330.082	184
$I.A_{96}^{i.c.}(0.75)$	0.3335530219	0.1439508032	180.7729773204	337.454	188
$I.A_{97}^{i.c.}(0.75)$	0.3589093395	0.1412419876	192.3672014822	337.454	188
$I.A_{98}^{i.c.}(0.75)$	0.3482666391	0.0790526219	181.9192922993	345.079	192
$I.A_{99}^{i.c.}(0.75)$	0.3048177204	0.0814603730	175.3772678451	366.643	204
$I.A_{100}^{i.c.}(0.75)$	0.1916766819	0.0708884572	148.9747140897	373.907	208
$I.A_{101}^{i.c.}(0.75)$	0.1906790534	0.1012235584	153.9336180218	380.799	212
$I.A_{102}^{i.c.}(0.75)$	0.1752778915	0.0747834674	186.0099259874	474.459	264

Table S. XV. Initial conditions and periods T of the periodic three-body orbits for class I.B in the case of  $\mathbf{r}_1(0)=(-1,0)=-\mathbf{r}_2(0),\ \dot{\mathbf{r}}_1(0)=(v_1,v_2)=\dot{\mathbf{r}}_2(0)$  and  $\mathbf{r}_3(0)=(0,0),\ \dot{\mathbf{r}}_3(0)=(-2v_1/m_3,-2v_2/m_3)$  when G=1 and  $m_1=m_2=1$  and  $m_3=0.75$  by means of the search grid  $4000\times 4000$  in the interval  $T_0\in[0,200]$ , where  $T^*=T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

Inferior   0.75   0.395011540   0.194606519   12.2220151916   17.855   10   1.18]**(0.75)   0.163925912   0.228900725   13.7490351066   31.883   18   18   10.5**(0.75)   0.20669008   0.228898318   16.657620550   31.876   18   11.5**(0.75)   0.4676333456   0.217941573   31.0357663010   31.879   18   11.5**(0.75)   0.4676333456   0.2017941573   31.0357663010   31.879   18   11.5**(0.75)   0.4676333456   0.2017941573   31.0357663010   31.879   18   11.5**(0.75)   0.4371771293   0.2657669634   33.6549023018   37.828   22   11.9**(0.75)   0.33412750   0.159655904   23.2572575707   39.449   22   11.5**(0.75)   0.363323250   0.2714983226   28.8310339812   45.873   26   11.5**(0.75)   0.433323250   0.204490251   49.4361887730   45.872   26   11.5**(0.75)   0.433323250   0.204490251   49.4361887730   45.872   26   11.5**(0.75)   0.4705908017   0.21965716984   49.3642077739   59.869   34   11.5**(0.75)   0.470601818   0.2121794524   66.856048178   59.855   34   11.5**(0.75)   0.3969003749   0.142041184   34.9624207763   61.021   34   11.5**(0.75)   0.3969003749   0.142041184   34.9624207763   67.616   38   11.5**(0.75)   0.3666299184   0.184221787   49.0361217055   75.177   42   11.5**(0.75)   0.3666299184   0.184221787   49.0361217055   75.177   42   11.5**(0.75)   0.3666299184   0.184221787   49.0361217055   75.177   42   11.5**(0.75)   0.3666299184   0.184221787   49.0361217055   75.177   42   11.5**(0.75)   0.3666299184   0.184221787   49.0361217055   75.177   42   11.5**(0.75)   0.3666299184   0.184221787   0.098023733   33.393218167   89.899   50   11.5**(0.75)   0.3666299184   0.184221787   0.79842173701   89.899   50   11.5**(0.75)   0.3666299184   0.184221787   0.79842173701   89.899   50   11.5**(0.75)   0.3666299184   0.184221787   0.79842173701   89.899   50   11.5**(0.75)   0.3666299184   0.184221787   0.79842173701   89.899   50   11.5**(0.75)   0.3666299184   0.184221787   0.79842173701   89.899   50   11.5**(0.75)   0.3666299187   0.098623753   39.9911527301   89.899   50   11.5**(0.75)   0.3666721828	Class and number	$v_1$	$v_2$	T	$T^*$	$L_f$
IBB_C						10
IB _{0.075 }	$I.B_2^{i.c.}(0.75)$					
IBB; (0.75)	$I.B_3^{i.c.}(0.75)$					
IB\$_6^*(0.75)	$I.B_4^{i.c.}(0.75)$					
IBB* (0.75)		0.4765383456	0.2017941573	31.0357663010		
IBB; (0.75)						
IBB** (0.75)						
Bis				23.2527357070		
$\begin{array}{c} \mathrm{IB}_{15}^{15}(0.75) & 0.4970901818 & 0.212794524 & 66.8568084178 & 59.855 & 34 \\ \mathrm{IB}_{15}^{15}(0.75) & 0.4970901818 & 0.212794524 & 66.8568084178 & 66.8568084178 \\ \mathrm{IB}_{15}^{15}(0.75) & 0.3960803749 & 0.1442941184 & 34.9624207563 & 61.021 & 34 \\ \mathrm{IB}_{15}^{15}(0.75) & 0.396784534 & 0.2136076584 & 45.588118422 & 67.616 & 38 \\ \mathrm{IB}_{15}^{15}(0.75) & 0.3456671473 & 0.0493751273 & 35.2949100577 & 68.304 & 38 \\ \mathrm{IB}_{15}^{15}(0.75) & 0.3456671473 & 0.0493751273 & 35.2949100577 & 68.304 & 38 \\ \mathrm{IB}_{15}^{15}(0.75) & 0.396529184 & 0.1814212787 & 46.08123795 & 75.177 & 42 \\ \mathrm{IB}_{15}^{15}(0.75) & 0.996438490 & 0.1814212787 & 46.08123795 & 75.177 & 42 \\ \mathrm{IB}_{15}^{15}(0.75) & 0.3567021441 & 0.1399199549 & 46.6652961512 & 82.653 & 46 \\ \mathrm{IB}_{15}^{15}(0.75) & 0.3471929158 & 0.0866324753 & 33.3493218167 & 82.633 & 46 \\ \mathrm{IB}_{15}^{15}(0.75) & 0.3471929158 & 0.086633513 & 46.9911527301 & 89.899 & 50 \\ \mathrm{IB}_{15}^{15}(0.75) & 0.3471929158 & 0.0866335513 & 46.9911527301 & 89.899 & 50 \\ \mathrm{IB}_{15}^{15}(0.75) & 0.3656724828 & 0.1347224452 & 58.3679719678 & 104.150 & 88 \\ \mathrm{IB}_{15}^{15}(0.75) & 0.3656724828 & 0.1347224452 & 58.3679719678 & 104.150 & 82 \\ \mathrm{IB}_{15}^{15}(0.75) & 0.3627539276 & 0.1894007450 & 67.9077026251 & 110.890 & 62 \\ \mathrm{IB}_{15}^{15}(0.75) & 0.3627539276 & 0.1894007450 & 67.907702651 & 110.890 & 62 \\ \mathrm{IB}_{15}^{15}(0.75) & 0.318123472 & 0.0779310702 & 58.6898518060 & 111.433 & 62 \\ \mathrm{IB}_{15}^{15}(0.75) & 0.318123472 & 0.0779310702 & 58.6898518060 & 111.433 & 62 \\ \mathrm{IB}_{15}^{15}(0.75) & 0.318123472 & 0.0779310702 & 58.6898518060 & 111.433 & 62 \\ \mathrm{IB}_{15}^{15}(0.75) & 0.36616101 & 0.2161876380 & 81.9972461438 & 117.371 & 66 \\ \mathrm{IB}_{15}^{15}(0.75) & 0.318123472 & 0.0779310702 & 58.6898518060 & 111.433 & 62 \\ \mathrm{IB}_{15}^{15}(0.75) & 0.36616101 & 0.2161876380 & 81.9972461438 & 117.371 & 66 \\ \mathrm{IB}_{15}^{15}(0.75) & 0.366163348 & 0.1782820664 & 80.8001762843 & 122.494 & 74 \\ \mathrm{IB}_{15}^{15}(0.75) & 0.3665163480 & 0.1727930850 & 75.65141753629 & 131.394 & 74 \\ \mathrm{IB}_{15}^{15}(0.75) & 0.366$	$I.B_9^{i.c.}(0.75)$			28.8310339842		
IBig (0.75)						
Bis						
Bit   (0.75)						
IBIg (0.75)						
Big (0.75)						
Big (0.75)						
Bibir (0.75)						
Bibir (0.75)						
IBB\$** (0.75)						
LB½ (0.75)						
Bbbr (0.75)	$I.B_{20}^{i.c.}(0.75)$					
$\begin{array}{c} LB_{35}^{\perp}(0.75) & 0.3568724828 & 0.1307224452 & 58.3679719678 & 104.150 & 58 \\ LB_{35}^{\perp}(0.75) & 0.3984106274 & 0.2071944713 & 73.0101745147 & 103.333 & 58 \\ LB_{35}^{\perp}(0.75) & 0.3627539276 & 0.1894607450 & 67.9677626251 & 110.890 & 62 \\ LB_{35}^{\perp}(0.75) & 0.4903485127 & 0.1991992446 & 114.2530489267 & 109.638 & 62 \\ LB_{35}^{\perp}(0.75) & 0.3179795403 & 0.262939702 & 67.9132993588 & 109.638 & 62 \\ LB_{35}^{\perp}(0.75) & 0.3181423472 & 0.0779310702 & 58.6898548060 & 111.433 & 62 \\ LB_{35}^{\perp}(0.75) & 0.3481423472 & 0.0779310702 & 58.6898548060 & 111.433 & 62 \\ LB_{35}^{\perp}(0.75) & 0.185605918 & 0.1204393057 & 48.2038106270 & 118.404 & 66 \\ LB_{35}^{\perp}(0.75) & 0.4066546101 & 0.2161876360 & 84.9972461438 & 117.371 & 66 \\ LB_{35}^{\perp}(0.75) & 0.4006546101 & 0.2161876360 & 84.9972461438 & 117.371 & 66 \\ LB_{35}^{\perp}(0.75) & 0.303648281 & 0.2705939680 & 75.6551445192 & 123.646 & 70 \\ LB_{35}^{\perp}(0.75) & 0.3558263642 & 0.1271499463 & 70.0711814814 & 125.713 & 70 \\ LB_{35}^{\perp}(0.75) & 0.3666436348 & 0.1782820654 & 80.8001762843 & 132.494 & 74 \\ LB_{35}^{\perp}(0.75) & 0.3487887762 & 0.0385642922 & 70.3896308512 & 132.995 & 74 \\ LB_{35}^{\perp}(0.75) & 0.4039901561 & 0.2225640639 & 97.5141753629 & 131.394 & 74 \\ LB_{35}^{\perp}(0.75) & 0.3487887762 & 0.0385642922 & 70.3896308512 & 132.995 & 74 \\ LB_{35}^{\perp}(0.75) & 0.3617854786 & 0.0717293035 & 81.7272612361 & 132.997 & 74 \\ LB_{35}^{\perp}(0.75) & 0.3617854786 & 0.1531862240 & 81.4797594757 & 139.923 & 78 \\ LB_{35}^{\perp}(0.75) & 0.4087169648 & 0.0717293035 & 81.7272612361 & 132.997 & 74 \\ LB_{35}^{\perp}(0.75) & 0.3617854786 & 0.1531862240 & 81.4797594757 & 139.923 & 78 \\ LB_{35}^{\perp}(0.75) & 0.408684802 & 0.2273814344 & 110.1366017255 & 145.408 & 82 \\ LB_{35}^{\perp}(0.75) & 0.408681230 & 0.1673452927 & 105.8649119993 & 147.299 & 82 \\ LB_{35}^{\perp}(0.75) & 0.436681230 & 0.167365297 & 105.8649119993 & 147.299 & 82 \\ LB_{35}^{\perp}(0.75) & 0.3668436349 & 0.2517165071 & 98.6763921324 & 159.383 & 90 \\ LB_{35}^{\perp}(0.75) & 0.3668436349 & 0.1251765071 & 98.6763921324 & 159.383 & 90 \\ LB_{35}^{\perp}(0.75)$						
$\begin{array}{c} LB_{32}^{LS}(0.75) & 0.3884106274 & 0.2071944713 & 73.0101745147 & 103.333 & 58 \\ LB_{32}^{LS}(0.75) & 0.3627539276 & 0.1894607450 & 67.9677626251 & 110.890 & 62 \\ LB_{32}^{LS}(0.75) & 0.4903485127 & 0.1991992446 & 114.2530489267 & 109.638 & 62 \\ LB_{32}^{LS}(0.75) & 0.3179795403 & 0.2629379702 & 67.9432993586 & 109.638 & 62 \\ LB_{32}^{LS}(0.75) & 0.4061419103 & 0.2382710400 & 84.7867751755 & 109.650 & 62 \\ LB_{32}^{LS}(0.75) & 0.4381423472 & 0.0779310702 & 58.6898548060 & 111.433 & 62 \\ LB_{32}^{LS}(0.75) & 0.18856050918 & 0.1204393057 & 48.2038106270 & 118.404 & 66 \\ LB_{32}^{LS}(0.75) & 0.4806546101 & 0.2161876360 & 84.9972461438 & 117.371 & 66 \\ LB_{32}^{LS}(0.75) & 0.4006546101 & 0.2161876360 & 84.9972461438 & 117.371 & 66 \\ LB_{32}^{LS}(0.75) & 0.3063648281 & 0.2705939680 & 75.6551445192 & 123.646 & 70 \\ LB_{32}^{LS}(0.75) & 0.3063648281 & 0.2705939680 & 75.6551445192 & 123.631 & 70 \\ LB_{32}^{LS}(0.75) & 0.365636429 & 0.1271496463 & 70.0711814814 & 125.713 & 70 \\ LB_{32}^{LS}(0.75) & 0.36564348 & 0.1782820654 & 80.801762843 & 132.494 & 74 \\ LB_{32}^{LS}(0.75) & 0.3686436488 & 0.1782820654 & 80.801762843 & 132.494 & 74 \\ LB_{32}^{LS}(0.75) & 0.3487887762 & 0.0835642922 & 70.3896308512 & 132.995 & 74 \\ LB_{32}^{LS}(0.75) & 0.3988539052 & 0.2035444203 & 97.5124252288 & 139.046 & 78 \\ LB_{32}^{LS}(0.75) & 0.3988539052 & 0.2035444203 & 97.7542252288 & 139.046 & 78 \\ LB_{32}^{LS}(0.75) & 0.4068544802 & 0.0717293035 & 81.7272612361 & 132.997 & 74 \\ LB_{32}^{LS}(0.75) & 0.366486130 & 0.1531862240 & 81.4797594757 & 139.923 & 78 \\ LB_{32}^{LS}(0.75) & 0.366391145 & 0.1949996882 & 81.427535972 & 146.599 & 82 \\ LB_{32}^{LS}(0.75) & 0.36530121154 & 0.1949996882 & 81.427535972 & 146.599 & 82 \\ LB_{32}^{LS}(0.75) & 0.3552976675 & 0.1245404438 & 81.7748046257 & 147.275 & 82 \\ LB_{32}^{LS}(0.75) & 0.3664862640 & 0.1717732793 & 89.5915066227 & 154.085 & 86 \\ LB_{32}^{LS}(0.75) & 0.36648626470 & 0.183372430 & 10.32412757962 & 168.211 & 94 \\ LB_{32}^{LS}(0.75) & 0.3664862640 & 0.1481927024 & 93.1815353179 & 161.494 &$						
$\begin{array}{c} LB_{3c}^{\perp}(0.75) & 0.3627539276 & 0.1894607450 & 67.9677626251 & 110.890 & 62 \\ LB_{2d}^{\perp}(0.75) & 0.4903485127 & 0.1991992446 & 114.2530489267 & 109.638 & 62 \\ LB_{2c}^{\perp}(0.75) & 0.3179795403 & 0.2629379702 & 67.9432993586 & 109.638 & 62 \\ LB_{3c}^{\perp}(0.75) & 0.4061419103 & 0.2382710400 & 84.7867751755 & 109.650 & 62 \\ LB_{3c}^{\perp}(0.75) & 0.3481423472 & 0.0779310702 & 58.6898548060 & 111.433 & 62 \\ LB_{3c}^{\perp}(0.75) & 0.1856050918 & 0.1204393057 & 48.2038106270 & 118.404 & 66 \\ LB_{3c}^{\perp}(0.75) & 0.4806546101 & 0.2161876360 & 84.9972461438 & 117.371 & 66 \\ LB_{3c}^{\perp}(0.75) & 0.4006546101 & 0.2161876360 & 84.9972461438 & 117.371 & 66 \\ LB_{3c}^{\perp}(0.75) & 0.3063648281 & 0.2705939680 & 75.6551445192 & 123.631 & 70 \\ LB_{3c}^{\perp}(0.75) & 0.3558263642 & 0.1271496463 & 70.0711814814 & 125.713 & 70 \\ LB_{3c}^{\perp}(0.75) & 0.4039901561 & 0.2225640639 & 97.5141753629 & 131.394 & 74 \\ LB_{3c}^{\perp}(0.75) & 0.3666436348 & 0.1782820654 & 80.8001762843 & 132.494 & 74 \\ LB_{3c}^{\perp}(0.75) & 0.3487887762 & 0.0835642922 & 70.3896308512 & 132.995 & 74 \\ LB_{3c}^{\perp}(0.75) & 0.4037169648 & 0.0717293035 & 81.7276213361 & 132.997 & 74 \\ LB_{3c}^{\perp}(0.75) & 0.3617854786 & 0.1531862240 & 81.4797594757 & 139.923 & 78 \\ LB_{4c}^{\perp}(0.75) & 0.1879766977 & 0.0838158904 & 58.391568318 & 147.365 & 82 \\ LB_{4c}^{\perp}(0.75) & 0.4068544802 & 0.2273814344 & 110.1366017255 & 145.408 & 82 \\ LB_{4c}^{\perp}(0.75) & 0.353021154 & 0.1949096882 & 88.1427535972 & 146.599 & 82 \\ LB_{4c}^{\perp}(0.75) & 0.3552976675 & 0.1245404438 & 81.7748946257 & 147.275 & 82 \\ LB_{4c}^{\perp}(0.75) & 0.3528780404 & 0.1717732793 & 89.591506627 & 154.085 & 86 \\ LB_{4c}^{\perp}(0.75) & 0.3528780404 & 0.1717732793 & 89.591506627 & 154.085 & 86 \\ LB_{4c}^{\perp}(0.75) & 0.3660526470 & 0.1839372430 & 103.2412757962 & 168.211 & 94 \\ LB_{5c}^{\perp}(0.75) & 0.3660526470 & 0.1839372430 & 103.2412757962 & 168.211 & 94 \\ LB_{5c}^{\perp}(0.75) & 0.364847369 & 0.1645612931 & 104.5220019814 & 175.668 & 98 \\ LB_{5c}^{\perp}(0.75) & 0.364847369 & 0.1645612931 & 104.5220019814 & 175.668 & 98 \\ LB_{5c}^{\perp}(0.$						
$\begin{array}{c} LB_{bc}^{LS}(0.75) & 0.4903485127 & 0.1991992446 & 114.2530489267 & 109.638 & 62 \\ LB_{bc}^{LS}(0.75) & 0.3179795403 & 0.2629379702 & 67.9432993586 & 109.638 & 62 \\ LB_{bc}^{LS}(0.75) & 0.4061419103 & 0.2382710400 & 84.7867751755 & 109.650 & 62 \\ LB_{bc}^{LS}(0.75) & 0.3481423472 & 0.0779310702 & 58.6898548060 & 111.433 & 62 \\ LB_{bc}^{LS}(0.75) & 0.1856050918 & 0.1204393057 & 48.2038106270 & 118.404 & 66 \\ LB_{bc}^{LS}(0.75) & 0.4006546101 & 0.2161876360 & 84.9972461438 & 117.371 & 66 \\ LB_{bc}^{LS}(0.75) & 0.4112452436 & 0.2412703817 & 98.1488111526 & 123.646 & 70 \\ LB_{bc}^{LS}(0.75) & 0.3063642881 & 0.270593680 & 75.6551445192 & 123.631 & 70 \\ LB_{bc}^{LS}(0.75) & 0.3053642881 & 0.270593680 & 75.6551445192 & 123.631 & 70 \\ LB_{bc}^{LS}(0.75) & 0.3058648281 & 0.270593680 & 97.5141753629 & 131.394 & 74 \\ LB_{bc}^{LS}(0.75) & 0.366643648 & 0.1782820654 & 80.8001762843 & 132.494 & 74 \\ LB_{bc}^{LS}(0.75) & 0.366643648 & 0.1782820654 & 80.8001762843 & 132.494 & 74 \\ LB_{bc}^{LS}(0.75) & 0.36663648 & 0.0717293055 & 81.7272612361 & 132.995 & 74 \\ LB_{bc}^{LS}(0.75) & 0.4037169648 & 0.0717293055 & 81.7272612361 & 132.997 & 74 \\ LB_{bc}^{LS}(0.75) & 0.3617854786 & 0.1531862240 & 81.4797594757 & 139.923 & 78 \\ LB_{bc}^{LS}(0.75) & 0.3617854786 & 0.1531862240 & 81.4797594757 & 139.923 & 78 \\ LB_{bc}^{LS}(0.75) & 0.406851430 & 0.167452927 & 105.864911903 & 147.299 & 82 \\ LB_{bc}^{LS}(0.75) & 0.3608544902 & 0.2273814344 & 110.1366017255 & 145.408 & 82 \\ LB_{bc}^{LS}(0.75) & 0.3552976675 & 0.124540438 & 81.47758972 & 146.599 & 82 \\ LB_{bc}^{LS}(0.75) & 0.352873044 & 0.1717732793 & 89.591566227 & 154.085 & 86 \\ LB_{bc}^{LS}(0.75) & 0.352978049 & 0.2115568486 & 109.2687466676 & 153.092 & 86 \\ LB_{bc}^{LS}(0.75) & 0.3660526470 & 0.1839372430 & 103.2412757962 & 168.211 & 94 \\ LB_{bc}^{LS}(0.75) & 0.3660526470 & 0.1839372430 & 103.2412757962 & 168.211 & 94 \\ LB_{bc}^{LS}(0.75) & 0.3664847369 & 0.1645612391 & 104.5220019814 & 175.668 & 98 \\ LB_{bc}^{LS}(0.75) & 0.3660526470 & 0.1839372430 & 103.2412757962 & 168.211 & 94$						
$\begin{array}{c} LB_{22}^{LC}(0.75) & 0.3179795403 & 0.2629379702 & 67.9432993586 & 109.638 & 62 \\ LB_{22}^{LC}(0.75) & 0.4061419103 & 0.2382710400 & 84.7867751755 & 109.650 & 62 \\ LB_{22}^{LC}(0.75) & 0.3481423472 & 0.0779310702 & 58.6898548060 & 111.433 & 62 \\ LB_{32}^{LC}(0.75) & 0.1856050918 & 0.1204393057 & 48.2038106270 & 118.404 & 66 \\ LB_{32}^{LC}(0.75) & 0.4006546101 & 0.2161876360 & 84.9972461438 & 117.371 & 66 \\ LB_{32}^{LC}(0.75) & 0.4112452436 & 0.2412703817 & 98.1488111526 & 123.646 & 70 \\ LB_{32}^{LC}(0.75) & 0.3063648281 & 0.2705939680 & 75.6551445192 & 123.631 & 70 \\ LB_{32}^{LC}(0.75) & 0.3558263642 & 0.1271496463 & 70.0711814814 & 125.713 & 70 \\ LB_{32}^{LC}(0.75) & 0.3558263642 & 0.1271496463 & 70.0711814814 & 125.713 & 70 \\ LB_{32}^{LC}(0.75) & 0.3666436348 & 0.1782820654 & 80.8001762843 & 132.494 & 74 \\ LB_{32}^{LC}(0.75) & 0.3487887762 & 0.0835642922 & 70.3896308512 & 132.995 & 74 \\ LB_{32}^{LC}(0.75) & 0.3487887762 & 0.0835642922 & 70.3896308512 & 132.995 & 74 \\ LB_{32}^{LC}(0.75) & 0.3487887762 & 0.0835642922 & 70.3896308512 & 132.995 & 74 \\ LB_{32}^{LC}(0.75) & 0.3617854786 & 0.0717293035 & 81.7272612361 & 132.997 & 74 \\ LB_{32}^{LC}(0.75) & 0.3617854786 & 0.1531862240 & 81.4797594757 & 139.923 & 78 \\ LB_{32}^{LC}(0.75) & 0.3617854786 & 0.1531862240 & 81.4797594757 & 139.923 & 78 \\ LB_{32}^{LC}(0.75) & 0.4068544802 & 0.2273814344 & 110.1366017255 & 145.408 & 82 \\ LB_{32}^{LC}(0.75) & 0.3530121154 & 0.1949906882 & 88.1427559972 & 146.599 & 82 \\ LB_{32}^{LC}(0.75) & 0.3530121154 & 0.1949906882 & 88.1427559972 & 146.599 & 82 \\ LB_{32}^{LC}(0.75) & 0.3527830404 & 0.1717732793 & 89.5915066227 & 154.085 & 86 \\ LB_{32}^{LC}(0.75) & 0.3660526470 & 0.1839372430 & 103.2412757962 & 168.211 & 94 \\ LB_{32}^{LC}(0.75) & 0.3660526470 & 0.1839372430 & 103.2412757962 & 168.211 & 94 \\ LB_{32}^{LC}(0.75) & 0.364847369 & 0.124550462391 & 104.5220019814 & 175.668 & 98 \\ LB_{32}^{LC}(0.75) & 0.364847369 & 0.12725506762 & 93.4790465144 & 168.836 & 94 \\ LB_{32}^{LC}(0.75) & 0.364847369 & 0.1645612391 & 104.5220019814 &$						
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$						
$\begin{array}{c} LB_{35}^{\perp}(0.75) & 0.3481423472 & 0.0779310702 & 58.6898548060 & 111.433 & 62 \\ LB_{35}^{\perp}(0.75) & 0.1856050918 & 0.1204393057 & 48.2038106270 & 118.404 & 66 \\ LB_{35}^{\perp}(0.75) & 0.4006546101 & 0.2161876360 & 84.9972461438 & 117.371 & 66 \\ LB_{35}^{\perp}(0.75) & 0.4112452436 & 0.2412703817 & 98.1488111526 & 123.646 & 70 \\ LB_{35}^{\perp}(0.75) & 0.3063648281 & 0.2705939680 & 75.6551445192 & 123.631 & 70 \\ LB_{35}^{\perp}(0.75) & 0.3063648281 & 0.2705939680 & 75.6551445192 & 123.631 & 70 \\ LB_{35}^{\perp}(0.75) & 0.3558263642 & 0.1271496463 & 70.0711814814 & 125.713 & 70 \\ LB_{35}^{\perp}(0.75) & 0.3666436348 & 0.1728220664 & 80.8001762843 & 132.494 & 74 \\ LB_{36}^{\perp}(0.75) & 0.3666436348 & 0.1782820664 & 80.8001762843 & 132.494 & 74 \\ LB_{35}^{\perp}(0.75) & 0.3487887762 & 0.0836642922 & 70.3896308512 & 132.995 & 74 \\ LB_{35}^{\perp}(0.75) & 0.3487887762 & 0.083642922 & 70.3896308512 & 132.995 & 74 \\ LB_{35}^{\perp}(0.75) & 0.3487887762 & 0.083642922 & 70.3896308512 & 132.997 & 74 \\ LB_{36}^{\perp}(0.75) & 0.3617854786 & 0.0717293035 & 81.7272612361 & 132.997 & 74 \\ LB_{36}^{\perp}(0.75) & 0.3617854786 & 0.1531862240 & 81.47979497877 & 139.923 & 78 \\ LB_{46}^{\perp}(0.75) & 0.3617854786 & 0.1531862240 & 81.47979497877 & 139.923 & 78 \\ LB_{46}^{\perp}(0.75) & 0.3618547860 & 0.2273814344 & 110.1366017255 & 145.408 & 82 \\ LB_{46}^{\perp}(0.75) & 0.4068544802 & 0.2273814344 & 110.1366017255 & 145.408 & 82 \\ LB_{46}^{\perp}(0.75) & 0.3530121154 & 0.1949096882 & 88.1427535972 & 146.599 & 82 \\ LB_{46}^{\perp}(0.75) & 0.35302976675 & 0.1245404438 & 81.47748946257 & 147.275 & 82 \\ LB_{46}^{\perp}(0.75) & 0.352976675 & 0.1245404438 & 81.47748946257 & 147.275 & 82 \\ LB_{46}^{\perp}(0.75) & 0.352830404 & 0.1717732793 & 89.515066227 & 154.085 & 86 \\ LB_{46}^{\perp}(0.75) & 0.3669452650 & 0.1481927024 & 93.1815353179 & 161.494 & 90 \\ LB_{56}^{\perp}(0.75) & 0.3669452650 & 0.1481927024 & 93.1815353179 & 161.494 & 90 \\ LB_{56}^{\perp}(0.75) & 0.3669452650 & 0.1481927024 & 93.1815353179 & 161.494 & 90 \\ LB_{56}^{\perp}(0.75) & 0.3669452650 & 0.1481927024 & 93.1815353179 & 161.494 & 90 \\ LB_{56}^{\perp}($						
$\begin{array}{c} LB_{30}^{1,0}(0,75) & 0.1856050918 & 0.1204393057 & 48.2038106270 & 118.404 & 66 \\ LB_{31}^{1,0}(0,75) & 0.4006546101 & 0.2161876360 & 84.9972461438 & 117.371 & 66 \\ LB_{35}^{1,0}(0,75) & 0.4112452436 & 0.2412703817 & 98.1488111526 & 123.646 & 70 \\ LB_{35}^{1,0}(0,75) & 0.3063648281 & 0.2705939680 & 75.6551445192 & 123.631 & 70 \\ LB_{35}^{1,0}(0,75) & 0.3558263642 & 0.1271496463 & 70.0711814814 & 125.713 & 70 \\ LB_{36}^{1,0}(0,75) & 0.4039901561 & 0.2225640639 & 97.5141753629 & 131.394 & 74 \\ LB_{36}^{1,0}(0,75) & 0.3666436348 & 0.1782820654 & 80.8001762843 & 132.494 & 74 \\ LB_{36}^{1,0}(0,75) & 0.3487887762 & 0.0835642922 & 70.3896308512 & 132.995 & 74 \\ LB_{36}^{1,0}(0,75) & 0.3487887762 & 0.0835642922 & 70.3896308512 & 132.997 & 74 \\ LB_{36}^{1,0}(0,75) & 0.3988539052 & 0.2035444203 & 97.7542523288 & 139.046 & 78 \\ LB_{36}^{1,0}(0,75) & 0.3617854786 & 0.1531862240 & 81.4797594757 & 139.923 & 78 \\ LB_{46}^{1,0}(0,75) & 0.1887966977 & 0.0838158904 & 58.9391568318 & 147.365 & 82 \\ LB_{46}^{1,0}(0,75) & 0.4068544802 & 0.2273814344 & 110.366017255 & 145.408 & 82 \\ LB_{46}^{1,0}(0,75) & 0.3530121154 & 0.1949096882 & 88.1427535972 & 146.599 & 82 \\ LB_{46}^{1,0}(0,75) & 0.352976675 & 0.1245404438 & 81.748946257 & 147.279 & 82 \\ LB_{46}^{1,0}(0,75) & 0.352976675 & 0.1245404438 & 81.748946257 & 147.275 & 82 \\ LB_{46}^{1,0}(0,75) & 0.352976675 & 0.1245404438 & 81.748946257 & 147.275 & 82 \\ LB_{46}^{1,0}(0,75) & 0.352976675 & 0.1245404438 & 81.748946257 & 147.275 & 82 \\ LB_{46}^{1,0}(0,75) & 0.352976675 & 0.1245404438 & 81.748946257 & 147.275 & 82 \\ LB_{46}^{1,0}(0,75) & 0.352976675 & 0.124504388 & 81.748946257 & 147.275 & 82 \\ LB_{46}^{1,0}(0,75) & 0.352976675 & 0.124504388 & 81.748946257 & 147.275 & 82 \\ LB_{46}^{1,0}(0,75) & 0.3605452650 & 0.1481927024 & 93.1815353179 & 161.494 & 90 \\ LB_{56}^{1,0}(0,75) & 0.3605452650 & 0.1481927024 & 93.1815353179 & 161.494 & 90 \\ LB_{56}^{1,0}(0,75) & 0.3604487369 & 0.2517165071 & 96.6763921324 & 159.383 & 90 \\ LB_{56}^{1,0}(0,75) & 0.346010783 & 0.0901110425 & 93.479046541$						
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$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$1.B_{35}^{i.c.}(0.75)$					
$\begin{array}{c} 1.\mathrm{B}_{38}^{1.c}(0.75) & 0.4037169648 & 0.0717293035 & 81.7272612361 & 132.997 & 74 \\ 1.\mathrm{B}_{36}^{1.c}(0.75) & 0.3988539052 & 0.2035444203 & 97.7542523288 & 139.046 & 78 \\ 1.\mathrm{B}_{40}^{1.c}(0.75) & 0.3617854786 & 0.1531862240 & 81.4797594757 & 139.923 & 78 \\ 1.\mathrm{B}_{41}^{1.c}(0.75) & 0.1897966977 & 0.0838158904 & 58.9391568318 & 147.365 & 82 \\ 1.\mathrm{B}_{42}^{1.c}(0.75) & 0.4068544802 & 0.2273814344 & 110.1366017255 & 145.408 & 82 \\ 1.\mathrm{B}_{42}^{1.c}(0.75) & 0.4408681230 & 0.1057452927 & 105.8649119093 & 147.299 & 82 \\ 1.\mathrm{B}_{42}^{1.c}(0.75) & 0.3530121154 & 0.1949096882 & 88.1427535972 & 146.599 & 82 \\ 1.\mathrm{B}_{42}^{1.c}(0.75) & 0.3532976675 & 0.1245404438 & 81.7748946257 & 147.275 & 82 \\ 1.\mathrm{B}_{46}^{1.c}(0.75) & 0.3990215187 & 0.2115568486 & 109.2687466676 & 153.092 & 86 \\ 1.\mathrm{B}_{42}^{1.c}(0.75) & 0.1863329193 & 0.1272516263 & 63.1308826408 & 154.153 & 86 \\ 1.\mathrm{B}_{42}^{1.c}(0.75) & 0.3527830404 & 0.1717732793 & 89.5915066227 & 154.085 & 86 \\ 1.\mathrm{B}_{49}^{1.c}(0.75) & 0.3265683649 & 0.2517165071 & 98.6763921324 & 159.383 & 90 \\ 1.\mathrm{B}_{50}^{1.c}(0.75) & 0.3660526470 & 0.1839372430 & 103.2412757962 & 168.211 & 94 \\ 1.\mathrm{B}_{52}^{1.c}(0.75) & 0.3660526470 & 0.1839372430 & 103.2412757962 & 168.211 & 94 \\ 1.\mathrm{B}_{52}^{1.c}(0.75) & 0.3549037937 & 0.1225506762 & 93.4790465414 & 168.836 & 94 \\ 1.\mathrm{B}_{52}^{1.c}(0.75) & 0.3997914828 & 0.2009996337 & 122.7109281615 & 174.757 & 98 \\ 1.\mathrm{B}_{55}^{1.c}(0.75) & 0.3644847369 & 0.1645612391 & 104.5220019814 & 175.668 & 98 \\ 1.\mathrm{B}_{55}^{1.c}(0.75) & 0.3297350406 & 0.1911985943 & 102.569898873 & 182.305 & 102 \\ 1.\mathrm{B}_{55}^{1.c}(0.75) & 0.3297350406 & 0.1911985943 & 102.569898873 & 182.305 & 102 \\ 1.\mathrm{B}_{56}^{1.c}(0.75) & 0.4035305311 & 0.2218058040 & 134.0109575462 & 181.148 & 102 \\ 1.\mathrm{B}_{56}^{1.c}(0.75) & 0.4035305311 & 0.2218058040 & 134.0109575462 & 181.148 & 102 \\ 1.\mathrm{B}_{56}^{1.c}(0.75) & 0.4101378717 & 0.1341894173 & 121.0976361440 & 183.067 & 102 \\ 1.\mathrm{B}_{50}^{1.c}(0.75) & 0.4101378717 & 0.1341894173 & 121.0976361440 & 183.067 \\ 1.02$	$1.B_{36}^{i.c.}(0.75)$					
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$\begin{array}{c} \text{I.B}_{40}^{\text{i.c.}}(0.75) & 0.3617854786 & 0.1531862240 & 81.4797594757 & 139.923 & 78 \\ \text{I.B}_{41}^{\text{i.c.}}(0.75) & 0.1897966977 & 0.0838158904 & 58.9391568318 & 147.365 & 82 \\ \text{I.B}_{43}^{\text{i.c.}}(0.75) & 0.4068544802 & 0.2273814344 & 110.1366017255 & 145.408 & 82 \\ \text{I.B}_{43}^{\text{i.c.}}(0.75) & 0.4408681230 & 0.1057452927 & 105.8649119093 & 147.299 & 82 \\ \text{I.B}_{44}^{\text{i.c.}}(0.75) & 0.3530121154 & 0.1949096882 & 88.1427535972 & 146.599 & 82 \\ \text{I.B}_{46}^{\text{i.c.}}(0.75) & 0.3552976675 & 0.1245404438 & 81.7748946257 & 147.275 & 82 \\ \text{I.B}_{46}^{\text{i.c.}}(0.75) & 0.3990215187 & 0.2115568486 & 109.2687466676 & 153.092 & 86 \\ \text{I.B}_{46}^{\text{i.c.}}(0.75) & 0.1863329193 & 0.1272516263 & 63.1308826408 & 154.153 & 86 \\ \text{I.B}_{46}^{\text{i.c.}}(0.75) & 0.3527830404 & 0.1717732793 & 89.5915066227 & 154.085 & 86 \\ \text{I.B}_{46}^{\text{i.c.}}(0.75) & 0.3265683649 & 0.2517165071 & 98.6763921324 & 159.383 & 90 \\ \text{I.B}_{56}^{\text{i.c.}}(0.75) & 0.3605452650 & 0.1481927024 & 93.1815353179 & 161.494 & 90 \\ \text{I.B}_{56}^{\text{i.c.}}(0.75) & 0.3600526470 & 0.1839372430 & 103.2412757962 & 168.211 & 94 \\ \text{I.B}_{56}^{\text{i.c.}}(0.75) & 0.3549037937 & 0.1225506762 & 93.4790465414 & 168.836 & 94 \\ \text{I.B}_{56}^{\text{i.c.}}(0.75) & 0.3644847369 & 0.1645612391 & 104.5220019814 & 175.668 & 98 \\ \text{I.B}_{56}^{\text{i.c.}}(0.75) & 0.3496110783 & 0.0901110425 & 93.7904600164 & 176.119 & 98 \\ \text{I.B}_{56}^{\text{i.c.}}(0.75) & 0.3297350406 & 0.1911985943 & 102.5698998873 & 182.305 & 102 \\ \text{I.B}_{56}^{\text{i.c.}}(0.75) & 0.4010378717 & 0.1341894173 & 121.0976361440 & 183.067 & 102 \\ \text{I.B}_{56}^{\text{i.c.}}(0.75) & 0.401378717 & 0.1341894173 & 121.0976361440 & 183.067 & 102 \\ \text{I.B}_{56}^{\text{i.c.}}(0.75) & 0.4010378717 & 0.1341894173 & 121.0976361440 & 183.067 & 102 \\ \text{I.B}_{56}^{\text{i.c.}}(0.75) & 0.4010378717 & 0.1341894173 & 121.0976361440 & 183.067 & 102 \\ \text{I.B}_{56}^{\text{i.c.}}(0.75) & 0.4010378717 & 0.1341894173 & 121.0976361440 & 183.067 & 102 \\ \text{I.B}_{56}^{\text{i.c.}}(0.75) & 0.4010378717 & 0.1341894173 & 121.0976361440 & 183.067 & 102 \\ \text{I.B}_{56}^{i.c.$						
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$\begin{array}{c} \textbf{I.B}_{43}^{4.c.}(0.75) & 0.4408681230 & 0.1057452927 & 105.8649119093 & 147.299 & 82 \\ \textbf{I.B}_{44}^{4.c.}(0.75) & 0.3530121154 & 0.1949096882 & 88.1427535972 & 146.599 & 82 \\ \textbf{I.B}_{45}^{4.c.}(0.75) & 0.3552976675 & 0.1245404438 & 81.7748946257 & 147.275 & 82 \\ \textbf{I.B}_{46}^{4.c.}(0.75) & 0.3990215187 & 0.2115568486 & 109.2687466676 & 153.092 & 86 \\ \textbf{I.B}_{47}^{4.c.}(0.75) & 0.1863329193 & 0.1272516263 & 63.1308826408 & 154.153 & 86 \\ \textbf{I.B}_{48}^{4.c.}(0.75) & 0.3527830404 & 0.1717732793 & 89.5915066227 & 154.085 & 86 \\ \textbf{I.B}_{49}^{4.c.}(0.75) & 0.3265683649 & 0.2517165071 & 98.6763921324 & 159.383 & 90 \\ \textbf{I.B}_{50}^{4.c.}(0.75) & 0.3605452650 & 0.1481927024 & 93.1815353179 & 161.494 & 90 \\ \textbf{I.B}_{51}^{5.c.}(0.75) & 0.3660526470 & 0.1839372430 & 103.2412757962 & 168.211 & 94 \\ \textbf{I.B}_{52}^{5.c.}(0.75) & 0.3549037937 & 0.1225506762 & 93.4790465414 & 168.836 & 94 \\ \textbf{I.B}_{54}^{5.c.}(0.75) & 0.3549037937 & 0.1225506762 & 93.4790465414 & 168.836 & 94 \\ \textbf{I.B}_{54}^{5.c.}(0.75) & 0.3644847369 & 0.1645612391 & 104.5220019814 & 175.668 & 98 \\ \textbf{I.B}_{55}^{5.c.}(0.75) & 0.3496110783 & 0.0901110425 & 93.7904060164 & 176.119 & 98 \\ \textbf{I.B}_{55}^{5.c.}(0.75) & 0.3297350406 & 0.1911985943 & 102.5698998873 & 182.305 & 102 \\ \textbf{I.B}_{55}^{5.c.}(0.75) & 0.4035305311 & 0.2218058040 & 134.0109575462 & 181.148 & 102 \\ \textbf{I.B}_{55}^{5.c.}(0.75) & 0.401378717 & 0.1341894173 & 121.0976361440 & 183.067 & 102 \\ \textbf{I.B}_{55}^{5.c.}(0.75) & 0.4101378717 & 0.1341894173 & 121.0976361440 & 183.067 & 102 \\ \textbf{I.B}_{55}^{5.c.}(0.75) & 0.4101378717 & 0.1341894173 & 121.0976361440 & 183.067 & 102 \\ \textbf{I.B}_{55}^{5.c.}(0.75) & 0.4101378717 & 0.1341894173 & 121.0976361440 & 183.067 & 102 \\ \textbf{I.B}_{55}^{5.c.}(0.75) & 0.4101378717 & 0.1341894173 & 121.0976361440 & 183.067 & 102 \\ \textbf{I.B}_{55}^{5.c.}(0.75) & 0.4101378717 & 0.1341894173 & 121.0976361440 & 183.067 & 102 \\ \textbf{I.B}_{55}^{5.c.}(0.75) & 0.4101378717 & 0.1341894173 & 121.0976361440 & 183.067 & 102 \\ \textbf{I.B}_{55}^{5.c.}(0.75) & 0.4101378717 & 0.1341894173 & 121.0976361440 & 183.067 $						
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$\begin{array}{c} \text{I.B}_{50}^{\text{i.c.}}(0.75) & 0.3605452650 & 0.1481927024 & 93.1815353179 & 161.494 & 90 \\ \text{I.B}_{51}^{\text{i.c.}}(0.75) & 0.3660526470 & 0.1839372430 & 103.2412757962 & 168.211 & 94 \\ \text{I.B}_{52}^{\text{i.c.}}(0.75) & 0.4010926159 & 0.2171880829 & 121.4505154029 & 167.125 & 94 \\ \text{I.B}_{53}^{\text{i.c.}}(0.75) & 0.3549037937 & 0.1225506762 & 93.4790465414 & 168.836 & 94 \\ \text{I.B}_{54}^{\text{i.c.}}(0.75) & 0.3997914828 & 0.2009996337 & 122.7109281615 & 174.757 & 98 \\ \text{I.B}_{55}^{\text{i.c.}}(0.75) & 0.3644847369 & 0.1645612391 & 104.5220019814 & 175.668 & 98 \\ \text{I.B}_{56}^{\text{i.c.}}(0.75) & 0.3496110783 & 0.0901110425 & 93.7904060164 & 176.119 & 98 \\ \text{I.B}_{57}^{\text{i.c.}}(0.75) & 0.3297350406 & 0.1911985943 & 102.5698998873 & 182.305 & 102 \\ \text{I.B}_{56}^{\text{i.c.}}(0.75) & 0.4035305311 & 0.2218058040 & 134.0109575462 & 181.148 & 102 \\ \text{I.B}_{59}^{\text{i.c.}}(0.75) & 0.4101378717 & 0.1341894173 & 121.0976361440 & 183.067 & 102 \\ \end{array}$						
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$I.B_{59}^{i.c.}(0.75)$ 0.4101378717 0.1341894173 121.0976361440 183.067 102						

Table S. XVI. Initial conditions and periods T of the periodic three-body orbits for class I.B in the case of  $\boldsymbol{r}_1(0)=(-1,0)=-\boldsymbol{r}_2(0),\ \dot{\boldsymbol{r}}_1(0)=(v_1,v_2)=\dot{\boldsymbol{r}}_2(0)$  and  $\boldsymbol{r}_3(0)=(0,0),\ \dot{\boldsymbol{r}}_3(0)=(-2v_1/m_3,-2v_2/m_3)$  when G=1 and  $m_1=m_2=1$  and  $m_3=0.75$  by means of the search grid  $4000\times4000$  in the interval  $T_0\in[0,200]$ , where  $T^*=T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

Class and number	$v_1$	$v_2$	T	$T^*$	$L_f$
$I.B_{61}^{i.c.}(0.75)$	0.3654676797	0.2013570084	114.7433059587	182.305	102
$I.B_{62}^{i.c.}(0.75)$	0.3449318936	0.0361030611	94.1918485647	183.345	102
$I.B_{63}^{i.c.}(0.75)$	0.3415958742	0.1781262476	108.0595164221	189.810	106
$I.B_{64}^{i.c.}(0.75)$	0.4877782970	0.2017718810	193.6578545086	187.395	106
$I.B_{65}^{i.c.}(0.75)$	0.3984752813	0.2084246515	133.7314062042	188.808	106
$I.B_{66}^{i.c.}(0.75)$	0.3665861978	0.1770429252	115.5336619786	189.810	106
$I.B_{67}^{i.c.}(0.75)$	0.3545991926	0.1209830667	105.1835820780	190.398	106
$I.B_{68}^{i.c.}(0.75)$	0.2542763689	0.0906969814	86.6765732076	197.702	110
$I.B_{69}^{i.c.}(0.75)$	0.4385158820	0.0640412131	136.5399121956	197.702	110
$I.B_{70}^{i.c.}(0.75)$	0.3498884353	0.0921885496	105.4909744219	197.680	110
$I.B_{71}^{i.c.}(0.75)$	0.3195165115	0.2668419229	126.3377271874	201.388	114
$I.B_{72}^{i.c.}(0.75)$	0.3588402297	0.1409427418	116.5906134693	204.629	114
$I.B_{73}^{i.c.}(0.75)$	0.1882107801	0.0759993295	84.3610994604	212.092	118
$I.B_{74}^{i.c.}(0.75)$	0.3543567239	0.1197162571	116.8884609087	211.959	118
$I.B_{75}^{i.c.}(0.75)$	0.3789094561	0.2014477498	142.9781422625	218.012	122
$I.B_{76}^{i.c.}(0.75)$	0.3263199440	0.2689451474	138.1775267185	215.378	122
$I.B_{77}^{i.c.}(0.75)$	0.3861811965	0.1515735137	136.6224402902	218.823	122
$I.B_{78}^{i.c.}(0.75)$	0.3501113687	0.0938162205	117.1915230963	219.241	122
$I.B_{79}^{i.c.}(0.75)$	0.3368216321	0.1814605343	127.4112970385	225.529	126
$I.B_{80}^{i.c.}(0.75)$	0.3582187899	0.1382234521	128.2933049194	226.194	126
$I.B_{81}^{i.c.}(0.75)$	0.3462687542	0.0578449911	117.5796866317	226.478	126
$I.B_{82}^{i.c.}(0.75)$	0.2922389186	0.1233632111	112.1079121569	233.528	130
$I.B_{83}^{i.c.}(0.75)$ $I.B_{84}^{i.c.}(0.75)$	0.3650176895	0.1669814530	139.2710431601	$232.987 \\ 240.971$	130 134
$I.B_{85}^{i.c.}(0.75)$	0.1892866683 $0.3467696508$	0.0221911859 $0.0639486684$	$94.4736496681 \\ 129.2760551437$	248.043	134
$I.B_{86}^{i.c.}(0.75)$ $I.B_{86}^{i.c.}(0.75)$	0.3665370880	0.0039480084 $0.1763780053$	150.2600860454	247.126	138
$I.B_{87}^{i.c.}(0.75)$	0.4083790203	0.0486715665	153.0421639861	248.051	138
$I.B_{88}^{i.c.}(0.75)$ $I.B_{88}^{i.c.}(0.75)$	0.3690686913	0.1623965482	153.0450189807	254.568	142
$I.B_{89}^{i.c.}(0.75)$ $I.B_{89}^{i.c.}(0.75)$	0.3539953094	0.1023903482 $0.1177951302$	140.2991413854	255.081	142
$I.B_{90}^{i.c.}(0.75)$	0.3504472774	0.0962027008	140.5923548014	262.363	146
$I.B_{91}^{i.c.}(0.75)$	0.3607597004	0.1491004927	151.3989526999	261.966	146
$I.B_{92}^{i.c.}(0.75)$	0.3238354084	0.1371451748	140.0376600674	269.324	150
$I.B_{93}^{i.c.}(0.75)$	0.3572663963	0.1339427411	151.6983734688	269.323	150
$I.B_{94}^{i.c.}(0.75)$	0.3538571400	0.1170498280	152.0049074323	276.642	154
$I.B_{95}^{i.c.}(0.75)$	0.1558167527	0.0551420353	105.7114373172	276.818	154
$I.B_{96}^{i.c.}(0.75)$	0.3116737151	0.2686682600	172.3506257247	279.145	158
$I.B_{97}^{i.c.}(0.75)$	0.3601419243	0.1464946578	163.1073152563	283.535	158
$I.B_{98}^{i.c.}(0.75)$	0.3666366005	0.1799469189	172.8962301998	282.847	158
$I.B_{99}^{i.c.}(0.75)$	0.1903231208	0.1095157688	115.2343315677	283.684	158
$I.B_{100}^{i.c.}(0.75)$	0.3954025458	0.0884270772	171.7557975683	283.926	158
$I.B_{101}^{i.c.}(0.75)$	0.2741018535	0.2554039554	158.4790653853	286.886	162
$I.B_{102}^{i.c.}(0.75)$	0.3475551472	0.0723444026	152.6716275805	291.172	162
$I.B_{103}^{i.c.}(0.75)$	0.2818186464	0.1243991912	140.4484779108	298.229	166
$I.B_{104}^{i.c.}(0.75)$	0.3626205078	0.1566624854	174.4937355907	297.722	166
$I.B_{105}^{i.c.}(0.75)$	0.3027698354	0.0259686608	139.3368817906	298.387	166
$I.B_{106}^{i.c.}(0.75)$	0.1894542670	0.0321508176	120.0636229967	305.697	170
$I.B_{107}^{i.c.}(0.75)$	0.2539168728	0.0672679036	135.4481207778	312.762	174
$I.B_{108}^{i.c.}(0.75)$	0.1902702982	0.0922368820	125.6625896697	312.635	174
$I.B_{109}^{i.c.}(0.75)$	0.3619799733	0.1540068560	186.2125683041	319.296	178
$I.B_{110}^{i.c.}(0.75)$	0.3663329795	0.1827419446	195.3412841176	318.565	178
$I.B_{111}^{i.c.}(0.75)$	0.3660548095	0.1724827017	196.9056013811	326.034	182
$I.B_{112}^{i.c.}(0.75)$	0.3507844479	0.0985240185	175.6925290041	327.046	182
$I.B_{113}^{i.c.}(0.75)$	0.3044228733	0.1358113832	166.1997019758	334.023	186
$I.B_{114}^{i.c.}(0.75)$	0.2542619684	0.1152551288	152.1697588812	341.537	190
$I.B_{115}^{i.c.}(0.75)$	0.3560451335	0.1282099353	198.5101580823	355.575	198
$I.B_{116}^{i.c.}(0.75)$	0.2804847900	0.1262187708	170.7534255757	362.934	202
$I.B_{117}^{i.c.}(0.75)$	0.3534701962	0.1149289573	198.8305779481	362.885	202
$I.B_{118}^{i.c.}(0.75)$	0.2555646230	0.0547119924	184.8709866273	427.814	238
$I.B_{119}^{i.c.}(0.75)$	0.2195606596	0.0417898885	192.3469350978	471.042	262

Table S. XVII. Initial conditions and periods T of the periodic three-body orbits for class I.C, II.A, II.B and II.C in the case of  $\mathbf{r}_1(0) = (-1,0) = -\mathbf{r}_2(0)$ ,  $\dot{\mathbf{r}}_1(0) = (v_1,v_2) = \dot{\mathbf{r}}_2(0)$  and  $\mathbf{r}_3(0) = (0,0)$ ,  $\dot{\mathbf{r}}_3(0) = (-2v_1/m_3, -2v_2/m_3)$  when G=1 and  $m_1=m_2=1$  and  $m_3=0.75$  by means of the search grid  $4000\times 4000$  in the interval  $T_0\in[0,200]$ , where  $T^*=T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

Class and number	$v_1$	$v_2$	T	$T^*$	$L_f$
$I.C_1^{i.c.}(0.75)$	0.3492568012	0.0873658756	164.1798151269	309.114	172
II.A <sub>1</sub> <sup>i.c.</sup> $(0.75)$	0.0457217669	0.4155107550	35.7782521925	56.700	32
$II.A_2^{i.c.}(0.75)$	0.3662039254	0.1902792904	177.3768529401	286.154	160
$II.B_1^{i.c.}(0.75)$	0.4381285967	0.3220362889	49.4213483013	43.320	26
$II.B_2^{i.c.}(0.75)$	0.0101896437	0.4481073728	38.7358240153	55.005	30
$II.B_3^{\bar{i}.c.}(0.75)$	0.4307203115	0.3088925608	60.0883362355	57.396	34
$II.B_4^{i.c.}(0.75)$	0.4948733005	0.2916775784	89.3424667418	62.744	38
$II.B_5^{i.c.}(0.75)$	0.2400776886	0.2653522221	84.1773067006	159.382	90
$II.B_6^{i.c.}(0.75)$	0.4923254515	0.2163677765	181.8030296771	165.584	94
$II.B_7^{i.c.}(0.75)$	0.3303505354	0.2593875979	119.0524138670	187.395	106
$II.B_8^{i.c.}(0.75)$	0.2917672411	0.2642502660	130.2350131232	223.133	126
$\text{II.C}_{1}^{i.c.}(0.75)$	0.4035774909	0.4506146940	40.5609005922	21.662	14
$II.C_2^{i.c.}(0.75)$	0.1324227283	0.5620538107	36.1934395342	24.808	14
$II.C_3^{i.c.}(0.75)$	0.1341111080	0.4453950464	24.2244479627	32.110	18
II. $C_4^{i.c.}(0.75)$	0.4517904481	0.3984638333	48.3454418351	26.479	18 20
$II.C_5^{i.c.}(0.75)$ $II.C_6^{i.c.}(0.75)$	0.2393043211 $0.1116511450$	0.5033839293 $0.2453018959$	42.5998505567 15.2915537330	34.028 $34.906$	20
$II.C_6 (0.75)$ $II.C_7^{i.c.}(0.75)$	0.3480362654	0.2453018959 $0.3387435791$	28.8665011010	34.911	20
II. $C_8^{i.c.}(0.75)$	0.0786239428	0.3387433791	14.4096938296	34.911	20
$II.C_8 (0.75)$ $II.C_9^{i.c.}(0.75)$	0.0780239428	0.2174331233	14.6316164321	39.529	20
	0.5663074233	0.0801318300 $0.2564527847$	81.3382389484	36.201	22
II. $C_{10}^{i.c.}(0.75)$ II. $C_{11}^{i.c.}(0.75)$	0.3133201544	0.2667139185	24.0027591665	38.878	22
$II.C_{12}^{i.c.}(0.75)$	0.2776333523	0.3754936101	27.4777277295	36.138	22
II. $C_{13}^{i.c.}(0.75)$	0.3979273741	0.4420643033	52.2403095602	30.782	22
II. $C_{14}^{i.c.}(0.75)$	0.2337646460	0.5184766402	56.5821385868	41.552	$\frac{22}{24}$
$II.C_{15}^{i.c.}(0.75)$	0.3880716012	0.2990924298	41.2462573723	48.876	28
$II.C_{16}^{i.c.}(0.75)$	0.4865292170	0.3089091578	66.3906894676	45.926	28
$II.C_{17}^{i.c.}(0.75)$	0.1104500350	0.0419264385	19.8549898630	54.017	30
$II.C_{18}^{i.c.}(0.75)$	0.4882350663	0.2165892200	56.7379512652	52.865	30
$II.C_{19}^{i.c.}(0.75)$	0.4740333642	0.2953568360	63.5896788482	50.380	30
II. $C_{20}^{i.c.}(0.75)$	0.1657977156	0.2455083068	26.0997008776	56.742	32
$II.C_{21}^{i.c.}(0.75)$	0.1029343451	0.0959035704	21.4416761259	57.375	32
$II.C_{22}^{i.c.}(0.75)$	0.3642933566	0.2731827113	43.3607662662	59.855	34
$II.C_{23}^{\tilde{i}.\tilde{c}.}(0.75)$	0.4263629526	0.2972052504	56.5834451288	57.398	34
$II.C_{24}^{\vec{i}.\vec{c}.}(0.75)$	0.2599355931	0.2563418408	34.3131840064	63.752	36
$II.C_{25}^{\overline{i.c.}}(0.75)$	0.1081605506	0.2250738084	27.0471407539	63.765	36
$II.C_{26}^{i.c.}(0.75)$	0.3345621166	0.2805925385	45.0497509698	66.843	38
$II.C_{27}^{i.c.}(0.75)$	0.4559070521	0.2389218004	64.0813709053	66.847	38
$II.C_{28}^{i.c.}(0.75)$	0.4200713008	0.2503863362	56.1760930747	66.862	38
$II.C_{29}^{i.c.}(0.75)$	0.1657156665	0.1155208590	27.0714448548	68.140	38
$II.C_{30}^{i.c.}(0.75)$	0.2108168025	0.4566831086	61.2686739131	68.034	40
$II.C_{31}^{i.c.}(0.75)$	0.3297304506	0.2585199450	44.7971548880	70.759	40
$II.C_{32}^{i.c.}(0.75)$	0.4597689840	0.1105992605	56.0808786052	71.891	40
$II.C_{33}^{i.c.}(0.75)$	0.4455245438	0.2894791201	70.8609796509	67.167	40
$II.C_{34}^{i.c.}(0.75)$	0.2177623204	0.4365092717	54.8016637683	65.608	40
$II.C_{35}^{i.c.}(0.75)$	0.3610750806	0.4162413089	81.4888700221	68.038	40
$II.C_{36}^{i.c.}(0.75)$	0.3211861550	0.1095009097	38.0767038800	75.462	42
$II.C_{37}^{i.c.}(0.75)$	0.4799903735	0.2090171894	78.3383579606	77.757	44
$II.C_{38}^{i.c.}(0.75)$	0.3984873246	0.3859118015	95.4378576535	77.672	46
$II.C_{39}^{i.c.}(0.75)$	0.3419100775	0.1964415407	48.0914105413	82.226	46
$II.C_{40}^{i.c.}(0.75)$	0.1831280973	0.2527284011	38.7565386929	81.609	46
II. $C_{41}^{i.c.}(0.75)$	0.4668339454	0.2196009821	81.7805347369	84.753	48
$II.C_{42}^{i.c.}(0.75)$	0.2695654045	0.1653865705	41.2047988880	86.007	48
$II.C_{43}^{i.c.}(0.75)$	0.2471716099	0.4591801620	84.6911160699	85.058	50
$II.C_{44}^{i.c.}(0.75)$	0.2348993023	0.0496701443	37.5752092509	89.884	50
$II.C_{45}^{i.c.}(0.75)$	0.4607930537	0.2258237872	87.1992395234	91.746	52
II. $C_{46}^{i.c.}(0.75)$	0.4935658319	0.2099521465	99.8457220227	91.744	52
$II.C_{47}^{i.c.}(0.75)$	0.2312703709	0.2651474356	49.7439541861	95.631	54
$II.C_{48}^{i.c.}(0.75)$	0.4311991946	0.3298943041	101.2913661831	89.267	54
$II.C_{49}^{i.c.}(0.75)$	0.2786464975	0.4912442516	122.3430901926	92.589	54 5c
$II.C_{50}^{i.c.}(0.75)$	0.2591764961	0.1464220096	46.3658598456	100.521	56

Table S. XVIII. Initial conditions and periods T of the periodic three-body orbits for class II.C in the case of  $\boldsymbol{r}_1(0)=(-1,0)=-\boldsymbol{r}_2(0),\ \dot{\boldsymbol{r}}_1(0)=(v_1,v_2)=\dot{\boldsymbol{r}}_2(0)$  and  $\boldsymbol{r}_3(0)=(0,0),\ \dot{\boldsymbol{r}}_3(0)=(-2v_1/m_3,-2v_2/m_3)$  when G=1 and  $m_1=m_2=1$  and  $m_3=0.75$  by means of the search grid  $4000\times 4000$  in the interval  $T_0\in[0,200]$ , where  $T^*=T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

Class and number	$v_1$	$v_2$	T	$T^*$	$L_f$
$II.C_{51}^{i.c.}(0.75)$	0.3292245081	0.2700636997	64.0015597083	98.738	56
$II.C_{52}^{i.c.}(0.75)$	0.1761148939	0.2525176515	46.8228354487	99.462	56
$II.C_{53}^{i.c.}(0.75)$	0.2832288095	0.2029755275	51.6398129899	100.115	56
$II.C_{54}^{i.c.}(0.75)$	0.3734550631	0.2016974843	64.5586481759	100.079	56
$II.C_{55}^{i.c.}(0.75)$	0.1079453802	0.1017385147	37.6522550231	100.118	56
$II.C_{56}^{i.c.}(0.75)$	0.2893615743	0.1732851961	52.1564228215	103.868	58
$II.C_{57}^{i.c.}(0.75)$	0.2739490611	0.1112476104	49.5347587742	107.810	60
$II.C_{58}^{i.c.}(0.75)$	0.3143025441	0.4537167635	123.0075052602	102.059	60
$II.C_{59}^{i.c.}(0.75)$	0.4442192619	0.2378705824	95.6646185864	105.733	60
$II.C_{60}^{i.c.}(0.75)$	0.1121048529	0.0933447023	40.3882383150	107.616	60
$II.C_{61}^{i.c.}(0.75)$	0.3654261757	0.4156535144	124.2929450326	102.061	60
$II.C_{62}^{i.c.}(0.75)$	0.2834284459	0.2714433145	63.7610535892	109.638	62
$II.C_{63}^{i.c.}(0.75)$	0.4748987889	0.2113333814	108.1264470442	109.638	62
$II.C_{64}^{i.c.}(0.75)$	0.2368845535	0.1168010406	49.9219991102	114.999	64
$II.C_{65}^{i.c.}(0.75)$	0.4039679707	0.2577569632	90.4576937114	112.726	64
$II.C_{66}^{i.c.}(0.75)$	0.3284638906	0.2771383238	74.0903119133	112.719	64
$II.C_{67}^{i.c.}(0.75)$	0.4954331403	0.2139970413	125.2599481230	112.719	64
$II.C_{68}^{i.c.}(0.75)$	0.3256664816	0.1877076128	65.3726386438	117.933	66
$II.C_{69}^{i.c.}(0.75)$	0.2177444543	0.4489972741	98.5606081526	111.692	66
$II.C_{70}^{i.c.}(0.75)$	0.1116166364	0.0787235985	45.4896538661	122.120	68
$II.C_{71}^{i.c.}(0.75)$	0.2166794086	0.2627405139	60.9820654333	120.507	68
$II.C_{72}^{i.c.}(0.75)$	0.4505347794	0.0407260752	87.5074099143	122.244	68
$II.C_{73}^{i.c.}(0.75)$	0.3236624960	0.4321844272	132.5563445697	119.080	70
$II.C_{74}^{i.c.}(0.75)$	0.3100482475	0.4328437530	126.4841220008	119.076	70
$II.C_{75}^{i.c.}(0.75)$	0.4520015173	0.2260302455	112.7189121601	123.633	70
$II.C_{76}^{i.c.}(0.75)$	0.3784298834	0.2495376710	88.8298126856	123.634	70
$II.C_{77}^{i.c.}(0.75)$	0.3002068606	0.2595660581	75.1530170603	127.505	72
$II.C_{78}^{i.c.}(0.75)$	0.5034392071	0.1881587075	139.7265179198	127.518	72
$II.C_{79}^{i.c.}(0.75)$	0.2792793082	0.1452471143	63.4672809336	132.888	74
$II.C_{80}^{i.c.}(0.75)$	0.1704946320	0.4652537474	109.7290454265	126.546	74
$II.C_{81}^{i.c.}(0.75)$	0.3886930945	0.1996476392	91.6231246943	135.788	76
$II.C_{82}^{i.c.}(0.75)$	0.2980200648	0.2673702020	80.1527850060	134.516	76
$II.C_{83}^{i.c.}(0.75)$	0.2092067208	0.4812270767	130.5165320395	128.641	76
$II.C_{84}^{i.c.}(0.75)$	0.1206988814	0.0968566597	51.5317621978	136.261	76
$II.C_{85}^{i.c.}(0.75)$	0.2017659812	0.1315138428	57.0335074764	136.280	76
$II.C_{86}^{i.c.}(0.75)$	0.4493814446	0.0747851880	101.6265270381	140.162	78
$II.C_{87}^{i.c.}(0.75)$	0.4800092637	0.2196511819	142.2301391511	137.618	78
$II.C_{88}^{i.c.}(0.75)$	0.4984728341	0.2029987070	151.5260848913	137.617	78
$II.C_{89}^{i.c.}(0.75)$	0.1132373526	0.0848401539	52.3492264971	139.989	78
$II.C_{90}^{i.c.}(0.75)$	0.2119580098	0.2619382262	72.9469032636	145.383	82
$II.C_{91}^{i.c.}(0.75)$	0.3555624393	0.2519592295	99.7762660659	148.518	84
$II.C_{92}^{i.c.}(0.75)$	0.3320163863	0.2697721730	98.9557389721	151.603	86
$II.C_{93}^{i.c.}(0.75)$	0.3390466919	0.2772682633	102.6797843211	151.602	86
$II.C_{94}^{i.c.}(0.75)$	0.3923869301	0.1982688683	104.7040889511	153.642	86
$II.C_{95}^{i.c.}(0.75)$	0.1133073970	0.0721736877	57.4443319839	154.480	86
$II.C_{96}^{i.c.}(0.75)$	0.3087791621	0.1727360704	80.5689065010	154.124	86
$II.C_{97}^{i.c.}(0.75)$	0.1935268473	0.2576335629	75.8206239259	156.235	88
$II.C_{98}^{i.c.}(0.75)$	0.2937269662	0.1310053585	76.6069327690	158.064	88
$II.C_{99}^{i.c.}(0.75)$	0.3119641579	0.4353298592	165.7961110787	153.101	90
$II.C_{100}^{i.c.}(0.75)$	0.2560099766	0.4488404136	148.3975005786	153.097	90
$II.C_{101}^{i.c.}(0.75)$	0.3282610510	0.4300507768	171.6008711131	153.101	90
$II.C_{102}^{i.c.}(0.75)$	0.1212986423	0.0683081873	61.6823006908	165.249	92
$II.C_{103}^{i.c.}(0.75)$	0.3201104564	0.2612000474	103.3021060602	166.396	94
$II.C_{104}^{i.c.}(0.75)$	0.3279000291	0.2763797657	108.4818233700	165.584	94
$II.C_{105}^{i.c.}(0.75)$	0.3084839268	0.1629915673	87.0805961214	168.638	94
$II.C_{106}^{i.c.}(0.75)$	0.2688296146	0.1181832994	78.9575107005	172.516	96
$II.C_{107}^{i.c.}(0.75)$	0.4152991663	0.1197329112	116.6820260011	175.958	98
$II.C_{108}^{i.c.}(0.75)$	0.2864563062	0.2740950034	103.9050178198	176.499	100
$II.C_{109}^{i.c.}(0.75)$	0.3120705685	0.2720598023	109.8857400002	176.499	100
$II.C_{110}^{i.c.}(0.75)$	0.3393296129	0.2554121073	116.4797850100	180.397	102

Table S. XIX. Initial conditions and periods T of the periodic three-body orbits for class II.C in the case of  $\mathbf{r}_1(0) = (-1,0) = -\mathbf{r}_2(0)$ ,  $\dot{\mathbf{r}}_1(0) = (v_1,v_2) = \dot{\mathbf{r}}_2(0)$  and  $\mathbf{r}_3(0) = (0,0)$ ,  $\dot{\mathbf{r}}_3(0) = (-2v_1/m_3, -2v_2/m_3)$  when G=1 and  $m_1=m_2=1$  and  $m_3=0.75$  by means of the search grid  $4000\times4000$  in the interval  $T_0\in[0,200]$ , where  $T^*=T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

Class and number	$v_1$	$v_2$	T	$T^*$	$L_f$
$II.C_{111}^{i.c.}(0.75)$	0.1181895504	0.0758977642	68.4251332328	183.131	102
$II.C_{112}^{i.c.}(0.75)$	0.3762237073	0.2525106949	131.7379537609	183.494	104
$II.C_{113}^{i.c.}(0.75)$	0.3960217517	0.1959815422	130.1263669788	189.352	106
$II.C_{114}^{i.c.}(0.75)$	0.2794461019	0.0678830585	86.2018484103	190.525	106
$II.C_{115}^{i.c.}(0.75)$	0.2193806640	0.2606198586	96.8102097868	191.261	108
II. $C_{115}^{i.c.}(0.75)$ II. $C_{116}^{i.c.}(0.75)$	0.3584363302	0.2496690110	131.0937464396	194.391	110
$II.C_{117}^{i.c.}(0.75)$	0.2555380955	0.2505369048	104.7207733533	198.265	112
$\text{II.C}_{118}^{i.c.}(0.75)$	0.2717717477	0.1211826752	94.4766709178	204.869	114
$II.C_{119}^{i.c.}(0.75)$	0.4557116917	0.2243651046	186.1251013074	201.389	114
$II.C_{120}^{i.c.}(0.75)$	0.3368046655	0.2601961372	130.4398908126	201.388	114
$II.C_{121}^{i.c.}(0.75)$	0.4716148265	0.2169359149	197.8478047433	201.390	114
$II.C_{122}^{i.c.}(0.75)$	0.1983164515	0.1289228288	85.0121650660	204.420	114
$II.C_{123}^{i.c.}(0.75)$	0.4314296428	0.1085814064	144.8128259248	208.309	116
$II.C_{124}^{i.c.}(0.75)$	0.3984768264	0.0833001471	126.8834769024	208.462	116
$II.C_{125}^{i.c.}(0.75)$	0.3508369213	0.1499116351	119.2495101416	211.730	118
$II.C_{126}^{i.c.}(0.75)$	0.2382029257	0.4600002384	199.3101782590	204.138	120
II. $C_{126}^{i.c.}(0.75)$ II. $C_{127}^{i.c.}(0.75)$	0.3539713336	0.1844517695	127.4592957316	214.729	120
$II.C_{128}^{i.c.}(0.75)$	0.4679065922	0.0492370802	169.3015724927	219.318	122
$II.C_{129}^{i.c.}(0.75)$	0.2897587650	0.1322568131	107.2426867012	222.768	124
II. $C_{130}^{i.c.}(0.75)$	0.3194350927	0.1437653927	117.2090308828	226.198	126
$II.C_{131}^{i.c.}(0.75)$	0.2856067895	0.2664601284	129.0520293629	223.134	126
$II.C_{132}^{i.c.}(0.75)$	0.3172180539	0.2641989427	140.3245705513	226.273	128
$II.C_{133}^{i.c.}(0.75)$	0.3500093713	0.1856549777	136.8114267533	232.586	130
$II.C_{134}^{i.c.}(0.75)$	0.2402496725	0.0392897276	99.6934119722	237.292	132
$II.C_{135}^{i.c.}(0.75)$	0.3685221643	0.0304592777	131.2899349360	240.865	134
II. $C_{136}^{i.c.}(0.75)$	0.2976587430	0.2715721913	144.3157440128	240.266	136
$II.C_{137}^{i.c.}(0.75)$	0.3818757096	0.1993653265	162.7392471190	246.680	138
$II.C_{138}^{i.c.}(0.75)$	0.3749650745	0.1872646291	158.6249794643	250.442	140
$II.C_{139}^{i.c.}(0.75)$	0.2533117212	0.0765118733	110.8909227112	255.234	142
II. $C_{140}^{i.c.}(0.75)$	0.2938548621	0.1099787648	123.3013725819	258.731	144
II. $C_{141}^{i.c.}(0.75)$	0.3418869995	0.1513320020	144.4057666321	261.966	146
$II.C_{142}^{i.c.}(0.75)$	0.3568986704	0.1789162975	157.3345205939	264.987	148
$II.C_{143}^{i.c.}(0.75)$	0.3507048994	0.1879172283	158.6337077719	268.298	150
$II.C_{144}^{i.c.}(0.75)$	0.4289912118	0.0740929532	183.2158749766	273.162	152
$II.C_{145}^{i.c.}(0.75)$	0.2052527461	0.0562709352	110.0586247060	273.334	152
II. $C_{146}^{i.c.}(0.75)$	0.4153025220	0.1261299710	184.5272181346	276.437	154
II. $C_{147}^{i.c.}(0.75)$	0.3985285632	0.1078946305	171.5880123205	276.644	154
$II.C_{148}^{i.c.}(0.75)$	0.2587721852	0.2603689252	155.2429995575	286.883	162
$II.C_{149}^{i.c.}(0.75)$	0.3015928568	0.0717923667	137.6823840835	291.173	162
II. $C_{150}^{i.c.}(0.75)$	0.4240317938	0.0881010520	193.6334447174	291.079	162
$II.C_{151}^{i.c.}(0.75)$	0.3829944898	0.1198045039	176.0142373504	294.549	164
II. $C_{152}^{i.c.}(0.75)$	0.1313939229	0.0902579269	114.5276827160	301.563	168
$II.C_{153}^{i.c.}(0.75)$	0.4089860758	0.0422384137	188.4670965240	305.576	170
II. $C_{154}^{i.c.}(0.75)$	0.3176605344	0.0885482935	152.5558686604	309.114	172
$II.C_{155}^{i.c.}(0.75)$	0.3712251533	0.1994085280	198.5463547977	311.052	174
$II.C_{156}^{i.c.}(0.75)$	0.3918996873	0.1432248649	196.4917875525	312.203	174
$II.C_{157}^{i.c.}(0.75)$	0.2050985427	0.1005778254	128.5282060869	312.635	174
$II.C_{158}^{i.c.}(0.75)$	0.3419116730	0.1826902734	182.6440041642	318.564	178
$II.C_{159}^{i.c.}(0.75)$	0.2830434789	0.0744751508	147.7926007568	323.523	180
$II.C_{160}^{i.c.}(0.75)$	0.2776335848	0.1338443741	154.0847561202	327.078	182
$II.C_{161}^{i.c.}(0.75)$	0.2865358925	0.1200209400	161.6167188878	341.339	190
$II.C_{162}^{i.c.}(0.75)$	0.2700170285	0.1122748834	162.4518900554	355.781	198
$II.C_{163}^{i.c.}(0.75)$	0.3047927194	0.0555637745	176.7118459185	373.871	208
II. $C_{164}^{i.c.}(0.75)$	0.1380664200	0.0670078477	142.5493314030	377.360	210
$II.C_{165}^{i.c.}(0.75)$	0.2454841019	0.1186795750	166.2145766431	377.360	210
$II.C_{166}^{i.c.}(0.75)$	0.3174952187	0.0732714608	191.6755720430	391.823	218
$II.C_{167}^{i.c.}(0.75)$	0.1696261275	0.1004200178	171.6162456989	434.470	242
$II.C_{168}^{i.c.}(0.75)$	0.2044935780	0.0801337875	178.1873542624	438.633	244
$II.C_{169}^{i.c.}(0.75)$	0.2196248784	0.0651566054	196.7913913300	478.244	266

Table S. XX. Initial conditions and periods T of the periodic three-body orbits for class I.A in the case of  $\mathbf{r}_1(0) = (-1,0) = -\mathbf{r}_2(0)$ ,  $\dot{\mathbf{r}}_1(0) = (v_1,v_2) = \dot{\mathbf{r}}_2(0)$  and  $\mathbf{r}_3(0) = (0,0)$ ,  $\dot{\mathbf{r}}_3(0) = (-2v_1/m_3, -2v_2/m_3)$  when G=1 and  $m_1=m_2=1$  and  $m_3=2$  by means of the search grid  $4000 \times 4000$  in the interval  $T_0 \in [0,200]$ , where  $T^* = T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

Class and number	$v_1$	$v_2$	T	$T^*$	$L_f$
$\overline{\mathrm{I.A}_{1}^{i.c.}(2)}$	0.6649107583	0.8324167864	12.6489061509	42.121	8
$I.A_2^{i.c.}(2)$	0.6656250225	0.4965289014	11.7760601476	64.923	12
$I.A_3^{i.c.}(2)$	0.6825287273	0.6174444064	36.6762382587	172.376	32
$I.A_4^{i.c.}(2)$	0.6994683581	0.3498807082	36.5607892892	216.851	40
$I.A_5^{i.c.}(2)$	0.4085452441	0.0399781234	28.1061456733	238.731	44
$I.A_6^{i.c.}(2)$	0.6925444835	0.7038306689	63.1111348894	256.991	48
$I.A_7^{i.c.}(2)$	0.6852621866	0.6403262957	61.6549217313	279.758	52
$I.A_8^{i.c.}(2)$	0.6786880593	0.5712663380	60.3945847774	302.292	56
$I.A_9^{i.c.}(2)$	0.7520356911	0.5826701574	68.5259154653	302.310	56
$I.A_{10}^{i.c.}(2)$	0.7731160189	0.4217868038	68.4901771349	346.809	64
$I.A_{11}^{i.c.}(2)$	0.6258530965	0.6849876905	78.6876130536	364.377	68
$I.A_{12}^{i.c.}(2)$	0.6687176921	0.3113447440	58.5101436950	368.722	68
$I.A_{13}^{i.c.}(2)$	0.5219331515	0.1537833421	50.5549444042	390.548	72
$I.A_{14}^{i.c.}(2)$	0.6484066991	0.5971552405	81.0318562446	409.726	76
$I.A_{15}^{i.c.}(2)$	0.6807496308	0.5512579156	84.6308430084	432.167	80
$I.A_{16}^{i.c.}(2)$	0.6508103868	0.6834247439	105.2312919998	471.737	88
$I.A_{17}^{i.c.}(2)$	0.6869542921	0.6559982684	111.7375242360	494.496	92
$I.A_{18}^{i.c.}(2)$	0.4665900313	0.2785470858	67.3569385398	520.654	96
$I.A_{19}^{i.c.}(2)$	0.6534853531	0.1865830089	80.1996471047	542.402	100
$I.A_{20}^{i.c.}(2)$	0.6640665882	0.6805220686	131.1248770198	579.098	108
$I.A_{21}^{\stackrel{20}{i.c.}}(2)$	0.6597778792	0.4513736509	104.8697669510	606.485	112
$I.A_{22}^{i.c.}(2)$	0.6471520650	0.4020332252	103.0148018894	628.563	116
$I.A_{23}^{\stackrel{2}{i.c.}}(2)$	0.6626286520	0.2839544976	104.4563049755	672.449	124
$I.A_{24}^{i.c.}(2)$	0.5253644523	0.2725303838	90.8016748047	672.470	124
$I.A_{25}^{\overset{24}{i.c.}}(2)$	0.5466058934	0.5202407423	112.2826008711	691.899	128
$I.A_{26}^{i.c.}(2)$	0.4524709149	0.1937913189	86.2950694409	694.359	128
$I.A_{27}^{\stackrel{20}{i.c.}}(2)$	0.7385582230	0.4623589255	143.0327822206	736.360	136
$I.A_{28}^{i.c.}(2)$	0.5178453979	0.4523792202	109.8866781782	736.361	136
$I.A_{29}^{i.c.}(2)$	0.6895973623	0.6795598105	186.6182866360	793.825	148
$I.A_{30}^{i.c.}(2)$	0.7417737283	0.5576355792	172.6862188871	799.409	148
$I.A_{31}^{i.c.}(2)$	0.6787848022	0.3324577477	130.4345522210	802.428	148
$I.A_{32}^{i.c.}(2)$	0.4942916225	0.3122096699	107.6378558600	802.502	148
$I.A_{33}^{i.c.}(2)$	0.6616192451	0.2773985935	127.4910900115	824.310	152
$I.A_{34}^{i.c.}(2)$	0.4972412503	0.2031457421	108.9018118241	846.169	156
$I.A_{35}^{i.c.}(2)$	0.4457731294	0.1076713105	105.3479284426	867.997	160
$I.A_{36}^{i.c.}(2)$	0.5807922587	0.4312476531	138.4192610066	888.319	164
$I.A_{37}^{i.c.}(2)$	0.6450450089	0.3979743783	148.4035753878	910.372	168
$I.A_{38}^{i.c.}(2)$	0.6059803235	0.5138426153	163.3555963264	951.591	176
$I.A_{39}^{i.c.}(2)$	0.6668050678	0.5220530211	177.2812672926	951.589	176
$I.A_{40}^{i.c.}(2)$	0.5306754259	0.3039555306	131.3126893593	954.331	176
$I.A_{41}^{i.c.}(2)$	0.6720090611	0.3203793706	152.7891565944	954.299	176
$I.A_{42}^{i.c.}(2)$	0.4750713117	0.2602297809	126.1180059947	976.273	180
$I.A_{43}^{i.c.}(2)$	0.6349837177	0.4678756830	169.5497785789	996.045	184
$I.A_{44}^{i.c.}(2)$	0.5469768158	0.2106173499	134.0431589510	997.999	184
$I.A_{45}^{i.c.}(2)$	0.4363598168	0.2067947916	123.2142494342	998.179	184
$I.A_{46}^{i.c.}(2)$	0.5105790195	0.4381247712	149.3966262626	1018.209	188
$I.A_{46}^{i.c.}(2)$ $I.A_{47}^{i.c.}(2)$	0.4826350873	0.1356931812	127.6057515007	1019.813	188
$I.A_{48}^{i.c.}(2)$	0.6581637706	0.4419205771	174.3387691062	1018.181	188
$I.A_{48}$ (2) $I.A_{49}^{i.c.}$ (2)	0.6208957754	0.3798674379	166.4674004829	1062.279	196
$I.A_{49}^{i.c.}(2)$ $I.A_{50}^{i.c.}(2)$	0.5283765904	0.5013997243	169.5876212839	1002.279	200
±±50 (±)	0.0200100004	0.0010331240	100.0010212000	1001,401	200

Table S. XXI. Initial conditions and periods T of the periodic three-body orbits for class I.A and I.B in the case of  $\mathbf{r}_1(0) = (-1,0) = -\mathbf{r}_2(0)$ ,  $\dot{\mathbf{r}}_1(0) = (v_1,v_2) = \dot{\mathbf{r}}_2(0)$  and  $\mathbf{r}_3(0) = (0,0)$ ,  $\dot{\mathbf{r}}_3(0) = (-2v_1/m_3, -2v_2/m_3)$  when G=1 and  $m_1=m_2=1$  and  $m_3=2$  by means of the search grid  $4000\times 4000$  in the interval  $T_0\in[0,200]$ , where  $T^*=T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

Class and number	$v_1$	$v_2$	T	$T^*$	$L_f$
$\overline{\text{I.A}_{51}^{i.c.}(2)}$	0.5485296197	0.4651956166	174.5302500004	1125.911	208
$I.A_{52}^{i.c.}(2)$	0.5048487414	0.2581606824	148.9271778598	1128.085	208
$I.A_{53}^{i.c.}(2)$	0.6605145396	0.2693390090	173.5846701186	1128.030	208
$I.A_{54}^{i.c.}(2)$	0.4653563186	0.2118208325	144.9873979687	1149.975	212
$I.A_{55}^{i.c.}(2)$	0.5563868856	0.4227642462	176.9274017831	1170.145	216
$I.A_{56}^{i.c.}(2)$	0.6360544648	0.3001819188	191.2539223702	1258.032	232
$I.A_{57}^{i.c.}(2)$	0.6601780173	0.2666833376	196.6368084515	1279.890	236
$I.A_{58}^{i.c.}(2)$	0.5619933924	0.1668615748	177.7791904073	1323.487	244
$I.A_{59}^{i.c.}(2)$	0.4301622650	0.0900627168	161.2579898217	1345.457	248
$I.A_{60}^{i.c.}(2)$	0.5707073620	0.3140095729	198.9345552114	1388.021	256
$I.A_{61}^{i.c.}(2)$	0.4475731896	0.2162152621	181.3252835399	1453.790	268
$I.A_{62}^{i.c.}(2)$	0.4807274686	0.1737864193	186.0049759663	1475.446	272
$I.B_1^{i.c.}(2)$	0.4136863353	0.9410539017	12.5429961223	46.245	10
$I.B_2^{i.c.}(2)$	0.3534308283	0.8508079166	12.2692512978	57.560	12
$I.B_3^{i.c.}(2)$	0.0413741936	0.9956426300	15.5793823937	62.100	16
$I.B_4^{i.c.}(2)$	0.7096842091	0.7687592147	27.3003058481	95.893	18
$I.B_5^{i.c.}(2)$	0.6799645758	0.5887672136	24.2551964847	118.671	22
$I.B_6^{i.c.}(2)$	0.6378164212	0.3886052513	22.6313155247	140.904	26
$I.B_7^{i.c.}(2)$	0.6699852230	0.7085344248	48.5438772348	203.302	38
$I.B_8^{i.c.}(2)$	0.6843384814	0.6317419448	49.1655025482	226.070	42
$I.B_9^{i.c.}(2)$	0.6919910882	0.5460326131	49.1547189071	248.549	46
$I.B_{10}^{i.c.}(2)$	0.6587938097	0.4454867422	46.5359882210	270.778	50
$I.B_{10}^{i.c.}(2)$ $I.B_{11}^{i.c.}(2)$	0.6746780747	0.3259416999	47.1768704466	292.788	54
$I.B_{12}^{i.c.}(2)$	0.4644102263	0.3239410999 $0.1242585136$	38.7788080029	314.634	58
$I.B_{13}^{i.c.}(2)$	0.6916935954	0.6979815204	75.4612881629	310.678	58
$I.B_{13}^{13}$ (2) $I.B_{14}^{i.c.}$ (2)	0.6858800687	0.6461043683	74.1577191496	333.444	62
$I.B_{15}^{i.c.}(2)$	0.7655652032	0.5318113366	82.4365268312	378.439	70
$I.B_{16}^{i.c.}(2)$ $I.B_{16}^{i.c.}(2)$	0.6615614523	0.4629426722	70.1179389107	400.634	70 74
$I.B_{16}^{i.c.}(2)$ $I.B_{17}^{i.c.}(2)$	0.6909413108	0.4029420722	100.2197830082	418.047	74 78
	0.6660110017	0.3013230386	69.9586100682	444.655	82
$I.B_{18}^{i.c.}(2)$ $I.B_{19}^{i.c.}(2)$					86
	0.5841885389	0.1723383387	64.0297299896	466.471	
$I.B_{20}^{i.c.}(2)$	0.7431942127	0.5700610999	106.8202805388	485.913	90
$I.B_{21}^{i.c.}(2)$	0.5876675319	0.5574609749	85.3743868704	485.912	90
$I.B_{22}^{i.c.}(2)$	0.5795678824	0.4658293684	84.8335185161	530.488	98
$I.B_{23}^{i.c.}(2)$	0.6903638351	0.6860300463	124.9278103447	525.413	98
$I.B_{24}^{i.c.}(2)$	0.6528942111	0.4180607376	92.2682840912	552.599	102
$I.B_{25}^{i.c.}(2)$	0.7613651698	0.3029365683	106.3455407448	596.558	110
$I.B_{26}^{i.c.}(2)$	0.4956617783	0.2749768423	78.7423557565	596.558	110
$I.B_{27}^{i.c.}(2)$	0.5322323384	0.5465974633	101.0708650395	615.814	114
$I.B_{28}^{i.c.}(2)$	0.6796765126	0.5536965825	120.7390738313	615.783	114
$I.B_{29}^{i.c.}(2)$	0.6677734178	0.5160072240	118.3019983217	638.107	118
$I.B_{30}^{i.c.}(2)$	0.4935620999	0.4634807883	97.3610217793	660.367	122
$I.B_{31}^{i.c.}(2)$	0.5632072523	0.5622457909	124.3998006645	723.269	134
$I.B_{32}^{i.c.}(2)$	0.7355327264	0.2899738941	127.7436324479	748.388	138
$I.B_{33}^{i.c.}(2)$	0.6897103691	0.6805154515	174.2881413969	740.143	138
$I.B_{34}^{i.c.}(2)$	0.5586512240	0.2714718822	103.9534809964	748.388	138
$I.B_{35}^{i.c.}(2)$	0.5333261360	0.4979744794	120.6208513685	767.971	142
$I.B_{36}^{i.c.}(2)$	0.6546992766	0.2061185669	114.7714470452	770.194	142
$I.B_{37}^{i.c.}(2)$	0.6637383695	0.4797530605	140.7636176347	790.183	146
$I.B_{38}^{i.c.}(2)$	0.6877572827	0.0842798120	118.9127597836	791.939	146
$I.B_{39}^{i.c.}(2)$	0.6498528395	0.4085255224	137.8351993001	834.412	154
$I.B_{40}^{i.c.}(2)$	0.7624410411	0.3440126143	160.8845261259	878.414	162

Table S. XXII. Initial conditions and periods T of the periodic three-body orbits for class I.B, I.C, II.A and II.B in the case of  $\mathbf{r}_1(0) = (-1,0) = -\mathbf{r}_2(0)$ ,  $\dot{\mathbf{r}}_1(0) = (v_1,v_2) = \dot{\mathbf{r}}_2(0)$  and  $\mathbf{r}_3(0) = (0,0)$ ,  $\dot{\mathbf{r}}_3(0) = (-2v_1/m_3, -2v_2/m_3)$  when G=1 and  $m_1 = m_2 = 1$  and  $m_3 = 2$  by means of the search grid  $4000 \times 4000$  in the interval  $T_0 \in [0,200]$ , where  $T^* = T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

Class and number	$v_1$	$v_2$	T	$T^*$	$L_f$
$I.B_{41}^{i.c.}(2)$	0.6879826248	0.6653325096	199.6963836583	870.272	162
$I.B_{42}^{i.c.}(2)$	0.6240256618	0.5323918480	156.2803805272	875.506	162
$I.B_{43}^{i.c.}(2)$	0.4563106153	0.2611707573	114.8148902213	900.372	166
$I.B_{44}^{i.c.}(2)$	0.6612633816	0.2749034316	139.0125537029	900.240	166
$I.B_{45}^{i.c.}(2)$	0.5221782655	0.2071155435	121.1696096366	922.081	170
$I.B_{46}^{i.c.}(2)$	0.5567455518	0.5648247735	164.5620595602	960.621	178
$I.B_{47}^{i.c.}(2)$	0.6790474390	0.4222695611	166.7970949155	964.291	178
$I.B_{48}^{i.c.}(2)$	0.6790840621	0.5778537084	193.5243647671	960.603	178
$I.B_{49}^{i.c.}(2)$	0.6791350775	0.5558668618	193.0705870639	983.015	182
$I.B_{50}^{i.c.}(2)$	0.5552591136	0.5217230932	164.7814492815	1005.379	186
$I.B_{51}^{i.c.}(2)$	0.6758170233	0.5099997641	191.3878276034	1027.651	190
$I.B_{52}^{i.c.}(2)$	0.6701256280	0.3155625312	164.1342676824	1030.233	190
$I.B_{53}^{i.c.}(2)$	0.6642332036	0.4838180744	187.8413617236	1049.878	194
$I.B_{54}^{i.c.}(2)$	0.6607235336	0.2709395969	162.0595886706	1052.100	194
$I.B_{55}^{i.c.}(2)$	0.5730352235	0.2138677593	147.7800306334	1073.921	198
$I.B_{56}^{i.c.}(2)$	0.5024476446	0.1455936846	139.4317930155	1095.727	202
$I.B_{57}^{i.c.}(2)$	0.6603346018	0.2679312271	185.1104590104	1203.960	222
$I.B_{58}^{i.c.}(2)$	0.4894090722	0.4358021895	176.1804659126	1224.076	226
$I.B_{59}^{i.c.}(2)$	0.5813604720	0.4153081963	192.0257778397	1246.108	230
$I.B_{60}^{i.c.}(2)$	0.4448123384	0.1589471380	152.8745723698	1247.733	230
$I.B_{61}^{i.c.}(2)$	0.5415992006	0.1607640861	164.4006039689	1247.564	230
$I.B_{62}^{i.c.}(2)$	0.4192735320	0.0717710277	150.8160159945	1269.555	234
$I.B_{63}^{i.c.}(2)$	0.4417759692	0.1037595968	171.9555602120	1421.360	262
$I.B_{64}^{i.c.}(2)$	0.4886058039	0.3067872636	195.0016721994	1464.084	270
$I.B_{65}^{i.c.}(2)$	0.4950064193	0.1779001099	197.8654949145	1551.356	286
$I.C_1^{i.c.}(2)$	0.4463851548	0.2849973719	113.7803515858	889.512	164
$I.C_2^{i.c.}(2)$	0.6539824310	0.1949540976	183.4448758069	1236.666	228
$II.A_1^{i.c.}(2)$	0.6563704629	0.7210981740	35.7193985834	149.610	28
$II.A_2^{\overline{i.c.}}(2)$	0.7043804200	0.2995216841	85.7364518416	520.587	96
$II.A_3^{i.c.}(2)$	0.6277885723	0.5727334717	101.0259827620	539.638	100
$II.A_4^{i.c.}(2)$	0.6705929917	0.5931586426	179.3690438002	884.414	164
$\text{II.B}_{1}^{i.c.}(2)$	0.0488561235	0.8544148215	11.9723950175	63.308	16
$\text{II.B}_{2}^{\overline{i}.c.}(2)$	0.5050095375	0.8532423197	30.3164175587	122.281	28
$II.B_3^{i.c.}(2)$	0.6912520905	0.8072079793	53.7137845333	180.218	34
$\text{II.B}_{4}^{i.c.}(2)$	0.7165670952	0.7256175703	133.4837704301	502.525	94
$II.B_5^{i.c.}(2)$	0.6928070373	0.6049731873	148.9269113715	700.779	130
$II.B_6^{i.c.}(2)$	0.4965910911	0.4930949987	135.9238691725	897.850	166
$\mathrm{II.B}_{7}^{i.c.}(2)$	0.4302338078	0.2044723241	113.3177648518	922.286	170

Table S. XXIII. Initial conditions and periods T of the periodic three-body orbits for class II.C in the case of  $\mathbf{r}_1(0) = (-1,0) = -\mathbf{r}_2(0)$ ,  $\dot{\mathbf{r}}_1(0) = (v_1,v_2) = \dot{\mathbf{r}}_2(0)$  and  $\mathbf{r}_3(0) = (0,0)$ ,  $\dot{\mathbf{r}}_3(0) = (-2v_1/m_3, -2v_2/m_3)$  when G=1 and  $m_1=m_2=1$  and  $m_3=2$  by means of the search grid  $4000 \times 4000$  in the interval  $T_0 \in [0,200]$ , where  $T^* = T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

Class and number	$v_1$	$v_2$	T	$T^*$	$L_f$
$\overline{\mathrm{II.C_1^{i.c.}(2)}}$	0.3419254129	0.6225042846	8.1277419699	53.018	10
$II.C_2^{\overline{i.c.}}(2)$	0.4183048895	0.8832477662	10.6835513193	44.526	10
$II.C_3^{\overline{i.c.}}(2)$	0.2315589634	0.5397849981	8.4364344582	62.741	12
$II.C_4^{i.c.}(2)$	0.3482359951	0.7977005516	10.6521916661	54.931	12
$II.C_5^{i.c.}(2)$	0.1747155595	0.9537742434	11.2448496818	47.676	12
$II.C_6^{i.c.}(2)$	0.1876415041	0.4973714491	9.2174482315	71.945	14
$II.C_7^{i.c.}(2)$	0.8585382523	0.3992014522	16.9140385366	75.336	14
$II.C_8^{i.c.}(2)$	0.6504217297	0.7534481524	34.5441872395	138.069	26
$II.C_9^{i.c.}(2)$	0.4066977900	0.2322976437	18.5607296551	151.910	28
$II.C_{10}^{i.c.}(2)$	0.7592503197	0.5681249718	41.3559042212	183.635	34
$II.C_{11}^{i.c.}(2)$	0.2066719875	0.9430283237	33.4271615500	143.057	36
$\operatorname{II.C}_{12}^{\overline{i.c.}}(2)$	0.4146350243	0.3221010514	27.6479764521	216.939	40
$II.C_{13}^{\widetilde{i.c.}}(2)$	0.7609558501	0.2581080751	39.6352947801	227.807	42
$II.C_{14}^{i.c.}(2)$	0.5254523354	0.5562034750	40.9263974560	248.593	46
$II.C_{15}^{i.c.}(2)$	0.4618794775	0.3060780263	38.2278117270	292.838	54
$II.C_{16}^{i.c.}(2)$	0.5661937162	0.2399247352	41.9294344254	303.721	56
$II.C_{17}^{i.c.}(2)$	0.5603872773	0.4510742855	52.0486804950	335.710	62
$II.C_{18}^{i.c.}(2)$	0.7141078611	0.7225193662	92.8206248892	352.914	66
$II.C_{19}^{i.c.}(2)$	0.4649115785	0.3402622958	47.6265126218	357.844	66
$II.C_{20}^{i.c.}(2)$	0.6167367951	0.5539133064	66.4559860020	367.232	68
$II.C_{21}^{i.c.}(2)$	0.5048320727	0.2952984074	49.4689463532	368.745	68
$\text{II.C}_{22}^{\widetilde{i.c.}}(2)$	0.7436868264	0.4653634325	78.6445804901	400.652	74
$II.C_{23}^{\overline{i.c.}}(2)$	0.5441086738	0.5425314916	71.4678190796	432.178	80
$II.C_{24}^{\overline{i.c.}}(2)$	0.4885885152	0.3251789493	58.3003419715	433.754	80
$\text{II.C}_{25}^{\tilde{i.c.}}(2)$	0.7413837805	0.3131678015	77.5137279648	444.662	82
$\operatorname{II.C}_{26}^{\overset{\circ}{i.c.}}(2)$	0.7045220682	0.7108410127	116.6767631628	460.295	86
$II.C_{27}^{\overline{i.c.}}(2)$	0.7275403415	0.4376751471	89.1194892943	476.631	88
$\operatorname{II.C}_{28}^{\overline{i.c.}}(2)$	0.4858219061	0.3440221728	67.5630284675	498.750	92
$II.C_{29}^{\overline{i.c.}}(2)$	0.7303738305	0.5310663433	104.5878736683	508.262	94
$II.C_{30}^{i.c.}(2)$	0.6947435987	0.4176207645	97.1791580997	552.600	102
$II.C_{31}^{i.c.}(2)$	0.7275761133	0.1015030833	87.4537840006	553.278	102
$II.C_{32}^{i.c.}(2)$	0.5070848684	0.4982410822	87.9447232560	573.200	106
$II.C_{33}^{\widetilde{i.c.}}(2)$	0.7735256256	0.3869085928	110.3782010705	574.666	106
$II.C_{34}^{i.c.}(2)$	0.7472783026	0.5729156410	134.2794021508	604.591	112
$II.C_{35}^{i.c.}(2)$	0.4578292895	0.2350514143	76.7938750498	607.518	112
$II.C_{36}^{i.c.}(2)$	0.5277658520	0.4270090654	91.2353195797	617.546	114
$II.C_{37}^{i.c.}(2)$	0.4898331715	0.4967647615	96.3577454631	638.141	118
$II.C_{38}^{i.c.}(2)$	0.7273677980	0.3285959518	114.1715748005	661.513	122
$II.C_{39}^{i.c.}(2)$	0.4892532461	0.4325728753	97.9924501867	682.497	126
$II.C_{40}^{i.c.}(2)$	0.4195342908	0.2326473489	84.1841967872	683.533	126
$II.C_{41}^{i.c.}(2)$	0.5695732932	0.4135235691	105.4982596512	693.508	128
$II.C_{42}^{i.c.}(2)$	0.4280215163	0.1399822283	85.1290496907	705.284	130
$II.C_{43}^{i.c.}(2)$	0.7172059976	0.4819919772	139.1173195582	725.261	134
$II.C_{44}^{i.c.}(2)$	0.4718680015	0.2668205044	96.7216297615	748.464	138
$II.C_{45}^{i.c.}(2)$	0.7232215799	0.4151840324	138.3471806807	758.459	140
$II.C_{46}^{i.c.}(2)$	0.5404093775	0.5230173790	122.4016509480	756.840	140
$II.C_{47}^{i.c.}(2)$	0.7525655073	0.2564337465	130.4554783876	759.328	140
$II.C_{48}^{i.c.}(2)$	0.6936999256	0.5700462542	160.6270068068	788.200	146
$II.C_{49}^{i.c.}(2)$	0.4843126014	0.0844399577	98.3846313825	791.988	146
$II.C_{50}^{i.c.}(2)$	0.4621277678	0.2887580227	105.3750210612	813.495	150

Table S. XXIV. Initial conditions and periods T of the periodic three-body orbits for class II.C and II.D in the case of  $\mathbf{r}_1(0) = (-1,0) = -\mathbf{r}_2(0)$ ,  $\dot{\mathbf{r}}_1(0) = (v_1,v_2) = \dot{\mathbf{r}}_2(0)$  and  $\mathbf{r}_3(0) = (0,0)$ ,  $\dot{\mathbf{r}}_3(0) = (-2v_1/m_3, -2v_2/m_3)$  when G = 1 and  $m_1 = m_2 = 1$  and  $m_3 = 2$  by means of the search grid  $4000 \times 4000$  in the interval  $T_0 \in [0,200]$ , where  $T^* = T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

Class and number	$v_1$	$v_2$	T	$T^*$	$L_f$
$\overline{\mathrm{II.C}_{51}^{i.c.}(2)}$	0.7409842733	0.2548024080	141.1189791990	835.242	154
$II.C_{52}^{i.c.}(2)$	0.5527686761	0.4114559610	124.7340630824	834.417	154
$II.C_{53}^{i.c.}(2)$	0.4411825868	0.2014775073	104.6229338883	846.269	156
$II.C_{54}^{i.c.}(2)$	0.5903490824	0.5542274770	149.8355193390	853.145	158
$II.C_{55}^{i.c.}(2)$	0.5095309361	0.1076273234	110.2367826286	867.906	160
$II.C_{56}^{i.c.}(2)$	0.6476911727	0.6592310093	186.5647808498	870.284	162
II. $C_{56}^{i.c.}(2)$ II. $C_{57}^{i.c.}(2)$	0.7542236731	0.2870913910	157.4602317313	900.278	166
$\text{II.C}_{58}^{i.c.}(2)$	0.7225383025	0.5182878319	180.0585926431	897.807	166
$II.C_{59}^{i.c.}(2)$	0.6829265182	0.3949227774	155.0007436922	910.373	168
$II.C_{60}^{i.c.}(2)$	0.4944962109	0.3354960370	126.5930724317	932.507	172
$II.C_{61}^{i.c.}(2)$	0.5731579573	0.5228404784	157.1449116997	940.441	174
$II.C_{62}^{i.c.}(2)$	0.4230035790	0.1230593685	113.2202463977	944.017	174
$II.C_{63}^{i.c.}(2)$	0.5348626093	0.1239575520	122.6812047920	943.827	174
$II.C_{64}^{i.c.}(2)$	0.7365486620	0.5209890777	197.7892719451	962.741	178
$II.C_{65}^{i.c.}(2)$	0.5405892439	0.4105686806	144.0844304603	975.325	180
$II.C_{66}^{i.c.}(2)$	0.5365593027	0.2631585954	132.5304152818	976.192	180
$II.C_{67}^{i.c.}(2)$	0.6963830870	0.5384939358	198.4069997112	1005.363	186
$II.C_{68}^{i.c.}(2)$	0.5612174596	0.1365651140	135.9019435025	1019.753	188
$II.C_{69}^{i.c.}(2)$	0.6047899899	0.5655352150	185.3068878768	1025.552	190
$II.C_{70}^{i.c.}(2)$	0.4608970442	0.0782609721	125.8586099496	1030.711	190
$II.C_{71}^{i.c.}(2)$	0.5090586559	0.4172474858	150.1955209051	1040.279	192
$II.C_{72}^{i.c.}(2)$	0.5153130739	0.2807750264	139.9401650184	1041.217	192
$II.C_{73}^{i.c.}(2)$	0.5711368382	0.4009174040	158.7702675567	1051.278	194
$II.C_{74}^{i.c.}(2)$	0.7330058203	0.2797227895	178.0148431024	1052.111	194
$II.C_{75}^{i.c.}(2)$	0.4930200888	0.2105623302	138.1032522068	1073.977	198
$II.C_{76}^{i.c.}(2)$	0.7071493167	0.3380296379	185.1011957070	1095.217	202
$II.C_{77}^{i.c.}(2)$	0.4843031700	0.3104804420	145.6710898949	1095.339	202
$II.C_{78}^{i.c.}(2)$	0.7021624722	0.1513651325	169.6462689466	1095.681	202
II. $C_{78}^{i.c.}(2)$ II. $C_{79}^{i.c.}(2)$ II. $C_{80}^{i.c.}(2)$	0.5309569319	0.4100676515	163.4279841939	1116.232	206
$II.C_{80}^{i.c.}(2)$	0.5863327862	0.2631956974	160.1895523872	1128.034	208
$II.C_{81}^{i.c.}(2)$	0.5042933121	0.4522661483	167.6967610047	1137.013	210
$II.C_{82}^{i.c.}(2)$	0.5127148367	0.2132236730	150.2626044139	1149.888	212
$II.C_{83}^{\widetilde{i.c.}}(2)$	0.4203013342	0.1117122153	141.3424549265	1182.749	218
$II.C_{84}^{i.c.}(2)$	0.5567049092	0.4018329071	177.6958840052	1192.184	220
$II.C_{85}^{i.c.}(2)$	0.7135436004	0.1993230096	197.0626213511	1236.670	228
$II.C_{86}^{i.c.}(2)$	0.6251526217	0.3253104492	188.2411712699	1236.133	228
$II.C_{87}^{i.c.}(2)$	0.5237375027	0.4094955579	182.8253593805	1257.140	232
$II.C_{88}^{i.c.}(2)$	0.4819316406	0.4834824592	192.6723508500	1298.529	240
$II.C_{89}^{i.c.}(2)$	0.6074378575	0.1960385648	185.5428302100	1312.598	242
$II.C_{90}^{i.c.}(2)$	0.4839930914	0.2707143755	175.6526252637	1345.022	248
$II.C_{91}^{i.c.}(2)$	0.5837038254	0.3300225890	199.9369167063	1366.100	252
$II.C_{92}^{i.c.}(2)$	0.4690144649	0.2161480199	174.4018734709	1377.782	254
$II.C_{93}^{i.c.}(2)$	0.5003326814	0.1974567430	188.6007498773	1464.529	270
$II.C_{94}^{i.c.}(2)$	0.5678878694	0.1486900134	199.7846561418	1486.225	274
$II.C_{95}^{i.c.}(2)$	0.5017212512	0.1238005807	199.1773956809	1573.086	290
$\text{II.D}_1^{i.c.}(2)$	0.7583850283	0.9342270211	20.3253125540	41.295	4
$\text{II.D}_2^{i.c.}(2)$	0.3057224330	0.5215124257	8.8237067653	64.567	12

Table S. XXV. Initial conditions and periods T of the periodic three-body orbits in the case of  $\mathbf{r}_1(0) = (-1,0) = -\mathbf{r}_2(0)$ ,  $\dot{\mathbf{r}}_1(0) = (v_1,v_2) = \dot{\mathbf{r}}_2(0)$  and  $\mathbf{r}_3(0) = (0,0)$ ,  $\dot{\mathbf{r}}_3(0) = (-2v_1/m_3, -2v_2/m_3)$  when G = 1 and  $m_1 = m_2 = 1$  and  $m_3 = 4$  by means of the search grid  $4000 \times 4000$  in the interval  $T_0 \in [0,200]$ , where  $T^* = T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

Class and number	$v_1$	$v_2$	T	$T^*$	$L_f$
$I.A_1^{i.c.}(4)$	0.6886546484	1.5771573890	16.3239594553	133.417	12
$I.A_2^{i.c.}(4)$	0.9911981217	0.7119472124	17.6507807837	276.852	24
$I.A_3^{\overline{i.c.}}(4)$	0.9714516067	0.9749602268	54.7446177671	736.894	64
$I.A_4^{i.c.}(4)$	0.9854347005	0.8533367754	53.9920673975	783.838	68
$I.A_5^{i.c.}(4)$	0.9956481414	0.5378987979	51.9746508441	877.069	76
$I.A_6^{i.c.}(4)$	0.9081405883	0.2793470372	48.3394213297	923.388	80
$I.A_7^{i.c.}(4)$	0.7820517397	0.9985331599	79.6878556097	1196.730	104
$I.A_8^{i.c.}(4)$	0.8303276771	0.8619453491	80.6381542416	1290.761	112
$I.A_9^{i.c.}(4)$	0.9875993951	0.7994230783	89.2570135846	1337.585	116
$I.A_{10}^{i.c.}(4)$	0.9941229185	0.6131386486	87.2715581867	1430.813	124
$I.A_{11}^{i.c.}(4)$	0.9962878480	0.4962235209	86.3035106391	1477.247	128
$I.A_{12}^{i.c.}(4)$	0.8893383967	0.3498550324	80.0242474362	1523.553	132
$I.A_{13}^{\overline{i.c.}}(4)$	0.9737611115	0.9412093990	127.0151385872	1750.771	152
$I.A_{14}^{i.c.}(4)$	0.8483553587	0.8712965762	114.1831544918	1797.704	156
$I.A_{15}^{i.c.}(4)$	0.8494308475	0.7648099341	113.0730029261	1891.278	164
$I.A_{16}^{i.c.}(4)$	0.9933657595	0.6428233219	122.5711891715	1984.535	172
$I.A_{17}^{i.c.}(4)$	0.9951416390	0.5658643691	121.5966724212	2031.019	176
$I.A_{18}^{i.c.}(4)$	0.9975784126	0.3688413653	119.6830441188	2123.741	184
$I.A_{19}^{i.c.}(4)$	0.9982910210	0.2119134110	118.7483807712	2169.985	188
$I.A_{20}^{i.c.}(4)$	0.9739258865	0.9354381836	163.1207379635	2257.704	196
$I.A_{21}^{i.c.}(4)$	1.0006092513	0.9001922281	165.7348113442	2304.647	200
$I.A_{22}^{\overline{i.c.}}(4)$	0.9871838469	0.8086143731	160.8692729101	2398.304	208
$I.A_{23}^{i.c.}(4)$	0.9892584028	0.7616755213	159.8521879007	2445.023	212
$I.A_{24}^{i.c.}(4)$	0.9103595990	0.6516159482	149.0219605514	2538.245	220
$I.A_{25}^{\tilde{i.c.}}(4)$	0.9929178705	0.6587699243	157.8716678998	2538.249	220
$I.A_{26}^{i.c.}(4)$	0.9739648867	0.9317358197	199.2150728253	2764.636	240
$I.A_{27}^{\overline{i.c.}}(4)$	0.8889807236	0.8833502365	184.6126736271	2811.576	244
$I.A_{28}^{\overline{i.c.}}(4)$	0.9853196522	0.8652680791	198.4147566034	2858.458	248
$I.A_{29}^{\overline{i.c.}}(4)$	0.8671151375	0.8139325244	180.3491172929	2905.263	252
$I.A_{30}^{\overline{i.c.}}(4)$	0.8593214437	0.7428413082	178.2861322510	2998.700	260
$I.A_{31}^{i.c.}(4)$	0.9926227094	0.6687318678	193.1725286382	3091.960	268
$I.A_{32}^{i.c.}(4)$	0.9938888216	0.6227285557	192.1926675081	3138.490	272
$I.A_{33}^{i.c.}(4)$	0.9949961979	0.5732651491	191.2190326488	3184.966	276
$I.B_1^{i.c.}(4)$	0.9686212391	1.0035538399	36.6553191032	483.414	42
$I.B_2^{i.c.}(4)$	0.9866754572	0.8199780135	35.8070111326	530.358	46
$I.B_3^{i.c.}(4)$	0.9947340612	0.5860022689	34.8112412541	576.973	50
$I.B_4^{i.c.}(4)$	0.9854028827	0.8697357271	72.2201261804	1037.310	90
$I.B_5^{i.c.}(4)$	0.9890031845	0.7676837385	71.1011788123	1084.083	94
$I.B_6^{i.c.}(4)$	0.9931162203	0.6518383667	70.1106814873	1130.696	98
$I.B_7^{i.c.}(4)$	0.9960573144	0.5122336176	69.1389009393	1177.159	102
$I.B_8^{i.c.}(4)$	0.9978825808	0.3190517769	68.1889247106	1223.481	106
$I.B_9^{i.c.}(4)$	0.9735740630	0.9455048919	108.9564778767	1497.304	130
$I.B_{10}^{i.c.}(4)$	0.8392318736	0.8671916976	97.2973539376	1544.233	134
$I.B_{11}^{i.c.}(4)$	0.8458015116	0.7418783799	96.5606171482	1637.758	142
$I.B_{12}^{i.c.}(4)$	0.9925099954	0.6724319595	105.4115215859	1684.407	146
$I.B_{13}^{i.c.}(4)$	0.9964353852	0.4852677552	103.4683068013	1777.334	154
$I.B_{14}^{i.c.}(4)$	0.9976499399	0.3583646437	102.5181725853	1823.655	158
$I.B_{15}^{i.c.}(4)$	0.8319211104	0.9612129024	130.6811566596	1957.203	170
$I.B_{16}^{i.c.}(4)$	0.9738675697	0.9379697056	145.0696448675	2004.238	174
$I.B_{17}^{i.c.}(4)$	0.9856690147	0.8450868125	143.7791552562	2098.037	182
$I.B_{18}^{i.c.}(4)$	0.9878345000	0.7942119413	142.7087243978	2144.810	186
$I.B_{19}^{i.c.}(4)$	0.9901313441	0.7402972658	141.7024169129	2191.502	190
$I.B_{20}^{i.c.}(4)$	0.8549437310	0.6783963636	128.6681308892	2238.064	194

Table S. XXVI. Initial conditions and periods T of the periodic three-body orbits in the case of  $\mathbf{r}_1(0) = (-1,0) = -\mathbf{r}_2(0)$ ,  $\dot{\mathbf{r}}_1(0) = (v_1,v_2) = \dot{\mathbf{r}}_2(0)$  and  $\mathbf{r}_3(0) = (0,0)$ ,  $\dot{\mathbf{r}}_3(0) = (-2v_1/m_3, -2v_2/m_3)$  when G = 1 and  $m_1 = m_2 = 1$  and  $m_3 = 4$  by means of the search grid  $4000 \times 4000$  in the interval  $T_0 \in [0,200]$ , where  $T^* = T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

Class and number	$v_1$	$v_2$	T	$T^*$	$L_f$
$\overline{\text{I.B}_{21}^{i.c.}(4)}$	0.8844781422	0.4726698354	128.6082000866	2377.481	206
$I.B_{22}^{i.c.}(4)$	0.9966129597	0.4712277307	137.7981863183	2377.505	206
$I.B_{23}^{i.c.}(4)$	0.9429283989	0.3750820300	132.1379728542	2423.826	210
$I.B_{24}^{i.c.}(4)$	0.8923470368	0.9533088376	170.1093582167	2464.159	214
$I.B_{25}^{i.c.}(4)$	0.8775052711	0.8805399726	166.4296589791	2558.109	222
$I.B_{26}^{i.c.}(4)$	0.9920009809	0.6884914607	176.0140436201	2791.821	242
$I.B_{27}^{i.c.}(4)$	0.9967158849	0.4626061527	172.1282612303	2977.676	258
$I.B_{28}^{i.c.}(4)$	0.8634040830	0.5193955710	191.6136346322	3531.367	306
$II.A_1^{i.c.}(4)$	0.8193973154	0.9283001929	80.5646820232	1243.829	108
$II.A_2^{i.c.}(4)$	0.8723346349	0.9762996085	118.0177898281	1703.721	148
$II.B_1^{\tilde{i}.c.}(4)$	0.9597587596	0.1504623356	33.0556151005	623.296	54
$II.B_2^{i.c.}(4)$	0.8028948361	0.9363733297	63.8749643029	990.355	86
$II.B_3^{\tilde{i}.c.}(4)$	0.8810167233	0.3087921811	159.1584664346	3070.198	266
$II.C_1^{i.c.}(4)$	0.8711838791	0.4362455003	15.9419555705	300.076	26
$II.C_2^{i.c.}(4)$	0.9037352027	0.7742282374	50.1414328025	807.223	70
$II.C_3^{\overline{i}.c.}(4)$	0.8360259826	0.7607468289	64.1746649704	1084.046	94
$II.C_4^{i.c.}(4)$	0.8741665029	0.6579865343	81.0825827121	1407.540	122
$II.C_5^{i.c.}(4)$	0.9222679496	0.2364217777	81.0640765674	1546.686	134
$II.C_6^{i.c.}(4)$	0.7727255378	0.9813722168	109.9022712350	1680.144	146
$II.C_7^{i.c.}(4)$	0.8493658092	0.8187152772	113.5938429128	1844.540	160
$II.C_8^{i.c.}(4)$	0.8864141076	0.7338054599	115.1717551193	1914.642	166
$II.C_9^{i.c.}(4)$	0.8572285903	0.5716988114	111.8713143932	2030.945	176
$II.C_{10}^{i.c.}(4)$	0.9564378250	0.5203631283	117.8780060823	2054.229	178
$II.C_{11}^{i.c.}(4)$	0.7920712457	0.9627630869	142.7572749740	2187.130	190
$II.C_{12}^{i.c.}(4)$	0.8735053580	0.6463061859	129.5549674540	2261.370	196
$II.C_{13}^{i.c.}(4)$	0.9164332753	0.2533703569	129.3612607156	2470.074	214
$II.C_{14}^{i.c.}(4)$	0.7800111798	0.9764870056	173.2559877508	2646.981	230
$II.C_{15}^{i.c.}(4)$	0.9103776228	0.5004553050	147.0538184568	2654.401	230
$II.C_{16}^{i.c.}(4)$	0.8581304222	0.7667302298	162.3271446419	2698.506	234
$II.C_{17}^{i.c.}(4)$	0.9287736223	0.8013022448	187.7523645240	2928.662	254
$II.C_{18}^{i.c.}(4)$	0.9040085960	0.6611792882	181.5941857827	3091.954	268
$II.C_{19}^{i.c.}(4)$	0.9415767476	0.5157985737	183.4012996390	3231.388	280

Table S. XXVII. Initial conditions and periods T of the periodic three-body orbits in the case of  $\mathbf{r}_1(0) = (-1,0) = -\mathbf{r}_2(0)$ ,  $\dot{\mathbf{r}}_1(0) = (v_1,v_2) = \dot{\mathbf{r}}_2(0)$  and  $\mathbf{r}_3(0) = (0,0)$ ,  $\dot{\mathbf{r}}_3(0) = (-2v_1/m_3, -2v_2/m_3)$  when G = 1 and  $m_1 = m_2 = 1$  and  $m_3 = 5$  by means of the search grid  $4000 \times 4000$  in the interval  $T_0 \in [0,200]$ , where  $T^* = T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

Class and number	$v_1$	$v_2$	T	$T^*$	$L_f$
$I.A_1^{i.c.}(5)$	0.9216454928	0.8533067851	53.9388331345	1287.788	88
$I.A_2^{i.c.}(5)$	0.8829781875	0.4605442830	87.3008518130	2401.081	164
$I.A_3^{i.c.}(5)$	0.9894241760	0.7387990271	128.2645973796	3103.339	212
$I.A_4^{i.c.}(5)$	0.9318317248	0.7011947975	160.3140070092	4040.338	276
$I.A_5^{i.c.}(5)$	0.9521954098	0.6380537638	160.8351666149	4099.305	280
$I.A_6^{i.c.}(5)$	0.9265055362	0.9102619607	199.1483651043	4623.496	316
$I.A_7^{i.c.}(5)$	0.9414948707	0.7972769172	198.2855107755	4800.760	328
$I.B_1^{i.c.}(5)$	0.9832102827	0.2610181545	35.4899632829	966.423	66
$I.B_2^{i.c.}(5)$	0.9328981538	0.9308414732	72.7716713885	1667.838	114
$I.B_3^{i.c.}(5)$	0.9272174116	0.7235415686	71.2641428314	1785.849	122
$I.B_4^{i.c.}(5)$	0.9299236101	0.7420212724	107.1531608781	2664.097	182
$I.B_5^{i.c.}(5)$	0.9364988380	0.4466549140	105.3306842497	2840.543	194
$I.B_6^{i.c.}(5)$	0.9185948030	0.9105452579	144.4229235715	3365.161	230
$I.B_7^{i.c.}(5)$	0.9451359068	0.8566746195	145.2514450303	3424.391	234
$I.B_8^{i.c.}(5)$	0.9785669623	0.4482186236	142.8066562282	3777.715	258
$I.B_9^{i.c.}(5)$	0.9275282956	0.9587654523	182.0138798215	4125.181	282
$I.B_{10}^{i.c.}(5)$	0.9473861305	0.8410107115	181.4678831986	4302.643	294
$I.B_{11}^{i.c.}(5)$	0.9750842086	0.5298658837	179.0967959379	4656.080	318
$II.B_1^{i.c.}(5)$	0.9093722173	0.9517890604	108.3964675729	2486.840	170
$II.C_1^{i.c.}(5)$	0.9557672644	0.7750957916	18.1014388854	439.118	30
$II.C_2^{i.c.}(5)$	0.9440503938	0.4093724806	70.2927303619	1903.491	130
$II.C_3^{i.c.}(5)$	0.9379933241	0.9019570416	90.9039033637	2106.988	144
$II.C_4^{i.c.}(5)$	0.9796113137	0.4175042061	89.1520279744	2372.076	162
$II.C_5^{i.c.}(5)$	0.9495004417	0.5575440062	106.5164092109	2781.860	190
$II.C_6^{i.c.}(5)$	0.9541817480	0.3673032901	105.6505055608	2869.923	196
$II.C_7^{i.c.}(5)$	0.9375025806	0.8054315397	144.0759759853	3483.402	238
$II.C_{\circ}^{i.c.}(5)$	0.9589010451	0.9949666993	166.9876407055	3656.438	250
II. $C_{10}^{i.c.}(5)$ II. $C_{10}^{i.c.}(5)$	0.9120508681	0.9598994535	162.8143855219	3715.515	254
$II.C_{10}^{i.c.}(5)$	0.9740372231	0.9339142055	167.0634447329	3745.309	256
$II.C_{11}^{i.c.}(5)$	0.9899851594	0.8685305652	167.1856631406	3834.035	262
$II.C_{12}^{i.c.}(5)$	0.9940186923	0.9576808872	188.1862788803	4125.284	282
$II.C_{13}^{i.c.}(5)$	0.9709140296	0.6030586716	161.6989567743	4128.753	282
$II.C_{14}^{i.c.}(5)$	0.9481881607	0.8352390313	199.5740996229	4741.767	324
$II.C_{15}^{i.c.}(5)$	0.9625713434	0.7097859647	198.5700198646	4918.722	336
$II.C_{16}^{i.c.}(5)$	0.9736869452	0.5566185906	197.2313607831	5095.245	348

Table S. XXVIII. Initial conditions and periods T of the periodic three-body orbits in the case of  $\mathbf{r}_1(0) = (-1,0) = -\mathbf{r}_2(0)$ ,  $\dot{\mathbf{r}}_1(0) = (v_1,v_2) = \dot{\mathbf{r}}_2(0)$  and  $\mathbf{r}_3(0) = (0,0)$ ,  $\dot{\mathbf{r}}_3(0) = (-2v_1/m_3, -2v_2/m_3)$  when G = 1 and  $m_1 = m_2 = 1$  and  $m_3 = 8$  by means of the search grid  $4000 \times 4000$  in the interval  $T_0 \in [0,200]$ , where  $T^* = T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

Class and number	$v_1$	$v_2$	T	$T^*$	$L_f$
$\overline{\mathrm{I.A}_{1}^{i.c.}(8)}$	0.9423354093	0.7626700078	64.9545993260	3647.042	152
$I.A_2^{i.c.}(8)$	0.9787845584	0.9658431710	110.1464503910	5854.357	244
$I.A_3^{i.c.}(8)$	0.9362611866	0.9596217871	153.3805946371	8253.495	344
$I.A_4^{i.c.}(8)$	0.9533667422	0.8663093755	196.1893235979	10749.369	448
$I.B_1^{i.c.}(8)$	0.8766620953	0.9033499768	44.2205415861	2446.480	102
$I.B_2^{i.c.}(8)$	0.9457785062	0.9086191760	87.3950286043	4750.723	198
$I.B_3^{i.c.}(8)$	0.8915770541	0.8430352261	86.7011183501	4845.690	202
$II.B_1^{i.c.}(8)$	0.9932653784	0.7033211462	173.7686930189	9742.249	406
$II.C_1^{i.c.}(8)$	0.9097066120	0.8777115746	21.7197890174	1199.563	50
$II.C_2^{i.c.}(8)$	0.9581798536	0.6904382924	43.1758329286	2447.457	102
$II.C_3^{i.c.}(8)$	0.9847000702	0.8980796629	65.8092926553	3551.151	148
$II.C_4^{i.c.}(8)$	0.9767252634	0.6098206184	64.6224044091	3695.308	154
$II.C_5^{i.c.}(8)$	0.9326547771	0.8868649644	86.6725399311	4750.725	198
$II.C_6^{i.c.}(8)$	0.9488792658	0.5640461916	86.1048324315	4990.663	208
$II.C_7^{i.c.}(8)$	0.9932645266	0.5364010933	107.5645277200	6190.987	258
$II.C_8^{i.c.}(8)$	0.9300079407	0.6918905419	129.5219662492	7389.860	308
$II.C_9^{i.c.}(8)$	0.9215017059	0.7440903890	172.8653044238	9789.022	408

Table S. XXIX. Initial conditions and periods T of the periodic three-body orbits for class I.A in the case of  $\mathbf{r}_1(0) = (-1,0) = -\mathbf{r}_2(0)$ ,  $\dot{\mathbf{r}}_1(0) = (v_1, v_2) = \dot{\mathbf{r}}_2(0)$  and  $\mathbf{r}_3(0) = (0,0)$ ,  $\dot{\mathbf{r}}_3(0) = (-2v_1/m_3, -2v_2/m_3)$  when G=1 and  $m_1=m_2=1$  and  $m_3=10$  by means of the search grid  $4000 \times 4000$  in the interval  $T_0 \in [0, 200]$ , where  $T^* = T|E|^{3/2}$  is its scale-invariant period,  $L_f$  is the length of the free group element.

Class and number	$v_1$	$v_2$	T	$T^*$	$L_f$
$I.A_1^{i.c.}(10)$	0.8854191796	0.4245045883	24.1711064176	2056.279	68
$I.A_2^{i.c.}(10)$	0.9689819274	0.2200633851	71.2682278153	6049.841	200
$I.A_3^{i.c.}(10)$	0.9262313425	0.3318793235	119.5012751111	10162.520	336
$I.B_1^{i.c.}(10)$	0.9399260818	0.3073705720	47.7043741335	4053.099	134
$II.A_1^{i.c.}(10)$	0.9565884486	0.8490970249	24.2568013892	1935.907	64
$II.B_1^{i.c.}(10)$	0.9986145269	0.5935168720	47.9360511650	3932.658	130
$II.C_1^{i.c.}(10)$	0.9639216986	0.9744362737	48.1316203628	3751.177	124
$II.C_2^{i.c.}(10)$	0.9254840675	0.9517296411	49.8685206945	3931.244	130
$II.C_3^{i.c.}(10)$	0.9637413393	0.4820091574	95.6185104257	7985.803	264

Table S. XXX. The free group elements for the periodic three-body orbits.

Bub		
0.55 (0.55) (0.5	Class and number	
0.55 (0.55) (0.5	$\mathrm{I.A}_1^{v.c.}(0.5)$	BabA
0.55 0.55 0.55 0.55 0.55 0.55 0.55 0.55	$\Gamma.A_2^{i.c.}(0.5)$	BaBabAbA
0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5)	$\mathrm{I.A}_{3}^{i.c.}(0.5)$	BaBababAbAbA
0.55 (0.55) (0.5	$\mathrm{I.A_4^{i.c.}(0.5)}$	BaBAbaBabAbaBAbA
0.55 (0.55) (0.5	$\mathrm{I.A_{5}^{i.c.}(0.5)}$	BaBAbAbaBabAbaBaBAbA
0.55 0.55 0.55 0.55 0.55 0.55 0.55 0.55	$\mathrm{I.A}_{6}^{i.c.}(0.5)$	BabAbAbABabABaBabA
0.55 0.55	$\mathrm{I.A}_7^{i.c.}(0.5)$	BAbAbAbabaBaBaBaBA
0.55 0.55	$\text{I.A}_8^{i.c.}(0.5)$	BAbAbaBABABaBABaBA
0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5)	$\text{I.A}_{q\cdot c\cdot}^{i.c.}(0.5)$	BabABaBabABabAbABabA
0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5)	$\mathrm{I.A}_{10}^{i.c.}(0.5)$	BabAbAbAbAbABaBaBaBaBaBabA
0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5)	$I.A_{11}^{i.c.}(0.5)$	BabAbaBAbABabABaBAbaBAbaBabA
0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5)	$\operatorname{I.A}_{12}^{i.c.}(0.5)$	BaBabAbaBaBAbABaBabAbABaBAbAbaBabAbA
0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5)	$I.A_{13}^{i.c.}(0.5)$	BabAbaBAbaBAbABabABaBAbaBAbaBAbaBabA
0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5)	$I.A_{14}^{i.c.}(0.5)$	BabABaBAbaBabABabABabAbaBAbaBAbABabA
0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5)	$I.A_{15}^{i.c.}(0.5)$	BAbAbaBaBaBabAbAbabaBaBAbabAbABAbaBaBA
0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5)	$I.A_{16}^{i.c.}(0.5)$	BabABAbaBAbabABabABabABabABabABabABabABa
0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5)	$\mathrm{I.A}_{17}^{i.c.}(0.5)$	BAbAbabABabABabABabABabaBaBABabABabABabaBaBA
0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5)	$I.A_{18}^{i.c.}(0.5)$	BabaBabABAbAbabABabABAbABabaBaBABabAbabA
0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5)	$\text{I.A}_{19}^{i.c.}(0.5)$	BabABaBAbaBAbaBAbaBabABabABabAbaBAbaBAba
0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5)	$\mathrm{I.A}_{20}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABAbaBAbaBAba
0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5)	$\mathrm{I.A}_{21}^{i.c.}(0.5)$	BabABabAbaBAbaBAbABabABabABabABaBAbaBAba
0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5)	$I.A_{22}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABAbaBAbaBAba
0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5)	$I.A_{23}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABabABabA
0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5)	$I.A_{24}^{i.c.}(0.5)$	BabABabaBabaBAbaBABabABabABabABAbABAbaBAba
0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5)	$I.A_{25}^{i.c.}(0.5)$	BabAbaBAbaBAbaBAbaBAbaBAbABabABaBAbaBAbABabABa
0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5)	$I.A_{26}^{i.c.}(0.5)$	BabABabAbaBAbaBAbaBAbABabABabABabABabABa
0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5)	$\mathrm{I.A}_{27}^{i.c.}(0.5)$	BabAbabABaBAbaBAbABAbABAbABabABabABabABa
0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5)	$I.A_{28}^{i.c.}(0.5)$	BAbAbaBABaBAbaBAbaBAbaBAbaBAbabaBAbabAbaBAbaB
0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5)	$I.A_{29}^{i.c.}(0.5)$	BabABabABabABabAbaBAbabABabABabABabABabA
0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5)	$I.A_{30}^{i.c.}(0.5)$	BAbAbabABaBAbabABAbaBabABAbAbabaBaBABabABab
0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5)	$I.A_{31}^{i.c.}(0.5)$	BabAbaBAbaBAbABabAbaBAbaBAbaBAbABabABaBAbaBAb
0.5) 0.5) 0.5) 0.5) 0.5) 0.5) 0.5)	$I.A_{32}^{i.c.}(0.5)$	BAbAbabABabABabABabABabABabABAbAbabaBaBABabABab
0.5) 0.5) 0.5) 0.5) 0.5) 0.5)	$I.A_{33}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABabABabA
0.5) 0.5) 0.5) 0.5) 0.5)	$\mathrm{I.A}_{34}^{i.c.}(0.5)$	BabABAbaBAbaBABabABabABabABabABabABabABa
0.5) 0.5) 0.5) 0.5) 0.5)	$I.A_{35}^{i.c.}(0.5)$	BabABabABabABaBABaBabABAbABabABabABabABa
0.5) 0.5) 0.5) 0.5)	$I.A_{36}^{i.c.}(0.5)$	BabABabABaBAbaBAbaBAbaBAbaBAbaBABabABabA
0.5) 0.5) 0.5)	$\mathrm{I.A}_{37}^{i.c.}(0.5)$	BabABabABabABaBAbaBAbaBAbaBAbabABabABabA
	$\mathrm{I.A}_{38}^{i.c.}(0.5)$	BabaBaBABAbAbabABabABabABabABabABabABabA
${\rm LA}_{40}^{i.c.}(0.5)$ ${\rm LA}_{40}^{i.c.}(0.5)$ ${\rm LA}_{40}^{i.c.}(0.5)$	$\mathrm{I.A}_{39}^{i.c.}(0.5)$	BabABaBAbaBAbaBabABabABaBAbaBAbaBabABabA
	$I.A_{40}^{i.c.}(0.5)$	BabABAbaBABabABabABabABabABabABabABabABa

Class and number	iber free group element
$I.A_{41}^{i.c.}(0.5)$	BAbAb
$\mathrm{I.A}_{42}^{i.c.}(0.5)$	BAbAbabABaBABaBAbabABAbaBabaBabABAbAbabaBaBABaBA
$I.A_{43}^{i.c.}(0.5)$	BabAbaBabAbaBabABabABabABaBAbaBAbABabABa
$I.A_{44}^{i.c.}(0.5)$	BabABabABaBAbaBAbaBAbaBAbaBAbaBABabABabA
$I.A_{45}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABabABabA
$I.A_{46}^{i.c.}(0.5)$	BabABabaBabaBAbaBAbaBAbaBABabABabABabABa
$I.A_{47}^{i.c.}(0.5)$	BabAbaBAbaBAbaBAbaBABaBAbaBAbaBAbABabABa
$I.A_{48}^{i.c.}(0.5)$	BabABaBAbaBAbaBAbABabABabAbaBAbaBAbaBABabABab
$I.A_{49}^{i.c.}(0.5)$	BabABAbaBAbaBAbaBAbabABabABAbaBAbaBAbabABabAB
$ ext{I.A}_{50}^{i.c.}(0.5)$	BaBAbAbaBabAbABaBAbABaBAbAbaBaBAbAbaBabAbAbaBabAbABaBAbABaBabAbABaBabAbABaBAbABaBabAbaBaBAbABaBabAbABABABAB
$I.A_{51}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABabABabA
$I.A_{52}^{i.c.}(0.5)$	BabAbaBAbaBAbaBAbABaBAbABaBAbABaBAbaBAbABabABa
$\mathrm{I.A}_{53}^{i.c.}(0.5)$	BabAbaBAbABabABabABaBAbaBabABabABaBAbaBAb
$\mathrm{I.A}_{54}^{i.c.}(0.5)$	BabABabAbaBAbaBAbaBAbaBAbaBAbABabABabABa
$\mathrm{I.A}_{55}^{i.c.}(0.5)$	BAbAbabABaBAbabAbaBAbaBabABAbABAbaBabABAbAbabaBABABabABab
$\mathrm{I.A_{56}^{i.c.}(0.5)}$	BAbAbaBABaBabABaBABaBaBABaBaBaBaBaBaBaBAbAbabaBABABABA
$I.A_{57}^{i.c.}(0.5)$	BabABaBAbaBAbaBAbABabABabAbaBAbaBAbaBabABabA
$I.A_{58}^{i.c.}(0.5)$	
$I.A_{59}^{i.c.}(0.5)$	BabABaBAbaBAbABaBAbaBAbaBAbaBabABabAbaBabABabA
$\mathrm{I.A}^{i.c.}_{60}(0.5)$	BAbAbabABABaBABaBABaBABabABABabABABABABA
$\mathrm{I.A}_{61}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABabABabA
$I.A_{62}^{i.c.}(0.5)$	BAbAbabABabABabABabABabABabABabABabABabA
$I.A_{63}^{i.c.}(0.5)$	BabABabABabABaBAbaBAbaBAbaBAbaBAbaBAbaBA
$1.A_{64}^{i.c.}(0.5)$	BabABabABabABabABabAbaBAbaBAbaBAbaBAbaBA
$\mathrm{I.A}^{i.c.}_{65}(0.5)$	BabABaBAbaBAbABabABabABaBAbABabABabABabA
$I.A^{i.c.}_{66}(0.5)$	BabAbaBAbaBAbaBAbaBAbaBAbaBAbABaBAbaBAba
${ m I.A}^{i.c.}_{67}(0.5)$	BabAbaBAbaBAbABabABabAbaBAbABabABabAbaBAbaB
$I.A_{68}^{i.c.}(0.5)$	BabABabABAbABAbABAbaBAbaBABaBABaBABabABaBABaB
$\mathrm{I.A}_{69}^{i.c.}(0.5)$	BabABAbaBAbaBAbaBABabABabABabABabABabABa
$\mathrm{I.A}_{70}^{i.c.}(0.5)$	BabABabAbaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBA
$I.A_{71}^{i.c.}(0.5)$	BabABaBAbaBAbaBAbaBABabABabABabABaBAbaBAb
$I.A_{72}^{i.c.}(0.5)$	BabAbaBAbaBAbABabAbaBAbABabABaBAbABaBAbABaBAbABaBAbABaBAbABaBAbABaBAbABaBABABABA
$I.A_{73}^{i.c.}(0.5)$	BAbAbabABabABabABabABabABabABabABabABabA
$I.A_{74}^{v.c.}(0.5)$	BabABabABabABabABabABabABabABabABabABabA
$I.A_{75}^{i.c.}(0.5)$	BAbAbabABaBABaBABaBABaBAbabAbABAbaBabaBAbABabaBAbABabaBAbABabaBaBABABabABaBABaB
	BabA
$I.A_{76}^{i.c.}(0.5)$	BabABabABAbABAbaBAbABABABABababAbaBABaBAB
	BabA
$\mathrm{I.A}_{77}^{v.c.}(0.5)$	BabABaBAbaBAbaBabABabABaBAbaBAbaBabABabA
$I.A_{78}^{i.c.}(0.5)$	BabABAbaBAbaBABabABabABabABabABabABabABa
	BabA
$\mathrm{I.A}_{79}^{i.c.}(0.5)$	BabAbaBAbaBAbaBAbaBAbaBAbaBABabABabABaBAbABabABa
T A i.C. (OF)	
$1.A_{80} (0.0)$	BADADADADADADADADADADADADADADADADADADAD
	UNITERIOR TO THE RESIDENCE TO

Class and number	iber free group element
$\overline{1.A_{81}^{i.c.}(0.5)}$	BabAB
$I.A_{82}^{i.c.}(0.5)$	bABabABabABabA BabAbaBAbaBAbaBABabABabAbaBAbaBAbABabABa
$I.A_{s3}^{i.c.}(0.5)$	baBAbaBAbaBabA BabABabABabABabABaBAbaBAbaBAbaBAbaBAbaBA
(0.0) 88	bABabABabA
${ m I.A}_{84}^{v.c.}(0.5)$	BabAbaBAbaBAbaBAbBabABabABabABabABabABab
$I.A_{85}^{i.c.}(0.5)$	BabABabABabABabAbaBAbaBAbabABabABabABabA
$I.A_{86}^{i.c.}(0.5)$	BAbAbabABabABabABabABabABabABabABabABabA
$\mathrm{I.A}^{i.c.}_{87}(0.5)$	BabABabaBabaBAbaBABaBAbabABabABabABAbaBAba
$\mathrm{I.A}_{88}^{i.c.}(0.5)$	BabABaBAbaBAbaBAbABabABabABabAbaBAbaBAba
$1 A_{i.c.}^{i.c.}(0.5)$	BabABaBAbaBAbaBAbABabA BahABahA BahABahA BAbaBA baBAbaBAbaBAbaBAbaBAbabA BabABabABabABabABabABabABabABabaBAbaBAba
(0.0) 68	BAbaBABabABabABabA
$1.A_{90}^{i.c.}(0.5)$	BabABabaBabABAbABAbaBabaBAbABAbaBAbabAbaBABaBAbabAbab
$I.A_{91}^{i.c.}(0.5)$	BabABAbaBAbaBAbabABabABabABabABabAbaBAbabABabAB
4	BabABabaBAbaBABabA
${ m I.A}_{92}^{i.c.}(0.5)$	BabABabABaBAbaBAbaBAbaBAbaBAbaBABabABabA
1 A i.c. (0 E)	baBAbaBAbaBAbaBAbABabABabA Dela dela le dele dale dale dela dela del
$1.A_{93}^{-1}(0.5)$	БарАБарАваБАваБАваБАваБАваБАБарАБарАБарАБарАваБАваБАваБАваБАваБАБарАБарАБавАБаБАваБАваБАваБАваБАваБАБарАБарА ВаБАваВАваВАваВаваАВарАВаВА
${ m I.A}_{94}^{i.c.}(0.5)$	BAbAbabABaBABaBABaBAbaBABaBAbaBABaBabaBAbABAbaBabaBAbABabaBabABBabABabA
$\Gamma A_{0\overline{b}}^{i,c}(0.5)$	babababAbaBAbaBabAbAbaBababa BahAbaBAbaBabAbaBAbaBabAbaBabAbaBAbABabABab
(0.0) 66	bABabAbaBAbaBAbaBabAbaBabA
${ m I.A}_{96}^{i.c.}(0.5)$	BabABabaBAbaBAbaBABabABabABabABabABabABa
$I.A_{97}^{i.c.}(0.5)$	babABabABabABaBAbaBAbaBABabABabABabABabA
$1.A_{66}^{i.c.}(0.5)$	BabABabABAbaBAbaBAbaBABabABabA BAbAbaBABabaBAbABabaBABABABABABABABABABA
(0.0) 06	bAbABAbaBAbAbABAbABAbABAbABABABABABABABA
${ m I.A}_{99}^{i.c.}(0.5)$	BabABabABabABabABabaBAbaBAbaBAbaBAbaBAba
$\mathrm{I.A}_{100}^{i.c.}(0.5)$	BabABabaBabaBAbaBABabABabABabaBabaBAbaBA
$I.A_{101}^{i.c.}(0.5)$	Brown
$I.A_{102}^{i.c.}(0.5)$	BabABabaBabABabABabABabABabABabABabABabA
T A i.c. (O.E.)	abABabABabABAbaBABabABabABabABabABabABab
$1.A_{103}^{+}(0.5)$	БарАБарА БАРаБАРаБА РаБА РаБА РаБА БарА БарА БарА БарА БарА БарА БарА Б
$\mathrm{I.A}_{104}^{i.c.}(0.5)$	BabABaBAbaBAbaBabABabABabABabABaBAbaBAba

Table S. XXXIII. The free group elements for the periodic three-body orbits.

Class and number	er free group element
$\mathrm{I.A}_{105}^{i.c.}(0.5)$	BAbAbabABaBABaBABaBABaBAbabAbaBABaBABaBA
${ m I.A}_{106}^{i.c.}(0.5)$	BAbAbabABabaBabABAbABAbaBAbaBAbABAbaBAba
$\mathrm{I.A}_{107}^{i.c.}(0.5)$	BabABabABabABabABabAbaBAbaBAbaBAbaBAbaBA
$\mathrm{I.A}_{108}^{i.c.}(0.5)$	BabaBabABAbaBABaBAbabABabaBabABABABABABababAbabA
$\mathrm{I.A}_{109}^{i.c.}(0.5)$	BabABabaBAbaBAbabABabABabABabABabABabAbaBAbaB
$I.A_{110}^{i.c.}(0.5)$	abritaningan nagarangan nagarangan nagarangan nagarangan BAbaBAbaBAbaBAbaBABaBABaBABabABABABABABA
$\mathrm{I.A}_{111}^{i.c.}(0.5)$	BabABaBAbaBabABabABaBAbaBAbaBAbaBAbaBAba
$\mathrm{I.A}_{112}^{i.c.}(0.5)$	BAbAbabABabABabABabABabABabABabABabABabA
$I.A_{113}^{i.c.}(0.5)$	BabABAbaBAbaBAbabABabABabABabABabABabABa
$I.A_{114}^{i.c.}(0.5)$	BAbAbabABaBABabABaBABaBAbaBABaBABaBABaBA
$I.A_{115}^{i.c.}(0.5)$	BabABabABabABabABAbABAbaBAbaBABabABABABabABab
$I.A_{116}^{i.c.}(0.5)$	BAbAbabABabABabABaBABabABaBABabABabABABAbaBABABABA
$I.A_{117}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABabABabA
$I.A_{118}^{i.c.}(0.5)$	BabaBAbABAbaBAbaBABabABabABabABabABabABa
$\mathrm{I.A}_{119}^{i.c.}(0.5)$	BabABabAbaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBA
$\mathrm{I.A}_{120}^{i.c.}(0.5)$	BabABabABabABabABabABaBAbaBAbaBAbaBAbaBA
$\mathrm{I.A}_{121}^{i.c.}(0.5)$	BabABAbaBAbaBAbabABabABabABabABaBAbaBAba
$\mathrm{I.A}_{122}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABabABabA
$I.A_{123}^{i.c.}(0.5)$	BabABabABabABabABabABaBAbaBAbaBAbaBAbaBA
$\mathrm{I.A}_{124}^{i.c.}(0.5)$	BAbABabaBabABAbaBAbaBAbaBAbaBAbaBAbaBAba
$I.A_{125}^{i.c.}(0.5)$	BabABabABabABabABabABabAbaBAbaBAbaBAbaBA

$I.A_{1.96}^{i.o.}(0.5)$	Dababababababababababababababababababab
1000	AbaBAbABabABabABaBAbaBAbaBAbaBAbABabABab
$1.A_{127}^{v.c.}(0.5)$	BabABabaBAbaBAbaBABabABabABabABabABabABa
$\mathrm{I.A}_{128}^{i.c.}(0.5)$	BAbAbabABabABabABabABabaBaBABAbAbabABaBABaB
$\mathrm{I.A}_{129}^{i.c.}(0.5)$	BabAbaBAbaBabABabABaBAbaBAbABabAbaBAbaBA
$I.A_{130}^{i.c.}(0.5)$	BAbAbabABabABabABabABabaBabABabABabABabA
$\mathrm{I.A}_{131}^{i.c.}(0.5)$	BabABabABabABabaBaBABAbabABabABabABabABa
$\mathrm{I.A}_{132}^{i.c.}(0.5)$	aby Division and The and The and The Structure of the Str
$I.A_{133}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABaBAbaBAbaBA
$I.A_{134}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABabABabA
$I.A_{135}^{i.c.}(0.5)$	BAbAbabABabABabABabABabABabABabABabABabA
$I.A_{136}^{i.c.}(0.5)$	BabABabABabABabABabABabABAbaBAbaBAbaBAba
$I.A_{137}^{i.c.}(0.5)$	BahABabaBabaBAbaBAbaBAbabAbabABabABabaBAbaBA
$I.A_{138}^{i.c.}(0.5)$	ADA BabaBabABAbaBAbaBAbaBABaBABaBABabAbabABabAB
$I.A_{139}^{i.c.}(0.5)$	Aba Baba Baba Baba Baba Baba Baba Baba B
$\mathrm{I.A}_{140}^{i.c.}(0.5)$	AbabAbabAbabAbabAbabaBabAbAbABAbaBabaBAbABAbaBabaBAbaBA
	ABAbABabABabABAbABabaBAbABAbaBabaBAbABAbA
$\mathrm{I.A}_{141}^{ic.}(0.5)$	BabABabaBAbaBAbaBAbabABabABabABabaBAbaBA
$\mathrm{I.A}_{142}^{i.c.}(0.5)$	BabABabaBabaBAbaBABabABabABabaBAbaBAbaBA
$\mathrm{I.A}_{143}^{i.c.}(0.5)$	BAbAbabABaBABabAbabABaBABabAbabAbaBABaBAbabAbaBABaBAB
$\mathrm{I.A}_{144}^{i.c.}(0.5)$	BabABabaBabABAbABabaBabABAbABabaBabABAbABabaBabABAbABabaBABABabABab
${ m I.A}_{145}^{i.c.}(0.5)$	BAbABabaBabaBAbabABabABabABabABabABAbBAbaBAba
$\mathrm{I.A}_{146}^{i.c.}(0.5)$	BabABabaBabABAbABAbaBAbaBAbABAbaBAbaBAba
$\mathrm{I.A}_{147}^{i.\mathrm{c.}}(0.5)$	BAbABabaBAbaBABaBABabABabABabABabABabABa

Table S. XXXV. The free group elements for the periodic three-body orbits.

5	, 1
Class and number	rree group element
$\Gamma \mathrm{B}_{1}^{v.c.}(0.5)$	BaBAbA
$\mathrm{I.B}_2^{i.c.}(0.5)$	BaBaBAbAbA
$\Gamma.B_3^{i.c.}(0.5)$	BAbAbABaBaBA
$\Gamma.\mathrm{B}_{4}^{i.c.}(0.5)$	BabAbaBAbaBabA
$\Gamma.\mathbf{B}^{i.c.}_{5}(0.5)$	BaBaBAbAbAbA
$\mathrm{I.B}^{i.c.}_{6}(0.5)$	BabABAbaBABabA
$\Gamma.B_7^{i.c.}(0.5)$	BaBabaBaBAbAbabAbA
$\mathrm{I.B_8^{i.c.}(0.5)}$	BaBaBaBabAbAbAbAbA
$\Gamma.B_0^{i.c.}(0.5)$	BabABaBAbaBAbABabA
$\Gamma.\mathrm{B}_{10}^{i.c.}(0.5)$	BAbAbabAbABaBaBaBA
$\Gamma.\mathrm{B}_{11}^{i.c.}(0.5)$	BaBabbbaBaBAbAbaaabAbA
$\Gamma.B_{12}^{i.c.}(0.5)$	BabaBabaBAbabAbAbabA
$\Gamma.B_{13}^{i.c.}(0.5)$	BabABabaBAbaBAbabABabA
$\Gamma. m B_{14}^{i.c.}(0.5)$	BabABabAbaBAbaBAbaBabABabA
$\Gamma.B_{15}^{i.c.}(0.5)$	BabABabABAbaBAbaBAbaBABabABabA
$\Gamma.B_{16}^{i.c.}(0.5)$	BaBAbAbaBabAbABaBAbABaBabAbaBaBAbA
$\Gamma.B_{17}^{i.c.}(0.5)$	BabABabaBAbaBAbaBAbabABabA
$\Gamma. m B_{18}^{i.c.}(0.5)$	BabABabABaBAbaBAbaBAbaBAbABabABabA
$\Gamma.B_{19}^{i.c.}(0.5)$	BAbAbabABaBAbABABaBaBAbABabaBABABABABABA
$\Gamma.B_{20}^{i.c.}(0.5)$	BabABabABabABaBAbaBAbaBAbaBAbabABabABabA
$\Gamma.\mathrm{B}_{21}^{i.c.}(0.5)$	BAbAbabABabaBAbabABABaBabaBAbabABabaBABABABA
$\mathrm{LB}_{22}^{i.c.}(0.5)$	BabABabABabAbaBAbaBAbaBAbaBAbaBabABabABa
$\Gamma.B_{23}^{i.c.}(0.5)$	BAbAbabABaBABaBAbabAbABABabaBAbABAbABabaBaBA
$\Gamma.\mathrm{B}_{24}^{i.c.}(0.5)$	BabABaBABaBABaBABaBAbaBAbABAbaBabABAbABabABa
$\mathrm{I.B}_{25}^{i.c.}(0.5)$	BabAbaBAbaBAbABabAbaBAbaBAbaBABabABaBAbaBAb
$1.\mathrm{B}_{26}^{i.c.}(0.5)$	BabABabABabABAbaBAbaBAbaBAbaBAbaBABabABab
$\Gamma.B_{27}^{i.c.}(0.5)$	BabABabABabABaBAbaBAbaBAbaBAbaBAbaBAbABabABa
$1.B_{28}^{i.c.}(0.5)$	BabABaBAbaBAbaBAbBaBABaBAbaBAbABabABabAB
$1.B_{29}^{i.c.}(0.5)$	BabABabABabABabAbaBAbaBAbaBAbaBAbaBAbaBA
$\sim$	BabABabABabABAbABAbaBAbaBAbaBAbaBAbaBABaBAB
$\Gamma.\mathrm{B}_{31}^{i.c.}(0.5)$	BabABabaBabABAbABAbaBabaBAbaBAbabAbabAba
$\Gamma.\mathrm{B}^{i.c.}_{32}(0.5)$	BAbAbabABaBABabAbaBABaBAbabABABaBabaBAbABABABAbABAb
$\overline{}$	BabABabABabABabAbaBAbaBAbaBAbaBAbaBAbaBA
$\overline{}$	BabABAbaBAbaBAbabABabABabABAbaBAbaBABabABab
$\sim$	BAbAbaBABaBaBAbABabaBaBABABABABABABabAbAbAbA
<u></u>	BabAbabABaBAbabAbabABaBAbabAbaBAbaBabaBAbABabaBaba
$\sim$	BabABabABabABabABAbaBAbaBAbaBAbaBAbaBAba
$\overline{}$	BADADabaBabABabABABaBADaBABaBabADABABABaBabABaBABABABABABABABABABABABABAB
1.539 (U.3) T R!:c.(0 द)	BabAbabAbabAbabAbabAbabAbabAbabAbabAbabA
1:240 (0:0)	

Table S. XXXVI. The free group elements for the periodic three-body orbits.

Close and mumber	find mount alamont
T Di.G. (O.F.)	
1.641(0.5)	BADADaBABABABABABABABABABABABABABABABABA
$I.B_{42}^{\iota.c.}(0.5)$	BabAbaBABaBaBABaBaBABaBaBABaBABaBaBABAbABABABAB
$1.\mathrm{B}_{43}^{i.c.}(0.5)$	BabABabABabABabABaBAbabAbaBAbaBAbaBAbaBA
$I.B_{44}^{i.c.}(0.5)$	BAbAbabABaBABabAbabAbaBABaBAbaABABABaBabaBAbABAbaBabaBAbABAbaBAba
$\mathrm{I.B}^{i.c.}_{45}(0.5)$	BabABabaBAbaBAbaBAbabABabaBAbaBAbaBAbaBA
$1.B_{46}^{i.c.}(0.5)$	BabABaBAbaBAbaBAbaBAbaBABabABaBAbaBAbABabABa
$1.B_{47}^{i.c.}(0.5)$	BabABabABabABabABabAbaBAbaBAbaBAbaBAbaBA
$1. m B_{48}^{i.c.}(0.5)$	BabABabABabABabABabAbaBAbaBAbaBAbaBAbaBA
$\mathrm{I.B}_{49}^{i.c.}(0.5)$	BabAbaBAbaBAbaBAbABabABabABabABabABabABa
$\mathrm{I.B}_{50}^{i.c.}(0.5)$	BabABabaBAbaBABaBABabABabABabaBAbaBAbaBA
$I.B_{51}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABabABaBAbaBABabABab
$I.B_{52}^{i.c.}(0.5)$	BabABabAbaBAbaBAbaBAbABabABabABabAbaBAbaB
$1.B_{53}^{i.c.}(0.5)$	BabABabABabABabABabABabABabAbaBAbaBAbaBA
$I.B_{54}^{i.c.}(0.5)$	BabABabABabABabABabABabABaBAbaBAbaBAbaBA
$I.B_{55}^{i.c.}(0.5)$	BabABabABabABabABabABaBAbaBAbaBAbaBAbaBA
$1.B_{56}^{i.c.}(0.5)$	BabAbaBAbaBAbABabAbaBAbaBAbABabABabAbaBAbaB
$1.\mathrm{B}_{57}^{i.c.}(0.5)$	BAbAbabABaBABabAbabAbaBABaBABaBABaBABaBA
$I.B_{58}^{i.c.}(0.5)$	BabABabABaBABabABabAbaBABaBABaBAbaBAbaBA
$1.B_{59}^{i.c.}(0.5)$	BabABabAbaBAbaBAbaBABabABabABabABabABabA
$1.\mathrm{B}_{60}^{ic.}(0.5)$	BabaBABABABABABABABABABABABABABABABABABA
$I.B_{61}^{ic.}(0.5)$	BabAbaBAbaBAbABabAbaBAbABaBAbaBAbaBAbaBA
$I.B_{62}^{i.c.}(0.5)$	
$I.B_{63}^{i.c.}(0.5)$	BAbAbabABaBABabAbAbaBabABAbAbabABaBAbabABABABAB
$1.\mathrm{B}^{i.c.}_{64}(0.5)$	BabABabABabABabABabABabAbaBAbaBAbaBAbaBA
$1.\mathrm{B}_{65}^{i.c.}(0.5)$	BAbAbabABaBABaBABaBABaBAbabAbaBABaBAbabAbABABABaBabaBAbABABABAB
$I.B_{66}^{i.c.}(0.5)$	BabABaBAbaBAbaBAbaBABaBAbaBAbaBabABabABa
	BabAbaBAbaBabABabABabAbaBabABabABabABabA
$I.B_{68}^{i.c.}(0.5)$	BabABabABabABabABabABabAbaBABaBAbaBAbaBA
$I.B_{69}^{i.c.}\left(0.5 ight)$	BabABabABabABabAbaBAbaBABabABabABabABAbABA
$\mathrm{I.B}_{70}^{i.c.}(0.5)$	BabABabABaBAbaBAbaBAbaBAbABabABabABabABa
$I.B_{71}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABaBAbaBAb
$1.B_{72}^{i.c.}(0.5)$	BabaBAbaBAbaBABabABabABabABabABAbABAbABA
$1.B_{73}^{ic.}(0.5)$	BabABAbaBAbaBAbaBAbaBAbaBAbaBAbabABabABa
$1.\mathrm{B}_{74}^{i.c.}(0.5)$	BabABabABabABabABabABabABaBAbaBAbaBAbaBA
$1.B_{75}^{i.c.}(0.5)$	BabABabABaBABaBAbaBAbaBAbaBAbBabABabABab
$1.B_{76}^{i.c.}(0.5)$	BabABabABabABabABabABabABAbaBAbaBAbaBAba
$I.B_{77}^{ic.}(0.5)$	BabABabABabABabABabABabABabABabABabABabA
$I.B_{78}^{i.c.}(0.5)$	BAbABabaBAbABAbABAbABAbABAbaBabaBAbABAbA
$1.B_{79}^{i.c.}(0.5)$	BabABAbaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBAba
$1. m B_{80}^{i.c.}(0.5)$	BabABabABabABabABabABabAbaBABabABaBABabABaBAbaBAb

Table S. XXXVII. The free group elements for the periodic three-body orbits.

Class and number	er free group element
$I.B_{81}^{i.c.}(0.5)$	BAbAB babABa
$I.B_{82}^{ic.}\left(0.5 ight)$	BabABabABabABabABabABabABabAbaBAbaBAbaBA
$I.B_{83}^{i.c.}(0.5)$	BabAbabABaBAbabAbABAbaBabaBAbABabaBAbABAbA
$I.B_{84}^{i.c.}(0.5)$	BabAbaBAbaBAbABabABabABaBAbaBabABabABaBAbaBAb
${ m I.B}_{85}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABabAbaBAbaB
$I.B_{86}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABabAbaBAbaB
$I.B_{87}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABaBAbaBAb
$I.B_{88}^{i.c.}(0.5)$	BabABaBAbaBAbaBAbaBabABabABabABabAbaBAbaB
$I.B_{89}^{i.c.}(0.5)$	BabAbaBAbaBAbaBABABABABABABABABABABABABA
$I.B_{90}^{i.c.}(0.5)$	BabABabABabABabABaBAbaBAbaBAbaBAbaBABabABab
$I.B_{91}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabaBAbaBAba
$I.B_{92}^{i.c.}(0.5)$	BabABabaBabaBAbaBABaBABabABabaBabaBAbaBA
$I.B_{93}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabAbaBAbaBA
$I.B_{94}^{i.c.}(0.5)$	BAbAbABABABABABABABABABABABABABABABABAB
$I.B_{95}^{i.c.}(0.5)$	BAbAbabABabABabABabABabABaBABabAbaBABaBAbaBAb
${ m I.B}_{96}^{i.c.}(0.5)$	BabABabaBabaBAbABAbaBAbabAbaBABaBABabAbabABabAB
$I.B_{97}^{i.c.}(0.5)$	BabABabABaBAbaBAbaBAbaBAbaBAbaBAbABabABa
$I.B_{98}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABAbaBAbaBAba
$I.B_{99}^{i.c.}(0.5)$	BabABabABabAbaBAbaBAbaBAbaBAbaBAbABabABa
$\mathrm{I.B}_{100}^{i.c.}(0.5)$	BabaBabABabABaBABaBABabABabABAbABAbABAbaBabaBAbaBA

Table S. XXXVIII. The free group elements for the periodic three-body orbits.

Class and number	nber free group element
$I.B_{101}^{i.c.}(0.5)$	1, , ,
$\mathrm{I.B}_{102}^{i.c.}(0.5)$	BabAbaBAbaBabABabABaBAbaBAbABabABaBAbaBAb
$\mathrm{I.B}^{i.c.}_{103}(0.5)$	BabABabABAbaBAbaBABaBABabABabABabABabABa
$\mathrm{I.B}_{104}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABaBAbaBAb
$\mathrm{I.B}_{105}^{i.c.}(0.5)$	BabABAbaBAbaBABabABabABabABabABabABabABa
$I.B_{106}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABabABabBAbaBAba
$I.B_{107}^{i.c.}(0.5)$	BabABabAbaBAbaBAbaBAbaBAbABABABABABABABA
${ m I.B}_{108}^{i.c.}(0.5)$	Trond Trong Baba BAba BABaba BABab BABab Bab Bab BAB Baba Baba
$\mathrm{I.B}^{i.c.}_{109}(0.5)$	BabABabABabABabABabABabABabABabABaBABaBA
$\mathrm{I.B}_{110}^{i.c.}(0.5)$	BabABAbaBAbaBAbabABabABabABabAbaBAbaBABabABab
$\mathrm{I.B}_{111}^{i.c.}(0.5)$	BAbAbabABaBAbaBAbaBAbaBAbaBAbaBAbaBAbABabABa
$\mathrm{I.B}^{i.c.}_{112}(0.5)$	BabABabABAbaBAbaBAbaBABabABabABabABabABa
$\mathrm{I.B}_{113}^{i.c.}(0.5)$	BabABabABaBAbaBAbaBAbaBAbaBAbaBAbABabABa
$\Gamma.\mathrm{B}^{i.c.}_{114}(0.5)$	BabABabaBAbaBAbaBABabABabABabABabABabABa
$I.B_{115}^{i.c.}(0.5)$	BabABabaBabaBAbaBABaBABabABabABabABAbABA
$I.B_{116}^{i.c.}(0.5)$	BabABaBAbaBAbABabABabABaBABaBABaBABaBABa
$I.B_{117}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABabABaBAbaBAb
$I.B_{118}^{i.c.}(0.5)$	Baharan Bahab Bahaba Baharan Bahab B
$I.B_{119}^{i.c.}(0.5)$	BabABabaBabABAbABAbaBAbaBABaBABabAbabABabAB
$I.B_{120}^{ic.}(0.5)$	BabABabABabABaBAbaBAbaBAbaBAbaBAbaBAbaBA

Table S. XXXIX. The free group elements for the periodic three-body orbits.

I.Bi <sup>i,c.</sup> <sub>1.25</sub> (0.5)	BAbbababaBabABabABabABabABabABabABabABaBABaB
	ABabABabABabABabABabAbaBAbaBAbaBAbaBAbaB
	baBAbABabABabABaBAbaBAbaBAbaBAbaBAbABabABa
	$\verb ABABABABABABABABABABABABABABABABABABAB$
	AbaBAbabABabABAbaBAbaBAbabABabABabABAbaBAba
	BabABabABaBAbaBAbaBAbaBAbABabABabABabABa
	BabABabABabABabABabABabABabABabABabABabA
	BAbAbabABaBABaBABaBABaBaBaBAbABABaBabaBab
	BabABabAbaBAbaBAbaBAbaBAbABabABabABabABa
aBABabABabABabABabABabABabABabABabABabAB	BAbABabaBabABAbABAbaBAbaBAbaBAbaBAbaBAba
$\mathrm{I.B}^{i.c.}_{135}(0.5)$ BabABabABabABabABah aBAbaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBAb	BabABabABabABabABabABabABabABabABabABabA
I.B $_{136}^{i.c.}(0.5)$ BabABabABabABabABabABabABabABabABabABabA	BabABabABabABabABabABaBAbaBAbaBAbaBAbaBA
$I.B_{137}^{i.c.}(0.5)$ BabABabABabABabAbaBAba ABabABabABabAbaBAba	BabABabABaBAbaBAbaBAbaBAbaBabABabABabABa
$\mathrm{I.B_{138}^{i.c.}(0.5)}$ $BAbABabaBabABABABabABabABabABabABabABabA$	BAbABabABabABabABabABabABabABabABAbaBAba
I.B <sub>139</sub> (0.5) BabABabABabABabABabABa AbaBAbaBAbaBAbaBAbaBAbaBAb	BabABabABabABabABabABabABabABabABabABabA
$\begin{array}{ll} \text{L.B}_{140}^{i.c.}(0.5) & \text{BabABabABabABabABab} \\ & \text{aBAbaBAbaBAbaBAbaBAbaBAbaBA} \end{array}$	BabABabABabABabABabABabABabABabABabABabA

Table S. XL. The free group elements for the periodic three-body orbits.

Class alld Iluliber	January Januar
${ m I.B}_{141}^{v.c.}(0.5)$	BabABabABabABabABAbaBAbaBAbaBAbaBAbaBAba
$I.B_{142}^{i.c.}(0.5)$	BabABabaBabaBAbaBAbaBAbabABabABaBABabABab
$\mathrm{I.B}_{143}^{i.c.}(0.5)$	BAbAbabABaBABabABABABABABABABABABABABABA
$\mathrm{I.B}_{144}^{i.c.}(0.5)$	BabABabABabABABABABABABABABABABABABABABA
$I.\mathrm{B}^{i.c.}_{145}(0.5)$	BabABabaBAbaBAbaBABabABabABabaBAbaBAbaBA
$\Gamma B_{146}^{i.c.}(0.5)$	BAbAbabABaBAbabAbaBABabAbaBABaBabaBAbABAbaBABabaBAbABabaBabABAbABabABa
$\Gamma \mathrm{B}_{147}^{i.c.}(0.5)$	BabABabaBabaBAbaBABaBAbabABabABabABAbABA
$\Gamma B_{148}^{i.c.}(0.5)$	BabABabaBabABAbABabaBabaBAbABAbaBabaBAbaBA
$\Gamma.\mathrm{B}^{i.c.}_{149}(0.5)$	BAbAbabABBABABABABABABABABABABABABABABAB
$\mathrm{I.B}^{i.c.}_{150}(0.5)$	BABAbabaBABBABABBABBABBABBABBABBABBABBABBABBABB
$\mathrm{I.B}_{151}^{i.c.}(0.5)$	BAbAbabaBabaBAbABABABABABABABABABABABABA
$\mathrm{I.B}^{i.c.}_{152}(0.5)$	BAbAbabaBabABAbABAbaBAbaBAbaBABaBABaBAbabABaBABaB
$II.A_1^{i.c.}(0.5)$	BabAbaBAbABabABabAbaBabA
$\Pi.\mathrm{A}_{2}^{\mathrm{co}}(0.5) \ \Pi.\mathrm{A}_{3}^{i.c.}(0.5)$	bababababababababababababababababababa
$\mathrm{II.A_4^{i.c.}(0.5)}$	BabAbaBAbaBabABaBAbaBAbABaBAbABaBAbABabAbaBAbaB
$ ext{II.A}_6^{i.c.}(0.5)  ext{II.A}_6^{i.c.}(0.5)$	BADADaDADaBABaBABABABADABADADBABABABABAB
$\mathrm{II.A}_7^{i.c.}(0.5)$	BabABabABaBAbaBAbaBAbaBabABabABabABabABa
$\Pi.\mathrm{B}_{1}^{i.c.}(0.5)$	BaBAbAbaBabAbABaBabAbaBabAbaBabAbaBabAbaBabAba
$\Pi.B_{3}^{\iota.c.}(0.5)$ $\Pi.B_{3}^{\iota.c.}(0.5)$	BabAbaBAbaBabABabABaBAbaBAbABabABaBAbaBAb
$\mathrm{II.B}_4^{i.c.}(0.5)$	BabABaBAbaBabABaBAbaBAbaBabABabABabABabA

Table S. XLI. The free group elements for the periodic three-body orbits.

Class and number	free group element
$\Pi(C^{i,c}(0.5))$	BaBAbaBAbA
$\Pi.C^{b.c.}(0.5)$	BAbAbaBaBA
·.	BAbAbaBaBA
,	BAbAbaBaBaBA
$\sim$	BaBaBAbaBAbAbA
	${ m BAbAbabaBabaB}$
$\Pi.C_7^{i.c.}(0.5)$	${\bf BabAbaBabAbaBabA}$
$\operatorname{II.C}_8^{i.c.}(0.5)$	${ m BabABaBAbaBAbABabA}$
	${ m BabABaBabAbABabA}$
$\Pi.C_{10}^{i.c.}(0.5)$	${ m BabABaBaBAbAbABabA}$
$\overline{}$	BaBaBaBababAbAbAbA
$\Pi.C^{i.c.}_{12}(0.5)$	${ m BaBAbaBabAbaBAbA}$
$\Pi.C_{13}^{i.c.}(0.5)$	${ m BaBaBabAbABaBaBabAbAbA}$
	${ m BabAbAbaBAbaBAbaBabA}$
$\Pi.C^{i.c.}_{15}(0.5)$	${\bf BabAbAbABabABabABaBaBabA}$
$\Pi.C^{i.c.}_{16}(0.5)$	BabAbabABaBababAbABabaBabA
$\operatorname{II.C}_{17}^{i,c}(0.5)$	${ m BAbAbabABAbaBABaBaBaBA}$
$\Pi.C_{18}^{i.c.}(0.5)$	BaBAbAbaBabAbaBabAbaBabAbA
$\Pi.C_{19}^{i.c.}(0.5)$	BAbAbabAbabABabaBaBaBaBA
	${ m BabABabABAbAbabaBaBABabABabA}$
$\Pi.\mathrm{C}^{\mathrm{i.c.}}_{21}(0.5)$	BAbAbaBABaBAbaBAbaBAbABAbABABBABA
	${\bf BabAbaBABaBaBababAbABABABaBabA}$
$\operatorname{II.C}_{23}^{i.c.}(0.5)$	BAbAbAbAbAbaBAbaBAbaBaBaBaBA
$\operatorname{II.C}_{24}^{i.c.}(0.5)$	${\bf BabABaBAbaBAbaBAbaBAbaBAbABabA}$
$\overline{}$	${\bf BaBAbABaBAbAbaBabAbaBaBAbABaBAbA}$
	BAbAbaBabABabABabABabAbaBABaBA
$\Pi.\mathrm{C}_{27}^{i.c.}(0.5)$	BAbAbABABABaBaBAbaBAbABABABaBaBABABABABA
	BAbAbAbAbABaBaBaBabAbAbAbABabaBaBA
$\Pi.C_{29}^{i.c.}(0.5)$	BAbAbABAbaBabABabABabABaBABaBABABABABABA
$\mathrm{II.C}_{30}^{i.c.}(0.5)$	BabAbaBAbABabABaBAbaBAbABabABaBAbaBabA
$\sim$	BAbAbAbAbaBaBABaBAbaBAbaBAbABABaBaBABABABA
$\overline{}$	${\it BabABabABaBAbabABABABABaBabaBAbABabABabA$
$\sim$	${\it BabaBabABABABABaBababAbABABaBABabAbabA}$
$\overline{}$	BAbAbabAbAbAbAbaBAbaBAbaBABaBaBaBaBaBABA
	BabABabABabABabaBaBABAbAbAbabABabABabABa
$\operatorname{II.C}_{36}^{i.c.}(0.5)$	${\bf BabABabABaBAbaBAbaBAbaBAbaBAbaBAbABabABa$
	${\it BabABabABabABaBABaBABabABABABabABabABabA$
$\text{II.C}_{38}^{v.c}(0.5)$	BAbAbABAbaBaBABabAbABABABABABAbAbABABaBaBA
$\Pi.C^{3.6.}_{-0.0}(0.5)$	BaBabAbaBaBAbABaBabAbaBabAbABaBAbAbaBabAbA RAFAFARAFARAFARAFARAFARARARARARARARARA
11.040 (0.9)	Adababardabardabardabardabardabardabarda

Table S. XLII. The free group elements for the periodic three-body orbits.

Olone and windows	face marine alamant
Class and number	nee group element
$\Pi.C_{41}^{v.c.}(0.5)$	${\tt BabAbaBAbaBAbaBAbABabABabAbaBAbaBAbaBAba$
$\mathrm{II.C}_{42}^{ic.}(0.5)$	BAbAbabABabABabABabABabABabABabABAbABAbA
$\operatorname{II.C}_{43}^{i.c.}(0.5)$	BabABabABaBABabAbaBABababAbABAbaBabABAbABabABa
$\operatorname{II.C}_{44}^{i.c.}(0.5)$	BabAbaBabAAbaBabAbaBabAbaBAbaBabAbaBabAbaBabAbaBabA
$\Pi.\mathrm{C}_{45}^{i.c.}(0.5)$	BAbAbAbaBABaBaBAbabAbABABaBaBaBabAbAbABABaBaBABABABA
$\operatorname{II.C}_{46}^{i.c.}(0.5)$	BAbABabaBAbABAbABAbaBAbaBAbaBABaBABabAbabAb
$\mathrm{II.C}_{47}^{i.c.}(0.5)$	BabAbabABaBabaBAbABabABabABabABaBABaBAbabABabAB
$\Pi.\mathrm{C}^{i.c.}_{48}(0.5)$	BabABabABabABabABabAbabaBaBABabABabABabA
	BAbAbAbabABaBaBaBabAbAbABABABaBaBABabAbAbAbA
$\mathrm{II.C}_{50}^{i.c.}(0.5)$	BaBAbAbaBabAbABaBAbABaBAbABaBAbABaBAbABaBAbABaBAbABaBAbABaBABABABA
$\operatorname{II.C}_{51}^{i.c.}(0.5)$	BAbAbaBABaBabaBAbAbaBAbaBAbABabaBaBAbaBAb
$\operatorname{II.C}_{52}^{i.c.}(0.5)$	BAbAbaBAbABaBaBaBaBABaBAbaBAbAbAbAbAbAbA
$\text{II.C}_{53}^{i.c.}(0.5)$	BabABabABabABabAbaBAbaBAbaBAbaBAbaBabABabA
$\Pi.C^{i.c.}_{54}(0.5)$	BAbAbabABabaBAbaBAbaBAbaBAbaBABaBaBaBaBABabABab
$\text{II.C}_{55}^{i.c.}(0.5)$	BAbbaBAbaBAbaBABabABaBAbaBAbaBAbABabABab
$\Pi.C_{56}^{i.c.}(0.5)$	BabAbaBAbaBAbaBAbABaBAbaBAbaBAbABabABabA
$\Pi.C^{i.c.}_{57}(0.5)$	BabaBABABABABABABABABABABABABABABABABABA
$\text{II.C}_{58}^{i.c.}(0.5)$	BAbAbabABabABabABabABaBABabABAbAbabaBaBABabABab
$\Pi.C_{59}^{i.c.}(0.5)$	BAbAbabABabABabABabABabABabABabABabABabA
$\Pi.C_{60}^{i.c.}(0.5)$	BabAbaBAbaBAbaBAbABabABabABaBAbaBAbABabABa
$\Pi.\mathrm{C}^{i.c.}_{61}(0.5)$	BabABabABabABabABaBAbabAbABABaBabaBAbABabABa
$\Pi.\mathrm{C}^{i.c.}_{62}(0.5)$	BAbABabaBAbABAbABAbaBAbaBAbaBAbaBAbabAbaBABaBAB
$\Pi.\mathrm{C}^{i,.c.}_{63}(0.5)$	BabAbaBAbaBAbABaBAbaBAbABabABabABaBAbaBAb
$\operatorname{II.C}^{i.c.}_{64}(0.5)$	BabABabABabABabAbaBAbaBAbaBAbaBAbaBAbaBA
$\Pi.\mathrm{C}^{i,c.}_{65}(0.5)$	BabAbAbabaBaBaBaBABabAbAbaBaBaBabAbAbAbA
$\mathrm{II.C}^{i.c.}_{66}(0.5)$	BabABabABabABabABaBABabAbaBABababAbABAbaBabABAbABA
$\Pi.\mathrm{C}^{i.c.}_{67}(0.5)$	BAbAbaBABaBAbaBAbaBAbaBabABabABabABabABa
$\mathrm{II.C}_{68}^{i.c.}(0.5)$	BabABabaBabABAbABabaBabaBAbABAbaBababAbaBABaBAB
$\text{II.C}_{69}^{v.c.}(0.5)$	BAbAbabABabABabABabABabaBABabABabABabABa
$\text{II.C}_{70}^{e.c.}(0.5)$	BabABabABabABaBAbaBABaBABaBABabAbabaBabABABABAB
$\mathrm{II.C}_{71}^{v.c.}(0.5)$	BabABaBABABABaBABaBABabABabABabaBaBABAbAbabABabAB
$\mathrm{II.C}_{72}^{v.c.}(0.5)$	BAbAbabABabABabABabABabABabABabABabABabA
$\mathrm{II.C}_{73}^{v.c.}(0.5)$	BabAbaBAbaBabABabAbaBAbaBAbABaBAbaBAbABaBAbABabABa
$\prod_{i \in \mathcal{C}_{74}^{r}} (0.5)$	BabABabABabABabABabABaBABaBABaBababAbABABABababABABABAB
$\text{II.C}_{75}^{v.c.}(0.5)$	BAbAbabABabaBabABAbabaBabABabABabABAbAbabaBaBABABABabABab
$\Pi.C_{76}^{ec}(0.5)$	BabABaBAbaBAbaBabABabABaBAbaBAbaBAbaBAbABabABa
11. C77 (0.5)	BabAbaBAbaBABaBAbaBAbaBAbaBAbaBaBAbaBabAbaBabAbaBABaBAB
$11.C_{78} (0.3)$ $11.C_{78} (0.5)$	DAOADADADADADADADADADAOADAOADADADADAOA
II. C79 (0.3)	BabABabABabABabABaBAbaBAbaBAbaBAbaBAbaBA
(2.0) 00	

Table S. XLIII. The free group elements for the periodic three-body orbits.

Class and number	oer free group element  Bride identidation in the partition of the partition in the identidation in the identical interpretation in the identidation in the identical interpretation in the identidation in the identical interpretation in the identity in the identi
$\Pi.C_{81}^{i.c.}(0.5)$ $\Pi.C_{82}^{i.c.}(0.5)$	DabAbabAbabAbabAbabAbabAbabAbaBAbaBAbABabABa
$II.C_{83}^{i.c.}(0.5)$	BabABabABaBAbaBAbaBAbaBAbABabABabABabABa
$\text{II.C}_{84}^{i.c.}(0.5)$	BAbAbAbaBABaBaBAbabAbaBABaBaBAbabAbaBAbaB
$\Pi_{\rm C_{85}^{i.c.}}(0.5)$	BabABabABabABabABabaBAbABAbaBabaBAbABABABAB
$\mathrm{II.C}^{i.c.}_{86}(0.5)$	BAbAbabABabABAbABAbaBabaBAbABAbaBabaBAbabAbaBABaBAB
$\Pi.C_{87}^{i.c.}(0.5)$	BabaBabABAbaBAbabABabABabABAbABAbaBababAbaBABaBAB
$\text{II.C}_{88}^{i.c.}(0.5)$	BAbAbabABabABabABabABabABabABabABabABabA
$11.C_{89}^{\circ}(0.5)$	ΒαρΑΒαρΑΒαΒΑβαΒΑβαΒΑβαρΑΒαρΑΒαρΑΒαρΑβαρΑβαΒΑβαΒΑβαΒΑβαβΑβαβΑβαρΑΒαρΑβαρΑβαβΑβαβΑβαβΑβαρΑΒαρΑ Βόμ η Βουραμορική Βουμαρομαμου Αμουλημαρομαμος Αμουλαμουμαρομαρομαρομαρομαρομαρομαρομαρομαρομα
$\Pi_{C_{0}}^{i,c}(0.5)$	Dayandanananananananananananananananan dayananan manananan mananananananananananan
$\Pi.C_{92}^{i.c.}(0.5)$	BabABabABabABabABabABabAbaBAbaBAbaBAbaBA
$^{i.c.}_{93}(0.5)$	BabABabABabABabABabABabABabABaBABaBABaBA
$II.C_{94}^{i.c.}(0.5)$	BabAbaBAbaBAbABaBAbABabABabAbaBAbABabAbaBAbaB
$II.C_{95}^{i.c.}(0.5)$	BabaBABABABABABABABABABABABABABABABABABA
$\mathrm{II.C}^{i.c.}_{96}(0.5)$	BabAbaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBA
$II.C_{97}^{i.c.}(0.5)$	BAbAbabABaBABaBAbaBAbABAbaBabaBAbABAbABabaBABABAbABabABABABAB
$II.C_{98}^{i.c.}(0.5)$	BAbAbabABabABabABabABaBABaBABABABABABABA
$^{i.c.}_{99}(0.5)$	BabABabABaBAbaBAbaBAbaBAbaBAbaBabABabABa
$\mathrm{II.C}_{100}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABabABabA
$ ext{II.C}^{i.c.}_{101}(0.5)$	BabABabaBAbaBAbaBAbaBAbaBABaBABabAbabABaBABABABA
$\mathrm{II.C}_{102}^{i.c.}(0.5)$	BabABAbaBAbaBABabABabABabABabABabABabABa
$\mathrm{II.C}_{103}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABabABabA
$\mathrm{II.C}_{104}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABabABabA
$\mathrm{II.C}_{105}^{i.c.}(0.5)$	BAbABabaBAbABAbABAbaBAbaBAbaBAbaBABabABab
$\mathrm{II.C}^{i.c.}_{106}(0.5)$	BAbAbabABabABabABabABabABAbABabaBaBABabABab
$\Pi.C_{107}^{i.c.}(0.5)$	BabABabABabABaBABaBABaBABaBABaBABaBABaBA
$ ext{II.C}^{i.e.}_{108}(0.5)$	BabAbaBAbaBAbaBabABabAbaBAbaBAbaBABabABab
	BabAbabABaBABaBABaBABABABABABABABABABABA
:5;	BabABabABabABabABabABabABabABabABabaBabABabA
$\text{II.C}_{111}^{irr}(0.5)$	BabABabABabAbaBAbaBAbaBAbaBAbaBAbaBAbABabABa
$II.C_{112}^{(1)}(0.5)$	БарАВарАВарАВарАВарАВарАВарАВарАВаВАВаВАВ
$\Pi.C_{113}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABabABabA
$\text{II.C}_{114}^{i.c.}(0.5)$	BabABabABabABaBAbaBAbaBAbaBABabABabABabA
$\Pi.\mathrm{C}_{115}^{i.e.}(0.5)$	BabaBaBABAbabABabABabABabaBaBABabABabABa
$\text{II.C}_{116}^{v.c.}(0.5)$	BAbAbabABaBABaBABABABABABABABABABABABABA
$\text{II.C}_{117}^{i.c.}(0.5)$	BabABabAbaBAbaBAbaBAbaBAbaBABabABabABaBAbaBAb
$\text{II.C}_{118}^{v.c.}(0.5)$	BAbAbabABabABabABabABabABabABabABabABabA
$\Pi.C_{119}^{i.c.}(0.5)$	BAbAbabaBAbaBAbaBAbaBAbaBAbaBABaBABaBAbabAbab
$II.C_{120}^{ic.}(0.5)$	BabABabABabABAbaBAbaBAbaBAbaBABabAbabAba

Table S. XLIV. The free group elements for the periodic three-body orbits.

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Class and number	free group element
$\Pi.\mathrm{C}_{121}^{i.c.}(0.5)$	BAbABabaBabaBAbaBABaBABabABabABabABabABa
$\mathrm{II.C}^{i.c.}_{122}(0.5)$	BabA BAbABabaBabaBAbABAbabAbabABabABabABabABa
	bABaBA
$\mathrm{II.C}_{123}^{i.c.}(0.5)$	BAbAbabABabABabABabABAbaBabaBAbABAbaBabaBAbABAbA
$ ext{II.C}_{124}^{i.c.}(0.5)$	BabABabABabABAbABabaBabABAbABabABAbABabABa
$\Pi.\mathrm{C}_{125}^{i.c.}(0.5)$	BabABabABAbABabaBaBABAbAbabABabABabABabA
$\Pi.C^{i.c.}_{126}(0.5)$	BabABabABabAbabaBaBABabABabABabABabABabA
$\mathrm{II.C}_{127}^{i.c.}(0.5)$	BabAbaBAbaBAbABabABabAbaBAbABabABabAbaBAbaB
$\Pi.\mathrm{C}_{128}^{i.c.}(0.5)$	BabABaBABabAbABAbaBabaBabABAbAbabaBaBABabAbabAb
$\Pi.\mathrm{C}^{i.c.}_{129}(0.5)$	BabABabABaBAbaBAbaBAbaBAbaBAbABabABabABa
$\Pi.C_{130}^{i.c.}(0.5)$	BabABabABabABaBABaBAbabABabaBabABabABabA
$\Pi.\mathrm{C}^{i.c.}_{131}(0.5)$	BabABabABabABabABabABabABabaBAbaBAbaBAba
$\mathrm{II.C}^{\mathrm{i.c.}}_{132}(0.5)$	BabABabABabABabABabABabABabABabABabABabA
$\Pi.\mathrm{C}^{\mathrm{i.c.}}_{133}(0.5)$	BabABabAbaBAbaBAbaBAbABabABabABaBAbaBAba
$\Pi.\mathrm{C}^{i.c.}_{134}(0.5)$	BabABabABaBAbaBAbaBAbaBAbaBabABabABabABa
$\mathrm{II.C}_{135}^{i.c.}(0.5)$	BaBAbAbaBabAbaBaBAbABaBAbAbaBabAbaBaBAbABaBAbAbaBabAbABaBAbABaBAbABaBabAbaBaBAbABaBAbABaBAbAbaBaBAbABaBAbA baBabAbaBaBAbA
$\mathrm{II.C}_{136}^{i.c.}(0.5)$	BAbABabaBabaBAbABAbaBAbabAbabABaBABabABab
$\mathrm{II.C}_{137}^{i.c.}(0.5)$	BabABabABabABAbABAbaBAbaBAbaBAbaBAbabAbaBABaBAB
$\mathrm{II.C}_{138}^{i.c.}(0.5)$	BAbAbabABabABabABAbAbabaBaBABabABabABabA
$\Pi.C_{139}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABabABabA
$\Pi.\mathrm{C}_{140}^{i.c.}(0.5)$	BabABabABAbABAbABAbaBAbaBAbaBAbaBAbaBABABABA

Table S. XLV. The free group elements for the periodic three-body orbits.

Class and number	free group element
$\overline{\mathrm{II.C}}_{141}^{i.c.}(0.5)$	BAbABabaBabaBAbaBABabABabABabABabABAbABA
$\mathrm{II.C}_{142}^{i.c.}(0.5)$	BabABAbaBAbaBAbabABabABAbABAbaBAbabABabAB
$\mathrm{II.C}_{143}^{i.c.}(0.5)$	BAbAbabABabABabABabABabABabABAbABabaBabABAbAbabaBabABabA
$ ext{II.C}_{144}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABAbaBAba
$ ext{II.C}^{i.c.}_{145}(0.5)$	BabABabABabABabABaBAbaBAbaBAbaBAbaBAbaBA
$\mathrm{II.C}_{146}^{i.c.}(0.5)$	BabAbaBAbaBAbaBabABabABabAbaBAbaBAbaBAba
$\mathrm{II.C}_{147}^{i.c.}(0.5)$	BAbAbabABabABabABabABabaBabABAbABabaBabABabA
$\Pi.C^{i.c.}_{148}(0.5)$	BAbABabaBabaBAbaBAbaBAbabABabABabABAbABA
$\Pi.C^{i.c.}_{149}(0.5)$	BAbABabaBabaBAbaBAbaBAbaBABaBABaBABabABab
$\mathrm{II.C}_{150}^{i.c.}(0.5)$	BabAbaBAbABabABabABaBAbaBAbaBAbaBAbaBAba
$\mathrm{II.C}_{151}^{i.c.}(0.5)$	BabABabABabABAbaBAbaBAbaBAbaBAbaBAbaBAba
$\mathrm{II.C}_{152}^{i.c.}(0.5)$	BabAbabABaBAbaBAbABAbaBabABAbABabaBaBABabAbaBABabABab
$\mathrm{II.C}_{153}^{i.c.}(0.5)$	BabABabABAbaBAbaBAbaBAbaBABabABabABabABa
$\mathrm{II.C}_{154}^{i.c.}(0.5)$	BAbAbaBABaBabaBAbABabaBaBABaBABaBaBaBABABABA
$\mathrm{II.C}_{155}^{i.c.}(0.5)$	BabABaBAbaBAbABabABabABabAbaBAbABabABaBAbaBAb
$\mathrm{II.C}_{156}^{i.c.}(0.5)$	BabaBaBABAbAbabABaBABaBABAbAbabaBaBABabABab
$\Pi.C_{157}^{i.c.}(0.5)$	BabABabABabABabaBAbaBAbaBAbaBAbaBAbaBAba
$\mathrm{II.C}_{158}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabaBabABAbABabaBabABabA
$\mathrm{II.C_{159}^{i.c.}(0.5)}$	BAbAbabABabABabABabABabABabABabABabABABABAB
$\mathrm{II.C}^{i.c.}_{160}(0.5)$	BabAbabABabABabABabABabABabABabABabABaBAbaBAb

Table S. XLVI. The free group elements for the periodic three-body orbits.

$\begin{array}{ccc} \Pi.C_{161}^{i.c.}(0.5) & B_{8} \\ B_{1}.C_{162}^{i.c.}(0.5) & B_{2}. \end{array}$	
	BabAbabABaBABabAbabABaBABabABaBABaBABaBA
	BAbABabaBabABAbABabaBabABAbABabaBabABAbABabaBabABAbABabaBabABabA
$\mathrm{II.C}_{163}^{i.c.}(0.5)$ $\mathrm{B}_{6}$	BabABabABabABabABaBABabAbaBAbaBAbaBAbaBA
II. $C_{164}^{i.c.}(0.5)$ Be	BabABabaBabaBABaBABaBABabABabaBAbaBAbaBA
$\text{II.C}_{165}^{i.c.}(0.5)$ Be	BabAbaBAbaBAbaBabABabABabABabABabABabABa
$\text{II.C}_{166}^{i.c.}(0.5) \qquad \begin{array}{c} \text{E.} \\ \text{B.} \\ \text{b.} \end{array}$	BAbABabaBabaBAbABAbaBAbabABabABabABabABa
$\operatorname{II.C}_{167}^{i.c.}(0.5)$ Be	BabaBaBABAbAbabABabABabABAbAbabaBaBABabABAbAbabaBaBABabABab
$\mathrm{II.C_{168}^{i.c.}(0.5)}$ $\mathrm{Be}$	BabABabABabABaBAbaBAbaBAbaBAbaBAbaBAbaBA
II. $C_{169}^{i.c.}(0.5)$ Be	
$\mathrm{II.C}_{170}^{i.c.}(0.5)$ $\mathrm{Be}$	BabaBabABabABaBABabABabaBABabABaBABaBABa
$II.C_{171}^{i.c.}(0.5)$ Be	BabABabABaBAbaBAbaBAbaBAbaBAbABabABabABa
II.C $^{i.c.}_{172}(0.5)$ Be	BabaBAbaBAbaBABabABabABabaBAbaBABabABabA
$\text{II.C}_{173}^{i.c.}(0.5)$ B <sub>6</sub>	BabABabABabABabABabABabABabABabABabABabA
$II.C_{174}^{i.c.}(0.5)$ Be	BabABabABaBAbaBAbABAbaBAbABabABabABabABa
$II.C_{175}^{i.c.}(0.5)$ Be	BabAbaBAbaBabABaBAbaBAbABabABaBAbaBAbABabAbaBAbaB
II. $C_{176}^{i.c.}(0.5)$ Be	BabABabABabABabABabABabABabABabABabABabA
$\operatorname{II.C}_{177}^{i.c.}(0.5)$ Be	BabABabABabABAbABabABabABabABabABabABabA
$\mathrm{II.C}^{i.c.}_{178}(0.5)$ Be	BabABabABabABabABabABabABabABabABabABabA
$II.C^{i.c.}_{179}(0.5)$ Ba	BabABabABabABabABabABabABabABabaBAbaBAba
$\text{II.C}_{180}^{i.c.}(0.5)$ B <sub>6</sub>	BabABaBAbaBAbaBabABabABabAbaBAbaBAbaBAbABabABa

Table S. XLVII. The free group elements for the periodic three-body orbits.

Class and number	free group element
$\Pi.\mathrm{C}^{i.c.}_{181}(0.5)$	BabABabaBabaBAbaBAbaBAbabABabABabABabABAbaBAba
$\mathrm{II.C}_{182}^{i.c.}(0.5)$	BAbAbabaBabaBAbaBABabABabABabABabABabABa
$\mathrm{II.C}_{183}^{i.c.}(0.5)$	BabAbaBAbaBabABabAbaBAbaBAbABabABabABaBAbABabABa
$\mathrm{II.C}^{i.c.}_{184}(0.5)$	BabaBabABAbABAbaBAbaBAbaBABabABabABabABa
$\mathrm{II.C}_{185}^{i.c.}(0.5)$	BabABabABabABabABabAbaBAbaBAbaBAbaBAbABabABa
$\mathrm{II.C}_{186}^{i.c.}(0.5)$	BAbAbabABaBABaBABaBABaBaBaBaBABABABABABA
$\mathrm{II.C}_{187}^{i.c.}(0.5)$	BabABabaBAbaBAbaBABabABabABabABAbaBAbaBA
$\mathrm{II.C}_{188}^{i.c.}(0.5)$	BabABabABabABAbaBAbaBAbaBAbaBAbaBAbaBABabABab
$\mathrm{II.C}_{189}^{i.c.}(0.5)$	BabABabAbaBAbaBAbaBAbABabABabABabAbaBAbaB
$\mathrm{II.C}_{190}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABabABabA
$\mathrm{II.C}_{191}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABabABabA
$\mathrm{II.C}^{i.c.}_{192}(0.5)$	BabAbaBAbaBAbABabABabAbaBAbaBAbABabABaBAbaBAb
$\Pi.\mathrm{C}^{i.c.}_{193}(0.5)$	BabABabABabABabABabAbaBAbaBAbaBAbaBAbaBA
$\mathrm{II.C}^{i.c.}_{194}(0.5)$	BAbAbabABabABabABabABabABabABabABabABAbABabABAbABabABa
$\mathrm{II.C}_{195}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABabABabA
$\mathrm{II.C}_{196}^{i.c.}(0.5)$	BabABabABabABabABabABabABAbaBAbaBAbaBAba
$\mathrm{II.C}_{197}^{i.c.}(0.5)$	BabAbaBAbaBAbaBAbABabABabABabABabABabABa
$\mathrm{II.C}^{i.c.}_{198}(0.5)$	BabABaBAbaBAbaBAbABabABabABabABabABabABa
$\mathrm{II.C}^{i.c.}_{199}(0.5)$	BabABabaBabaBAbaBAbabAbabABabABabABabABa
$\mathrm{II.C}_{200}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABabABabA

Table S. XLVIII. The free group elements for the periodic three-body orbits.

Class and number	free group element
$\overline{\mathrm{II.C}}_{201}^{i.c.}(0.5)$	BabaBabABAbaBABaBABabABabABabABabABabABa
$\Pi.\mathrm{C}^{i.c.}_{202}(0.5)$	BabAbaBAbaBAbaBAbBabABabAbaBAbaBAbABabABa
$\Pi.C^{i.c.}_{203}(0.5)$	BabABabABabAbaBAbaBAbaBAbaBAbaBAbaBAbaBA
$\mathrm{II.C}_{204}^{i.c.}(0.5)$	BabABabABabABabABabABaBAbaBAbaBAbaBAbaBA
$\mathrm{II.C}^{i.c.}_{205}(0.5)$	BabABabABabABaBAbaBAbaBAbaBAbaBAbaBAbaBA
$\mathrm{II.C}^{i.c.}_{206}(0.5)$	BabABabABabABabABabABabABabABabABabABabA
$\mathrm{II.C}^{i.c.}_{207}(0.5)$	BabAbabABaBABaBABaBABaBAbabAbaBAbaBAbaBA
$\mathrm{II.C}^{i.c.}_{208}(0.5)$	BabABabABAbABabaBabABABABABaBababAbaBABaBAB
$\mathrm{II.C}^{i.c.}_{209}(0.5)$	BabABabaBAbaBAbaBABabABabABabaBAbaBABabABab
$\mathrm{II.C}^{i.c.}_{210}(0.5)$	BabAbaBAbaBabABabABabABabABabABabABabAbaBAbaB
$\Pi.\mathrm{C}^{i.c.}_{211}(0.5)$	BabABabAbaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBA
$\Pi.\mathrm{C}^{i.c.}_{212}(0.5)$	BAbAbabABabABabABAbAbabaBaBABabABAbAbabaBaBABabABab
$\Pi.\mathrm{C}^{i.c.}_{213}(0.5)$	BabABabABabABabABabABaBABabAbabABabAbabABAbaBabABAbABabABa
$\Pi.\mathrm{C}_{214}^{i.c.}(0.5)$	BAbAbabABabABabABabABabABabABAbAbabaBaBABabABab
$\mathrm{II.C}^{i.c.}_{215}(0.5)$	BabABabABAbaBAbaBAbaBAbaBAbaBAbaBAbaBAba
$\mathrm{II.C}^{i.c.}_{216}(0.5)$	BabAbaBAbaBAbABabABabABabABabABabABabAbaBAbaB
$ ext{II.C}^{i.c.}_{217}(0.5)$	BAbAbabABabABabABabABabABabABabABabABabA
$\mathrm{II.C}_{218}^{i.c.}(0.5)$	BAbAbabABaBABabABabABaBABabABabABabABabA
$\Pi.\mathrm{C}^{i.c.}_{219}(0.5)$	BAbABabaBabaBAbABAbaBAbabABabABabABabABa
$\mathrm{II.C}_{220}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABabABabA

Table S. XLIX. The free group elements for the periodic three-body orbits.

Class and number	free oron element
$\overline{\Pi.C^{i.c.}_{221}(0.5)}$	BabABabABabABabABabABabAbaBAbaBAbaBAbaBA
$\mathrm{II.C}_{222}^{i.c.}(0.5)$	BabAbabABaBABaBABabABaBAbabABaBABaBAbaBAb
$\mathrm{II.C}_{223}^{i.c.}(0.5)$	BabAbaBAbaBAbaBAbaBAbABabABabABaBAbaBAbABabABa
$\mathrm{II.C}_{224}^{i.c.}(0.5)$	BabABabAbaBAbaBAbaBAbABabABabABabABaBAbaBAb
$\mathrm{II.C}_{225}^{i.c.}(0.5)$	BabABabABabABAbABAbaBAbaBAbaBAbaBAbaBAba
$\Pi.C_{226}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABabABabA
$\Pi.C_{227}^{i.c.}(0.5)$	BabAbaBAbaBAbABabABabABabAbaBAbaBAbaBAba
$\Pi.C^{i.c.}_{228}(0.5)$	BabABabABAbaBAbaBAbaBAbaBABabABabABabABa
$\Pi.C^{i.c.}_{229}(0.5)$	BabABabaBAbaBAbaBABabABabABabABabABabABa
$\Pi.\mathrm{C}_{230}^{i.c.}(0.5)$	BabABabABaBAbaBAbaBAbaBAbABabABabABabABa
$\Pi.C^{i.c.}_{231}(0.5)$	BabABabABabABAbABabaBabABAbABAbaBabaBAbABAbaBabaBAbaBA
$\mathrm{II.C}_{232}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabaBAbaBAba
$\mathrm{II.C}_{233}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABabABabA
$\mathrm{II.C}_{234}^{i.c.}(0.5)$	BAbAbabABaBABabAbaBABabaBAbABabaBABABABA
$\Pi.C^{i.c.}_{235}(0.5)$	BabAbaBAbaBAbaBAbABabABabAbaBAbaBabABabA
$\mathrm{II.C}_{236}^{i.c.}(0.5)$	BabABabaBabABAbABabaBabABAbABabaBabABAbABabaBabABAbABabaBabABAbABabaBabABAbABabaBabABabaBabABAbABabABa
$\Pi.C_{237}^{i.c.}(0.5)$	BabABabABabABabABabABabABabABabABabABabA
$\Pi.C_{238}^{i.c.}(0.5)$	BabABabaBAbaBAbaBABabABabABabABAbaBAbaBA
$\mathrm{II.C}_{239}^{i.c.}(0.5)$	BAbAbabABaBABaBABabABaBAbabAbaBABaBAbaBABABABA
$\Pi.\mathrm{C}_{240}^{i.c.}(0.5)$	BAbAbabABaBABABABABABABABABABABABABABABA

Table S. L. The free group elements for the periodic three-body orbits.

Class and number	ber free group element
$ ext{II.C}^{i.c.}_{241}(0.5)$	BabABabABabABabABAbaBAbaBAbaBAbaBAbaBAba
$\text{II.C}_{242}^{i.c.}(0.5)$	BabABabABabABabABabABabABAbABabABabABAbABabaBabABabA
$\mathrm{II.C}_{243}^{ic.}(0.5)$	BabABabaBAbaBAbaBABabABabABabABabaBAbaBA
$ ext{II.C}^{i.c.}_{244}(0.5)$	BabABabaBAbaBAbaBABabABabABabABAbaBAbaBA
$\mathrm{II.C}_{245}^{i.c.}(0.5)$	BabABabaBAbaBAbaBAbabABabABabABabABAbaBAba
$\Pi.C_{246}^{i.c.}(0.5)$	BabABabABabABabABabAbaBAbaBABabABabABabA
$\Pi.C_{247}^{i.c.}(0.5)$	BabABabABabABabABabABabABAbABabaBAbABAbA
$\Pi.C_{248}^{i.c.}(0.5)$	BabABabaBAbaBAbaBABabABabABabABabaBAbaBA
$\Pi.C_{249}^{i.c.}(0.5)$	BabAbabABaBABabAbabABaBABabAbabABaBABabABab
$\Pi.C_{250}^{i.c.}(0.5)$	BabABabaBabaBAbaBAbaBAbaBABaBABabABabABa
$\text{II.C}_{251}^{i.c.}(0.5)$	BAbAbabABabABabABabaBaBABaBABabABabABabaBABABAbAbabABabaBABABABA
$\Pi.C_{252}^{i.c.}(0.5)$	BAbAbabABaBABaBAbabAbaBABaBabaBAbABAbaBabaBa
$\Pi.C_{253}^{i.c.}(0.5)$	BAbAbabABabABabABabABabABabABabABabABabA
$\Pi.C_{254}^{i.c.}(0.5)$	BAbAbabABabABabABabABabABabABabABabABabA
$ ext{II.C}_{255}^{i.c.}(0.5)$	BAbABabaBabaBAbaBABabABabABabABabABAbaBAba

Table S. LI. The free group elements for the periodic three-body orbits.

Class and number	free group element
$\overline{\mathrm{I.A_1^{i.c.}}}(0.75)$	BabA
$\mathrm{I.A}_2^{i.c.}(0.75)$	BaBabAbA
$1.A_3^{i.c.}(0.75)$	BabaBABabABabA
$\Gamma.A_4^{i.c.}(0.75)$	BaBAbaBabAbaBAbA
${ m I.A}_5^{i.c.}(0.75)$	BabAbABabABaBabA
$I.A_{6}^{i.c.}(0.75)$	BAbAbabaBaBaBA
$I.A_7^{i.c.}(0.75)$	BabAbaBAbABabABaBAbA
$1.A_8^{i.c.}(0.75)$	BabaBAbaBABabABabAbabA
$1.A_9^{c.c.}(0.75)$	BabaBABabaBABabABAbabABAbabA BAbaBABabaBabababAbabAbaBA
$1.A_{10}^{i.c.}(0.75)$	DAUGDADGUGDAUGUDAUGDAUGDA Roha Raha Raha Roha Raha Raha Raha Raha
$1.A_{13}^{i.c.}(0.75)$	BAbabABAbabABAbabaBABabaBABabaBA
40,6	BabaBAbabABAbaBABabABabaBAbabA
	BabaBABabaBABabaBABabABAbabABAbabABAbabABAbabABAbabABAbabABABAbabABABABAB
$I.A_{15}^{i.c.}(0.75)$	BabABAbaBAbaBAbaBAbaBAbaBAbaBABabA
$I.A_{16}^{i.c.}(0.75)$	BaBAbaBabABabABabAbaBAbABabABabABabAbaBAbA
40	BabaBAbabABabABAbabABAbabABabaBAbabABabaBAbabA
$I.A_{18}^{i.c.}(0.75)$	BAbabABAbabABAbabaBABabaBABabaBABabaBA
	BababABABABABABABABABABABABABABABABABABA
	BAbaBABabABabaBAbabaBAbabABabABabABabABABABA
	BabABabaBAbaBABabABabABabABabABabABabABa
	BabaBAbaBABabABabaBABabABAbabABabABabABa
	BabaBABabaBABabaBABabABAbabABAbabABAbabABAbabABAbabA
	BabaBABabABabABabaBABabaBABabaBABabaBABabaBABabABABabABABabAB
	BabABabaBAbaBAbaBABabABabABabABabABabAbaBAbaB
	BAbabABAbabABAbabABAbabABAbabBABabaBABabaBABabaBABabaBABabaBA
	BabABAbaBAbabABabaBAbabABabABabaBAbaBABabABab
	BabaBABabaBAbabABAbaBABabABAbabABAbabABAbaBABabaBABababABAbabA Edit Sidesi Edit Edit Edit Edit Sidesi Edit Edit Edit Edit Edit Edit Edit Edi
	BabABabABbaBAbaBAbaBAbabABabABabABabABab
	BabaBABABABBABABABABABABABABABABABABABAB
	BabaBABabaBABabaBABabaBABabABABabABAbabABAbabABABabABAbabABAbabABABabABABABAB
	BabaBAbabABabaBABabABabABABabABABabABABabABAbaBABabABab
$1.A_{33}^{i.c.}(0.75)$	BAbaBABababABabaBABabABABabaBABabaBABabABABABAB
$1.A_{34}^{\prime} (0.75)$ $1.A^{i.c.(0.75)}$	DabADADADADADADADADADADADADADADADADADADA
1.435(0.79)	Dabab Abab A BAbab A Babab Ababa B Ababa B A Baba B A B Abab A B Abab A B Abab A B Abab A B Ababa B A Baba B A
	BabaBABabaBAbabABAbabABAbaBABabaBABabABAbabABAbaBABabaBABabaBABabaBAbabABAbabA
	BabaBABabaBABabaBABabaBABabABabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABABAbabABABABAB
	BA bab ABA bab ABA bab ABA bab ABA bab ABA bab ABA bab BAB Baba BABaba BABaba BABaba BABaba BABaba BABaba BABA bab ABABA bab ABABA bab ABABABA BABABA BABABA BABABA BABABABAB
$I.A_{40}^{i.c.}(0.75)$	BabaBAbabABabaBAbabABabABAbaBABabABABaBABABABA
$I.A_{41}^{i.c.}(0.75)$	BabaBABabABAbabABabABAbabABabaBABabABabA
$I.A_{42}^{v.c.}(0.75)$	BabABAbaBAbaBABabABabABabABAbaBAbaBAbabABabAB
	BabaBAbabABAbaBABabABAbabABabaBAbabABAbaBABabABAbaBABabABAbaBABabaBABabABAbaBABabABAbaBABabABAbaBABAbabA
$1.A_{44}^{i.c.}(0.75)$	BabaBABabaBABabaBABabaBABabaBABabaBABabABABabABABabABABabABAbabABAbabABAbabABAbabABAbabABAbabA
$1.A_{45}^{i.c.}(0.75)$	BabaBABabABAbabABAbaBABabaBAbabABAbabABabAB
$1.A_{46}^{(0.75)}$ $1.A_{i.c.(0.75)}$	- Babab Abab Abab Abab Abab Abab Abab Aba
$1.A_{47} (0.73)$ $1.A_{i.c.}(0.75)$	DababADADabaDADabaDADabADADADADADADADADA
1.748 (0.79) $1.A^{i.c.}(0.75)$	BahABahaBAhaBAhabABahABahABahABAhaBAhaBAhaBABahABahABahAB
$I.A_{50}^{i.c.}(0.75)$	BabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabA

Table S. LII. The free group elements for the periodic three-body orbits.

Class and number	her free groin element
$1.A_{51}^{i.c.}(0.75)$	BabaB/
$I.A_{52}^{i.c.}(0.75)$	BabABAbaBAbaBABabABabABabABabABabABabABa
$1.A_{5.2}^{i.c.}(0.75)$	BabaBAbabABabaBAbabABabaBAbabABabABAbaBABabABAbaBABabABAbaBABabABAbaBABabABAbabABabaBAbabABabaBAbabA
$I.A_{54}^{i.c.}(0.75)$	BAbaBABabaBAbabaBAbabABabaBABabABABaBABa
$I.A_{55}^{i.c.}(0.75)$	BabaBABabaBABabaBABabaBABabABAbabABAbaBABabaBABabaBABabABAbabABAbabABabaBABabaBABabABABabaBABabaBABabABAbabA
$1.A_{56}^{i.c.}(0.75)$	BAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABABABA
$I.A_{57}^{i.c.}\left(0.75 ight)$	BabaBABabABabABabABabABabABabABabABabABa
$I.A_{58}^{i.c.}(0.75)$	baba BabaBAbaBABabABabaBAbaBABabABabaBAbaBABabABab
(0.76)	babA Boroe Raboro Raboro Rabora Rabora Rabora Rabora Raboro Rabora Rabora Rabora Rabora Raboro Raboro Raboro Rabora Rabor
1.759 (0.19)	
$I.A_{60}^{i.c.}(0.75)$	BA babABA BabaBABabaBAB abaBABA babABA babABABA babABA b
$I.A_{61}^{i.c.}(0.75)$	BAABAABAABAABAABAABABABABABABABABABABA
4 i.C. (0.11	
$1.A_{62}^{i}(0.75)$	БараБАБараБАБараБАБараБАБараБАБараБАБараБАБараБАБараБАБараБАБарАБАБарАБАБарАБАБарАБАБарАБАБарАБАБарАБАБар БАВАраЬА
$I.A_{63}^{ic.}(0.75)$	BabaBABAbaBAbaBABabaBABabaBABabaBABabaBABabaBABabaBAbabABABabABAbabABAbabABAbabABAbabABAbabABAbabABAba
$1.A_{64}^{i.c.}(0.75)$	BAbaBABabABabaABabaBABabaBABabaBABabaBABabABABABAB
I A i.c. (0 7E)	Lok A D A Lok A D A Lok A D A L
$1.A_{65}^{-}(0.75)$	БАрарАБАБараБАБАрарАБАрарАБАрарАБАрарАБАрарАБАрарАБАрарАБАрараБАБараБАБараБАБараБАБараБАБараБАБараБАБараБА ВараВАВараВА
$I.A_{66}^{i.c.}(0.75)$	BabaBABabABAbabABAbabABAbabABAbabABAbaBABabaBABabABab
	BAbaBABabABabA
${ m I.A}_{67}^{i.c.}(0.75)$	BAbabABAbabABAbabABABabaBABabaBABabaBABababABAbabABAbabABAbabBABabaBABabaBABabaBABabaBABabaBABAbabABABABAB
$I.A_{68}^{i.c.}(0.75)$	BAbaBABababABabaBABabABABabABAbabABababABabaBABABABA
	babaBababABAbaBA
$1.A_{69}^{ic.}(0.75)$	BababABAbabABAbabaBAbabaBABabaBABabaBABAbaBABAbabABAbabABABABAB
$\text{I.A}_{70}^{i.c.}(0.75)$	BabaBABabABAbabABAbaBABabaBABabaBABabaBABabaBABabABAbabABAbabABabAB
1	baBABabaBABabABAbabA
$I.A_{71}^{v.c.}(0.75)$	BabaBAbabABabaBAbabABAbaBABabABAbabABabaBAbabABAbaBABabABAbaBABabABab
$\text{I.A}_{72}^{i.c.}(0.75)$	BAbaBABababABabaBABAbaBABAbabABAbabABAbabABAbabaBAbabaBABabABABabaBAbabABABabABABabABAbabaBABABABA
1 1 2 2	
$1.A_{73}^{-1}(0.75)$	БараБАБараБАБарАБАраБАБараБАБараБАБараБАБараБАБараБАБараБАБараБАБараБАБарАБАрарАБараБАБараБАБараБАБарАБАБА аВАВараВАВарАВАрарАВАрарА
$I.A_{74}^{i.c.}(0.75)$	BabaBAbabABabABabABAbaBABabABabABabaBAbabABabaBAbabABabaBAbabABABabABABabABab
	AbaBABabABAbaBABabaBAbabA
$ ext{I.A}_{75}^{i.c.}(0.75)$	BAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABABabaBABABABA

Table S. LIII. The free group elements for the periodic three-body orbits.

Class and number	her free group element
$I.A_{76}^{i.c.}(0.75)$	BabaB/ A Baba
$\mathrm{I.A}_{77}^{i.c.}(0.75)$	BabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabABAbabABABAbabABABABAB
$I.A_{78}^{i.c.}(0.75)$	BabaBAbabABAbaBABabABAbabABabaBAbabABAbaBABabABAbabABabaBAbabABAbaBAba
$I.A_{79}^{i.c.}(0.75)$	BAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABABabaBABABABA
$I.A_{80}^{i.c.}(0.75)$	BAbaBABabaABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabABABabaBAbabABABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABABABAB
$I.A_{81}^{i.c.}(0.75)$	BabaBABabaBABabABAbabABAbabABAbaBABabaBABabaBABabaBABAbabABAbabABAbabABabaBABabaBABabABAbabABAbabABabaBABabaBAB abaBAbabABAbabABAbaBABabaBABabaBABabABAbabABAbabABAbabA
$I.A_{82}^{i.c.}(0.75)$	BAbabABabaABAbabABAbabaBABabaBABabaBABabaBABabaBABABABA
$I.A_{83}^{i.c.}(0.75)$	BabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabABAbabABABAbabABAbabABAbabABAbabABAbabABAbabABABABAB
$I.A_{84}^{i.c.}(0.75)$	BabaBABabABAbabABabaBAbabABAbabABabaBAbabABAbaBABABABA
$I.A_{85}^{i.c.}(0.75)$	BAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABABabaBABABabaBABABABA
$I.A_{86}^{i.c.}(0.75)$	BabaBABabABAbabABAbaBABabaBAbabABAbabABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabABABabABABabaBABABabaBABabaBABabaBABabaBABabaBABabaBABABABA
$I.A_{87}^{i.c.}(0.75)$	BAbaBABababABabaBABababABababABabaBABABABA
$I.A_{88}^{i.c.}(0.75)$	BAbaBABababABabaBABabaBABabaBABabaBABAbaBABabaBAbabABabaBABabABABabaBAbabABABabaBABabaBAbabABABAbabABABABAB
$I.A_{89}^{i.c.}(0.75)$	BAbabABAbabABAbabABABabaBABabaBABababABAbabABAbabABAbabaBABabaBABabaBABAbabABAbabABAbabABAbabABABabaBABabaBABabaBABababABAbabABAbabABABabaBABababABAbabABAbabaBABabaBABababABAbabABABabaBABababABAbabABABABAB
$I.A_{90}^{i.c.}(0.75)$	BAbaBABabaBAbabaBABabABABABABABababABabaBABabaBABABABA
$I.A_{91}^{i.c.}(0.75)$	BAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabBABabaBABABabaBABABABA
$I.A_{92}^{i.c.}(0.75)$	BabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBAbabABAbabABAbabABAbabABAbabABAbabABabaBABabaBABabaBABabABABabaBABABABA
$I.A_{93}^{i.c.}(0.75)$	BAbabABAbabABAbabABAbabABAbabABABabaBABabaBABabaBABabaBABabaBABababABAbabABAbabABAbabABAbabABAbabABAbabABABabaBABABabaBABABABA
$I.A_{94}^{i.c.}(0.75)$	BAbaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABABABA
$I.A_{95}^{i.c.}(0.75)$	BabaBABabaBABabABAbaBABabaBABabaBABabaBAbabABAbabABAbaBABabaBABabaBABabaBABabABAbabABabaBABabaBABabaBABabABABABAB
$I.A_{96}^{i.c.}(0.75)$	BAbaBABabaBABABabaBABABabaBABABABA
$I.A_{97}^{i.c.}(0.75)$	BabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabABAbabABAbabABAbabABABabaBABabaBABabABABABAB

Table S. LIV. The free group elements for the periodic three-body orbits.

Class and number	her free oron element
$\overline{\text{I.A}_{98}^{i.c.}(0.75)}$	BAbab <sub>2</sub>
$I.A_{99}^{i.c.}(0.75)$	BAbabABAbabABAbabABAbabABAbabABAbabABABabaBABabaBABabaBABabaBABabaBABABABA
$I.A_{100}^{i.c.}(0.75)$	BAbaBABabaABabaBABabaBABabaBABabaBABabaBABabaBABabaBABABABA
$I.A_{101}^{ic.}(0.75)$	BAbaBABabaABabaBABabABABabABABababABababABAbaBABABabaBABabaBAbabABAbabABAbabABAbabABabaBABabaBABabABABABAB
$ ext{I.A}_{102}^{i.c.}(0.75)$	BAbaBABababABabaBABAbabaBAbabaBABabABABabABAbabaBAbabABAbabABAbabABabaBABabaBABabaBABAbabABABabaBABabaBABABABA
$1.B_1^{i.c.}(0.75)$ $1.B_2^{i.c.}(0.75)$	BabaBAbabA BabAbaBAbaBabA
$1.B_3^{i.c.}(0.75)$	BAbABABABABABABA
$1.B_i^{i.c.}(0.75) \ 1.B_5^{i.c.}(0.75)$	BabaBabaBAbabAbabA BabaBabaBAbabAbabA
$\mathrm{I.B}^{i.c.}_{6}(0.75)$	BabAbAbaBAbaBabA
$1.B_7^{c.c.}(0.75)$ 1 $B_6^{c.c.}(0.75)$	BabABaBAbaBAbABabA BahaBABahaBAhahABAhahA
$1.B_9^{i.c.}(0.75)$	BabABAbABAbaBAbaBABabA
$1.B_{10}^{i.c.}(0.75)$	BabABabaBAbaBAbabABabA Bok ABok A Bako Bako Bako Bako Bako Abok Abok A
$I.B_{12}^{i.c.}(0.75)$	
$1.B_{13}^{i.c.}(0.75)$	BabaBABabaBABabaBABabABAbabA
$1.B_{14}^{i.c.}(0.75)$	BabaBAbaBABabABabABabABabABAbabA BAbabABAbabABAbabABABabaBABabaBA
$I.B_{16}^{i.c.}(0.75)$	BabABabaBabaBAbaBAbaBAbabAbabAbabABabA
$1.B_{17}^{i.c.}(0.75) \ 1.B_{17}^{i.c.}(0.75)$	BabaBABabABAbaBABabaBAbabABAbabABAbabABAbabA RAbaRA RababABaba RA RababA RA baba Rababa
$I.B_{19}^{i.c.}(0.75)$	BabaBABabaBABabaBAbabABABAbabABABABAB
$1.B_{20}^{i.c.}(0.75)$	BabABAbaBAbabABabABabaBABabaBABabaBABabA Palitabalitabalitabalitabalitaban bababababababababababababababababab
$1.\mathbf{B}_{21}^{i.c.}(0.75) \ 1.\mathbf{B}_{32}^{i.c.}(0.75)$	BAbabABabABabABAbaBABabABABABABABABABABA
$I.B_{23}^{i.c.}(0.75)$	BabaBABabaBABabaBABabaBABabABAbabABAbabABAbabABAbabABAbabABABABAB
$1.B_{24}^{i.c.}(0.75)$	BabaBAbaBABabABAbaBABabABabaBAbabABabABAbaBABAbaBAba
$1.B_{25}^{i.c.}(0.75)$	Dababababababababababababababababababab
$1.B_{27}^{i.c.}(0.75)$	BabABAbaBABaBABabABabABAbaBABabABabABAbABA
$1.B_{28}^{i.c.}(0.75)$ 1 $R_{2.c.}^{i.c.}(0.75)$	BabABAbaBAbaBAbabABabABabABAbaBABabABabA
$1.B_{20}^{i.c.}(0.75)$ $1.B_{30}^{i.c.}(0.75)$	BAbaBABabaBAbabaBABabaBABabABABABABABABA

Table S. LV. The free group elements for the periodic three-body orbits.

Class and number	oer free group element
$I.B_{31}^{i.c.}(0.75)$	BabaB/
$1.B_{32}^{i.c.}(0.75)$	BabABabaBAbaBAbaBABabABabABabABabABabABa
$1. m B_{33}^{i.c.}(0.75)$	BabABAbABAbaBABabABabABAbABAbaBAbaBABaBAB
$1. m B_{34}^{i.c.}(0.75)$	BabaBABabaBABabaBABabaBABabaBABabaBABabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABABABAB
$1.\mathrm{B}_{35}^{i.c.}(0.75)$	BabABAbaBAbabABabABabABabABabABabABabABa
$1.\mathrm{B}_{36}^{i.c.}(0.75)$	BabaBABabABAbabABAbaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabABABabABABabABABabABABABAB
$1.B_{37}^{i.c.}(0.75)$	BAbabABAbabABAbabABAbabABAbabABABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABAbabABAbabABAbabABABABAB
$1.B_{38}^{i.c.}(0.75)$	BababABAbabABAbabABAbabABAbabABAbabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaABABabaABABabaABABABAB
$1.B_{39}^{i.c.}(0.75)$	BabaBAbabABabABabABabABabABabABabABabABa
$1.B_{40}^{i.c.}(0.75)$	BabaBABabABAbabABAbaBABabaBABabaBABabaBABabABAbabABAbaBABabaBABabABAbabABAbabABAbabABAbabABABabABABABAB
$1.B_{41}^{i.c.}(0.75)$	BAbaBABabaABABABABABABABabaBABabABABabABABabABABabABABabABABaBABABABA
$I.B_{42}^{i.c.}(0.75)$	BabABAbaBAbaBAbaBAbaBAbaBAbaBAbaBAbaBABabABab
$I.B_{43}^{i.c.}(0.75)$	BabaBABAbaBABabaBABabaBABabaBABabaBABabaBAbabABABabABABabABAbabABAbabABAbabABAbaBABAbaBABAbaBABAbabA
$1.B_{44}^{i.c.}(0.75)$	BabaBAbabABAbaBABabABAbabABAbabABabaBAbabABabaBABabABABabABABABAB
$1.B_{45}^{i.c.}(0.75)$	BabaBABabaBABabaBABabaBABabaBABabaBABabaBAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabA
$\mathrm{I.B_{46}^{i.c.}}(0.75)$	BabaBABabABAbaBABabABabABabABabABabABabA
$1.B_{47}^{i.c.}(0.75)$	BAbaBABabABAbabABabaBAbabABAbaBABABABABA
$1.\mathrm{B}_{48}^{i.c.}(0.75)$	BabaBABabABAbabABAbaBAbaBABabaBABabaBABabaBABabaBABabABABABAB
$I.B_{49}^{i.c.}(0.75)$	BabABAbaBABabABAbABAbABAbABAbaBABabABabA
$1.B_{50}^{i.c.}(0.75)$	BabaBABabaBABabABAbabABAbabABAbaBABabaBABabaBABabaBAbabABAbabABAbaBABabaBABabABAbabABABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABABABAB
$1.B_{51}^{ic.}(0.75)$	BabaBABabABAbaBABabaBABabABabABABabABABabaBABabABAbabABAbabABAbabABabAB
$1.B_{52}^{ic.}(0.75)$	BabaBABabABabABabABabABabABabABabABabABa
$1.B_{53}^{ic.}(0.75)$	BabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABABABAB
$1.B_{54}^{i.c.}(0.75)$	BabaBAbabABAbaBABabABAbaBABabABAbaBAbaBA
$I.B_{55}^{i.c.}(0.75)$	BabaBABabaBABabABABabaBABabABABabABABabaBABabaBABabaBABabABABabABABabABABABAB
$\mathrm{I.B}_{56}^{i.c.}(0.75)$	BAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABABABA
$\mathrm{I.B}_{57}^{i.c.}(0.75)$	BabaBABabABAbaBABabABAbaBABabABAbaBABabABAbaBABabaBAbaBA
$I.B_{58}^{i.c.}(0.75)$	BabABAbaBAbabABabABabABabABabABAbaBAbabABabAB
$I.B_{59}^{ic.}(0.75)$	BabaBABabaBABabABAbabABAbabABAbabABAbaBABabaBABabaBAbabABAbabABAbabABAbaBABabaBABabaBABabaBABabABAbabABAbabABAbabABABABAB
$I.B_{60}^{ic.}(0.75)$	BAbaBABabaBAbabABABaBABABabABabABabaBABabaBABabABABABAB
$I.B_{61}^{ic.}(0.75)$	BabaBAbabABabaBAbabABabaBAbabABabaBAbabABabaBAbabABabaBAbabABabaBAbabABabaBAbabABAbabABabaBAbabABABABAB
$1.\mathrm{B}^{ic.}_{62}(0.75)$	BAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABABabaBABabaBABabaBABababABAbabABAbabABAbabABAbabABABabaBABabaBABABABA
$1.\mathrm{B}^{ic.}_{63}(0.75)$	BabaBABabABAbabABabaBAbabABabaBAbabABabaBAbabABAbaBABabaBABabaBABabABABabaBABabaBABabaBABabaBABabABABABAB
$1.\mathrm{B}^{i.c.}_{64}(0.75)$	BabaBabaBAbabAbabABabABabaBAbaBAbabABabAB
$1.\mathrm{B}^{i.c.}_{65}(0.75)$	BabaBABabABabABabABabaBAbabABabABabABabA
$I.B_{66}^{i.c.}(0.75)$	BabaBABabABAbabABAbaBABabABAbaBABabABAbaBABAbaBABAbaBABabABAbaBABabABAbaBABabABAbaBABABABA
$1.\mathrm{B}^{i.c.}_{67}(0.75)$	BabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABABABAB
$I.B_{68}^{i.c.}(0.75)$	BAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABABabABABABAB
	$_{ m baBA}$
$\mathrm{I.B}^{i.c.}_{69}(0.75)$	BababABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabaBABabABABabABABabaBABabaBABabaBABabaBABABABA
	babA
$I.B_{70}^{i.c.}(0.75)$	BAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABABaBABABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABa boba
	DaBA

Table S. LVI. The free group elements for the periodic three-body orbits.

Class and number	er free ordin element
$\overline{1.B_{71}^{i.c.}(0.75)}$	BabAB.
$I.B_{72}^{i.c.}(0.75)$	abAbabababa BabaBabaBABabaBABabaBABabABAbabABAbabABAbaBABabaBABabaBABabaBAbabABAbabABAbabABAbaBABabaBABabABABABAB
$I.B_{73}^{i.c.}(0.75)$	AbabABAbabA BAbaBABababABabaBABAbaBABababAbabaBABabABABABAB
$I.B_{74}^{i.c.}(0.75)$	abarbabanganan BabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBAbabABAbabABAbabABAbabABAbabABAbabABAbabABAb aba Rababa Rababa
$I.B_{75}^{i.c.}(0.75)$	abananananan BabaBAbabABabaBAbabABabaBAbabABabaBAbabABabaBAbabABabaBAbabABabaBAbabABabaBAbabABabaBAbabABabaBABabaBAbabAB abaBAbabABabaBAbabA
$I.B_{76}^{i.c.}(0.75)$	BabbaBABabABaBABaBABaBABabABabABabABabAB
$I.B_{77}^{i.c.}(0.75)$	ADADADADADADADADADA BabaBABabaBABabABAbabABAbaBABabaBABabABAbabABAbaBABabaBABabaBAbabABAbabABAbaBABabABABabABAbabABAbaBABab SPABSAABABAABAABAA
$I.B_{78}^{i.c.}(0.75)$	ababababababababa BAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABABabaBABabaBABabaBABabaBABabaBABabaBABabaB ABabaBABabaBABabaBA
$I.B_{79}^{i.c.}(0.75)$	BabaBABabABAbaBABabaBAbabABabaBABabaBABabaBABabABab
$I.B_{80}^{i.c.}(0.75)$	abababababababababababababababABabaBAbabABAbaBABabaBABabaBABabaBAbabABAbabABAbabABAbabABAbaBABabaBABabaBABabaB BabaBABabaBABabaBABabaBABabaBAbabA A Raba RababaRababa Rababa
$I.B_{81}^{i.c.}(0.75)$	ADSOLITION OF THE STANDARD OF
$I.B_{82}^{i.c.}(0.75)$	BARABABABABABABABABABABABABABABABABABAB
$I.B_{83}^{i.c.}(0.75)$	and the first of t
$I.B_{84}^{i.c.}(0.75)$	Ababah Babab
$I.B_{85}^{i.c.}(0.75)$	ABababABabaBABabaBABabaBABabaBABabaBABabaBABabaBABababABAbabABAbabABAbabABABabaBABabaBABabaBABababABAbabABAbab BAbabABAbabABAbabABAbabaBABabaBABabaBABabaBABababABAbabABAbabABAbabABABabaBABabaBABabaBABabaBABABABA
$1. m B_{86}^{i.c.}(0.75)$	abABAbabABAbabABabaBABabaBABabaBABabaBAbabABAbaBABabaBAbabABAbaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABab BabaBABabABAbabABabaBABabABAbabABabaBAbabABabAB
(0.75)	abABabaBABabABabaBABabaBABabABAbabA BohohA BAhoha BAhoha BABahon A ABahon BABahon BABAhohA BahoohABAhoha BAhoha BAhoha BABoha BABoha ABahon BABAhA
1.D87 (0.19)	bababababababababababababababababababa
$I.B_{88}^{i.c.}(0.75)$	BabaBABabaBAbabABAbaBABabABAbabABAbabABAbaBABabaBABabABAbabABabaBABabaBABabaBABabABab
$I.B_{89}^{i.c.}(0.75)$	BabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBAbabABABAbabABABABAB
$I.B_{90}^{i.c.}(0.75)$	abanangananganangananganangananganangan
$1.B_{91}^{i.c.}(0.75)$	Ababab Ababab Ababab Ababab Ababab Ababab Ababab Abab Abab ABAbab ABAbab ABabaB Abab ABAbab ABAbab ABAbab ABabaB ABabaB Abab ABaba BAbab ABAbab ABABABABAB ABABABABABABABABABABABABABA
I Di.c. (0.7E)	abABAbabABAbabABabaBABabaBABabABAbabABAbabA PALEDA DELE AADA BELEDA DELEAADALEAADALEAADALEAADALEADA LEBA DELEDA DELEDA DELEDA DELA DALEAADALEAADALEAADALEA
1.D <sub>92</sub> (U.73)	DAD&DAD&DADADADADADADADADADADADADADADAD
$1.\mathrm{B}_{93}^{i.c.}(0.75)$	BabaBABabaBABabaBABabABAbabABAbabABAbabABAbabABAbaBABabaBABabaBABabaBABabaBABabaBABabaBAbabABAbabABAbaBABAbaBABABABA
$I.B_{94}^{i.c.}(0.75)$	Ababa Baba Baba Baba Baba BABaba BABABABABA GOO A BABABA BABABA BABABA BABABA BABABABA
$1.\mathrm{B}_{95}^{i.c.}(0.75)$	aba bababababababababababababababababab

Table S. LVII. The free group elements for the periodic three-body orbits.

Class and number	r free oronn element
$\overline{1.B_{96}^{i.c.}(0.75)}$	BabAB. aBABa]
$I.B_{97}^{i.c.}(0.75)$	BabaBABabaBABabaBAbabABAbabABAbaBABabaBABabaBABabABAbabABAbabABabaBABabaBABabaBABabaBAbabABAbabABAbabABABABAB
$I.B_{98}^{i.c.}(0.75)$	BabaBABabABAbaBABabaBAbabABAbaBABabaBAbabABabAB
$I.B_{99}^{i.c.}(0.75)$	BAbaBABabaBAbabABABABABABABABABababAbabaBABABABA
$\mathrm{I.B}_{100}^{i.c.}(0.75)$	BababABAbaBABabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABABabABABabABABabABABabaBABabaBABabaBABaba BABahara Rahara ABahara ABaha
$\mathrm{I.B}_{101}^{i.c.}(0.75)$	DADABABABABABABABABABABABABABABABABABAB
$I.B_{102}^{i.c.}(0.75)$	DADADADADADADADADADADADADADADADADADADA
$\mathrm{I.B}_{103}^{i.c.}(0.75)$	BAbaBABababABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabABABabABABabABABABAB
$I.B_{104}^{i.c.}(0.75)$	BabaBABabaBAbabABAbaBABabaBABabABAbabABAbabABABabaBABabABAbabABAbabABAbabABabaBABabaBABabaBAbabABAbabABABabaBABABABA
$\mathrm{I.B}^{i.c.}_{105}(0.75)$	BAbabABAbabABABabaBABabaBABAbabABAbabABAbabABABabABABabaBABABABA
$\mathrm{I.B}^{i.c.}_{106}(0.75)$	BAbabABababABAbaBABAbabABababABAbabaBABAbabaBababABABABAB
$\mathrm{I.B}_{107}^{i.c.}(0.75)$	BAbabABababABAbabaBAbabaBABabABABabaBABabaBABABABabaBABababABababABAbabABAbabABAbabABAbabABAbabABABabaBABabaBABabaBABabaBABabaBABababABABABAB
$\mathrm{I.B}^{i.c.}_{108}(0.75)$	BAbaBABababABabaBABabaBABabaBAbabaBABaBAB
$\mathrm{I.B}_{109}^{i.c.}(0.75)$	BabaBABabaBABabABAbabABAbaBABabaBABabaBAbabABAbabABAbabABabaBABabABABabABAbabABAbabABabaBABabaBABabaBABabABABA baBABabaBABabaBABabABAbabABabaBABabaBABabaBABabaBABAbabABAbaBABabaBABabABAbabABAbabABAbabA
$\mathrm{I.B}_{110}^{i.c.}(0.75)$	BabaBABabABabaBABabaBABabaBABabaBABabaBABabaBABABABA
$I.B_{111}^{i.c.}(0.75)$	babaBABabABabaBABabaBABabaBABabABabABAbabABAbabABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabABABAB BabaBABabABAbabABabaBABabABAbabABABabABAbabABAbabABabAB
$\mathrm{I.B}_{112}^{i.c.}(0.75)$	BAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABABabaBABabaBABaba RABabaBA BabaBA BabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABABABA
$\mathrm{LB}_{113}^{i.c.}(0.75)$	DADabaharanan dabaharan dabaharan dabaharan dabaharan dabaharan dabaharan dabaharan dabaharan dabaharan dabahar BAbaBABabaBABabaBABabaBABabABAbabABAbabABAbabABAbabABAbabABAbabABABABAB
$1.\mathrm{B}_{114}^{i.c.}(0.75)$	BADARDAR BADAR BAD
$I.B_{115}^{i.c.}(0.75)$	DababAdadAbadAbadabAdadABadABAdadABAdadaBadadABAdabABAdadABadaBAdaBABABABABABABABABABABABAB
$\mathrm{LB}_{116}^{i.c.}(0.75)$	BAbaBABababABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBAbabaBABabABA RabaRAbabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBAbabaBABabaBABABABA
$I.B_{117}^{i.c.}(0.75)$	DabABABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABab BabaBABAbabABABABAB
$I.B_{118}^{i.c.}(0.75)$	BAbabABababABAbabABAbabaBABabaBABabaBABabaBABAbabABAbabABAbabABAbabABABabaBABabaBABabaBABababABababABABABAB
$I.B_{119}^{i.c.}(0.75)$	babABAbabaBAbabaBABabaBABabaBABabaBABABabaBABABabaBABABababABAbabABAbabABAbabABABabaBABABABA

Table S. LVIII. The free group elements for the periodic three-body orbits.

	Class and number	ber free group element
	$\mathrm{I.C}_1^{i.c.}(0.75)$	BAbabABAbabABAbabABAbabABAbabAlbabABAbabAl
	$II.A_1^{i.c.}(0.75)$	BabABaBAbaBabABabAbaBAbABabA
	$\Pi.A_2^{i.c.}(0.75)$	BabaBAbabABAbaBABabABabaBAbabABabaBAbabABabAB
		abaBABabABAbaBABabaBAbabABabaBABabaBABabABABABAB
	$\mathrm{II.B_1^{i.c.}(0.75)}$	BaBAbaBabABaBAbABabAbaBAbA
	$\text{II.B}_{2}^{i.c.}(0.75)$	BabABabAbaBAbaBAbaBabABabA
	$\text{II.B}_{3}^{i.c.}(0.75)$	BaBAbaBAbABabABaBAbABaBAbaBAbA
	$\text{II.B}_{4}^{i.c.}(0.75)$	BaBAbaBabAbaBabABaBAbABabAbaBabAbaBAbA
	$\text{II.B}_5^{i.c.}(0.75)$	BabABAbABAbABAbABAbABAbABAbABAbABAbABAbA
	$\text{II.B}_{6}^{i.c.}(0.75)$	BabABabaBAbaBAbaBAbabABabABabABabABabaBAbaBA
	$\mathrm{II.B}_7^{i.c.}(0.75)$	BabABAbaBAbaBAbaBABabABabABAbaBABaBABabABab
		aBABabA
	$\text{II.B}_8^{i\cdot c\cdot}(0.75)$	BabABAbaBAbaBABabABAbABABABABABABABABABA
		A Ba
	$\text{II.C}_{1}^{i.c.}(0.75)$	BaBaBAbaBAbAbA
	$II.C_2^{i.c.}(0.75)$	BaBAbaBAbaA
	$II.C_3^{i.c.}(0.75)$	BabAbaBAbaBAbaBabA
	$II.C_4^{i.c.}(0.75)$	BaBabababaBabababa
	$II.C_5^{i.c.}(0.75)$	BaBAbaBabABabAbaBabAba
	$\Pi.C_6^{i.c.}(0.75)$	BAbABabABabABabABaBA
	$\text{II.C}_7^{i.c.}(0.75)$	BaBABaBABABABAbA
	$II.C_8^{i.c.}(0.75)$	BAbABAbABabABaBABaBA
	$\mathrm{II.C_9^{i.c.}}(0.75)$	BAbABABabaBAbabABABaBA
	$\Pi.\mathrm{C}^{i.c.}_{10}(0.75)$	BaBAbaBabAbaBabAbaBAbA
	$II.C_{11}^{ic.}(0.75)$	BabABAbABABABABABABABA
	$\Pi.C_{12}^{i.c.}(0.75)$	BabAbABabAbaBabABaBabA
	$\mathrm{II.C_{13}^{ic.}(0.75)}$	BaBaBAbAbAbaBaBaBAbAbA
	$\Pi.C_{14}^{i.c.}(0.75)$	BaBAbaBAbABabABaBAbA
	$\Pi.\mathrm{C}_{15}^{i.c.}(0.75)$	BabABaBABabABabABabABabA
	$\Pi.C_{16}^{i.c.}(0.75)$	BaBAbABaBAbaBabAbABaBAbA
	$\text{II.C}_{17}^{i.c.}(0.75)$	BAbAbABababABababABababABaBA
	$II.C_{1.8}^{ic.}(0.75)$	BabABabaBabaBAbaBAbabAbabABabA
	$\mathrm{II.C}_{19}^{i.c.}(0.75)$	BaBAbaBabABaBAbABabAbaBAbA
	$\text{II.C}_{20}^{ic.}(0.75)$	BAbABAbABABaBABaBABABABABABABABABABABABA
	$\Pi.C_{21}^{i.c.}(0.75)$	BAbABABabaBAbabABabABaBA
	$\Pi.C_{22}^{i.c.}(0.75)$	BabABabABAbABABABABBABabABabA
	$II.C_{23}^{i.c.}(0.75)$	BabAbaBabABaBAbABabAbaBabAbaBabA
	$\mathrm{II.C}_{24}^{i.c.}(0.75)$	BabABAbaBABabABabABAbABAbaBABabA
	$\Pi.C_{25}^{ic.}(0.75)$	BAbABAbABAbABAbABAbABAbABABABABABABABAB
	$\text{II.C}_{26}^{i.c.}(0.75)$	BabABabABAbABAbaBAbaBABabABabABabA
	$\text{II.C}_{27}^{i.c.}(0.75)$	BabABabABabaBAbaBAbabABabABabABabA
0 0	$II.C_{28}^{i.c.}(0.75)$	BabABabABabABabABabABabABabABabABabABabA
(	$II.C_{29}^{i.c.}(0.75)$	BAbaBABabaBAbabABabaBAbabABAbaBABAbaBABAbabABAbaBABAbabABAbaBABAbabABABAbabABABABAB
	$II.C_{30}^{i.c.}(0.75)$	BabAbaBAbABabABabABabABaBABaBABabA

Table S. LIX. The free group elements for the periodic three-body orbits.

Class and number	free group element
$\overline{\Pi.C_{31}^{i.c.}(0.75)}$	BabABAbaBAbaBABabABAbABAbaBAbaBAbaBAbaBA
$\Pi.C_{0.6}^{i.c.}(0.75)$	${\it BabaBABAbaBAbabABababABABABABABABABABABA$
$\Pi_{1}(C_{0}^{0,c},(0.75))$	Bab Aba B Ab A Bab Aba Bab Aba Bab Aba Bab Aba Bab A
II (%:0: (0.75)	Bab Aba Bab Ab Aba Bab Ab Aba Bab Ab Aba Bab Aba Bab A
	Don A har Bar A har A har B A
	DaDADaDaDADADADADADADADADADADADADADADAD
	DA USBANDO A PARA DA UNA DA UN
11.(37 (0.19))	DabADabababAbabAbabAbabAbabababababAbabA
	Dad Anter A Da Alex A Day A A Day A A D A A A D A A A D A B A B A B A B A
	DabaDADabADADADADADADADADADADADADADADADA
11.040 (0.13)	DAUADAUADAUADAUADAUADAUADAUADAUADAUADAU
$11.\bigcirc_{41} (0.73)$	DabADababAbabAbabAbabADabABababababAbabAbabAbabAbabA DAL-DAD-LADAL-LAD-L-LAD-L-LAL-D-L-LAD-L-DAD-LADAL-DA
$11.C_{42}$ $(0.73)$	BAbab Abab Abab Ababab Babab Babab Ababab BAbab Abab
$\text{II.C}_{43}^{(43)}(0.75)$	BabAbaBAbABaBAbaBabAbaBabABaBABBABABABAB
$11.C_{44}^{c.c.}(0.75)$	BAbabABababABAbabABAbabaBABabaBABabaBABabaBABabaBABabaBA
	${\tt BabABabaBAbaBAbabAbabABabaBabaBAbaBAbaBA$
$\mathrm{II.C_{46}^{u.c.}(0.75)}$	BabABabaBabaBAbaBAbabABabABabaBAbaBAbabABabAB
$\Pi.\mathrm{C}_{47}^{i.c.}(0.75)$	BabABAbABAbaBABabABabABAbaBABabABabABabA
$\text{II.C}^{i.c.}_{48}(0.75)$	BaBAbaBabAbaBabABaBAbABabABaBAbABaBAbABaBAbaBAb
$\text{II.C}^{i.c.}_{49}(0.75)$	BaBAbaBAbABabABabAbaBAbaBAbaBAbABaBAbABabABa
$\text{II.C}_{50}^{i.c.}(0.75)$	BAbaBABabaBAbabABABABABABABABABABabaBAbabABABABAB
$\Pi.C_{51}^{i.c.}(0.75)$	BabABAbABAbaBABaBABaBABabABabABabABAbABA
$\Pi.C_{52}^{i.c.}(0.75)$	BAbABAbABAbaBABabABAbaBABabABabABAbaBABabABAbaBABABABA
$\Pi.C_{5.3}^{i.c.}(0.75)$	BAbaBABabABabaBAbabABababAbabABabaBAbabABabAB
	BabaBAbabABabaBAbabABabaBABabABAbabABabaBAbabABabaBAbabABabaBAbabA
	BAbABABabABAbaBABabABAbaBABabABAbaBABabABAbaBABabABABBBA
	BAbaBABabABAbaBabaBAbabaBababAbabaBAbabABAbabABAbaBABabABAbaBA
	BAbaBABababABabaBABAbaBABababABababABAbaBABababABAbabABAbabABABababABAbaBA
0.0	BaBAbaBAbABabAbaBAbABaBAbaBabABabAbaBAbABaBAbaBAb
$\text{II.C}_{59}^{i.c.}(0.75)$	BabABabABabABabAbaBAbabAbabABabaBabaBAbaBA
$\Pi.C^{i.c.}_{60}(0.75)$	BAbABABabaBAbabABabaBABabABAbabABabaBABabaBABabaBABabaBABaBAB
$\operatorname{II.C}_{\mathrm{cl.c.}}^{i.c.}(0.75)$	BaBAbaBabABaBAbaBAbABaBAbaBabABabAbaBAbABaBAbABaBAbABabAbaBAbA
	BabABAbABAbaBAbaBABabABabABAbaBAbaBABabABab
$\Pi.C^{i.c.}_{63}(0.75)$	BabABabaBAbaBAbabABabaBabaBAbabABabaBaba
	BAbaBABababABabaBABAbABABabaBAbabaBAbabABABABAB
$\operatorname{II.C}_{\mathrm{GF}^{i,c}}^{i,c}(0.75)$	BabABabABabaBAbaBAbaBAbaBABabABabABAbABA
$\Pi.C^{i.c.}_{66}(0.75)$	BabABabABabABaBABaBABabABabABabABAbABAbA
$\text{II.C}_{67}^{i.c.}(0.75)$	BabABabaBAbaBAbaBAbaBAbabABabABabaBabaBAbaBA
$\Pi.C_{68}^{i.c.}(0.75)$	BabaBABabABAbaBABabABAbaBABabABABabABABabABAbaBABabABAbaBABabABab
$ ext{II.C}^{i.c.}_{69}(0.75)$	BabAbaBAbABaBAbaBabAbaBabABaBAbABabABabA
$\mathrm{II.C}_{70}^{i.c.}(0.75)$	BAbABABabaBAbabABAbabABAbabABAbabABAbabABABabaBABabABAbabABAbabABABABAB
$\text{II.C}_{71}^{i.c.}(0.75)$	BabABAbABAbaBABabABAbaBAbaBABabABAbaBAba
$II.C_{72}^{i.c.}(0.75)$	BababABAbabABAbabABABabABAbabABAbabABABabABABABAB
	BaBAbaBabABabAbaBAbABaBAbaBAbABabAbaBabAbaBabABaBAbABaBAbABaBAbaBabABabA
$\text{II.C}_{74}^{i.c.}(0.75)$	BaBAbaBabABabABaBABaBAbaBabABabABabABabA
$\text{II.C}_{75}^{i.c.}(0.75)$	BabABabaBAbaBAbabABabABabaBabaBAbabABabAB
$\operatorname{II.C}_{76}^{v.c.}(0.75)$	BabABAbaBAbaBAbaBABabABabABAbABAbaBAbaBA
$\text{II.C}_{7.5}^{t.c.}(0.75)$	BabABAbaBAbaBABabABabABAbABAbBABABABABAB
II. 778 (0.75)	bababababAbabAbabABabABabaBAbabABabaBABabaBAbabABabAB
$\Pi:C_{79}^{r}(0.75)$	Bohaha Bahaba Baha Baha Bahah Baha Baha Ba
(61.6) (87.11	Described in the control of the cont

Table S. LX. The free group elements for the periodic three-body orbits.

Class and number	er free group element.
II Ci.c. (0.75)	BahaB
$\Pi \subset \mathbb{S}_1 \ (0.15)$	Baha Baha Baha Baha Baha Baha Baha Baha
$\text{II.C}_{83}^{i.c.}(0.75)$	BabAbaBabABaBAbaBabABaBAbABaBAbaBabAbaBAbABabABa
$\Pi.C_{84}^{i.c.}(0.75)$	BAbABABabaBABabABabABAbaBABAbaBABAbaBABAbaBABAbabABabaBABabaBABabaBABabaBABABABA
$\Pi.\mathrm{C}^{i.c.}_{85}(0.75)$	BAbaBABabaBABabABabABabABAbaBABAbaBABAbaBABAbabABabAB
$II.C_{86}^{i.c.}(0.75)$	BababABAbaBABababABabaBABAbabABababABabaBABabaBABAbaBABababABababABabaBABabaBABababA
$\Pi.C_{87}^{i.c.}(0.75)$	BabABabaBAbaBAbaBAbabABabABabABabABabABa
$\text{II.C}_{88}^{i.c.}(0.75)$	BabABabaBAbaBAbabABabABabABabaBAbabAbabA
$II.C_{89}^{i.c.}(0.75)$	BAbABABabaBAbabABAbabABAbabABabaBABabaBAbabABAbabABAbabABABabaBABabaBAbabABABABAB
$\Pi.\mathrm{C}^{i.c.}_{90}(0.75)$	BabABAbABAbaBABabABabABabABabABabABabABa
$\Pi.\mathrm{C}^{t.c.}_{91}(0.75)$	BabABAbaBABaBABabABabABAbaBAbaBABaBABBABABABA
$\text{II.C}_{92}^{i.c.}(0.75)$	BabABAbABAbaBAbaBABabABabABabABAbABAbaBAba
$11.C_{93}^{i.c.}(0.75)$	BabABabABAbABAbaBABaBABaBABabABabABaBAbaBAb
$11.C_{94} (0.75)$ $11 C^{i.c.} (0.75)$	BababAbabABababAbabABababAbabABabABabABa
$\Pi.C_{95}^{i.c.}(0.75)$	BAbaBABabABAbabABabaBABabaBABabaBABabABab
$II.C_{97}^{i.c.}(0.75)$	BAbABAbABAbaBABabABabABabABabABabABabABABABAB
$\text{II.C}_{98}^{i.c.}(0.75)$	BAbaBABabaBABabaBABabABABabaBABabaBABabABABabABAbabABABabABABabABABABAB
$II.C_{99}^{i.c.}(0.75)$	BaBAbaBabABabABabABaBAbaBAbABabABabABaBAbaBABABABA
$\mathrm{II.C}_{100}^{i.c.}(0.75)$	BabAbaBAbABaBAbaBAbABaBAbABaBAbaBAbABaBAbABabABa
$\Pi.C_{101}^{ic.}(0.75)$	BaBAbaBabABaBAbaBaAbaBabAbaBabAbaBabABaBAbaBAb
$\Pi.C_{102}^{i.c.}(0.75)$	BAbABABababABabaBABabABABabaBAbabABAbabABAbaBABABABA
$\Pi.C_{103}^{i.c.}(0.75)$	BabABAbaBABaBABabABAbABAbaBAbaBABabABabA
$\text{II.C}_{104}^{i.c.}(0.75)$	BabABabABabABabABaBABabABabABabABabABabA
$\Pi.\mathrm{C}_{105}^{i.c.}(0.75)$	BAbaBABabaBAbabABabaBABabABabABABabABAbabaBabaB
$\text{II.C}_{106}^{i.e.}(0.75)$	BAbaBABababABabaBABababABababABabaBABAbaBABababABababABABABAB
$\text{II.C}_{107}^{ior}(0.75)$	BabaBABabaBABabaBABabaBAbabaBAbabABAbabABAbabABabaBABabaBABabaBABabaBAbabABAbabABAbabABAbabABAbabA BalabaBABabaBABabaBABabaBABabaBABabABABabABABabaBABabaBABabaBABabaBABabaBABabaBABabABABABAB
$11.C_{108}(0.75)$	Babakabakabakabakabakabakabakabakabakaba
$11.C_{109}(0.73)$ $11.C_{i.c.}(0.75)$	Dau ADA UADA UADA UADA DADA DAU DAU DA UADA UADA UADA DAU ADA UADA UADA UADA DADA DAU DAU
$\Pi.C_{110}^{i.c.}(0.75)$	BAbABABabaBABabABAbabABAbabABabaBABabaBABABABA
$\Pi.C_{112}^{i.c.}(0.75)$	BabABAbaBAbaBABabABabABabABAbABAbaBAbaBA
	abA
$\Pi.\mathrm{C}_{113}^{v.c.}(0.75)$	BabaBAbabABabaBAbabABabaBAbabABabaBAbabABabaBAbabABAbaBABabaBABabaBABabABab
$\Pi(G_i^i;c_i(0.75))$	AbabA BAhahABAhahABAhahABAhahaBAhahaBABahaBABahaBABahaBABAhaBABAhahABAhahABAhahABAhahABAhahaBABahaBABahaBAB
( ) 114 ( )	abaBA
$\mathrm{II.C}^{i.c.}_{115}(0.75)$	BabABAbABAbaBABabABabABABaBABabABAbABAbA
	aBABabA
$ ext{II.C}^{i.c.}_{116}(0.75)$	BabABAbaBAbaBABabABAbABAbABAbaBAbaBABaBAB
$\Pi_{1}^{i,c}(0.75)$	incentrations.i. Baba Baba Baba Baba Baba Baba Baba Baba
( ) ) / / /	${\rm ABAbaBABabA}$
$\Pi.C_{118}^{i.c.}(0.75)$	BAbaBABababABabaBABAbaBABababABabaBABAbaBABababABabaBABABABA
11 Qi.c. (0.47)	ABababABAbaBA portopato pator altoranda no tona tona tona ta and tona tona de natoranda de natora portona a tona and tona de
$11.C_{119}(0.75)$	БарАБараБАрарАрарАрарАБарАБараБараБараБАрарАБарАБ
$\Pi.\mathrm{C}_{1.20}^{i.c.}(0.75)$	BabABAbaBAbaBABaBABabABabABabABAbaBAbaBA
	AbaBAbaBABabA

Table S. LXI. The free group elements for the periodic three-body orbits.

Class and number	free group element
$\overline{\Pi.\mathrm{C}_{121}^{i.c.}(0.75)}$	BabABabaBAbaBAbabABabABabABabaBAbaBAbaBA
$\mathrm{II.C}_{122}^{i.c.}(0.75)$	BAbaBABabaBAbabABabaBABabABABABABABABABA
$\mathrm{II.C}^{i.c.}_{123}(0.75)$	BabaBABABabaBABabaBABabaBABabABABabABABabABAbabABAbabABAbaBABabaBABabaBABabaBABabABABabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABABAbabABABABAB
$\mathrm{II.C}^{i.c.}_{124}(0.75)$	BababABAbaBABAbabABAbabABAbabABAbabABAbabABAbabABABabABABabABABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaB ABabaBABAbaBABababA
$\mathrm{II.C_{125}^{i.c.}(0.75)}$	BabaBABabaBABabABAbabABAbabABAbaBABabaBABabaBABabaBAbabABAbabABABabaBABabaBABabaBABabABAbabABAbaBABabaBABABabaBAB abaBABabABAbabABAbabA
$\mathrm{II.C}^{i.c.}_{126}(0.75)$	BabAbaBAbABaBAbaBAbABabAbaBabABaBAbaBAbABabABa
$\mathrm{II.C}^{i.c.}_{127}(0.75)$	BabaBABabABAbaBABabaBAbabABabaBABabaBABabABAbaBABabaBABabaBABabABABABAB
$\mathrm{II.C}^{i.c.}_{128}(0.75)$	BababABAbaBABABABABABABABBABBABBABABABAB
$\mathrm{II.C}^{i.c.}_{129}(0.75)$	BAbaBABabaBABabaBABabaBABabABAbabABAbabABABabABABABAB
$\mathrm{II.C}^{i.c.}_{130}(0.75)$	BAbaBABabaBABabABAbabABAbabABAbabABAbabABabaBABabaBABabaBABabaBABabaBAbabABABAbabABABABAB
$ ext{II.C}^{i.c.}_{131}(0.75)$	BabABAbABAbaBABabABabABabABabABabABAbaBAba
$\Pi.\mathrm{C}^{i.c.}_{132}(0.75)$	BabABAbaBAbaBABabABabABabABabABabABabABa
$ ext{II.C}^{i.c.}_{133}(0.75)$	BabaBABabABAbaBABabaBAbabABAbabABAbaBABabaBABabaBAbabABabaBABabABAbaBABabABAbabABABABAB
$\mathrm{II.C}^{i.c.}_{134}(0.75)$	BAbabABabaBABabABAbabaBAbabaBABabaBABababABAbabABAbaBABABABA
$ ext{II.C}^{i.c.}_{135}(0.75)$	BababABAbabABAbabaBABabaBABabaBABababABAbabABAbabaBABabaBABabaBABabaBABababABABAbabABABAbabABABABAB
$ ext{II.C}^{i.c.}_{136}(0.75)$	BabABAbABAbaBABabABabABAbaBABabABAbaBABabABab
$\mathrm{II.C}^{i.c.}_{137}(0.75)$	BabaBAbabABabaBAbabABabaBAbabABabaBAbabABabaBAbabABabaBABabABABabABABABAB
$ ext{II.C}^{i.c.}_{138}(0.75)$	BabaBAbabABAbaBABabaBABabaBABabaBABabaBAbabABAbabABabaBAbabABabaBAbabABabaBABabABAbabABABABAB
$\Pi.C^{i.c.}_{139}(0.75)$	BAbabABababABAbabABAbabaBAbabaBABabaBABabaBAbabaBABabaBABabaBABababABabaBABAbaBABababABababABababABababABababABabaBABABABA
$\mathrm{II.C}_{140}^{i.c.}(0.75)$	BA

Table S. LXII. The free group elements for the periodic three-body orbits.

Class and number  Three group of element  Three group		
BabaBABabaBABabaBAbaBABabaBABABabaBABABabaBABABabaBABABABA	Class and number	free group element
abaBAbabABAbabABabaBABABabaBABABabaBABABabaBABABabaBABABabaBABABABA	$\Pi.\mathrm{C}_{141}^{i.c.}(0.75)$	BabaBABabaBABabaBAbabABAbabABABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABABABA
BabaBABabABabaBAbabABAbabBABabaBabABABabaBBABabaBABabaBBABabaBABabaBBABabaBABBabaBABabaBABabaBABabaBABabaBABabaBABBABBABBABBABBABBABBABBABBABBABBABBAB		abaBAbabABAbaBABabaBABabaBAbabABAbabABAbabA
abABAbaBABabABAbaBABABABABABABABABABABAB	$\Pi.C_{142}^{i.c.}(0.75)$	BabaBABabABAbabABabaBAbabABAbaBABabaBAbabABAbaBABabABAbabABAbabABabAB
BababABabABabaBABabaBabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABABabaBABABABA		abABAbaBABabaBABabABabABabABabABabABAbabABAbabABAbabABAbabABAbabABABABAB
ABAbaBABabABAbabABabaBABabab BababABAbaBABAbabBABabab AbabaBABabaBABabaBABabal BAbabABABabaBABABABAbaB BabaBABABABABABABABABBABBABABABABABABABA	$\mathrm{II.C}^{i.c.}_{143}(0.75)$	BabaBABabABAbabABAbabABabaBAbabABabABabA
BabahABAbaBABAbabABABabab AbabaBABabaBABabaBABabal BAbabABabaBABabaBABAbal BabaBABabaBABabaBABabaB AbabABAbaBABABABABAB BabaBABAbaBABABABABAB BabaBABABABABABABABABAB BabaBABABABABABABABABAB BabaBABABABABABABABABABABABABABABABABABA		ABAbabABAbabABabaBAbabABabABAbABA
AbabaBABabaBABabaBABAbabaBABabaBABAbaBBABABABBABBABBABBABBABBABBABBABBABBABB	$\Pi.C^{i.c.}_{144}(0.75)$	BababABAbaBABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabBAbabABABABAB
BAbabABabaABAbabaBABAbal ABabABABabaBAbabaBABAba BabaBABabaBABabaBABabaB AbabABAbaBABABABABABBAB BabaBABABABABABABABABAB BabaBABABABABABABABABABABABABABABABABABA		
ABabABABabaBAbabaBABabaB BabaBABabaBABabaBABabaB AbabABAbaBABabaBABabaB BabaBABAbaBABabABABABABABABABABABABABABA	$\mathrm{II.C}^{i.c.}_{145}(0.75)$	BAbabABabaABABabABABabABABABABABABABABAB
BabaBABabaBABabaBABabaBABabaBAbaBABABABA		ABabABABabaBABAbabABABABababABabABABABAB
AbabABAbabABAbaBABabaB BabaBABAbabABabaBABabaB abABAbabABABABabABabAB abABABABABABABABABAba aBABaBABABABABABABAba BAbabABABABABABABABA ababABABABABABABABABAB BababABABABABABABABAB BababABABABABABABABAB BabaBABABABABABABABAB BABABABABABABABABABAB	$\Pi.\mathrm{C}^{i.c.}_{146}(0.75)$	BabaBABabaBABabaBABabaBAbabABAbabABAbaBABabaBABabaBABabaBABabaBABabABAbabABAbabABAbabABabaBABabABABABAB
BabaBABAbaBABabaBBabaBabaBabABabABabABab		A bab ABAbab ABAbaBABabaBABabaBAbabABABABAB
abABAbabABAbabABabABabAB BabABABabABabABabABabAB aBAbaBABabABAbabABAbal BAbabABABABABABABABABA aBABABABABABABABABABAB	$\mathrm{II.C}^{i.c.}_{147}(0.75)$	BabaBABAbaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabABABabABABabABABabABAbabABAbabABAbabABABABAB
BabABAbaBABabABabABabABabAB aBAbaBABabABabABAbaBAba BAbabABAbabABAbabABAbab aBABBABABABABABABABAB BababABABABABABABABABAB BabaBABABABABABABABABAB BabaBABabaBABABABABABAB BABABABABABABABABABABAB BABABABABABABABABABABABAB BA		
	$\mathrm{II.C}^{i.c.}_{148}(0.75)$	BabABAbaBAbaBABabABAbABAbaBABabABAbABAbaBAba
		aBAbaBABabABabABaBABaBABaBABaBABaBABaBAB
	$\mathrm{II.C}^{i.c.}_{149}(0.75)$	BAbabABAbabABAbabABAbabABAbabABABabaBABabaBABabaBABabaBABAbaBABAbabABAbabABAbabABAbabABAbabABABabaBABabaBABabaBABabaBABabaBABABABA
	$\mathrm{II.C}^{i.c.}_{150}(0.75)$	BababABAbaBABAbabABAbabABAbaBABAbabABAbabABAbaBABAbaBABAbaBABAbabABAbabABAbabABAbaBABAbabABAbaBABABABA
		ababABabaBABABAbabABabaABAbaBABAbaBABababABabaBABabaBABAbaBABABabaBABababA
	$\Pi.\mathrm{C}_{151}^{i.c.}(0.75)$	BabaBABabaBABabaBABabaBABabaBABabaBABabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbaBABABABA
		${\tt aBABabaBABabaBABabaBABabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABABabABABabABABabABABABAB$
	$\mathrm{II.C_{152}^{ic.}(0.75)}$	BAbABABabaBAbabABAbaBABABABABABABABABABA
		abaBAbabABABaBABaBABabaBAbabaBAbabABABABAB
	$\Pi.\mathrm{C}^{i.c.}_{153}(0.75)$	BababABAbabABAbabABAbabABABabaBABabaBABABABA
	$\Pi.C_{154}^{i.c.}(0.75)$	BAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABABabaBABabaBABabaBABabaBABabaBABabaBABabaBABababABAbabABAbabABAbabABABABAB
,		ABA babABA babABA babABA babABA babABA BABA babABA BABA B
ABahABAbaBABabABAbaBABabABAbabABAbabABabaBAbabABabaBAbabABabAB	$\mathrm{II.C}^{ic.}_{155}(0.75)$	BabaBAbabABabaBAbabABabaBABabABabaBAbabABabaBABabaBABabABab
		ABabABAbaBABabABAbaBABabABAbabABabABAbabABabAB

Table S. LXIII. The free group elements for the periodic three-body orbits.

II. Cfig. (0.75) Eabu B. Babah B. Rabah R. Rabah R. Babah B. Rabah R. Rabah	Class and number	free group element
	$\overline{\Pi.C^{i.e.}_{156}(0.75)}$	
	$\mathrm{II.C}_{157}^{i.c.}(0.75)$	BAbaBABababABabaBABabABABabaBAbabABABaBABABABA
	$\mathrm{II.C}_{158}^{\dot{v.c.}}(0.75)$	BabaBABabABAbaBABabaBAbabABabaBABabABAbabABabaBAbabABAbabABAbaBABabABAbabABabAB
	$\mathrm{II.C}_{159}^{\dot{v.c.}}(0.75)$	BAbabABAbabABAbabABAbabABAbabABABABABABA
	$\mathrm{II.C}_{160}^{i.c.}(0.75)$	BAbaBABabaBABabaBABabABABabABAbabABAbabABABabABABabABAbabaBABabABABabABAbabABabAB
	$\Pi.\mathrm{C}^{\mathrm{i.c.}}_{161}(0.75)$	BAbaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBABabaBAbabABABAbabABABABAB
	$\Pi.C_{162}^{i.c.}(0.75)$	BAbaBABababABabaBABAbaBABababABababABAbaBABAbabABababABAbaBABAbabABababABababABababABabaBABababABabaBABABABA
	$\Pi.C_{163}^{i.c.}(0.75)$	BAbabABAbabABAbabABABabaBABabaBABabaBABababABAbabABAbabABAbabABABabaBABabaBABabaBABabaBABABABA
	$II.C_{164}^{i.c.}(0.75)$	BAbaBABababAbabaBABabABABabaBAbabABABabABABabABAbabABabaBAbabABAbaBABAbaBABabaBABababABabaBABabABABabABABabABABabABABabABABabABABabABABabABABabABABabABABabABABabABABabABABabABABabABABABAB
	$\Pi.C_{165}^{i.c.}(0.75)$	BAbaBABababABabaBABABABABababAbabaBABabABABABAB
	$\text{II.C}_{166}^{i.c.}(0.75)$	BAbabABAbabABAbabABAbabABAbabABAbabABAbabABAbabABABabaBABABABA
	$\mathrm{II.C}_{167}^{i.c.}(0.75)$	BAbaBABabaBAbabABAbabABAbaBABabaBABababABabAB
BAbabABababABAbaBABAb ABABabABAbabaBAbabABA baBABABaBABababABababA	$\Pi.C_{168}^{i.c.}(0.75)$	BAbaBABabaBABabaBABabaBABabaBABabaBABAbaBABababAbabBABABABA
	$II.C_{169}^{i.c.}(0.75)$	BAbabABabaABAbaBABabaBABabaBABabABAbabABAbabaBABababABababABAbabABAbabABAbabABABabaBABABabABABabaBAbab ABABabaBABBABABABA

Table S. LXIV. The free group elements for the periodic three-body orbits.

Class and number	Hilber Tree group eighient
$I.A_1^{i.c.}(2)$	BAbabaBA
$I.A_5^{i.c.}(2)$	BABababABA
$I.A_3^{i.c.}(2)$	BABababaBABABababABABAbababABA
$1.A_{4}^{i.c.}(2)$	BABahahabaBABABABahahABABAbahahABA
$1.A_{i}^{i.c.}(2)$	BABAbabababababababaBABABABABABABA
$I.A_6^{i.c.}(2)$	BAbababABABAbababABABababaBABABababaBABABababaBA
$\mathrm{I.A}_7^{i.c.}(2)$	BABababaBABABabababABABababABABAbababABABABAB
$I.A_8^{i.c.}(2)$	BABababaBABABABABabababABABABAbababABABABAB
$\mathrm{I.A}_9^{i.c.}(2)$	BABababABABABABABABABABABABABABABABABABA
$I.A_{10}^{i.c.}(2)$	BABababaBABABABABABABABABABABABABABABABA
$I.A_{11}^{i.c.}(2)$	BAbababABABABABABAbababABABABABABABABABA
$\mathrm{I.A}_{12}^{i.c.}(2)$	BABababaBABABABABABABABABABABABABABABABA
$\mathrm{I.A}_{13}^{i.c.}(2)$	BABA babababABABABA BABABABA BABABA babababABABABABABABABABABABABABABABABABA
$\mathrm{I.A}_{14}^{i.c.}(2)$	BABababaBABABABABABABababaBABABababaBABABababaBABABABA
$I.A_{15}^{i.c.}(2)$	BABababABABABABABABABABABABABABABABABABA
$I.A_{16}^{i.c.}(2)$	BAbababABABABABABABAbababABABABAbababABABABAB
$I.A_{17}^{i.c.}(2)$	BABababaBABABAbababABABababaBABABABababaBABABabababABABABAB
$I.A_{18}^{i.c.}(2)$	BABA babababA BABA BABA BABA BABA BABA B
$I.A_{19}^{i.c.}(2)$	BABAbabababABABABAbabababABABABabababABABABAB
$\mathrm{I.A}_{20}^{i.c.}(2)$	BAbababABABABABABABAbababABABAbababABABAbababABABABAB
	baBA
$\mathrm{I.A}_{21}^{i.c.}(2)$	BABabababABABABABAbababaBABABAbababaBABABABA
T A 1.C. (O)	4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
$\Gamma. m A_{22}^{22}(2)$	БАБарарараБАБАБАБАрарарар БАБАБарарар БАБАБАБАБАБАБАБАБАБАБАБАБАБАБАБАБАБАБА
(7)	RARSHS TO SHORT BARARARA BARARA BARARA BARARA BARSHS TARARA BARARA BARA BARARA
1.723 (2)	Барабабарды жылы барабара жазы жылы жылы жылы жылы жылы жылы жылы жы
$I.A_{24}^{i.c.}(2)$	BABA bababa bABA BABA BABA BABA Bababa bABA BABA B
	babABABABabababaBABA
$\Gamma.\mathrm{A}_{25}^{i.c.}(2)$	BABababaBABABABABABABABABABABABABABABABA
	${ m BABAbababABABabababABA}$
$I.A_{26}^{i.c.}(2)$	BABAbabababABABABAbababababABABABAbababABABABAB
	${\it bababaBABABABABABA}$
$\mathrm{I.A}_{27}^{i.c.}(2)$	BABababaABABABABABABABABABABABABABABABAB
	${\it baba} {\it BABABABABABABABAA}$
$I.A_{28}^{i.c.}(2)$	BABabababABABABababaBABABABababaBABABABA
	babashAbAbabababAbABA
$\mathrm{LA}_{29}^{\circ\circ}(2)$	BAbababABABABababABABababABABABababABABABAB
$\mathrm{I.A}_{30}^{i.c.}(2)$	BaBababahABABababahABAbababahABAbababABABABAbababABABABABABABABABABAB
	BabababABABAbababaBABABababaBABABABababABA

Table S. LXV. The free group elements for the periodic three-body orbits.

Class and number f	ber free group element
$\overline{\mathrm{I.A}}_{31}^{i.c.}(2)$	BABabababaBABABABABAbababaBABABABABABABA
$\mathrm{I.A}_{32}^{i.c.}(2)$	BABAbababaBABABABabababaBABABABabababABABABAB
$I.A_{33}^{i.c.}(2)$	BABabababaBABABABABABABABABABABABABABABA
$\mathrm{I.A}_{34}^{i.c.}(2)$	BABAbabababABABABABAbabababABABABABAbabababABABABAB
$I.A_{35}^{i.c.}(2)$	BABAbababABABABABABABABABABABABABABABABA
$I.A_{36}^{i.c.}(2)$	BABabababABABABABABABABABABABABABABABABA
$I.A_{37}^{i.c.}(2)$	BABabababaBABABABabababABABABababaBABABABA
$\mathrm{I.A}_{38}^{i.c.}(2)$	BABabababABABABabababABABABababaBABABAbababABABABAB
$\mathrm{I.A}^{i.c.}_{39}(2)$	BABABABABABABABABABABABABABABABABABABA
$\mathrm{I.A}_{40}^{i.c.}(2)$	BABAbababaBABABABABabababaBABABABABABABA
$\mathrm{I.A}_{41}^{i.c.}(2)$	
$\mathrm{I.A}_{42}^{i.c.}(2)$	BABAbababABABABABABABABABABABABABABABABA
$\mathrm{I.A}_{43}^{i.c.}(2)$	BABabababABABABababaABABABABababaBABABAbababaBABABABA
$\mathrm{I.A}_{44}^{i.c.}(2)$	BABAbababABABABABABABABABABABABABABABABA
$\mathrm{I.A}_{45}^{i.c.}(2)$	BABAbababABABABABABABABABABABABABABABABA
$\mathrm{I.A}_{46}^{i.c.}(2)$	BABabababABABABABababaBABABAbababaBABABABabababABABABAB
$\mathrm{I.A}_{47}^{i.c.}(2)$	BABAbababABABABABABABABABABABABABABABABA
$\mathrm{I.A}_{48}^{i.c.}(2)$	BABabababABABABABABABABABABABABBABABABAB
$\mathrm{I.A}_{49}^{i.c.}(2)$	BABabababaBABABAbabababABABABabababaBABABABA
$\mathrm{I.A}_{50}^{i.c.}(2)$	BABababaBABABABABABABABABABABABABABABABA

Table S. LXVI. The free group elements for the periodic three-body orbits.

	BABabababABABABABABABABABABABABABABABABA
	ABABabababaBABABABABABABABABABABABABABAB
	ABABAbabababABABABAbabababABABABABABabababaBABABABA
	babaBABABABababaBBBBABABababaBABA
	BABAbabababABABABABababaBABABABababaBABABABA
	BABabababaBABABABababaBABABABABababABABABAbabababABABABabababABABABAB
I.A <sup>i.c.</sup> (2) BABAbababABAB bababABABABababa BABABabababaB.	BABAbabababABABABABABABABABABABABABABABA
	BABAbabababABABABABabababABABABAbababaBABABABA
I.B $^{1.c.}(2)$ BAbABABaBA I.B $^{1.c.}(2)$ RAbhARABaBA	
$\Gamma_{2}^{\text{ric}}(z)$ BaaBBaaBAbbAAbbA	bbA
	abaBA
$1.5_5^{\circ\circ\circ}(2)$ bABababaBABABABababABA $1.8_5^{\circ\circ\circ}(2)$ BABabababaBABABAbababABA	AbababABA ABAbababABA
	BAbabaABABABABababABABABababaBA
$1.B_{s.c.}^{c.}(2)$ BABababaBABABab $_{s.c.}(2)$ RABabababaBABABAB	BABababaBABABABABABABABABABABABABABABA BABabababa RABAbababaBA BABAbababaBA BABABababa ABA
	BABabababABABABABABABABABABABABABABABABA

Table S. LXVII. The free group elements for the periodic three-body orbits.

Class and number	Free group element
$[B_{i,c}^{i,c}(2)]$	BABab
$I.B_{1.5}^{ic.}(2)$	BABAbababABABABABABABABABABabababABABABAB
$\Gamma  ightarrow  ightar$	BA bababABABA BA bababABA BababaBABA BababaBABA BababaBA
$\Gamma  ightarrow  ightarrow  angle  ang$	BABabaBABABABABABABABABABABABABABABABABA
$I.B_{15}^{i.c.}(2)$	BABababABABABABABABABAbabaBABABABABABABA
$\Gamma B_{16}^{i.c.}(2)$	BABababABABABABABABABABAbabaBABABABABABA
$\mathrm{I.B}_{17}^{i.c.}(2)$	BA bababABABA BABA BABA BABA BABA BABA B
$\Gamma.\mathrm{B}_{18}^{i.c.}(2)$	BABababaBABABABababaBABABABababaBABABABA
$I.B_{19}^{i.c.}(2)$	BABAbababABABABABABABABABABABABABABABABA
$\mathrm{I.B}_{20}^{i.c.}(2)$	BABababABABABABABABABABABABABABABABABABA
$\mathrm{I.B}_{21}^{i.c.}(2)$	BABababaBABABABABABABababABABABABABABABA
$\mathrm{I.B}_{22}^{i.c.}(2)$	BABababABABABABABABABABAbababABABABABABA
$\mathrm{I.B}^{i.c.}_{23}(2)$	BAbababABABABABABABABABABABABABABABABABA
$\mathrm{I.B}_{24}^{i.c.}(2)$	BABababaBABABABABABABABABababaBABABABABA
$I.B_{25}^{i.c.}(2)$	BABababaBABABABABababaBABABABABABABABABA
	babABA
$\mathrm{I.B}_{26}^{i.c.}(2)$	BABAbabababABABABabababaBABABABABABABABA
	baBABA
$\mathrm{I.B}_{27}^{i.c.}(2)$	BABababaBABABABAbababaBABABABABABABABABA
	BAbababABA
$I.B_{28}^{i.c.}(2)$	BABababABABABABABABABABABABABABABABABababABABABAB
	BabababABA
$I.B_{29}^{ic.}(2)$	BABababaBABABABABABABABABABABABABABABABA
	BABABababABA
$I.B_{30}^{i.c.}(2)$	BABababaBABABABabababABABABababaBABABABA
	babABABABABABA
$I.B_{31}^{i.c.}(2)$	BABababaBABABABababaBABABababaBABABababaBABABABababABABABAB
	bababABABAbababaBABABAbababABA
$I.B_{32}^{i.c.}(2)$	BABababaBABABABABabababaBABABABababaBABABABA
	babABABABAbababABABABAbababABA
$\mathrm{I.B}^{i.c.}_{33}(2)$	BAbababABABABABABABABababABABABABABABABA
	${\rm a} BABABababaBABABababaBABababaBA}$
$I.B_{34}^{ic.}(2)$	BABA bababa bABA BABA BABA BABA BABA BAB
	${\it babABABABABABABABABABABABA}$
$I.B_{35}^{i.c.}(2)$	BABabababABABabababABABABabababABABAbababBABABABA
	BABABababABABABababaBABA
$I.B_{36}^{\iota.c.}(2)$	BABA bababA BABA BABA BABA BABA BABA BAB
( ) ( )	bababaBABABABABABABABABABABABABABABABABA
$1.B_{37}^{i.e.}(2)$	BABabababABABabababABABABabababABABABAbababABABABAB
T Di.C. (9)	UBDADDADBOORDAD AL LEELE OAD AND AL LEELE OAD AND AND ALCELE AND AND ALCELE AND AND AND ALCELE OAD AND AND AND AND AND AND AND AND AND A
1.D38 (2)	БАБА уаазалага БАБАБАБАБАБАБАБАБАБАБАБАБАБАБАБАБАБАБА
$\mathrm{I.B}^{i.c.}_{39}(2)$	BABababaBABABABABABABABABababaBABABABABA
	BAbababABABABABABABABABABABABABABABABABA
$\mathrm{I.B}_{40}^{i.c.}(2)$	BABababaBABABABabababABABABABAbabababABABABAB
	BABABABababaBABABABababaBABABABABABABABA

Table S. LXVIII. The free group elements for the periodic three-body orbits.

Class and nur	Class and number free group element
$\mathrm{I.B}_{41}^{ic.}(2)$	BABababaBABABababaBABABababaBABABababaBABABABababaBABABABababaBABABababaBABABababaBABABAbababABABABAB
$\mathrm{I.B}_{42}^{i.c.}(2)$	BABabababABABABABABABABABABABABABABABABA
$\mathrm{I.B}_{43}^{i.c.}(2)$	BABAbababABABABABABABABABABABABABABABABA
$\mathrm{I.B}_{44}^{i.c.}(2)$	BABabababaBABABABABABABABABABABABABABABA
$\mathrm{I.B}_{45}^{i.c.}(2)$	BABAbababABABABABAbababaBABABABABabababaBABABABA
$\mathrm{I.B}_{46}^{i.c.}(2)$	BABababaBABABABababaBABABABabababABABABababaBABABABabababABABABAB
$\mathrm{I.B}_{47}^{i.c.}(2)$	BABabababABABABABABABABABABABABABABABABA
$\mathrm{I.B}_{48}^{i.c.}(2)$	BABababaBABABABababaBABABabababABABAbababBABABababaBABABABA
$\mathrm{I.B}_{49}^{i.c.}(2)$	BABabababABABAbababaBABABAbababABABAbababABABABAbabaBABABABA
$\mathrm{I.B}^{i.c.}_{50}(2)$	BABabababABABABababaBABABAbababABABABAbababaBABABABababaBABABABabababABABABababABABABababaABABABabababABABABabababABABABabababABABABabababABABABabababABABABabababABABABabababABABABAB
$\mathrm{I.B}_{51}^{i.c.}(2)$	BABabababABABABABababaBABABababaBABABabababABABABababaBABABABabababABABABabababABABabababABABABababaBABABABA
$\mathrm{I.B}_{52}^{i.c.}(2)$	BABabababaBABABABabababaBABABABabababABABABAB
$\mathrm{I.B}^{i.c.}_{53}(2)$	BABabababABABABABABABABABABababaBABABABababaBABABABA
$\mathrm{I.B}_{54}^{i.c.}(2)$	BABabababaBABABABabababaBABABABabababABABABAB
$\mathrm{I.B}_{55}^{i.c.}(2)$	BABAbabababABABABABababababABABABAbabababABABABAB
$I.B_{56}^{i.c.}(2)$	BABAbababaBABABABABabababABABABAbababaBABABABA
$\mathrm{I.B}^{i.c.}_{57}(2)$	BABabababaBABABABabababaBABABABABABABABA
$\mathrm{I.B}^{i.c.}_{58}(2)$	BABabababABABABababaBABABABababaBABABABA
$\Gamma.\mathrm{B}_{59}^{i.c.}\left(2 ight)$	BABabababABABABABABABABABABABABABABABABA
$I.B_{60}^{i.c.}(2)$	BABAbababABABABABABABABABABABABababaBABABABA

Table S. LXIX. The free group elements for the periodic three-body orbits.

Class and number	ber free group element
$\overline{1.\mathrm{B}^{i.c.}_{61}(2)}$	BABAR bababal BABab
$\Gamma.\mathrm{B}^{ic.}_{62}(2)$	BABAbababABABABABAbabababBABABABabababABABABAB
$\Gamma.\mathrm{B}^{i.c.}_{63}(2)$	BABAbababABABABABABABABABABABababaBABABABA
$\Gamma.\mathrm{B}^{i.c.}_{64}(2)$	BABAbababaBABABABABababaBABABABABABABABA
$\mathrm{I.B}_{65}^{i.c.}(2)$	BABAbababABABABABABABABABABABABABABABABA
$I.C_1^{i.c.}(2)$	BABAbabababABABABAbabababABABABABABABABA
$I.C_2^{i.c.}(2)$	BABAbabababABABABABAbabababABABABAbabababABABABAB
$\Pi.A_1^{i.c.}(2)$ $\Pi.A_2^{i.c.}(2)$ $\Pi.A_3^{i.c.}(2)$ $\Pi.A_4^{i.c.}(2)$	BAbababABABABABABABABababaBA BABabababABABABABABABABABABABABABABABABA
$\Pi.B_{i}^{i.c.}(2)$	babABABAbababaBABABABABABABABABABABABABA
$\begin{array}{c} \Pi.B_{1}^{2} \cdot \cdot \cdot (2) \\ \Pi.B_{3}^{1} \cdot \cdot \cdot (2) \\ \Pi.B_{4}^{1} \cdot \cdot \cdot (2) \\ \Pi.B_{4}^{2} \cdot \cdot \cdot (2) \end{array}$	BAbaBAbABabABabABaaBABaaBA BAbabaBABABAbababABABAbabaBA BAbababABABABAbabaBABABababABABAbababABABAbababABABababaBABABABA
$\mathrm{II.B}_6^{i.c.}(2)$	baBABABABABABABABABababABABABABababABABABAB
$\mathrm{II.B}_7^{i.c.}(2)$	BABAbabababABABABAbababaBABABABAbabababABABABAB
$\text{II.C}_1^{i.c.}(2)$ $\text{II.C}_2^{i.c.}(2)$	BAbbabaaBA BAbbABaaBA
$\Pi.C_3^{i.c.}(2)$ $\Pi.C_4^{i.c.}(2)$	BAbbbabaaaBA BAbbbABaaaBA
$\Pi.C_5^{i.c.}(2)$	BabAAbaBBabA BALLLA LA L
$\begin{array}{c} \Pi.C_6 & (2) \\ \Pi.C_7^{i.c.}(2) \\ \vdots \\ \Omega & \Omega \end{array}$	BABubbabah BA BABubbabah BA
$egin{array}{ll} \Pi.C_8^{i.c.}(2) \ \Pi.C_9^{i.c.}(2) \ \Pi.C_9^{i.c.}(2) \end{array}$	BAbababABABAbababABABA BABAbababABABAbababaBABA BABohoba BABAbohob BABAbohobABA
11:010 (2)	DINDAGAGANI DINDAGAGAGANI DINDAGAGANI DIN

Table S. LXX. The free group elements for the periodic three-body orbits.

Class and number	aber free group element
$\overline{\mathrm{II.C}_{11}^{i.c.}(2)}$	BabAAbaBBabABaaBAbbABabAAbaBBabA
$\operatorname{II.C}_{12}^{i.c.}(2)$	BABAbababaBABABABABABABABABABABABABABABA
$\Pi.\mathrm{C}_{13}^{i.c.}(2)$	BABababaBABABABabababABABABABABABABABABA
$\Pi.\mathrm{C}_{14}^{i.c.}(2)$	BABababABABABABABABABABABABABABABABABABA
$\text{II.C}_{15}^{i.c.}(2)$	BABA bababaBABABABA BABABA BABABABABABABABABABABAB
$\Pi.\mathrm{C}_{16}^{i.c.}(2)$	BABA bababA BABA BABA BABA BABA BABA BAB
$\Pi.\mathrm{C}_{17}^{i.c.}(2)$	BABababABABABababaBABABABABABABABABABABA
$\Pi.\mathrm{C}_{18}^{i.c.}(2)$	BA bababABABA BabaBABABababABABA BababABA BABA babaBABA BABABA BababaBA BABABABABABABABABABABABABABABABABABA
$\Pi.\mathrm{C}_{19}^{ic.}(2)$	BABA bababaBABABABA BABABA bababaBABABABA BabababABABABABABABABABABA
$\Pi.\mathrm{C}_{20}^{i.c.}(2)$	BABababaBABABababaBABABababaBABABABABABA
$\Pi.\mathrm{C}^{ic.}_{21}(2)$	BABA babababABABABA BABABABABABABABABABABABABABABAB
$\Pi.\mathrm{C}^{ic.}_{22}(2)$	BABababABABABAbababABABABAbababABABABABA
$\text{II.C}_{23}^{i.c.}(2)$	BABababaBABABAbababABABABABABABABABABABA
$\Pi.C_{24}^{i.c.}(2)$	
$II.C_{25}^{ic.}(2)$	BABababaBABABABABababaBABABABabababABABABAB
$II.C_{26}^{i.c.}(2)$	BAbababABABABABABABABabaaBABABababABABABAB
$\Pi.\mathrm{C}_{27}^{i.c.}(2)$	BABababaBABABABababaBABABABababaABABABAB
$\text{II.C}_{28}^{i.c.}(2)$	BABA bababaBABABABABABABABABABABABABABABABABA
$\Pi.\mathrm{C}^{i.c.}_{29}(2)$	BABababABABABABABABABABABABABABABABABABA
$\Pi.\mathrm{C}_{30}^{i.c.}(2)$	BABababaBABABABAbababaBABABABABABABABABA
$\Pi.\mathrm{C}^{i.c.}_{31}(2)$	BABA bababa BABABA BABA BABABA BABA Bababa BABABA BABABA BabababA BABABA BabababA BABABA BABABA BABABA BABABABABA BABABA BABABABA BABABABA BABABABA BABABABA BABABABA BABABABABA BABABABABA BA
$\Pi.\mathrm{C}^{i.c.}_{32}(2)$	BABababaBABABababaBABABABABababaBABABABA
II (0).C. (0)	7 D
$\Pi.\overline{\bigcirc_{33}}(2)$	БАБарарараБАБАБАБарарарАБАБАБарараБАБАБАБА
$II.C_{34}^{i.c.}(2)$	BABababaABABABababaBABABababaABABABABababaBABABABA
$\mathrm{II.C}^{i.c.}_{35}(2)$	BABAbababABABABABABABABababaBABABABabababABABABAB
$\Pi.\mathrm{C}^{i.c.}_{36}(2)$	BABababaBABABABABABABABABABABABABABABABA
	ABabababABA
$\Pi.\mathrm{C}^{i.c.}_{37}(2)$	BABabababABABABabababABABabababABABABABA
11 Ci.c.(9)	ABABABababABA BABehered ABABABehered ABABAbeherek ABABAkerek ABABABABABABABABABABABARekerek BABABeherek BABABABABAKER
11: (38 (2)	радабабабарды жылып барарды жылып барарды жылып барарды жылып барарды жылып барарды жылып барарды жылып барард abABABABAbababABA
$\mathrm{II.C}^{i.c.}_{39}(2)$	BABabababABABABabababaBABABABAbababaBABABABA
$\mathrm{II.C}_{40}^{i.c.}(2)$	AbabababABABABABababaBABABABABABAbabababABABABAB
	abababABABabababaBABA

Table S. LXXI. The free group elements for the periodic three-body orbits.

Class and number	mber free group element
$\overline{\mathrm{II.C}_{41}^{i.c.}(2)}$	BABab ABAba
$\Pi.C_{42}^{i.c.}(2)$	BABAbababABABABABABABABABABABABABABABABA
$\text{II.C}_{43}^{i.c.}(2)$	BABababaABABABABABABababaABABABababaBABABABA
$\text{II.C}_{44}^{i.c.}(2)$	BABAbabababABABABABABABABABABABABABABABA
$\Pi.\mathrm{C}^{i.c.}_{45}(2)$	BABababaBABABABababaBABABABababaBABABABA
$\Pi.C_{46}^{i.c.}(2)$	BABababaABABABababaBABABABababaBABABABababaBABABababaBABABABababABABABababABABABababaBABABABababABABABAB
$\mathrm{II.C}_{47}^{i.c.}(2)$	BABababaBABABABABababaBABABABABABABABABA
$\mathrm{II.C_{48}^{i.c.}(2)}$	BABababaBABABAbababaBABABababaBABABABababaBABABABA
$\text{II.C}_{49}^{i.c.}(2)$	BABAbabababABABABABABABABABABABABABABABA
$\mathrm{II.C}_{50}^{i.c.}(2)$	BABAbabababABABABabababABABABABABABABABA
$\mathrm{II.C}_{51}^{i.c.}(2)$	BABababaBABABABABABABABABABABABABABABABA
$\Pi.C^{i.c.}_{52}(2)$	BABabababABABABABABABABABABABABABABABABA
$\Pi.C^{i.c.}_{53}(2)$	BABAbababABABABABABABABABABABABABABABABA
$\Pi.C^{i.c.}_{54}(2)$	BABababaBABABAbabaBABABabababABABABABababaABABABAB
$\mathrm{II.C}_{55}^{i.c.}(2)$	BABAbababABABABABABABABABABABABABABABABA
$\mathrm{II.C}_{56}^{i.c.}(2)$	BAbababaBABABababaBABABababaBABABababaBABABababaBABABABababaBABABababaBABABABA
$\mathrm{II.C}_{57}^{i.c.}(2)$	BABababaBABABABABababaBABABABABABABABABA
$\text{II.C}_{58}^{i.c.}(2)$	BABabababABABABabababABABabababABABABabababABABABAbababABABAbababaBABABABA
$\Pi.\mathrm{C}^{i.c.}_{59}(2)$	BABabababABABABabababABABABabababABABABAB
$\Pi.C_{60}^{i.c.}(2)$	BABAbababaBABABABABababaBABABABababaBABABABabababBABABABA

Table S. LXXII. The free group elements for the periodic three-body orbits.

Class and number f	Free group element
$\overline{\mathrm{II.C}}_{61}^{i.c.}(2)$	BABabababABABABababaBABABAbababaBABABAbababaBABABABababABABABabababABABABabababABABABabababABABABabababABABAbababaBABABABA
$\Pi.\mathrm{C}^{i.c.}_{62}(2)$	BABAbabababABABABAbabababABABABABABABABA
$\mathrm{II.C}^{i.c.}_{63}(2)$	BABAbabababABABABABabababABABABABabababABABABAB
$\Pi.\mathrm{C}^{i.c.}_{64}(2)$	BABabababABABABABababaBABABABabababABABABabababABABABAB
$\mathrm{II.C}^{i.c.}_{65}(2)$	BABabababABABABABABABABABABababaBABABABabababBABABABA
$\mathrm{II.C}^{i.c.}_{66}(2)$	BABAbabababABABABABABABABABABABABABABABababaBABABABabababaBABABABA
$\mathrm{II.C}^{i.c.}_{67}(2)$	BABabababABABAbababaBABABABABABABABABABA
$\mathrm{II.C}^{i.c.}_{68}(2)$	BABAbababaBABABABABABABABABABABABABABABA
$\mathrm{II.C}^{i.c.}_{69}(2)$	
$\Pi.\mathrm{C}^{i.c.}_{70}(2)$	BABAbabababABABABAbabababABABABABabababABABABAB
$\Pi.C_{71}^{i.c.}(2)$	
$\mathrm{II.C}_{72}^{i.c.}(2)$	BABAbabababABABABabababaBABABABabababaBABABABA
$\Pi.\mathrm{C}_{73}^{i.c.}(2)$	BABabababABABABAbababaBABABABabababABABABAB
$\Pi.\mathrm{C}_{74}^{i.c.}(2)$	BABababaBABABABABababaBABABABababaBABABABA
$\Pi.\mathrm{C}^{i.c.}_{75}(2)$	BABAbababaBABABABAbababaBABABABababaBABABABA
$\Pi.\mathrm{C}_{76}^{i.c.}(2)$	BABabababABABABABABABABABABABAbababABABABAbababABABABabababABABABAB
$\Pi.\mathrm{C}^{i.c.}_{77}(2)$	BABAbababaBABABABabababABABABAbababaBABABABA
$\Pi.C_{78}^{i.c.}(2)$	
$\Pi.C_{79}^{i.c.}(2)$	BABabababABABABABababaBABABABABabababABABabababaBABABABabababab
$\Pi.C_{80}^{i.c.}(2)$	BABAbabababABABABABABABABABABABABABABABA

Table S. LXXIII. The free group elements for the periodic three-body orbits.

abababa	
ABabababABABABABababababABABABABABABABAB	
DABADADADADADADADADADADADADADADADADADAD	$\text{II.C}_{95}^{i.c.}(2)$
BABAbabababABABAbabababABABABabababABABABAB	$\text{II.C}_{94}^{\circ}(2)$
abababaBABABABABabababABABABABABABABABAB	
BABAbabababABABABABABABABABABABABABABABA	$\Pi.\mathrm{C}^{i.c.}_{93}(2)$
BABADababaBABABABABABABABABABABABABABABABABAB	$11. \bigcirc_{92} (2)$
ABABABABABABABABABABABABABABABABABABAB	11 (6, (6, (6)
BABabababaBABABABababaBABABABababaBABABABA	$\Pi.C_{91}^{i.c.}(2)$
bADABABABABABABABABABABABABABABABABABABA	II. $\bigcirc 90 \ (2)$
ababababAbABABABABABABABABABABABABABABAB	3
babababAbabababABABAbabababABABABAbabababABABABAB	$\Pi.C_{89}^{i.c.}(2)$
BABabababABABABABABABABABABababABABABabababABABABababaBABABABA	$\Pi.C_{88}^{e.e.}(2)$
ABAbababaABABABABababaBABABABABABABABABA	
BABabababaBABABABababaBABABABabababABABABAB	$\text{II.C}^{i.c.}_{87}(2)$
aBABABABabababABABABAbababABABABABABABAB	98)
abedownshandanananananananananananananananananan	11 (2):0:0
BABAbabababABABABAbabababABABABAbabababABABABAB	$\Pi.C^{i.c.}_{85}(2)$
BABabababaBABABABababaBABABABababABABABAB	$\Pi.\mathrm{C}^{v.c.}_{84}(2)$
BABA bababa BABABA BABA BABABABABA BABABABABABABAB	$\text{II.C}_{83}^{i.c.}(2)$
BABAbabababABABABABabababABABABABABABABA	$\Pi.\mathrm{C}^{i.c.}_{82}(2)$
BABabababABABABabababABABABABAbababaBABABABA	$\mathrm{II.C}^{v.c.}_{81}(2)$
Imber free group element	Class and number

Table S. LXXV. The free group elements for the periodic three-body orbits.

Class and number	mber free group element
$\mathrm{I.A}_{27}^{i.c.}(4)$	
$\mathrm{I.A}_{28}^{i.c.}(4)$	babababaBABABABABababababaBABABA BABABAbababababABABABABABAbababababABABABAB
$\mathrm{I.A}_{29}^{i.c.}(4)$	BABABABABABABABABABABABABABABABABABABA
$\mathrm{I.A}_{30}^{i.c.}(4)$	DADADADADADADADADADADADADADADADADADA BABABAbababababaBABABABABABABAbabababaBABABABA
$\mathrm{I.A}_{31}^{i.c.}(4)$	BABABABABABABABABABABABABABABABABABABA
$I.A_{32}^{i.c.}(4)$	BABABABABABABABABABABABABABABABABABABA
$\mathrm{I.A}_{33}^{i.c.}(4)$	BABABAbabababABABABABABABABababababaBABABABA
$\Gamma B_1^{i.c.}(4)$ $\Gamma B_2^{i.c.}(4)$	BABABabababababABABABABABabababababABABA BABAbababababababABABABABABabababababaBABABA
$I.B_3^{i.c.}(4)$	BABABAbabababababABABABABABABababababaBABABA PAPAPALALALAPAPAPAPAPAPAPAPAPAPAPAPAPA
$I.B_4^{i.c.}(4) $ $I.B_5^{i.c.}(4)$	bABABABababababABABABABABABABABABABABABA
$egin{aligned}  ext{I.B}_{6}^{i.c.}(4) \  ext{I.B}_{7}^{i.c.}(4) \end{aligned}$	BABABAbabababababaBABABABABABABababababABABABAB
$\mathrm{I.B}_8^{i.c.}(4)$	BABABABABabababababaBABABABABABABabababab
$I.B_9^{i.c.}(4)$	BABABababababaBABABABABABABababababaBABABABA
$\mathrm{I.B}_{10}^{i.c.}(4)$	BABABABabababaBABABABABABABababababaBABABABA
$\mathrm{I.B}_{11}^{i.c.}(4)$	BABABAbababababaBABABABABABABabababababa
$\mathrm{I.B}_{12}^{i.c.}(4)$	BABABAbababababababaBABABABABABAbabababa
$\mathrm{I.B}_{13}^{i.c.}(4)$	BABABAbabababababABABABABABABAbababababa
$I.B_{14}^{i.c.}(4)$	BABABABabababababaBABABABABABABababababa
$\mathrm{I.B}_{15}^{i.c.}(4)$	BABABababababababaBABABABABAbabababababa

Table S. LXXVI. The free group elements for the periodic three-body orbits.

I.Bi²c² (4)       BABABababab         I.Bi²c² (4)       BABABABABA         I.Bi²c² (4)       BABABABABA         I.Bi²c² (4)       BABABABABAB         I.Bi²c² (4)       BABABABABAB         I.Bi²c² (4)       BABABABABAB         I.Bi²c² (4)       BABABABABB         I.Bi²c² (4)       BABABABABB         I.Bi²c² (4)       BABABABABAB         I.Bi²c² (4)       BABABABABAB         I.Bi²c² (4)       BABABABABAB         I.Bi²c² (4)       BABABABABAB         BABABABABABABABABABABABABABABABABABABA	BABARABABABABABABABABABABABABABABABABAB
	Ale Abeababababababababababababababababababa
	BABABABababababababababababababababababa
	Parababababababababababababababababababa
	bababababaBABABABABABABABABABABABABABABA
	bababababababababababababababababababa
	babababababaBABABABABABABABababababaBABABABA
	babababababaBABABABABABABABABABABABABABA
	abababaBABABABABABABABABABABABABABABABabababab
	bababababABABABABABABABababababaBABABABA
	STATEST AND A DATE A DATE A DEFENDANCE A DEFENDANCE OF A DATE A D
$1.B_{26}^{i.c.}(4)$ $BABABAbabal$ abababababab $b$	арарарараран АБАБАБАБАБАБАБАБАБАБАБАБАБАБАБАБАБАБАБ
$1.B_{27}^{i.c.}(4)$ BABABAbabal ABABABAbab BABABABabal	BABABABabababababaBABABABABABABababababa
$1.B_{28}^{i.c.}(4)$ BABABABaba ABABAbababa BAbabababa	BABABABababababababABABABABABABAbabababa
II. $A_1^{i.c.}(4)$ BABABababab ABABA	BABABababababababaBABABABABabababababaBABABABA
$II.A_2^{i.c.}(4)$ BABABabab ABABABAB	BABABababababababaBABABABABAbabababababABABABAB
	BABABABababababababaBABABABABABABABabababaBABABABA
$\mathrm{II}.B_{3}^{c.c}(4)$ $\mathrm{BABABababab}$ $\mathrm{II}.B_{3}^{d.c.}(4)$ $\mathrm{BABABABABababa}$ $ABABABABABABABABABABABABABABABABABABAB$	BABABabababababaBABABABABABabababaBABABABA
	BABABababababababababABABA
	BABABAbabababababaBABABABABababababababa
$\text{II.C}_4^{\circ, \cdots}(4)$ $\text{BABABA}$ bababababaB $\text{II.C}_4^{\circ, \cdots}(4)$ $\text{BABABA}$ babababababaBABABAAAAAAAAAAAAAAAAAAAAAA	BABABAbabababababaBABABABABABABabababaBABABABA

Table S. LXXVII. The free group elements for the periodic three-body orbits.

<ul> <li>II. C<sub>1</sub><sup>c.c.</sup> (4) BABABABababababababababababababababababa</li></ul>	BABABABabababababaBABABABABABABABABABABA
	3AbababababababABABABABABABABABABABabababab
	abababABABABABABABABABABABABABABABABABAB
	abababABABABABABABABABABABABABABABABABAB
	abababaBABABABABABAbabababababABABABABAB
	a bababababABABABABABABabababababaBABABABA
	BABABABababababababaBABABABABABababababa
	BABABAbababababababABABABABABABABabababab
	BABABabababababaBABABABABAbababababaBABABABA
	BABABAbabababababaBABABABABABabababababa
	BABABABabababababababaBABABABABABabababab
II. $C_{14}^{i,c}$ .(4) BABABababababaBABABABABABABAbabababababa	BABABababababaBABABABABABabababababABABABAB
II.C <sup>i,c.</sup> (4) BABABABabababababaBABAB BABABABababababababABABA ABABabababababABABABA	BABABABabababababababABABABABABABAbabababa
II.C <sub>16</sub> .(4) BABABAbababab babababaBABABABABABABABABABABABABABABABA	BABABAbabababababaBABABABABABAbababababa
II.C' <sub>17</sub> .(4) BABABAbababab bababaBABAB. ABABABabab	BABABAbababababababABABABABABABabababaBABABABA
$II.C_{18}^{i.c.}(4)$ BABABAbababab Babababababababababababababababa	BABABAbabababababaBABABABABABAbababababa
II. $C_{19}^{i.c.}(4)$ BABABAbababab BABABAbababab ABAbabababababababababababababababababab	BABABAbabababababABABABABABABAbababababa

Ologo pad sustain	
Class and number	per rree group element
$I.A_1^{t.c.}(5)$	BABABABababababababaBABABABABABABABabababab
$1.A_2$ (3)	DABABABABABABABABABABABABABABABABABABAB
$\mathrm{I.A}_3^{i.c.}(5)$	BABABABabababababababaBABABABABABABABAbabababa
$\Gamma A^{i.c.}(5)$	ababababABABABABABABABababababababababABABABAB
*	abababababABABABABABABABABabababababABABABAB
$\mathrm{I.A}_5^{i.c.}(5)$	BA
	ababababababABABABABABABABABabababababab
$\mathrm{I.A}_6^{i.c.}(5)$	BABABABabababababaBABABABABABABABABabababab
$\Gamma$ $\Lambda^{i.c.}(eta)$	bababababaBABABABABABABAbabababababababa
L.A.7 (9)	bADADADADADADADADADADADADADADADADADADAD
	DADAOBOBOBOBOBOBOBADADADADADADADADAOBOBOBOB
$\Gamma.\mathrm{B}_{1}^{i.c.}(5)$	BABABABAbabababababababABABABABABABABABabababab
$\text{L.B}_2^{}(5)$	БАБАБАБарарарарарараБАБАБАБАБАБАБАБАБарарарар
$\text{I.B}_3^{i.c.}(5)$	BABABABABabababababababABABABABABABABABabababab
$\mathrm{I.B}_4^{i.c.}(5)$	arangananan Angarah BABABABabababababababABABABABABABABABabababab
	ababababaBABABABABABABababababababaBABABABA
$\mathrm{LB}_5^{c.c.}(5)$	BABABABABababababababaBABABABABABABABABA
$\text{I.B}_6^{i.c.}(5)$	BABABABababababababABABABABABABABabababab
	ababAbAbAbAbAbababababababababAbAbAbAbA
$\mathrm{I.B}_7^{i.c.}(5)$	BABABABababababababababababababababababa
	ababaBABABABABABABabababababababaBABABABA
$I.B_8^{i.c.}(5)$	BABABABAbabababababababababaBABABABABABA
	ababababababababABABABABABABABABABabababab
$\mathrm{I.B}_9^{i.c.}(5)$	BABABABABABABABABABABABABABABABABABABA
	aBABABABABABABAbabababababababaBABABABAB
$\mathrm{I.B}_{10}^{i.c.}(5)$	BABABABababababababaBABABABABABABabababab
	ABabababababababaBABABABABABABababababab
$\Gamma.\mathrm{B}_{11}^{i.c.}(5)$	BABABABABababababababaBABABABABABABABABA
$\mathrm{II.B}_{1}^{i.c.}(5)$	${ m BABABABababababaBABABABABABABABababababa$

Table S. LXXIX. The free group elements for the periodic three-body orbits.

	BABABABABabababababaBABABABABABABABABABA
	ABabababababababaBABABABABABABABABABABAB
	АБАР ТЕТЕТ ТЕТ
	BAbabababababaBABABA
	BABABABABabababababABABABABABABABAbabababa
	bababababababababababaBABABABABABABABAbabababa
	BABABABABAbababababaBABABABABABABABABABA
	BABABABABabababababababaBABABABABABABABA
	BABABABABabababababababababababababababa
	ABABabababababababABABABABABABABabababab
	ababABABABABABABababababababABABA
$\mathrm{II.C_{9}^{occ}(5)}$ $\mathrm{BABABA}$ $\mathrm{baBABAA}$	BABABABababababababABABABABABABababababa
$\text{II.C}_{10}^{i.c.}(5)$ BABABA ababABA babababa	BABABABabababababababABABABABABABAbabababa
$\text{II.C}_{11}^{i.c.}(5)$ BABABA abababA Babababa	BABABABabababababababababaBABABABABABabababab
II. $C_{12}^{i.c.}(5)$ BABABA abaBABA babababa	BABABABabababababababABABABABABABabababab
$\text{II.C}_{13}^{i.c.}(5)$ BABABA abababab	BABABABABababababababaBABABABABABABABABA
II.C <sup>i.c.</sup> (5) BABABABab ababaBABA BABAbababal abABABABA	BABABABAbababababababaBABABABABABABabababab
II.C <sup>i.c.</sup> <sub>15</sub> (5) BABABA abababab BABABA abababab	BABABABabababababababABABABABABABABAbabababa
II.C $_{16}^{i.c.}(5)$ BABABA abababababababABABABABABABABABABABABABAB	BABABABABababababababaBABABABABABABABABabababab

$I.A_1^{i.c.}(8)$	
$\mathrm{I.A}_2^{i.c.}(8)$	ABABABABABABAbabababababababababababABABABAB
$I.A_3^{i.c.}(8)$	BABABABABABABbabababababababababababABABABAB
$\operatorname{I.A}_4^{i.c.}(8)$	BABABABABABABABABABABABABABABABABABABA
$\mathrm{LB}_1^{i.c.}(8)$ $\mathrm{LB}_2^{i.c.}(8)$	BABABABABABAbabababababababababababababABABABAB
$I.B_3^{i.c.}(8)$	BABABABABABababababababababababababababa
$ ext{II.B}_1^{i.c.}(8)$	BABABABABABABABABABABABABABABABABABABA
$\Pi.C_1^{i.c.}(8)$ $\Pi.C_2^{i.c.}(8)$ $\Pi.C_3^{i.c.}(8)$	BABABABABABAbabababababababababababababa
$\Pi.\mathrm{C}_4^{i.c.}(8)$	BABABABababababababababababababaBABABABA
$\mathrm{II.C}_5^{i.c.}(8)$	BABABABABABAbababababababababababababaBABABABA
$\mathrm{II.C}_6^{i.c.}(8)$	BABABABABABABabababababababababababababa
$\Pi.C_7^{i.c.}(8)$	BABABABABABAbabababababababababababababa
$\Pi.C_8^{i.c.}(8)$	BABABABABABAbabababababababababababababa
$\Pi.C_9^{i.c.}(8)$	BABABABABABABabababababababababababababa

Table S. LXXXI. The free group elements for the periodic three-body orbits.

Class and number	r free group element
$I.A_1^{i.c.}(10)$	BABABABABABABABababababababababababababa
$\mathrm{I.A}_2^{i.c.}(10)$	BABABABABABABABababababababababababababa
$\mathrm{I.A}_3^{i.c.}(10)$	BABABABABABABABABabababababababababababa
	ABABABABABABABABABABabababababababababab
$\mathrm{I.B}_1^{i.c.}(10)$	BABABABABABABABABabababababababababababa
$\mathrm{II.A}_{1}^{i.c.}(10)$	BABABABABABABABAbabababababababababababa
$\Pi.B_1^{i.c.}(10)$	BABABABABABABABABABababababababababababa
$\mathrm{II.C}_1^{i.c.}(10)$	BABABABABABABABABABABabababababababababa
$\mathrm{II.C}_2^{i.c.}(10)$	BABABABABABABABABABABABababababababababa
$\Pi.C_3^{i.c.}(10)$	BABABABABABABABABabababababababababababa