Please complete ALL parts of each of the problems below. Be sure to show your work or explain your process to get full credit on each problem. Much of this homework is coming from sections 2.1 and 2.4 if you would like to look up definitions or additional hints!

1. Among a group of 5 people, is it possible for everyone to be friends with exactly 2 of the people in the group? What about 3 of the people in the group? Explain why or why not and what this has to do with graph threory.

2. (a) Is it possible for two different (non-isomorphic) graphs to have the same number of vertices and the same number of edges? If so give an example, if not explain why not.

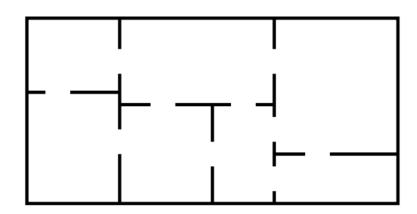
(b) What if the degrees of the vertices in the two graphs are the same (so both graphs have vertices with degrees 1, 2, 3, and 4, for example)? Draw two such graphs or explain why not.

3. For which n does  $K_n$ , the complete graph on n vertices, contain an Euler circuit? **Explain**.

4. For which m and n does the graph  $K_{m,n}$ , the complete bipartite graph with a set of m and a set of n vertices, contain an Euler trail? An Euler circuit? **Explain**.

5. For which n does  $K_n$ , the complete graph on n vertices, contain a Hamilton path? A Hamilton cycle? **Explain**.

6. Edward A. Mouse has just finished his brand new house. The floor plan is shown below:

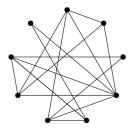


(a) Edward wants to give a tour of his home to a friend. Is it possible for them to walk through every doorway exactly once? If so, in which rooms must they begin and end the tour? **Explain**.

(b) Is it possible to tour the house visiting each room exactly once (not necessarily using every doorway)? **Explain**.

(c) Edward is now thinking about possible future remodeling. Is it possible for him to add doors in such a way that each room has an odd number of doors? **Explain**.

7. Below is a graph representing friendships between a group of students (each vertex is a student and each edge is a friendship). Is it possible for the students to sit around a round table in such a way that every student sits between two friends? **Explain**. (Hint: connect this question to Euler or Hamiltonian paths or circuits)



8. On the table rest 8 dominoes, as shown below. If you were to line them up in a single row, so that any two sides touching had matching numbers, what would the sum of the two end numbers be? **Explain**.

