# NOTES FOR KPZ EQUATION

## Based on the a survey by J.Quastel

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# 1 Mathematical Fundamentals

## 1.1 White Noise and Stochastic Integration in 1+1 dimensions

**Definiton 1.1.1.** (Time-Space White Noise)

White noise $\xi(t,x), t \geq 0, x \in \mathbb{R}$ is the distribution valued Gaussian process with mean zero and covariance

$$E[\xi(t,x)\xi(s,y)] = \delta(t-s)\delta(x-y)$$

which means we have a family of random variables

$$\left\{ \int \xi(t,x) f(t,x) dx dt \right\}_{f \in L^2(\mathbb{R}^+ \times \mathbb{R})}$$

**Proposition 1.1.1.** For an orthonormal basis $f_1, f_2, \cdots$ of $L^2(\mathbb{R}^+ \times \mathbb{R})$ and independent Gaussian random variables $Z_1, Z_2, \cdots$ each with mean zero and variance 1, and then

$$\xi(t,x) = \sum_{n=1}^{\infty} Z_n f_n(t,x)$$

is a time-space white noise required in above defintion.

# 2 Appendix

## 2.1 Functional Analysis

**Theorem 2.1.1.** (Existence of Orthonomal basis in $L^2$)

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