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## **NOTES FOR ANALYSIS AND PDES**

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**Based on the Math 719(Cole)/720(M.Ifrim),  
Folland and etc.**

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# 1 Sobolev Spaces

## 1.1 Sobolev Spaces

**Definiton 1.1.1.** Denote  $\mathcal{D}(U)$  to be the test functions. Let  $u \in \mathcal{D}'(U)$ , where  $U \subset \mathbb{R}^d$  is open. The  $k$ -th order  $L^p$ -based Sobolev norm of  $u$  is

$$\|u\|_{W^{k,p}(U)} := \sum_{|\alpha| \leq k} \|D^\alpha u\|_{L^p}, \quad 1 \leq p < \infty$$

where we are using the distributional derivative and assume that  $D^\alpha u$  is an  $L^p$  function.