Homework0 - Paquette

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Exercise.1

Show the Woodfury formulam for n-dimensional vectors U, V and a square amtrix A

$$R(z; A + UV^T) - R(z; A) = -\frac{R(z; A)UV^T R(z; A)}{1 + V^T R(z; A)U}$$

with $z \in \mathrm{Spec}(A + UV^T), \mathrm{Spec}(A)$.

Proof. Notice

$$(A + UV^T - zI)(R(z; A + UV^T) - R(z; A))(A - zI) = -UV^T$$

and hence

$$(A + UV^T - zI)(R(z; A + UV^T) - R(z; A)) = -UV^T R(z; A)$$

and it remains to show

$$R(z; A)U = (1 + U^T R(z; A)V)R(z; A + UV^T)U$$

assume $R(z; A)U = a$, $R(z; A + UV^T)U = b$ and we may have

$$b = a - bV^T a$$

and hence

$$b = a/(1 + V^T a)$$

and we are done. □

Exercise.2

Show the directional derivative of $R(z; A)$ in its A variable in the direction of V is

$$\lim_{\epsilon \to 0} \epsilon^{-1}(R(z; A + \epsilon))$$

which there fore gives us an expression for all partial derivatives in A.

Proof. We know

$$\epsilon^{-1}(R(z; A + \epsilon B) - R(z; A)) = -R(z; A + \epsilon B)BR(z : A)$$

and we are done. □

Exercise.3

Suppose that S is a symmetric matrix, G is GOE and set $A = SGS$. Show that for z with h

Proof. We know

$$\epsilon^{-1}(R(z; A + \epsilon B) - R(z; A)) = -R(z; A + \epsilon B)BR(z : A)$$

and we are done. □