NOTES FOR ANALYSIS AND PDES

Based on the Math 719(Cole)/720(M.Ifrim), Folland and etc.

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Contents

1 Stochastic Integrals

1.1 Wiener Integral

Let T be a set and $X := \{X(t)\}_{t\in T}$ a T-indexed stochastic process. We recall that X is a Gaussian random field (process when $T \subset \mathbb{R}$) if $(X_{t_1}, \cdots, X_{t_m})$ is a Gaussian random vector for all $t_1, \cdots, t_m \in T$.

Definiton 1.1.1. Let $\mathscr{L}(\mathbb{R}^m)$ denote the collection of all Borel-measurable subsets of $\mathbb{R}^m$ that have finite Lebesgue measure. White noise on $\mathbb{R}^m$ is a mean-zero set-indexed Gaussian random field $\xi(A)_{A\in\mathscr{L}(\mathbb{R}^m)}$ with covariance function

$$E[\xi(A_1)\xi(A_2)] := |A_1 \cap A_2| \quad \text{for all } A_1, A_2 \in \mathscr{L}(\mathbb{R}^m),$$

where $|\cdot|$ denotes the Lebesgue measure on $\mathbb{R}^m$ for every m.