TTMath School

Level 3: Algebra Test 1

Name (First and Last):

Instructions

1. Show **all** your work.
2. Calculators are NOT allowed.
3. Please leave your answers in exact form. For example. $\pi + \pi = 2\pi$ is correct while 6.28 is not.
4. Please simplify all your answers and rationalize your denominators.
   For example, $\frac{1}{\sqrt{12}}$ and $\frac{\sqrt{12}}{12}$ are both incorrect while $\frac{\sqrt{3}}{6}$ is correct.

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1. (a) [5 points] Simplify the expression

$$\frac{(6^3 a^{\frac{1}{2}} b^{\frac{2}{3}})(15ab)^{-\frac{1}{3}}}{(8^5 a^{-\frac{1}{2}} b^{\frac{1}{3}})}$$

(b) [5 points] Solve the equation

$$4^{2x^2+2x} = 4^x + 4^x$$

2. Rationalize and fully simplify the following expressions

(a) [3 points] $\frac{1}{\sqrt[3]{20736}}$

(b) [3 points] $\frac{\sqrt{12}-\sqrt{8}}{\sqrt{12}+\sqrt{8}}$

(c) [4 points] $\frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$

3. (a) [5 points] The gravitational constant is about $7 \times 10^{-11} \frac{\mathrm{m}^3}{\mathrm{kg}^1 \mathrm{s}^2}$. Convert this to $\frac{\mathrm{km}^3}{\mathrm{g}^1 \mathrm{min}^2}$.

(b) [5 points] Solve the equation $2ax + b = a^2x + a$ for all real values of x.

4. Fully factor the following expressions into factors with integers coefficients.

(a) [2 points] $x^3 + 7x^2 + 2x + 14$

(b) [2 points] $36x^3 + 33x^2 + 6x$

(c) [2 points] $2a^5b^2 + 8a^4b^3 + 8a^3b^4$

(d) [2 points] $a^3b - ab^3$

(e) [2 points] $a^{12} - b^{12}$

5. (a) [2 points] Fully factor the expression $x^4 - x^2 - 2$ into factors with integer coefficients.

(b) [3 points] Fully factor this expression $x^4 - x^2 - 2$ into factors with real coefficients.

(c) [5 points] Solve the equation $1 = 4x^2 + 2x^4$ for all real solutions. Do not de-nest radicals.

6. (a) [5 points] Solution Y is 30 percent liquid X and 70 percent water. If 2 kilograms of water evaporate from 8 kilograms of solutions Y and 2 kilograms of solution Y are added to the remaining 6 kilograms of liquid, what percent of this new liquid solution is liquid X?

   (b) [5 points] Jack drives from his house (J) to his Grandfather's house (G). Some of this road is on flat ground and some is downhill or uphill. His car travels downhill at 99 km/h, on flat ground at 77 km/h, and uphill at 63 km/h. It takes Jack 3 hours and 40 minutes to drive from J to G. It takes him 4 hours and 20 minutes to drive from G to J. Find the distance between J and G in kilometers.

7. (a) [2 points] Complete the square of the expression $-y^2 - y + 2$.

   (b) [8 points] Sketch the graph $2x = -y^2 - y + 2$. Remember to label the important points.

8. For this problem, all your answers must be of the form $x = 2y^2 + by + c$ where b and c are real number constants. Express your answer in standard form.

   (a) [2 points] Find the quadratic that has $y = 1 + \sqrt{11}$ and $y = 1 - \sqrt{11}$ as its roots.

   (b) [2 points] Find the quadratic that has its vertex at $(10, 0)$.

   (c) [3 points] Find the quadratic that has a vertex at $(-1, 2)$.

   (d) [3 points] Find the quadratic that passes through the points $(1, 1)$ and $(1, 3)$.

9. Consider the quadratic $y = 2x^2 + 4x + 5$ and let a and b be the roots. Solve this problem without explicitly finding the roots.

   (a) [2 points] Find the value of $\frac{1}{a} + \frac{1}{b}$.

   (b) [4 points] Find the value of $a^3 + b^3$.

   (c) [4 points] Find the value of $a^5 + b^5$.

10. Solve the following equations/system of equations

(a) [3 points] $\frac{x}{x-4} - \frac{4}{x+5} = \frac{36}{x^2+x-20}$

(b) [3 points] $\sqrt{3x-5} + \sqrt{x-1} = 2$

(c) [4 points] $2^{x-2} + 2^{3-x} = 3$

11. [5 points] Solve the equation $x^4 - 11x^3 + 24x^2 = 4x^2 - 44x + 96$.

12. [5 points] Find the real values of K for which the equation $x = K^2(x-1)(x-2)$ has real roots.

13. [5 points] Simplify the nested radical $\sqrt{8+\sqrt{63}}$.

14. [5 points] Find all values of m for which the zeros of $2x^2 - mx - 8$ differ by $m - 1$.

15. [5 points] Determine the possible values that $\frac{5x^2-4x+8}{x^2+1}$ can attains where x is a real number.