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sheet (1)

1] $f(t) = t^5$

$$F(s) = \frac{5!}{s^6}$$

2] $f(t) = e^{3t} \sinh(5t)$ $\qquad L[e^{\pm kt} f(t)] = F(s \mp k)$

$$F(s) = \frac{5}{(s-3)^2 - 5^2} = \frac{5}{s^2 - 6s - 16}$$

$\boxed{\text{or}}$

$$f(t) = e^{3t} \cdot \left[\frac{e^{5t} - e^{-5t}}{2}\right] = \frac{1}{2}\left[e^{8t} - e^{-2t}\right]$$

$$F(s) = \frac{1}{2}\left[\frac{1}{s-8} - \frac{1}{s+2}\right] = \frac{1}{2}\left[\frac{s+2-s+8}{s^2-6s-16}\right]$$

$$= \frac{5}{s^2 - 6s - 16}$$

3] $f(t) = t^2 e^{3t}$

$$F(s) = \frac{2!}{(s-3)^3}$$

$$= \frac{2}{(s-3)^3}$$

$\boxed{L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} F(s)}$

$$F(s) = (-1)^2 \cdot \frac{d^2}{ds^2} \frac{1}{s-3}$$

$$= 1 \cdot \frac{d}{ds} \frac{-1}{(s-3)^2}$$

$$= \frac{2}{(s-3)^3}$$

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Q3] $f(t) = \frac{\overset{\uparrow g(t)}{\cos(3t) - \cos(2t)}}{t}$ 3rd shift theorem

$$\lim_{t\to 0} \frac{\cos(3t) - \cos(2t)}{t} = \frac{0}{0}$$ undefined

l'hopital's rule

$$\lim_{t\to 0} \frac{-3\sin(3t) - 2\sin(2t)}{t} = \frac{0}{1}$$ ✓ limit exists

$$\therefore L[g(t)] = L[\cos(3t) - \cos(2t)] = \frac{s}{s^2+3^2} - \frac{s}{s^2-2^2}$$

$$= \frac{s}{s^2+9} - \frac{s}{s^2+4} = G(s)$$

$$\therefore L\left[\frac{g(t)}{t}\right] = \int_s^\infty \frac{s}{s^2+9} - \frac{s}{s^2+4}\, ds$$

$\boxed{\int \frac{1}{x}dx = \ln x}$

$$= \frac{1}{2}\ln(s^2+9) - \frac{1}{2}\ln(s^2+4)\Big|_s^\infty$$

$$= \frac{1}{2}\ln\frac{(s^2+9)}{(s^2+4)}\Big|_s^\infty = \frac{1}{2}\ln\frac{(s^2+9)}{(s^2+4)}\Big|^\infty - \frac{1}{2}\ln\frac{(s^2+9)}{(s^2+4)}\Big|_s$$

$\boxed{\lim_{x\to 0}\ln f(x) = \ln \lim_{x\to 0} f(x)}$

$$= \frac{1}{2}\lim_{s\to\infty}\ln\frac{(s^2+9)}{(s^2+4)} - \frac{1}{2}\ln\frac{(s^2+9)}{(s^2+4)}$$

$$= \frac{1}{2}\ln\lim_{s\to\infty}\frac{(s^2+9)}{(s^2+4)} - \frac{1}{2}\ln\frac{(s^2+9)}{s^2+4} = \frac{1}{2}\ln\lim_{s\to\infty}\left(\frac{\infty}{\infty}\right) - \frac{1}{2}\ln\frac{(s^2+9)}{(s^2+4)}$$

$$= \frac{1}{2}\ln\lim_{s\to\infty}\left(\frac{\cancel{2s}}{\cancel{2s}}\right)^1 - \frac{1}{2}\ln\frac{(s^2+9)}{(s^2+4)} = \cancel{\frac{1}{2}\ln(1)}^0 - \frac{1}{2}\ln\frac{(s^2+9)}{(s^2+4)}$$

$$= \ln\left\{\left(\frac{s^2+9}{s^2+4}\right)\right\}^{-1/2} = \ln\sqrt{\frac{s^2+4}{s^2+9}}$$ ⋇

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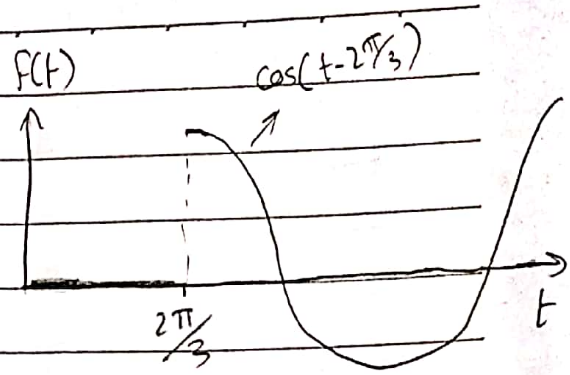
$\boxed{T=1}$. $\cos(t)$

5] $f(t) = H\left(t - \frac{2\pi}{3}\right) \cos(t - 2\pi/3)$

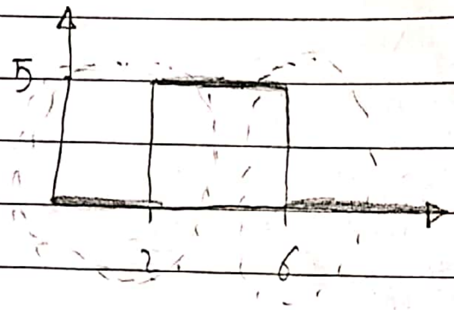
$F(s) = e^{-2\pi/3\, s} \cdot \frac{s}{s^2+1}$



6] $f(t) = H(t-3)\,(t-3)^3 \to t^3$

$F(s) = e^{-3s} \cdot \frac{3!}{s^4}$

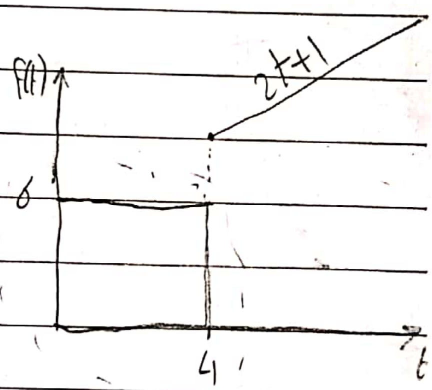
7] $f(t) = \begin{cases} 0 & t<2 \\ 5 & 2<t<6 \\ 0 & t>6 \end{cases}$



$f(t) = 5H(t-2) - 5H(t-6)$

$F(s) = \frac{5}{s} \cdot e^{-2s} - \frac{5}{s} e^{-6s}$

8] $f(t) = \begin{cases} 6, & 0<t<4 \\ 2t+1, & t>4 \end{cases}$



$f(t) = 6H(t) - 6H(t-4) + H(t-4).(2t+1)$

$= 6H(t) - 6H(t-4) + 2t.H(t-4) + H(t-4)$

$= 6H(t) - 6H(t-4) + [2(t-4)+8].H(t-4) + H(t-4)$

$= 6H(t) - 5H(t-4) + 2(t-4)H(t-4) + 8H(t-4)$

$= 6H(t) + 3H(t-4) + 2(t-4)H(t-4) = \frac{6}{s} e^{0s} + \frac{3}{s} e^{-4s} + \frac{2}{s^2} \cdot e^{-4s}$

$= \frac{6}{s} + \frac{3}{s} e^{-4s} + \frac{2}{s^2} \cdot e^{-4s}$

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1] $F(s) = \frac{5}{s^2-6s-16}$ partial fractions or complete squares

$F(s) = \frac{5}{(s-3)^2-9-16} = \frac{5}{(s-3)^2-5^2}$

$f(t) = e^{3t} \cdot sh(5t)$

2] $F(s) = \frac{s}{s^2-6s+13}$

$F(s) = \frac{s}{(s-3)^2+2^2} = \frac{(s-3)}{(s-3)^2+2^2} + \frac{3}{2} \cdot \frac{2}{(s-3)^2+2^2}$

$f(t) = e^{3t} \cdot \cos(2t) + \frac{3}{2} \cdot e^{3t} \cdot \sin(2t)$

3] $F(s) = \ln\sqrt{\frac{s^2+16}{s^2+25}}$ 2nd shift theorem

uses derivatives

use with ln

$F(s) = \frac{1}{2}\ln(s^2+16) - \frac{1}{2}\ln(s^2+25)$

$F'(s) = \frac{1}{2}\left[\frac{2s}{s^2+16} - \frac{2s}{s^2+25}\right]$

$\because L[t\,f(t)] = (-1)\frac{d}{ds}F(s)$ $\therefore L^{-1}[F'(s)] = -t\,f(t)$

$\therefore f(t) = \frac{(-)L^{-1}[F'(s)]}{t} = \frac{-1}{t}[\cos 4t - \cos 5t]$

$= \frac{\cos 5t - \cos 4t}{t}$

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4] $F(s) = \dfrac{2s+3}{s^3+3s^2+2s}$

$f(s) = \dfrac{2s+3}{s(s^2+3+2)} = \dfrac{2s+3}{s(s+1)(s+2)} = \dfrac{A}{s} + \dfrac{B}{s+1} + \dfrac{C}{s+2}$

$\therefore 2s+3 = A(s+1)(s+2) + B(s)(s+2) + C(s)(s+1)$

at $\boxed{s=0}$

$3 = 2A \Rightarrow \boxed{A = 3/2}$

at $\boxed{s=-1}$

$1 = -B \Rightarrow \boxed{B=-1}$

at $\boxed{s=-2}$

$-1 = 2C \Rightarrow C = -1/2$

$F(s) = \dfrac{3/2}{s} + \dfrac{-1}{s+1} + \dfrac{-1/2}{s+2}$

$f(t) = 3/2 - e^{-t} - \frac{1}{2} e^{-2t}$

$\boxed{or}$

Using Heaviside expansion

$L^{-1}\left[\dfrac{P(s)}{Q(s)}\right] = \dfrac{P(a_1)}{Q'(a_1)} \cdot e^{a_1 t} + \cdots$ for $a_1, a_2, a_3 \cdots a_n$

$Q'(s) = 3s^2 + 6s + 2$, $a_1 = 0$, $a_1 = -1$, $a_2 = -2$

$\therefore L^{-1}[F(s)] = \dfrac{P(0)}{Q'(0)} \cdot e^{0t} + \dfrac{P(-1)}{Q'(-1)} e^{-t} + \dfrac{P(-2)}{Q'(-2)} e^{-2t}$

$= \dfrac{3}{2} - (1) \cdot e^{-t} - \dfrac{1}{2} e^{-2t}$

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5] $F(s) = \dfrac{2s+3}{(s-2)^2(s+1)}$

$F(s) = \dfrac{A}{s+1} + \dfrac{B}{(s-2)} + \dfrac{C}{(s-2)^2}$

$\therefore \quad 2s+3 = A(s-2)^2 + B(s-2)(s+1) + C(s+1)$ — (eq.1)

at $\boxed{s=2}$

$7 = 3C \Rightarrow C = 7/3$

at $\boxed{s=-1}$

$1 = 9A \Rightarrow A = 1/9$

Coeff. of s^2 in both sides

$0s^2 = As^2 + Bs^2$

$\therefore \quad A = -B$

$B = -1/9$

differentiate eq. 1

$2 = A.2(s-2) + B(2s-1) + C$

at $\boxed{s=2}$

$2 = B(3) + 7/3$

$3B = 2 - 7/3$

$B = -1/9$

$F(s) = \dfrac{1/9}{s+1} - \dfrac{1/9}{s-2} + \dfrac{7/3}{(s-2)^2}$

$f(t) = \frac{1}{9} . e^{-t} - \frac{1}{9} e^{2t} + \frac{7}{3} . e^{2t} . t$

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7] $F(s) = \dfrac{s-1}{(s^2+2s+2)(s+3)}$

$F(s) = \dfrac{A}{s+3} + \dfrac{Bs+C}{(s^2+2s+2)}$

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8] $F(s) = \frac{s}{(s^2+a^2)^2}$

$G(s) = \frac{1}{s^2+a^2}$

$G'(s) = (-1)\frac{2s}{(s^2+a^2)^2} = -2\,F(s)$ $\quad 2^{nd}$ shift theorem

$\therefore L[t\,g(t)] = (-1)\ G'(s)$

$$\boxed{L^{-1}[G'(s)] = -t.\,g(t) = -2\,L^{-1}[F(s)]}$$

$g(t) = L^{-1}[G(s)] = L^{-1}\left[\frac{1}{a}.\frac{a}{s^2+a^2}\right]$

$= \boxed{\frac{1}{a}.\sin at}$ — ②

$L^{-1}[F(s)] = \frac{t}{2}\,g(t)$ — ③

from ②, ③

$f(t) = (-1)\frac{1}{2}.-t.\frac{1}{a}\sin at = \frac{t}{2a}.\sin at$

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1] $y'' + 2y' + y = 3te^{-t}$, $y(0) = 4$, $y'(0) = 2$

$L[y'] = sY(s) - y(0)$

$L[y''] = s^2 Y(s) - sy(0) - y'(0)$

$\therefore$ by taking Laplace of both sides

$\boxed{t \to \frac{1}{s^2}}$

$[s^2 Y(s) - sy(0) - y'(0)] + 2[sY(s) - y(0)] + Y(s) = \frac{3}{(s+1)^2}$

$Y(s)[s^2 + 2s + 1] - sy(0) - y'(0) - 2y(0) = \frac{3}{(s+1)^2}$

$Y(s)[s^2 + 2s + 1] = \frac{3}{(s+1)^2} + 4s + 10$

$Y(s) = \left[\frac{3}{(s+1)^2} + 4s + 10\right] \cdot \frac{1}{(s^2+2s+1)} \to (s+1)^2$

$Y(s) = \frac{3}{(s+1)^4} + \frac{4s+10}{(s+1)^2} = \frac{3}{(s+1)^4} + \frac{4(s+1) - 4 + 10}{(s+1)^2}$

$= \frac{1}{2} \frac{3 \cdot 2}{(s+1)^4} + \frac{4\cancel{(s+1)}}{(s+1)^{\cancel{2}}} + \frac{6}{(s+1)^2} \to \frac{1}{s^2} \to t$

$\therefore L^{-1}[Y(s)] = y(t) = \frac{1}{2} t^3 \cdot e^{-t} + 4 \cdot e^{-t} + 6 \cdot t \cdot e^{-t}$