HAI601I - Exercices de révisions

Benoît Huftier

2022

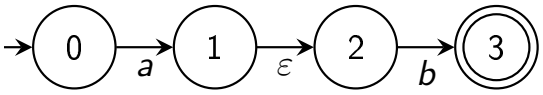
Construction d'un AFD à partir d'un AFN

Enoncé

Donner l'automate fini déterministe (AFD) de certaines des expressions régulières de l'exercice précédent.

1. ab
2. b*
3. a|b
4. ab*|c
5. ((a|b)|cc)*
6. b*a*|(cb)*
7. abc
8. (a|b)|c
9. a|(b|c)

Avec les AFD des expressions régulières (a|b)|c et a|(b|c), que peut-on en déduire sur la règle | ?



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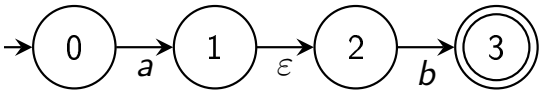
On rappelle que pour créer un AFD à partir d'un AFN, il faut créer des ε-fermetures d'ensemble d'états, en commençant par l'état de départ (ici l'état 0).

$$EpsilonFermeture(\{0\}) = \{0\}$$

En effet, l'ε-fermeture comprend tous les états de l'ensemble donné en paramètre (ici uniquement 0) et comprend également tous les états qui peuvent être atteint avec une ε-transition, il n'y en a aucune ici.



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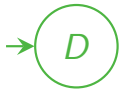

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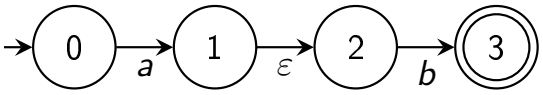
Voici donc D, notre nouvel état de départ, on l'ajoute à notre AFD :

$$D = \{0\}$$



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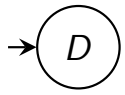

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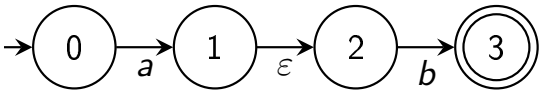
On part maintenant de D, pour créer notre AFD.



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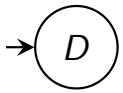
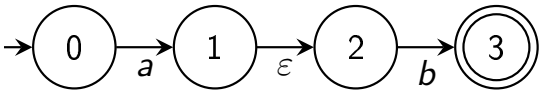


$D = \{0\}$

Etat actuel : $D = \{0\}$.



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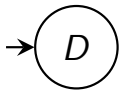

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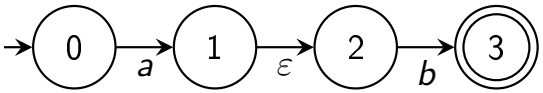
Etat actuel : $D = \{0\}$.
On va parcourir tout l'alphabet de notre vocabulaire et regarder les transitions depuis D.



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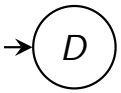

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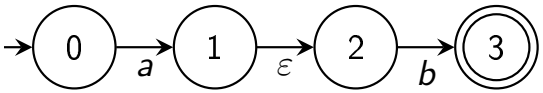
$$EpsilonFermeture(\{1\}) = E_1 = \{1, 2\}$$



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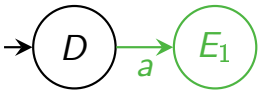

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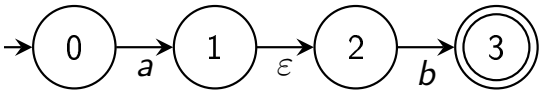

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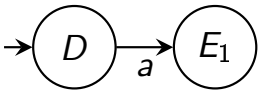

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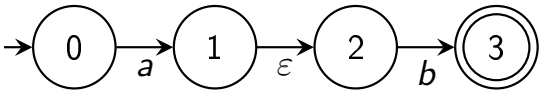

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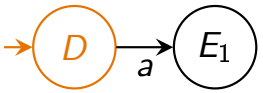

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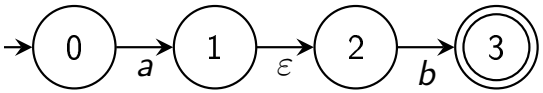

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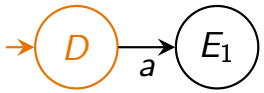

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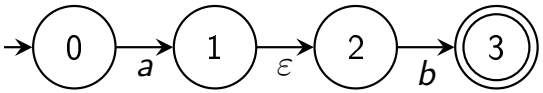
Etat actuel : $E_1 = \{1, 2\}$.



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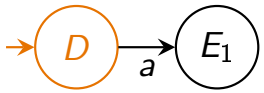

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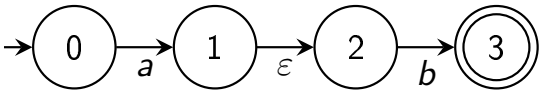

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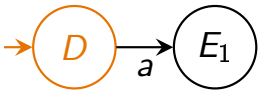

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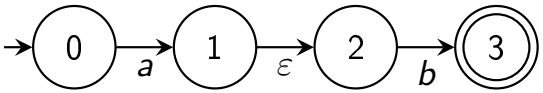

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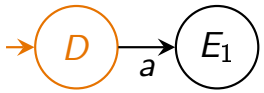

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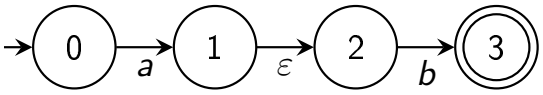
Pour x = b, il y a la transition 2b3, on calcule donc l'ε-fermeture de 3.

$$EpsilonFermeture(\{3\}) = E_2 = \{3\}$$



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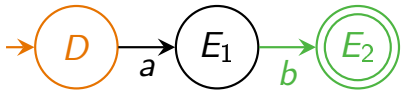

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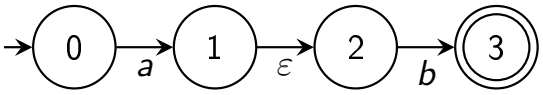

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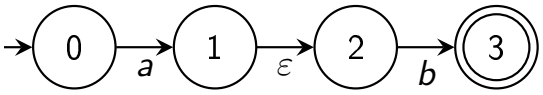

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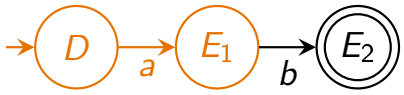

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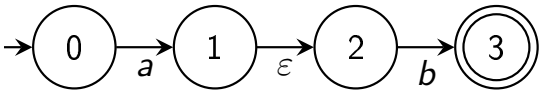
Etat actuel : $E_2 = \{3\}$.



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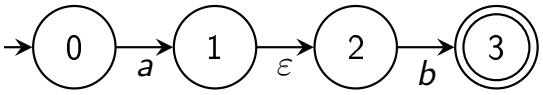
$D = \{0\}$
$E_1 = \{1, 2\}$
$E_2 = \{3\}$

Etat actuel : $E_2 = \{3\}$.
Il n'y a aucune transition depuis l'état 3 donc on marque directement l'état E_2.



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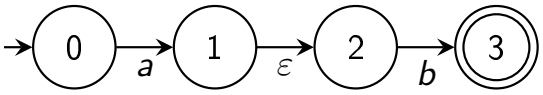
Etat actuel : $E_2 = \{3\}$.
Il n'y a aucune transition depuis l'état 3 donc on marque directement l'état E_2.
Tous les états sont maintenant marqués, l'AFD est terminé.



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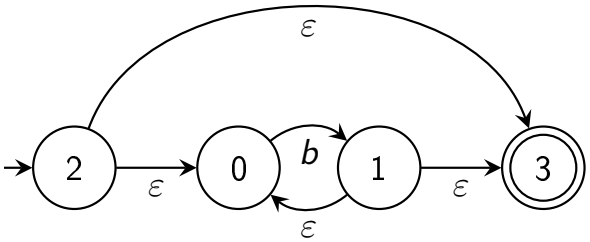
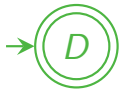
Notez que la correction a été expliquée pour cet exemple mais ne le sera pas pour les prochains.



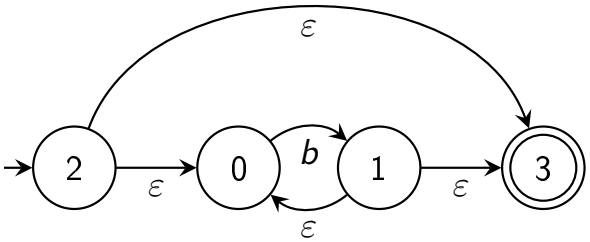
expr = b*



$D = \{0, 2, 3\}$

Calcul de l'état de départ :

$$D = EpsilonFermeture(\{2\}) = \{0, 2, 3\}$$



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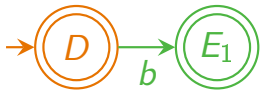
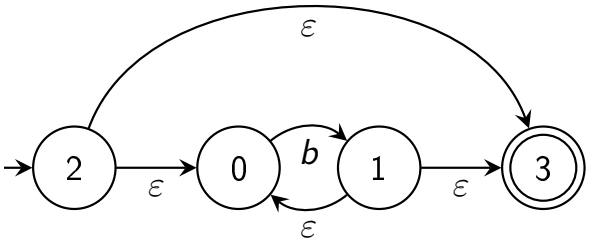


$D = \{0, 2, 3\}$

$E_1 = \{0, 1, 3\}$

Etat actuel : D

- x = b
- transitions : 0b1
- $EpsilonFermeture(\{1\}) = \{0, 1, 3\} = E_1$
- etat final : oui



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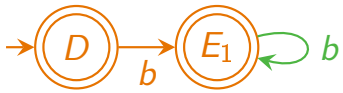

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Etat actuel : E_1

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- transitions : 0b1
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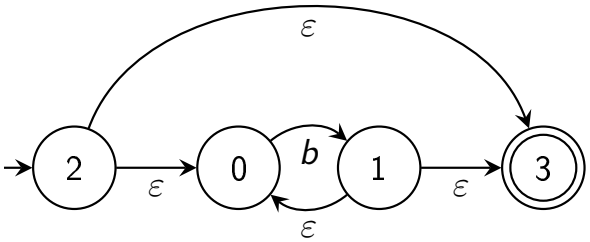
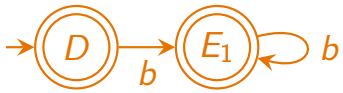


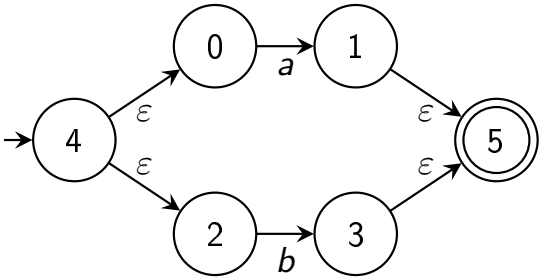
expr = b*



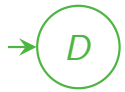
$D = \{0, 2, 3\}$

$E_1 = \{0, 1, 3\}$

AFD terminé, nombre d'états : 2



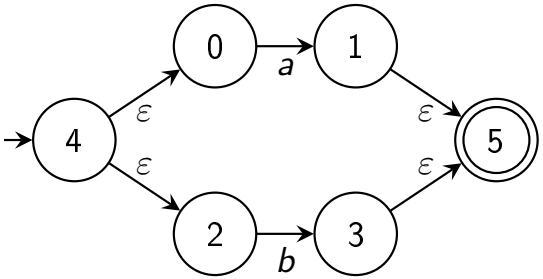
expr = a|b



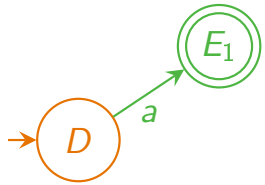
$D = \{0, 2, 4\}$

Calcul de l'état de départ :

$$D = EpsilonFermeture(\{4\}) = \{0, 2, 4\}$$



expr = a|b

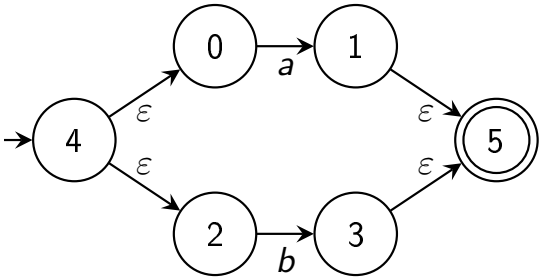


$D = \{0, 2, 4\}$

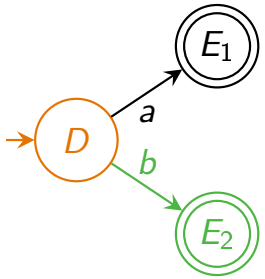
$E_1 = \{1, 5\}$

Etat actuel : D

- x = a
- transitions : 0a1
- $EpsilonFermeture(\{1\}) = \{1, 5\} = E_1$
- etat final : oui



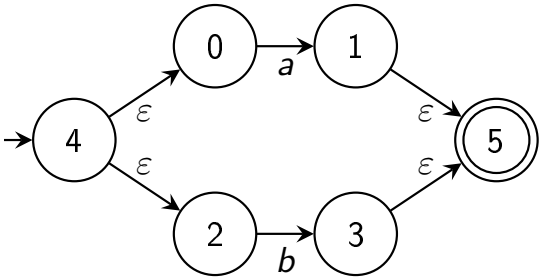
expr = a|b



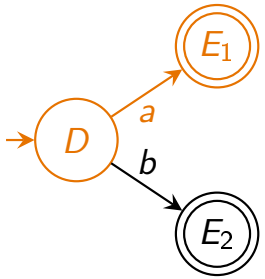
$D = \{0, 2, 4\}$
$E_1 = \{1, 5\}$
$E_2 = \{3, 5\}$

Etat actuel : D

- x = b
- transitions : 2b3
- $EpsilonFermeture(\{3\}) = \{3, 5\} = E_2$
- etat final : oui



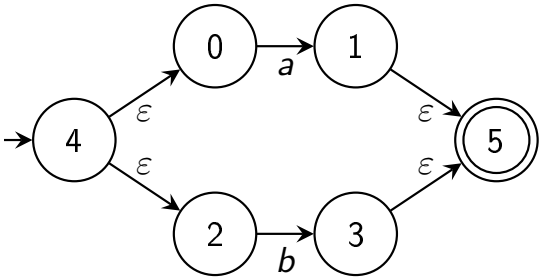
expr = a|b



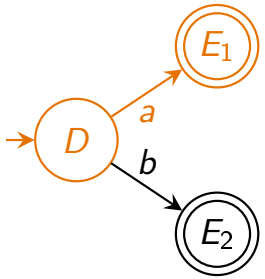
$D = \{0, 2, 4\}$
$E_1 = \{1, 5\}$
$E_2 = \{3, 5\}$

Etat actuel : E_1

- x = a
- transitions : aucune



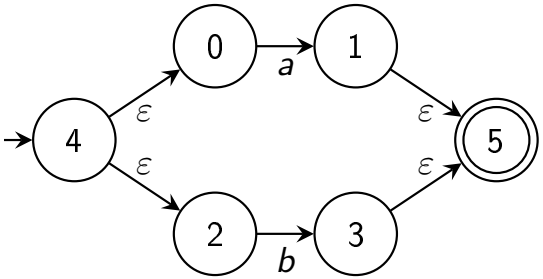
expr = a|b



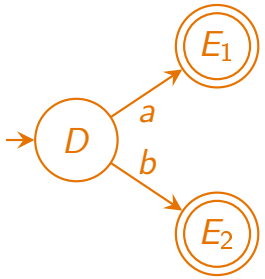
$D = \{0, 2, 4\}$
$E_1 = \{1, 5\}$
$E_2 = \{3, 5\}$

Etat actuel : E_1

- x = b
- transitions : aucune



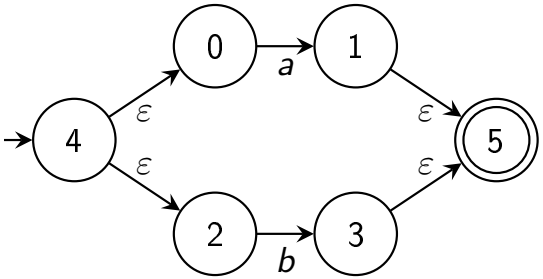
expr = a|b



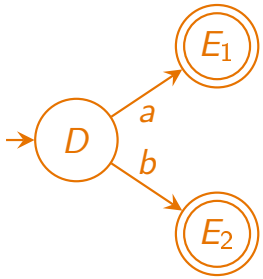
$D = \{0, 2, 4\}$
$E_1 = \{1, 5\}$
$E_2 = \{3, 5\}$

Etat actuel : E_2

- x = a
- transitions : aucune



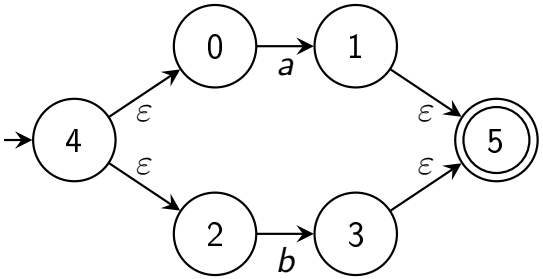
expr = a|b



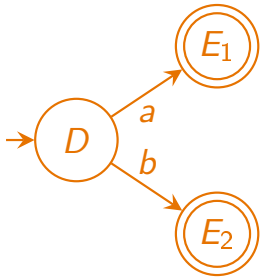
$D = \{0, 2, 4\}$
$E_1 = \{1, 5\}$
$E_2 = \{3, 5\}$

Etat actuel : E_2

- x = b
- transitions : aucune

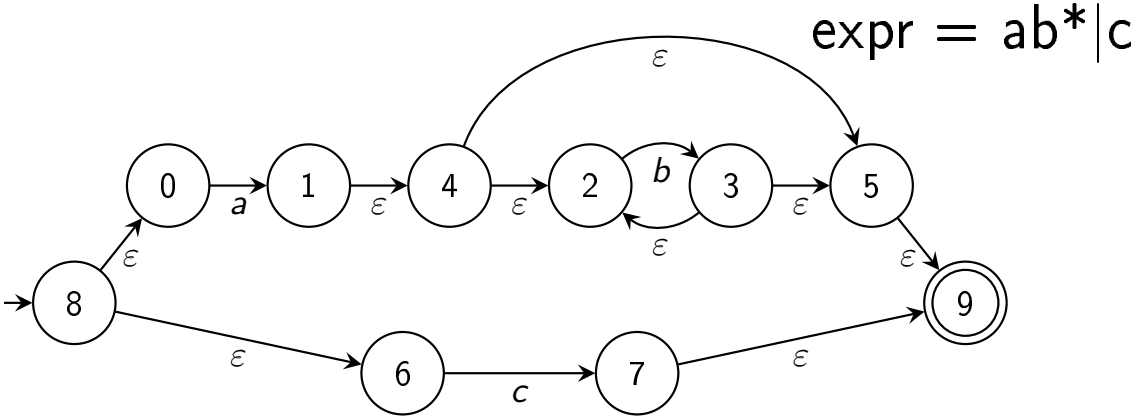


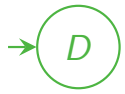
expr = a|b



$D = \{0, 2, 4\}$
$E_1 = \{1, 5\}$
$E_2 = \{3, 5\}$

AFD terminé, nombre d'états : 3

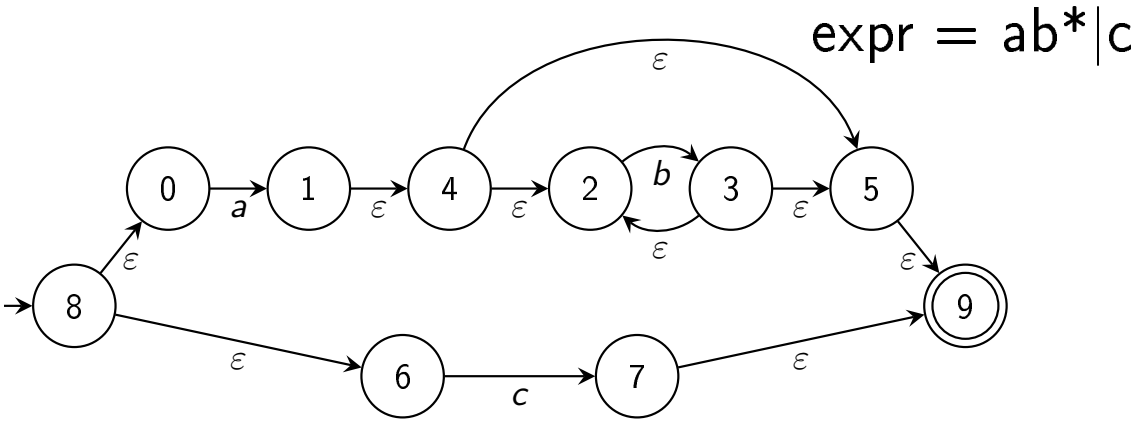


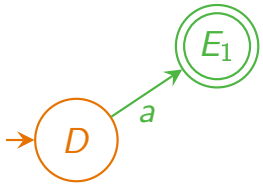


$D = \{0, 6, 8\}$

Calcul de l'état de départ :

$$D = EpsilonFermeture(\{8\}) = \{0, 6, 8\}$$



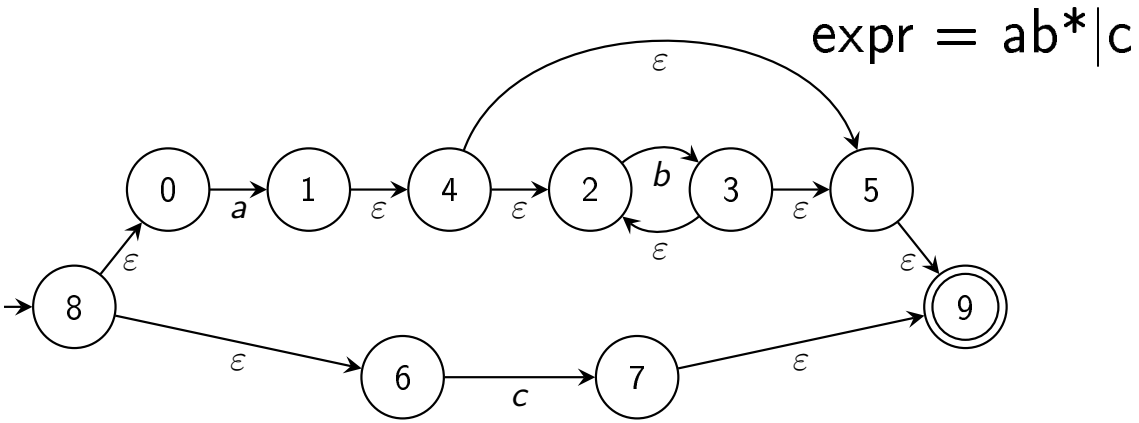


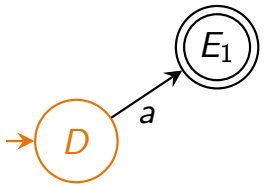
$D = \{0, 6, 8\}$

$E_1 = \{1, 2, 4, 5, 9\}$

Etat actuel : D

- x = a
- transitions : 0a1
- $EpsilonFermeture(\{1\}) = \{1, 2, 4, 5, 9\} = E_1$
- etat final : oui

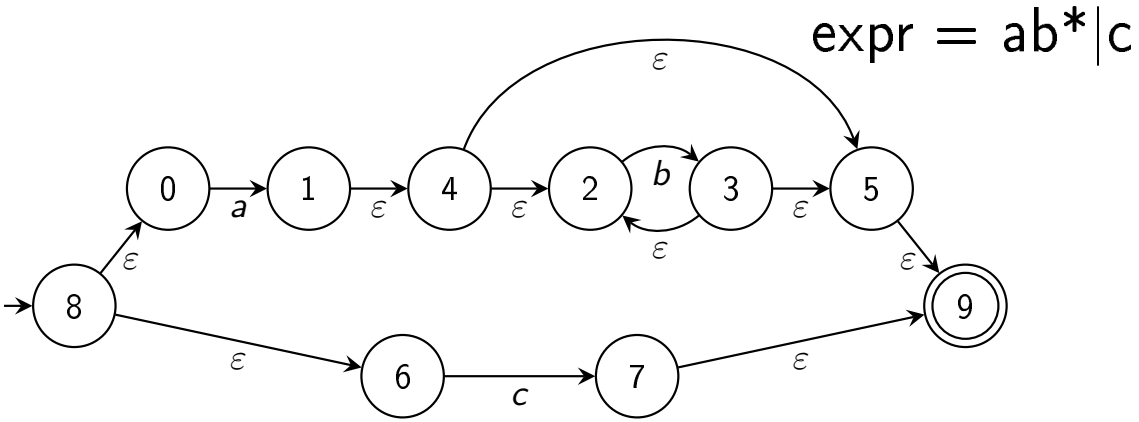


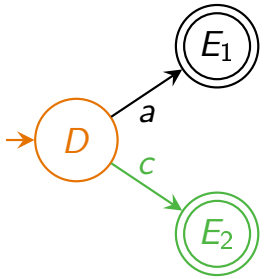


$D = \{0, 6, 8\}$
$E_1 = \{1, 2, 4, 5, 9\}$

Etat actuel : D

- x = b
- transitions : aucune





$D = \{0, 6, 8\}$

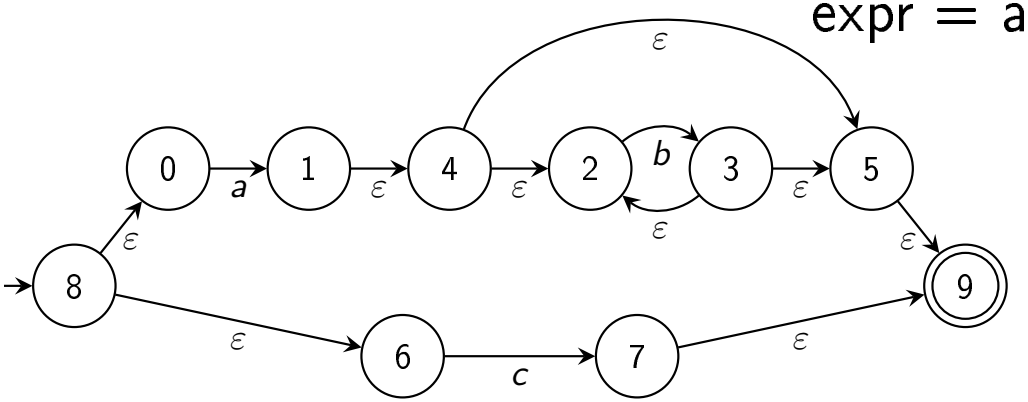
$E_1 = \{1, 2, 4, 5, 9\}$

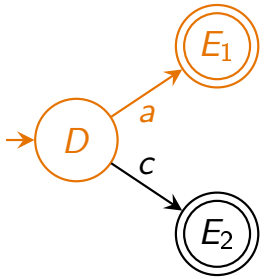
$E_2 = \{7, 9\}$

Etat actuel : D

- x = c
- transitions : 6c7
- $EpsilonFermeture(\{7\}) = \{7, 9\} = E_2$
- etat final : oui

expr = ab*|c

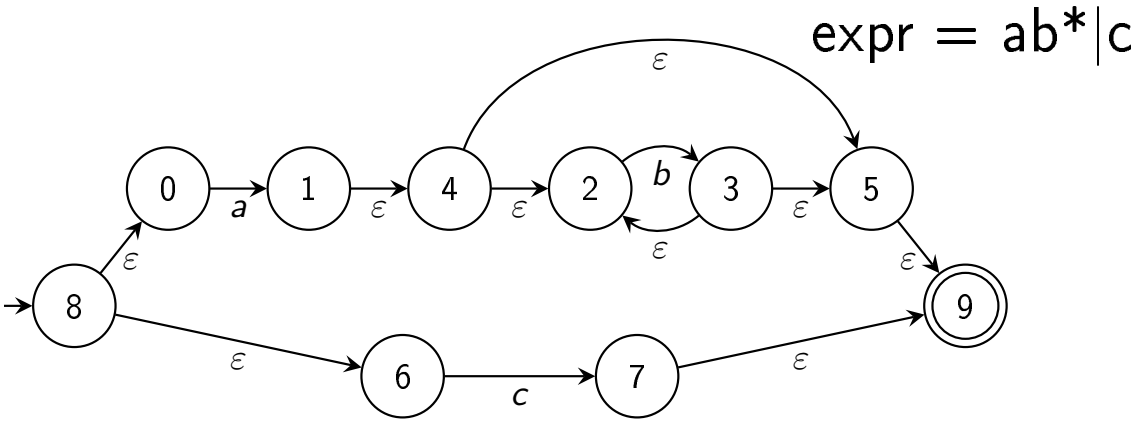


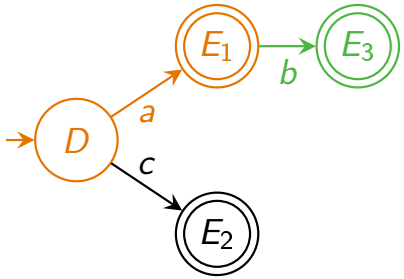


$D = \{0, 6, 8\}$
$E_1 = \{1, 2, 4, 5, 9\}$
$E_2 = \{7, 9\}$

Etat actuel : E_1

- x = a
- transitions : aucune



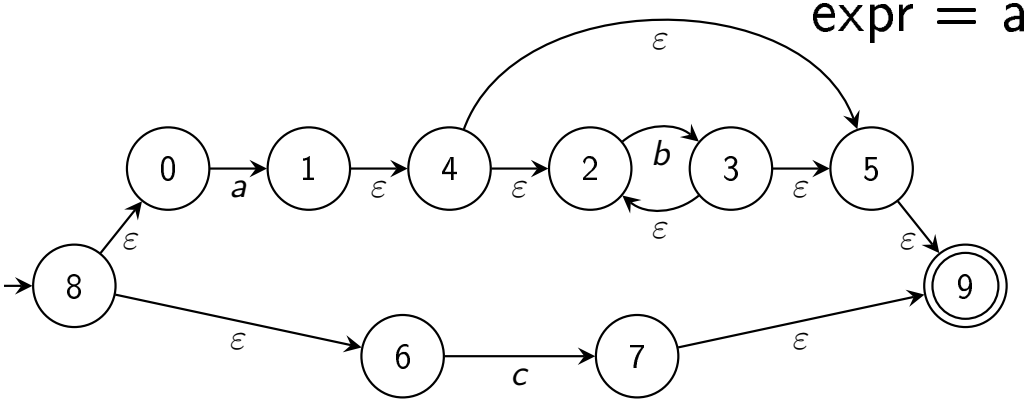


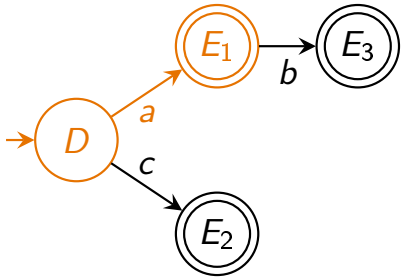
$D = \{0, 6, 8\}$
$E_1 = \{1, 2, 4, 5, 9\}$
$E_2 = \{7, 9\}$
$E_3 = \{2, 3, 5, 9\}$

Etat actuel : E_1

- x = b
- transitions : 2b3
- $EpsilonFermeture(\{3\}) = \{2, 3, 5, 9\} = E_3$
- etat final : oui

expr = ab*|c



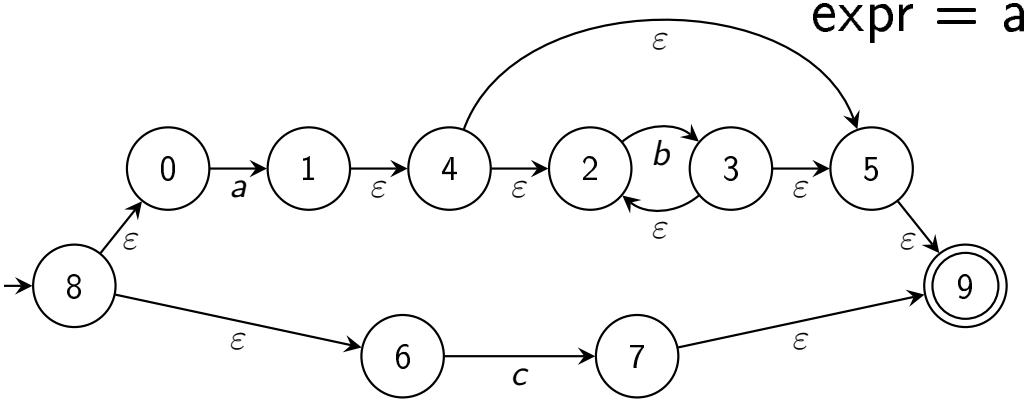


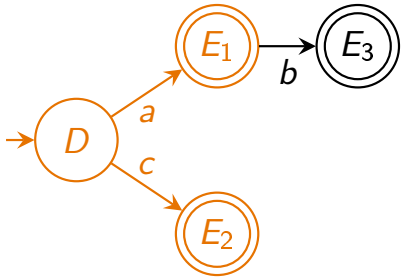
$D = \{0, 6, 8\}$
$E_1 = \{1, 2, 4, 5, 9\}$
$E_2 = \{7, 9\}$
$E_3 = \{2, 3, 5, 9\}$

Etat actuel : E_1

- x = c
- transitions : aucune

expr = ab*|c



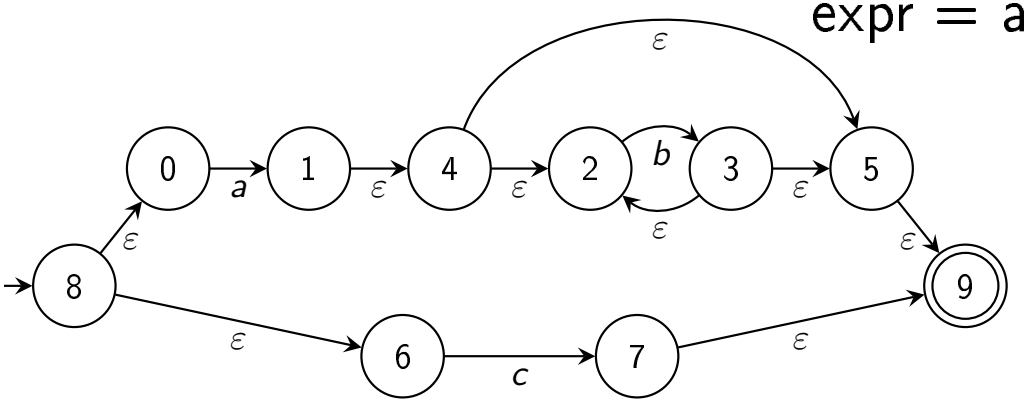


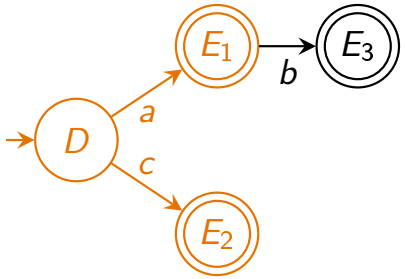
$D = \{0, 6, 8\}$
$E_1 = \{1, 2, 4, 5, 9\}$
$E_2 = \{7, 9\}$
$E_3 = \{2, 3, 5, 9\}$

Etat actuel : E_2

- x = a
- transitions : aucune

expr = ab*|c



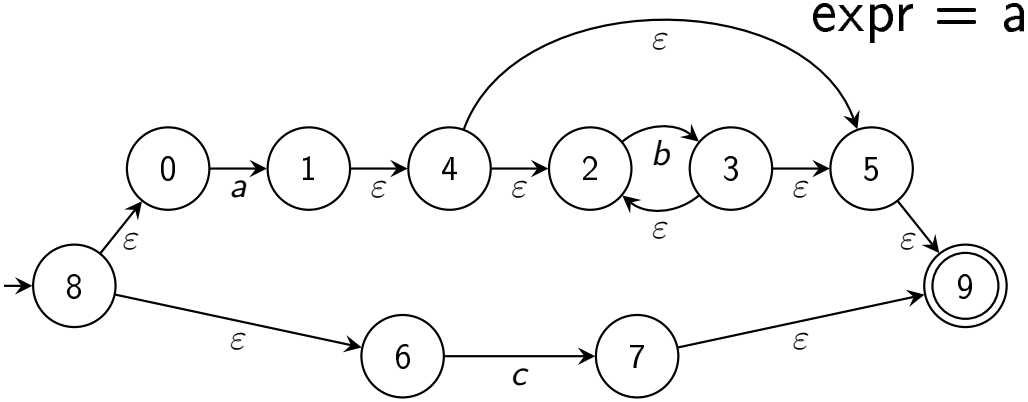


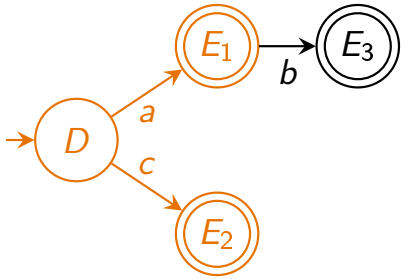
$D = \{0, 6, 8\}$
$E_1 = \{1, 2, 4, 5, 9\}$
$E_2 = \{7, 9\}$
$E_3 = \{2, 3, 5, 9\}$

Etat actuel : E_2

- x = b
- transitions : aucune

expr = ab*|c



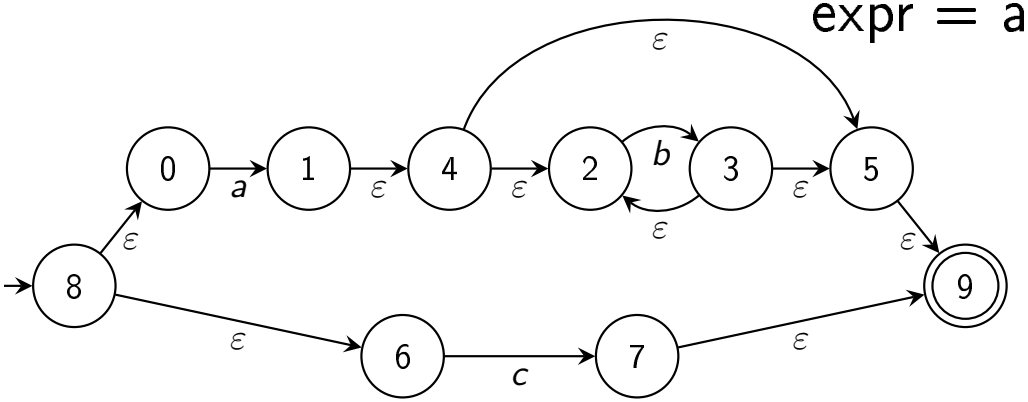


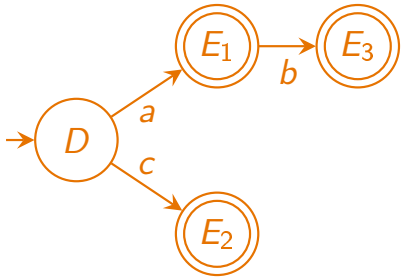
$D = \{0, 6, 8\}$
$E_1 = \{1, 2, 4, 5, 9\}$
$E_2 = \{7, 9\}$
$E_3 = \{2, 3, 5, 9\}$

Etat actuel : E_2

- x = c
- transitions : aucune

expr = ab*|c



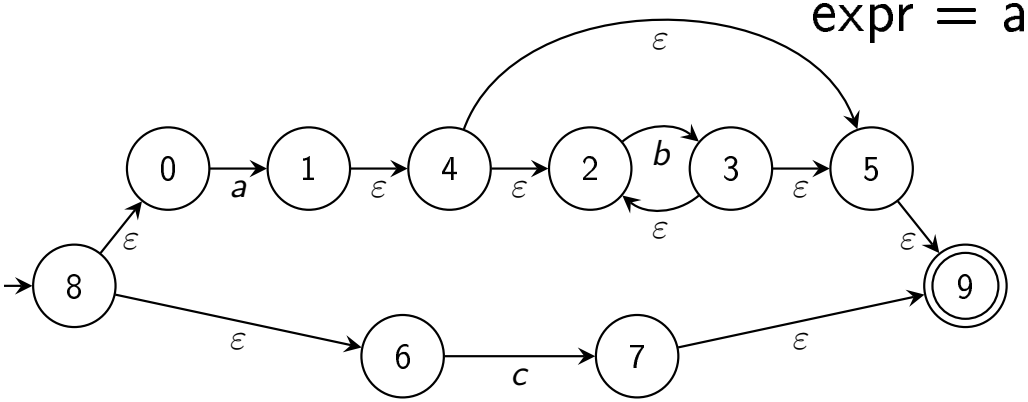


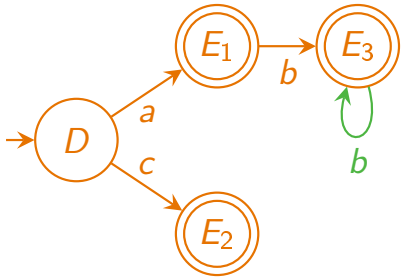
$D = \{0, 6, 8\}$
$E_1 = \{1, 2, 4, 5, 9\}$
$E_2 = \{7, 9\}$
$E_3 = \{2, 3, 5, 9\}$

Etat actuel : E_3

- x = a
- transitions : aucune

expr = ab*|c



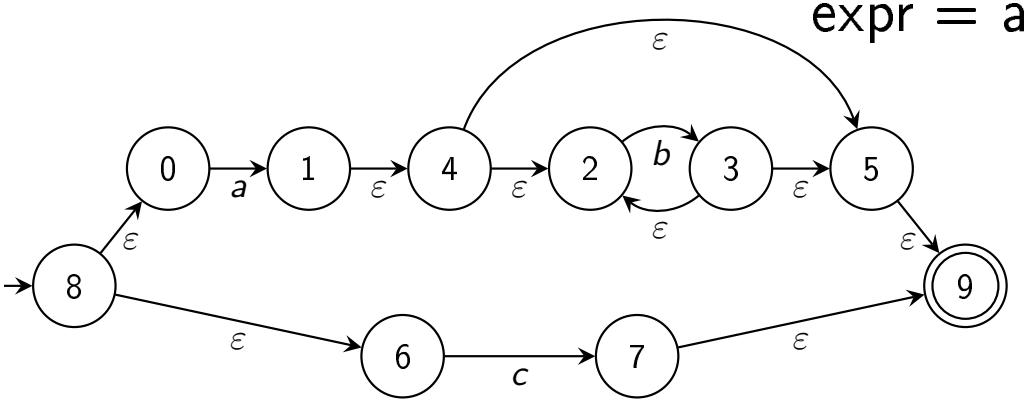


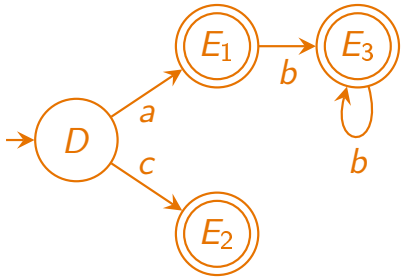
$D = \{0, 6, 8\}$
$E_1 = \{1, 2, 4, 5, 9\}$
$E_2 = \{7, 9\}$
$E_3 = \{2, 3, 5, 9\}$

Etat actuel : E_3

- x = b
- transitions : 2b3
- $EpsilonFermeture(\{3\}) = \{2, 3, 5, 9\} = E_3$
- etat final : oui

expr = ab*|c



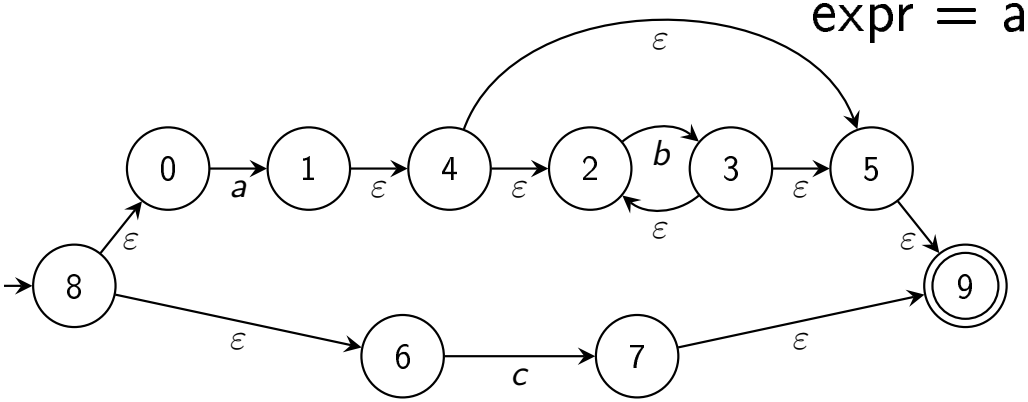


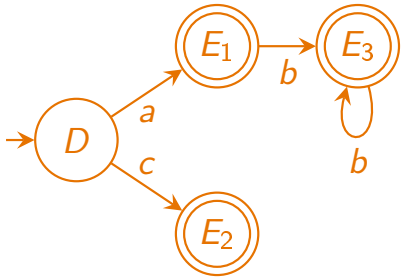
$D = \{0, 6, 8\}$
$E_1 = \{1, 2, 4, 5, 9\}$
$E_2 = \{7, 9\}$
$E_3 = \{2, 3, 5, 9\}$

Etat actuel : E_3

- x = c
- transitions : aucune

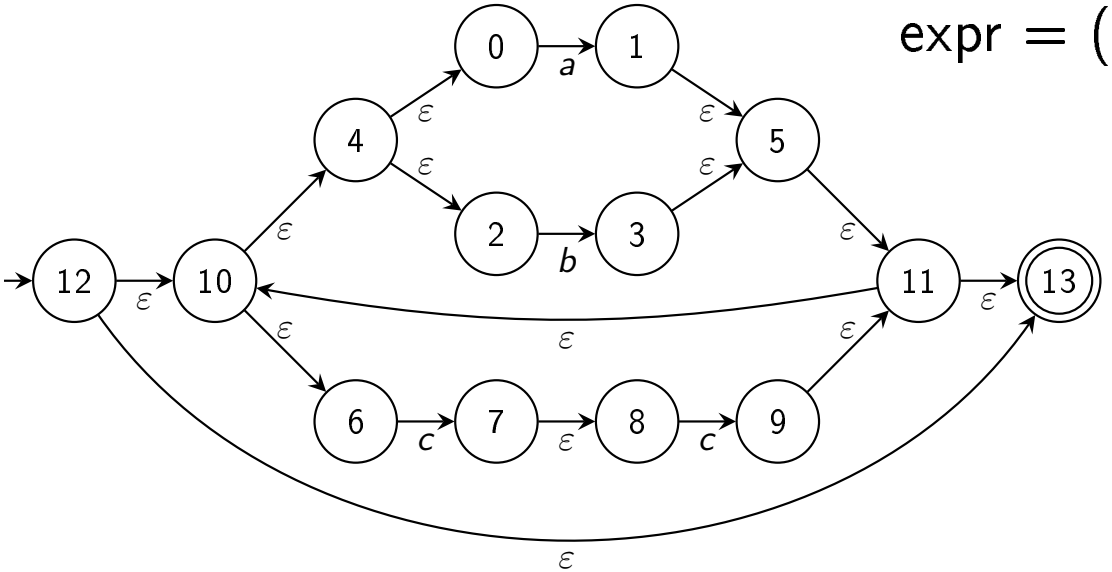
expr = ab*|c





$D = \{0, 6, 8\}$
$E_1 = \{1, 2, 4, 5, 9\}$
$E_2 = \{7, 9\}$
$E_3 = \{2, 3, 5, 9\}$

AFD terminé, nombre d'états : 4

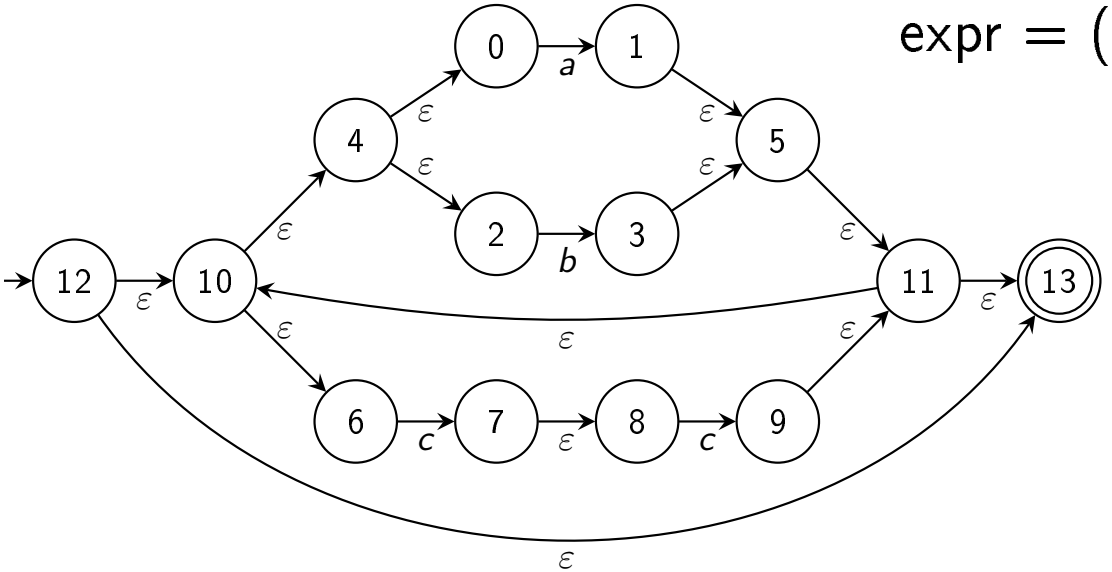


expr = ((a|b)|cc)*

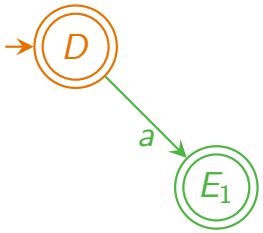
$D = \{0, 2, 4, 6, 10, 12, 13\}$

Calcul de l'état de départ :

$$D = EpsilonFermeture(\{12\}) = \{0,\ 2,\ 4,\ 6,\ 10,\ 12,\ 13\}$$



expr = ((a|b)|cc)*

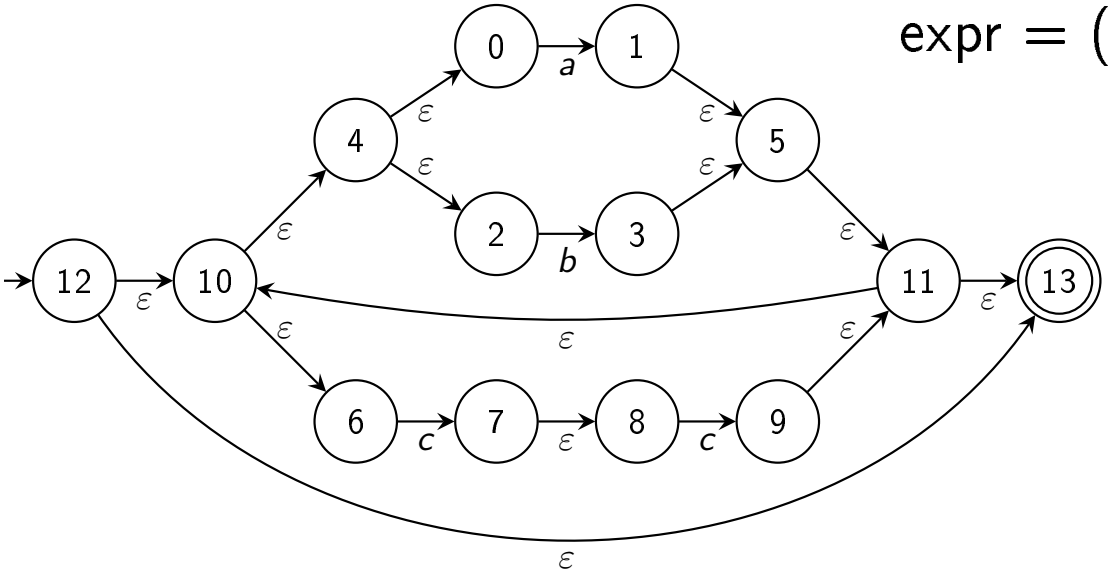


Etat actuel : D

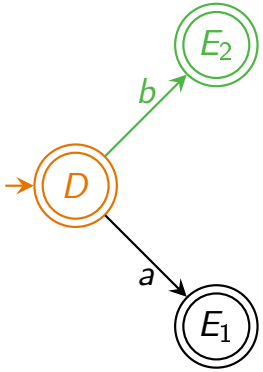
$D = \{0, 2, 4, 6, 10, 12, 13\}$

$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$

- x = a
- transitions : 0a1
- $EpsilonFermeture(\{1\}) = \{0, 1, 2, 4, 6, 5, 10, 11, 13\} = E_1$
- etat final : oui



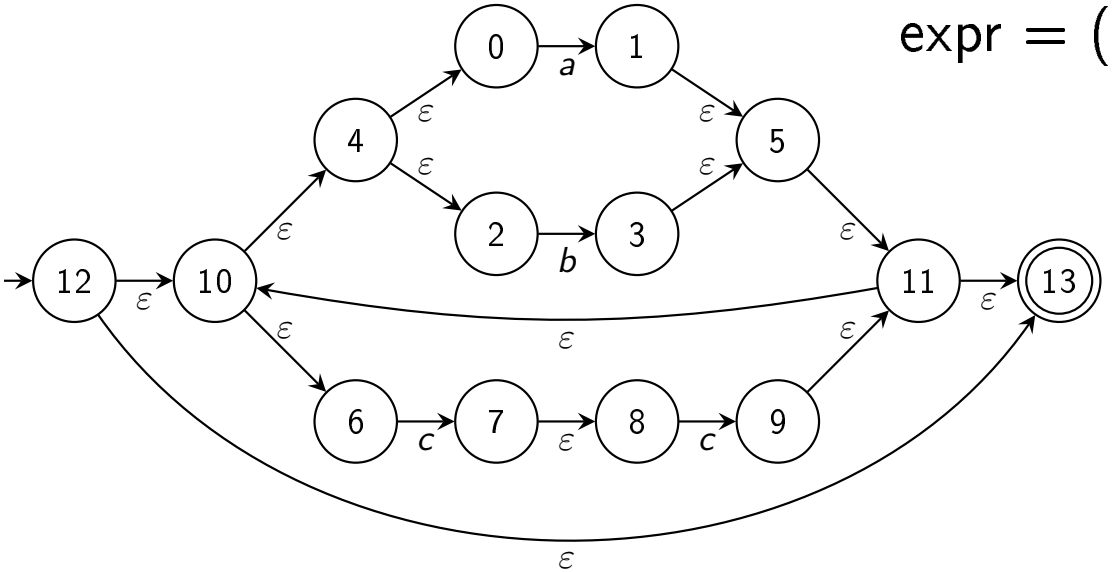
expr = ((a|b)|cc)*



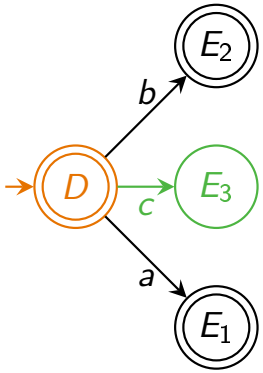
Etat actuel : D

$D = \{0, 2, 4, 6, 10, 12, 13\}$
$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$
$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$

- x = b
- transitions : 2b3
- $EpsilonFermeture(\{3\}) = \{0, 2, 3, 4, 6, 5, 10, 11, 13\} = E_2$
- etat final : oui



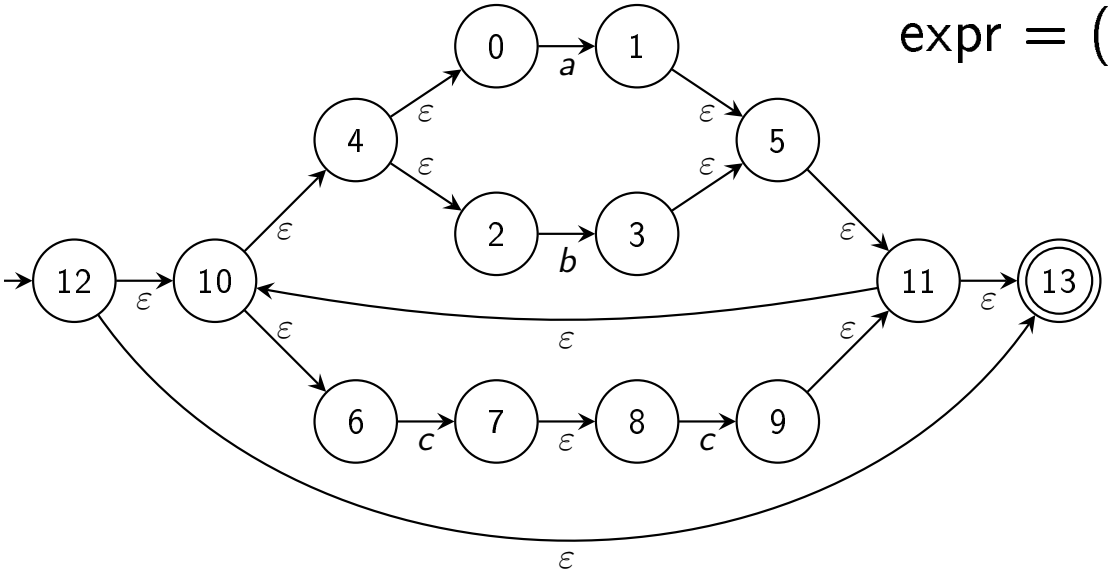
expr = ((a|b)|cc)*



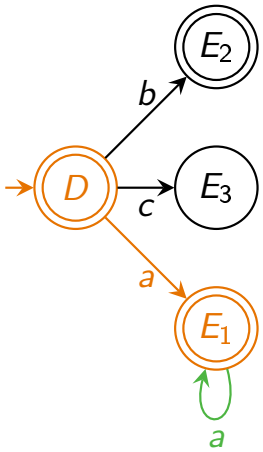
Etat actuel : D

$D = \{0, 2, 4, 6, 10, 12, 13\}$
$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$
$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$
$E_3 = \{7, 8\}$

- x = c
- transitions : 6c7
- $EpsilonFermeture(\{7\}) = \{7, 8\} = E_3$
- etat final : non



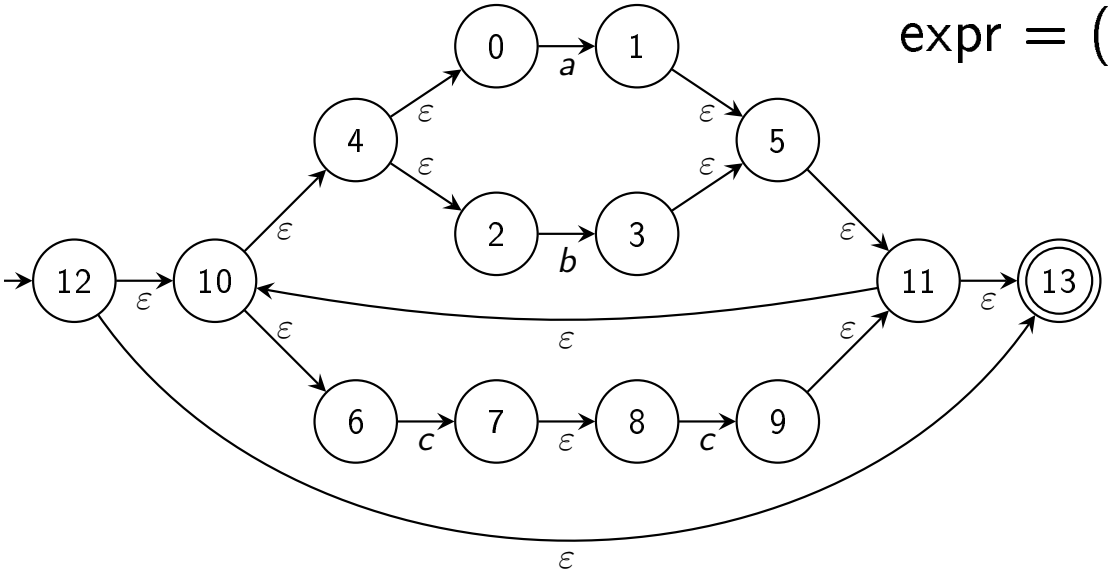
expr = ((a|b)|cc)*



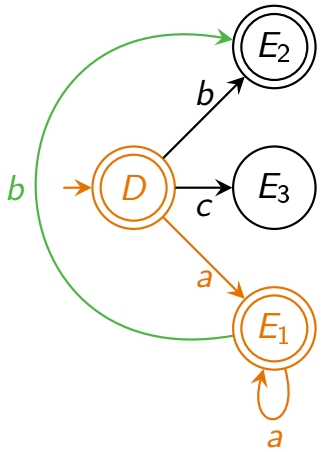
Etat actuel : E_1

$D = \{0, 2, 4, 6, 10, 12, 13\}$
$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$
$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$
$E_3 = \{7, 8\}$

- x = a
- transitions : 0a1
- $EpsilonFermeture(\{1\}) = \{0, 1, 2, 4, 6, 5, 10, 11, 13\} = E_1$
- etat final : oui



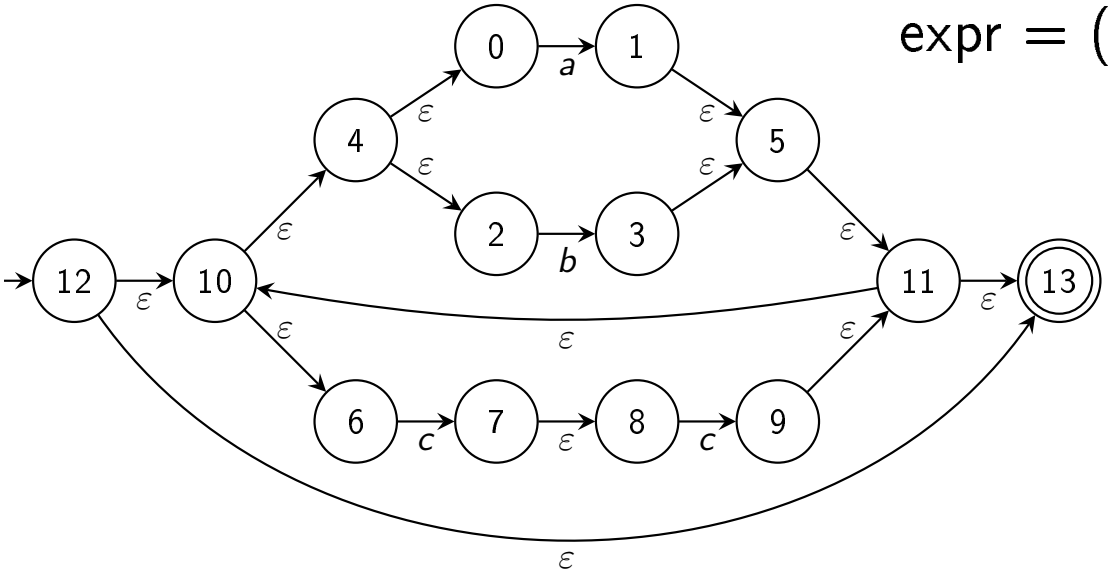
expr = ((a|b)|cc)*



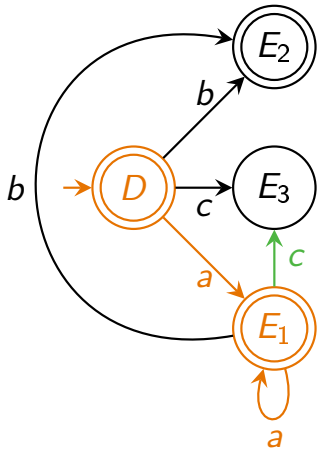
Etat actuel : E_1

$D = \{0, 2, 4, 6, 10, 12, 13\}$
$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$
$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$
$E_3 = \{7, 8\}$

- x = b
- transitions : 2b3
- $EpsilonFermeture(\{3\}) = \{0, 2, 3, 4, 6, 5, 10, 11, 13\} = E_2$
- etat final : oui



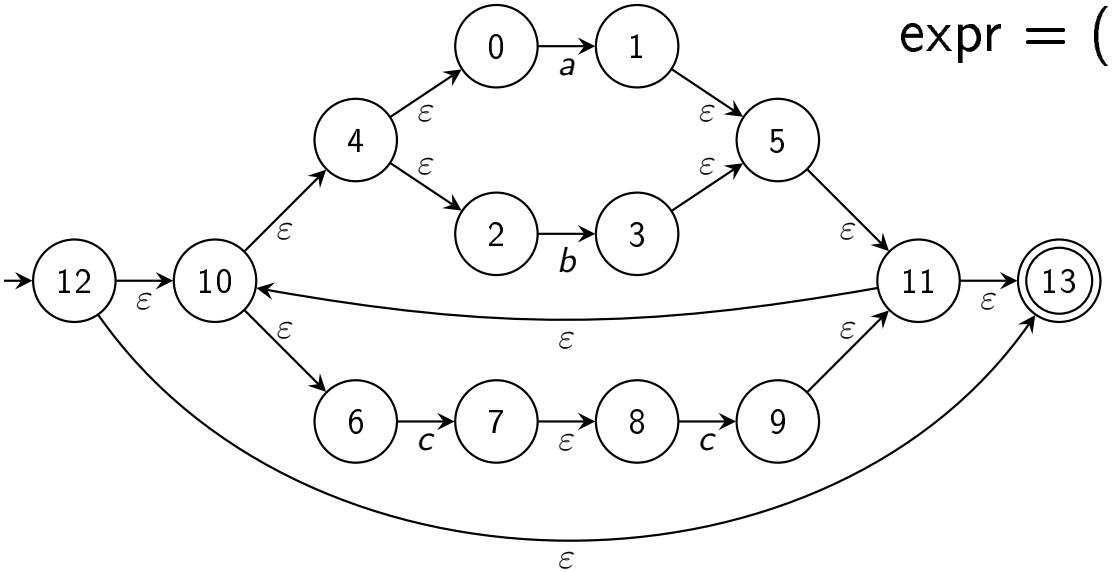
expr = ((a|b)|cc)*



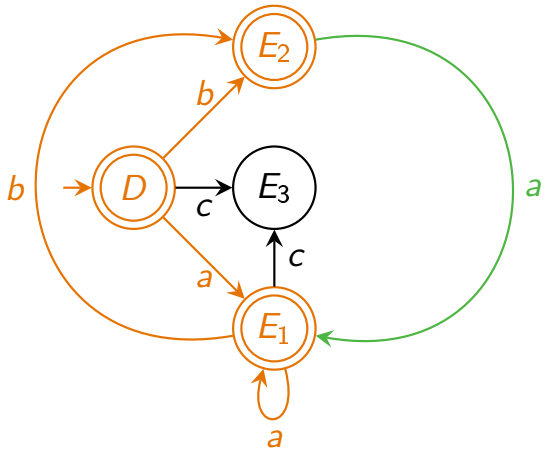
Etat actuel : E_1

$D = \{0, 2, 4, 6, 10, 12, 13\}$
$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$
$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$
$E_3 = \{7, 8\}$

- x = c
- transitions : 6c7
- $EpsilonFermeture(\{7\}) = \{7, 8\} = E_3$
- etat final : non



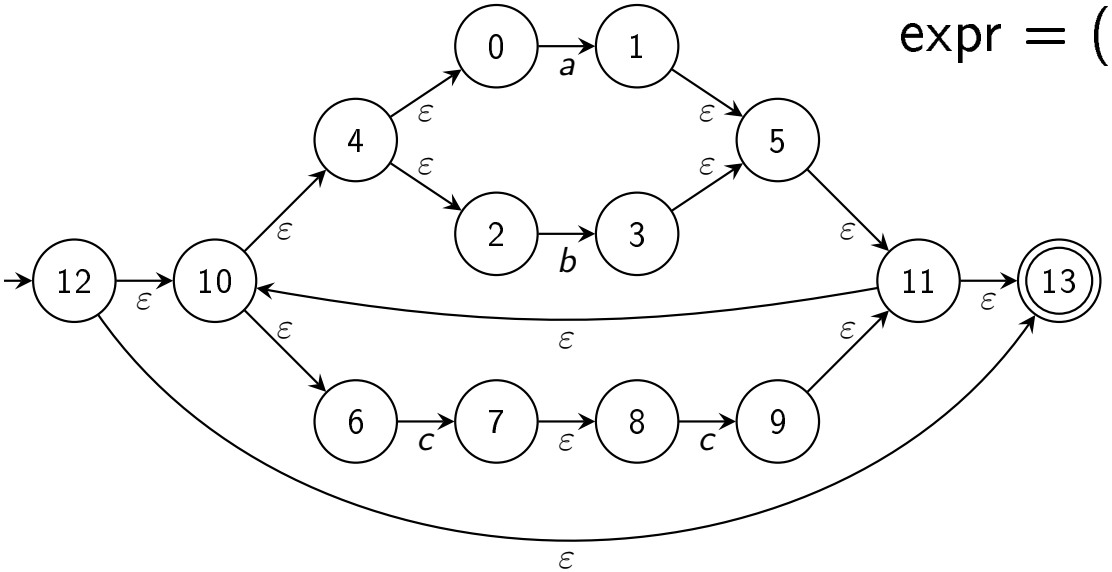
expr = ((a|b)|cc)*



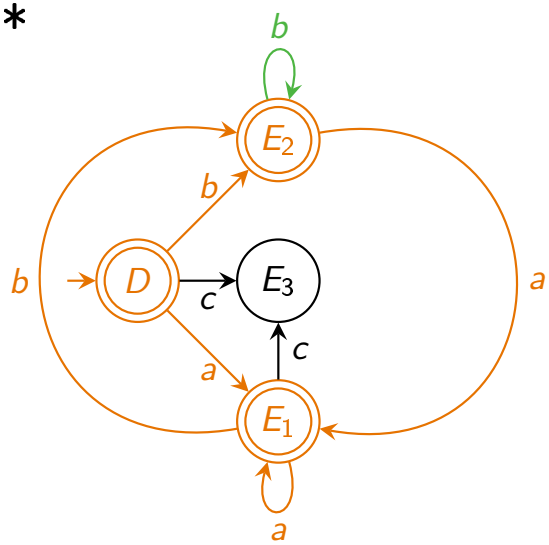
Etat actuel : E_2

$D = \{0, 2, 4, 6, 10, 12, 13\}$
$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$
$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$
$E_3 = \{7, 8\}$

- x = a
- transitions : 0a1
- $EpsilonFermeture(\{1\}) = \{0, 1, 2, 4, 6, 5, 10, 11, 13\} = E_1$
- etat final : oui



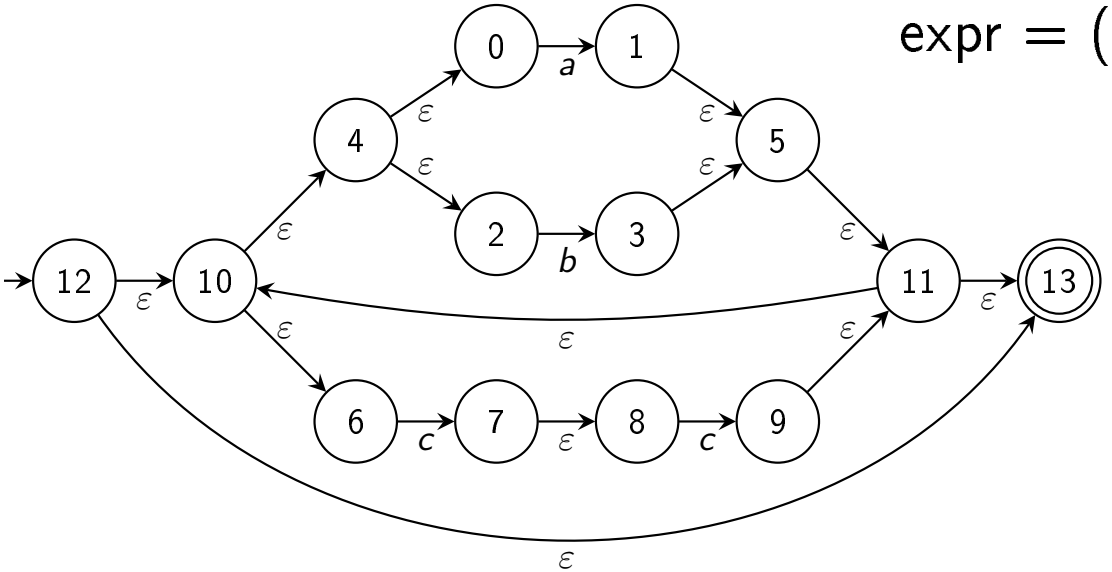
expr = ((a|b)|cc)*



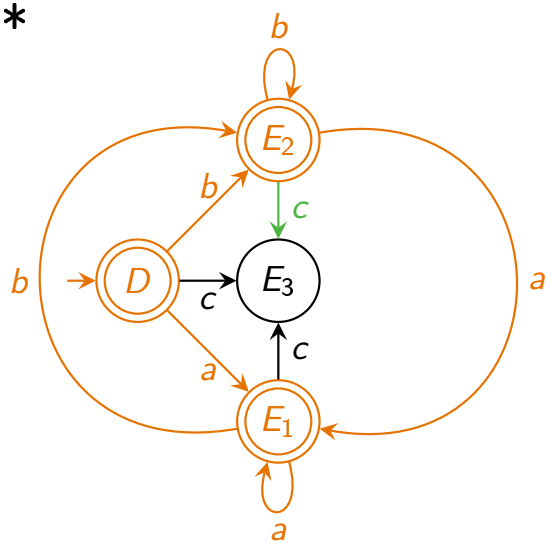
Etat actuel : E_2

$D = \{0, 2, 4, 6, 10, 12, 13\}$
$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$
$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$
$E_3 = \{7, 8\}$

- x = b
- transitions : 2b3
- $EpsilonFermeture(\{3\}) = \{0, 2, 3, 4, 6, 5, 10, 11, 13\} = E_2$
- etat final : oui



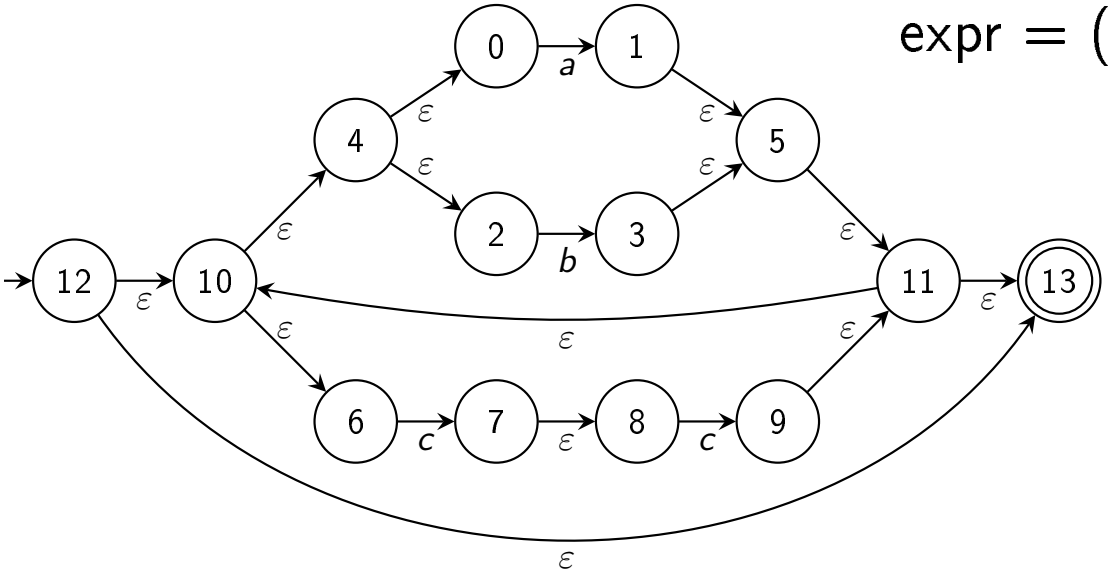
expr = ((a|b)|cc)*



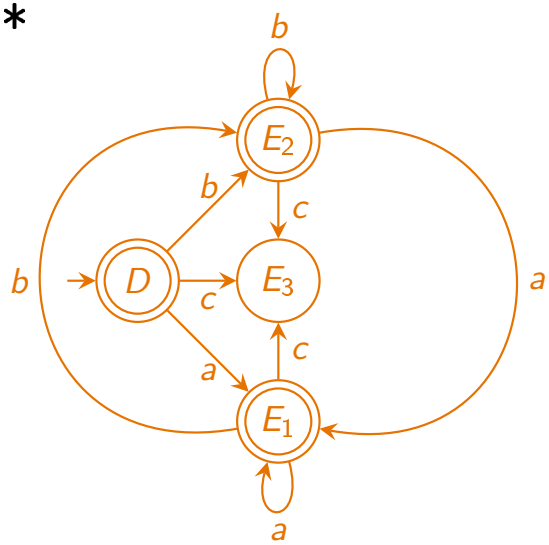
Etat actuel : E_2

- x = c
- transitions : 6c7
- $EpsilonFermeture(\{7\}) = \{7, 8\} = E_3$
- etat final : non

$D = \{0, 2, 4, 6, 10, 12, 13\}$
$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$
$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$
$E_3 = \{7, 8\}$



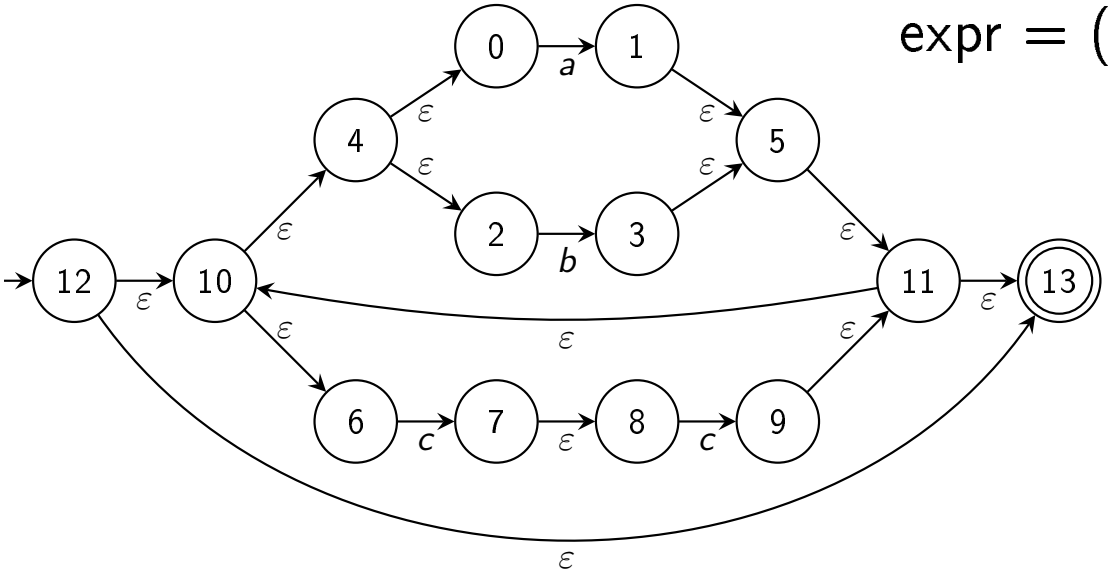
expr = ((a|b)|cc)*



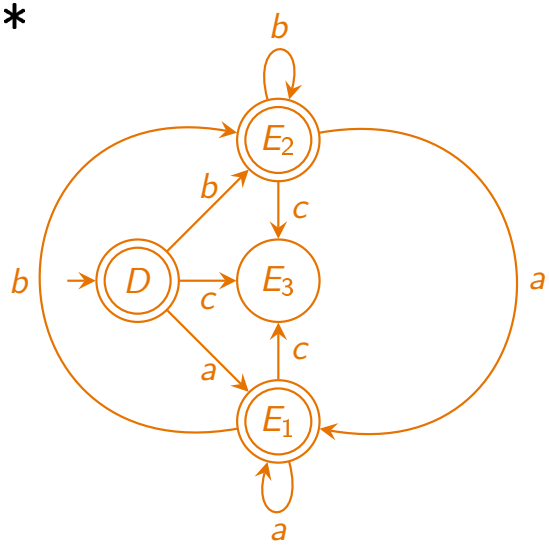
$D = \{0, 2, 4, 6, 10, 12, 13\}$
$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$
$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$
$E_3 = \{7, 8\}$

Etat actuel : E_3

- x = a
- transitions : aucune



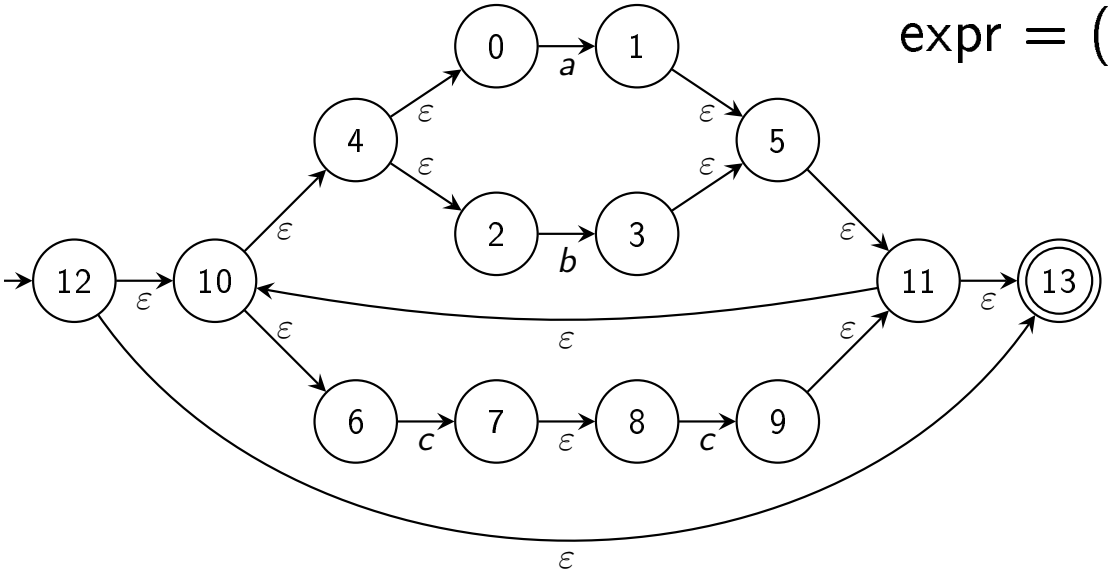
expr = ((a|b)|cc)*



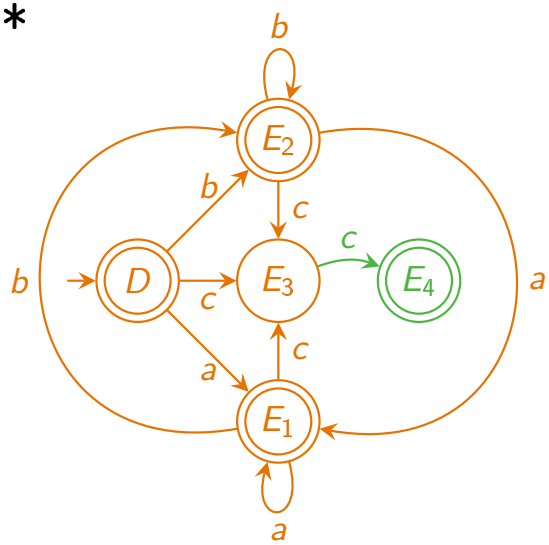
$D = \{0, 2, 4, 6, 10, 12, 13\}$
$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$
$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$
$E_3 = \{7, 8\}$

Etat actuel : E_3

- x = b
- transitions : aucune



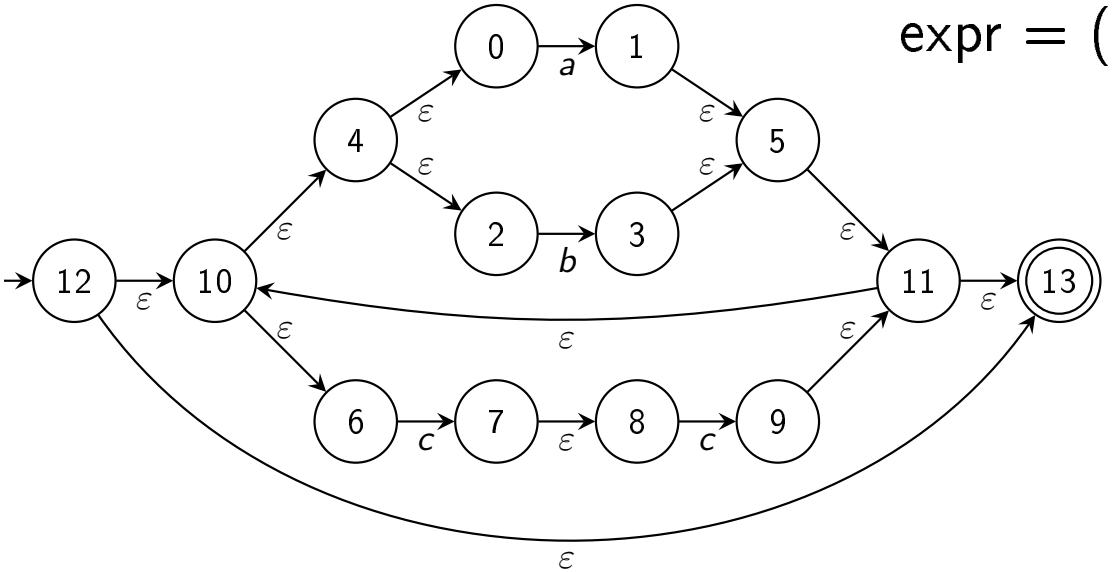
expr = ((a|b)|cc)*



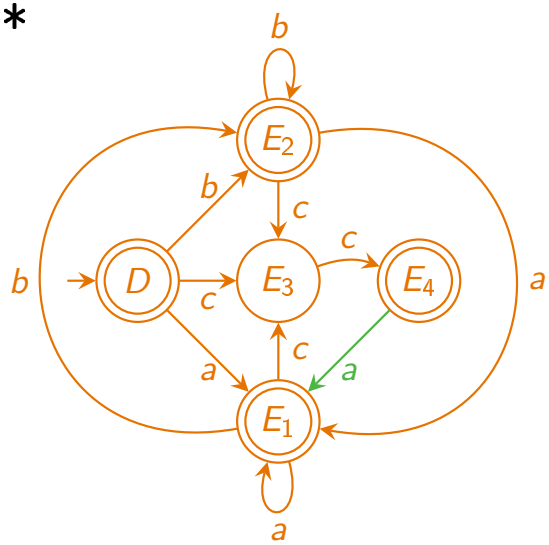
Etat actuel : E_3

$D = \{0, 2, 4, 6, 10, 12, 13\}$
$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$
$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$
$E_3 = \{7, 8\}$
$E_4 = \{0, 2, 4, 6, 9, 10, 11, 13\}$

- x = c
- transitions : 8c9
- $EpsilonFermeture(\{9\}) = \{0, 2, 4, 6, 9, 10, 11, 13\} = E_4$
- etat final : non



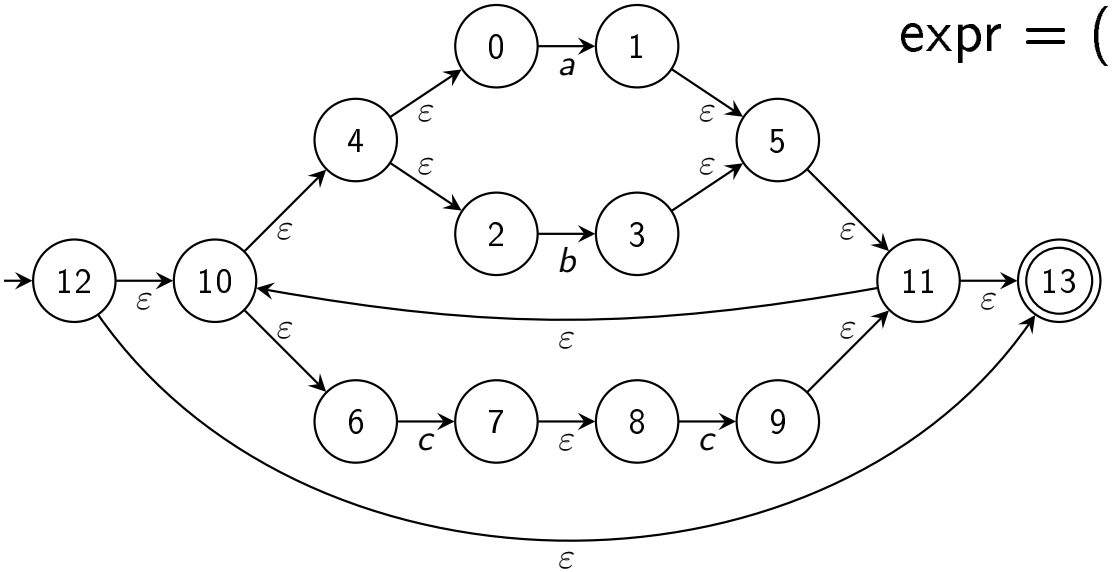
expr = ((a|b)|cc)*



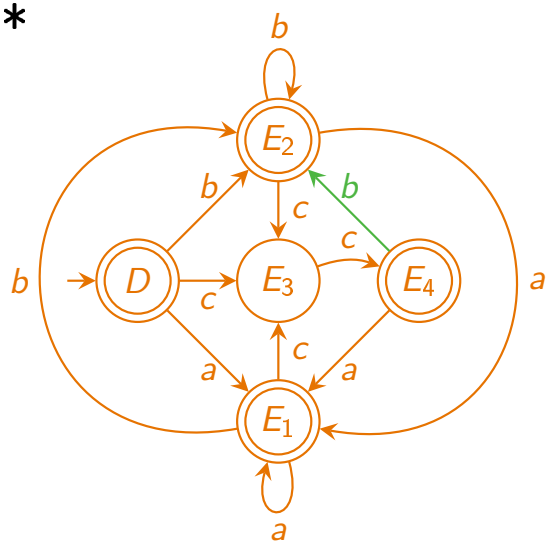
Etat actuel : E_4

$D = \{0, 2, 4, 6, 10, 12, 13\}$
$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$
$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$
$E_3 = \{7, 8\}$
$E_4 = \{0, 2, 4, 6, 9, 10, 11, 13\}$

- x = a
- transitions : 0a1
- $EpsilonFermeture(\{1\}) = \{0, 1, 2, 4, 6, 5, 10, 11, 13\} = E_1$
- etat final : oui



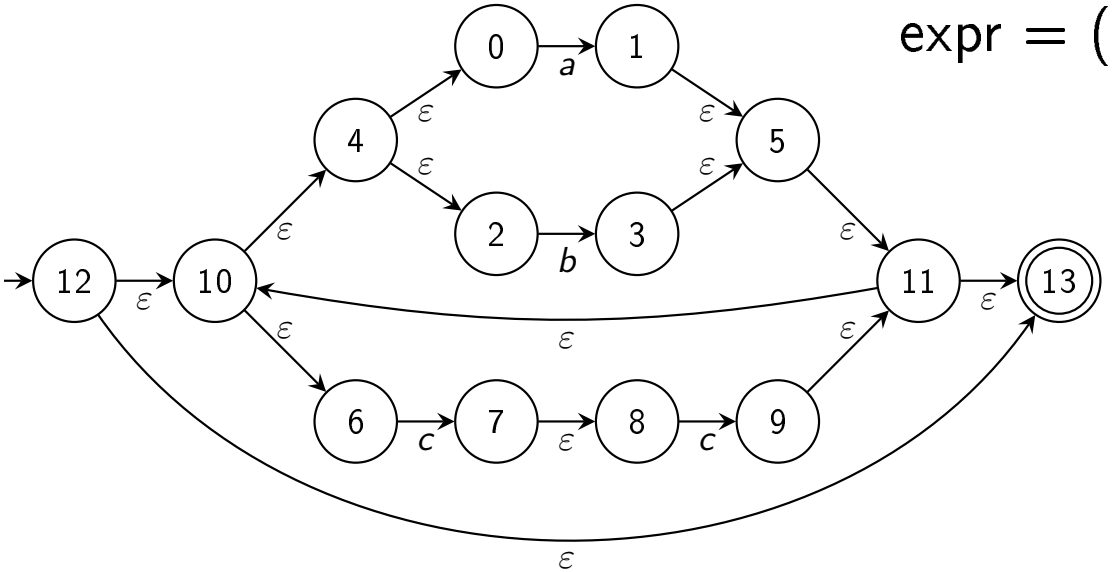
expr = ((a|b)|cc)*



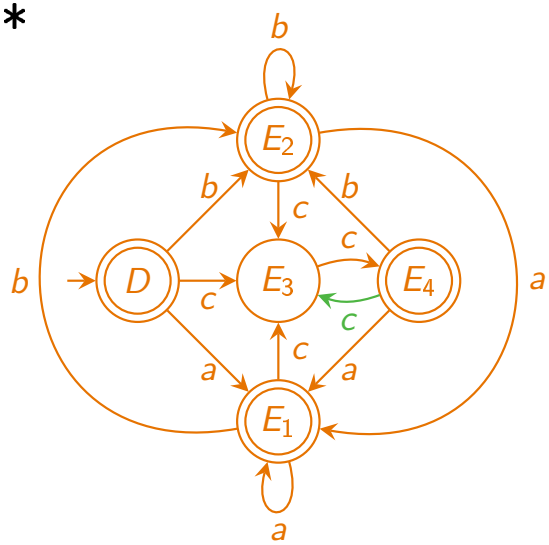
Etat actuel : E_4

$D = \{0, 2, 4, 6, 10, 12, 13\}$
$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$
$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$
$E_3 = \{7, 8\}$
$E_4 = \{0, 2, 4, 6, 9, 10, 11, 13\}$

- x = b
- transitions : 2b3
- $EpsilonFermeture(\{3\}) = \{0, 2, 3, 4, 6, 5, 10, 11, 13\} = E_2$
- etat final : oui



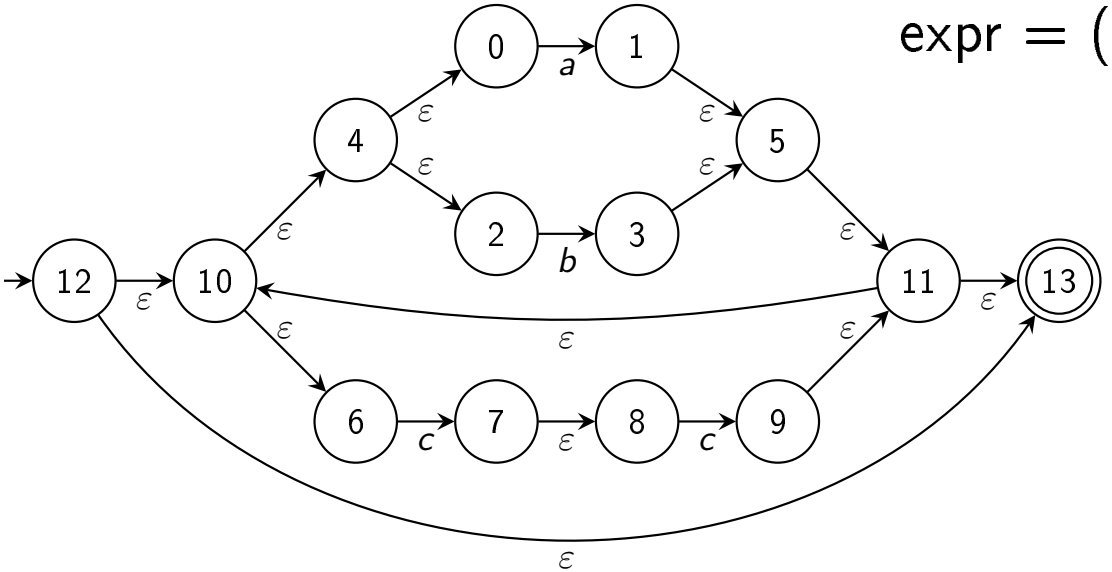
expr = ((a|b)|cc)*



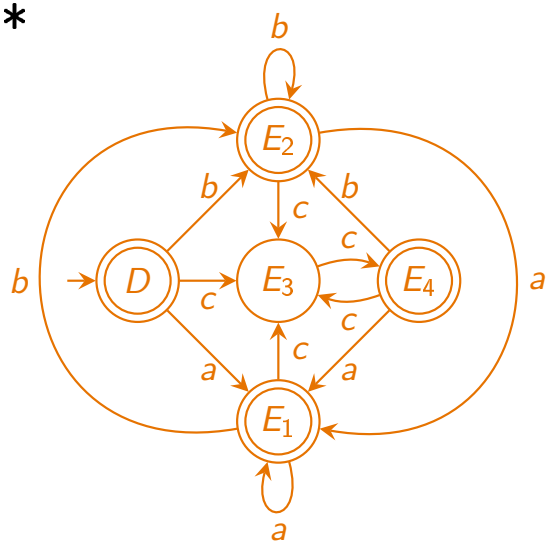
Etat actuel : E_4

- x = c
- transitions : 6c7
- $EpsilonFermeture(\{7\}) = \{7, 8\} = E_3$
- etat final : non

$D = \{0, 2, 4, 6, 10, 12, 13\}$
$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$
$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$
$E_3 = \{7, 8\}$
$E_4 = \{0, 2, 4, 6, 9, 10, 11, 13\}$



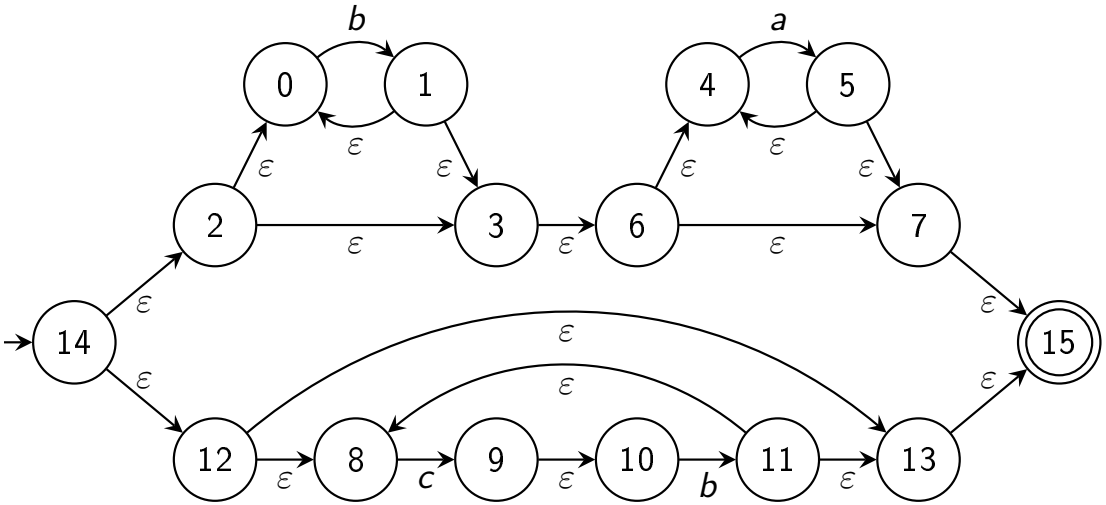
expr = ((a|b)|cc)*

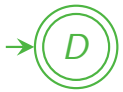


AFD terminé, nombre d'états : 5

$D = \{0, 2, 4, 6, 10, 12, 13\}$
$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$
$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$
$E_3 = \{7, 8\}$
$E_4 = \{0, 2, 4, 6, 9, 10, 11, 13\}$

expr = b*a*|(cb)*



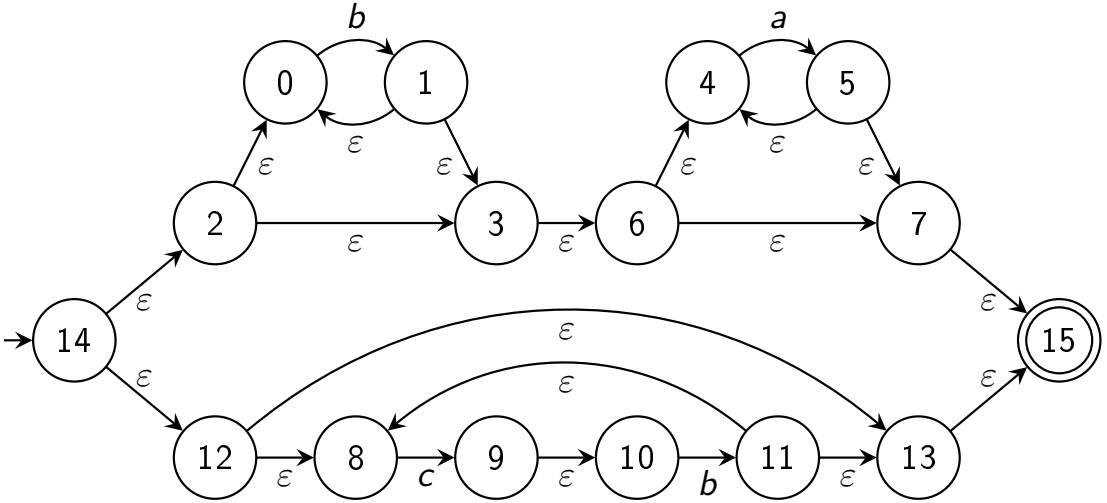


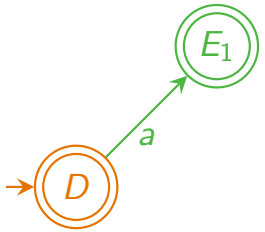
$D = \{0, 2, 3, 4, 6, 7, 8, 12, 13, 14, 15\}$

Calcul de l'état de départ :

$$D = EpsilonFermeture(\{14\}) = \{0,\ 2,\ 8,\ 12,\ 13,\ 14,\ 15\}$$

expr = b*a*|(cb)*





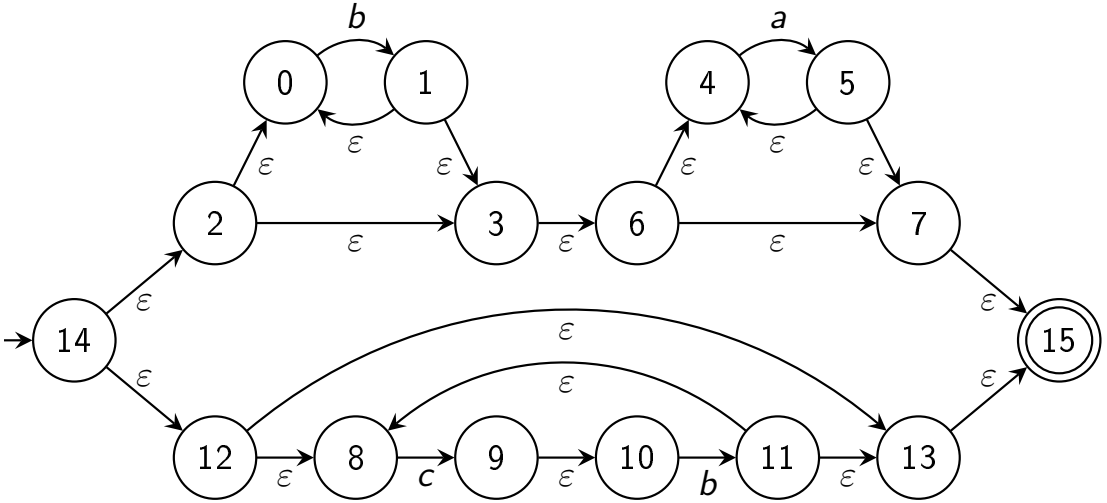
$D = \{0, 2, 3, 4, 6, 7, 8, 12, 13, 14, 15\}$

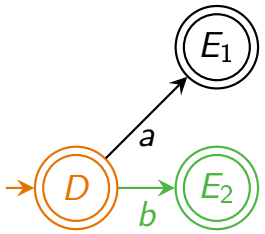
$E_1 = \{4, 5, 7, 15\}$

Etat actuel : D

- x = q
- transitions : 4a5
- $EpsilonFermeture(\{5\}) = \{4, 5, 7, 15\} = E_1$
- etat final : oui

expr = b*a*|(cb)*





$D = \{0, 2, 3, 4, 6, 7, 8, 12, 13, 14, 15\}$

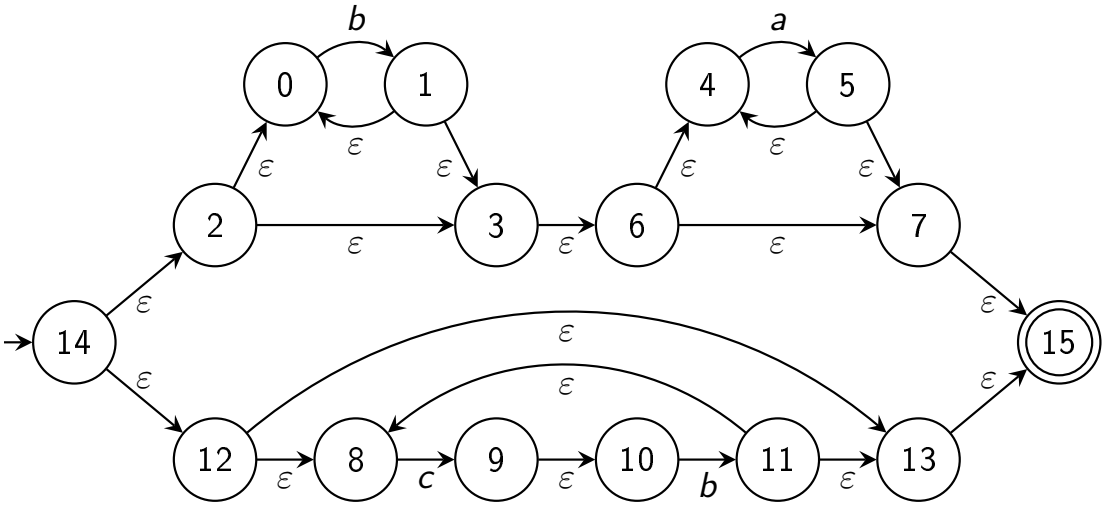
$E_1 = \{4, 5, 7, 15\}$

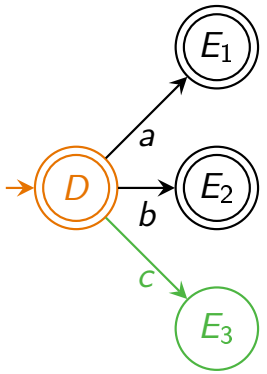
$E_2 = \{0, 1, 3, 4, 6, 7, 15\}$

Etat actuel : D

- x = b
- transitions : 0b1
- $EpsilonFermeture(\{1\}) = \{0, 1, 3, 4, 6, 7, 15\} = E_2$
- etat final : oui

expr = b*a*|(cb)*





$D = \{0, 2, 3, 4, 6, 7, 8, 12, 13, 14, 15\}$

$E_1 = \{4, 5, 7, 15\}$

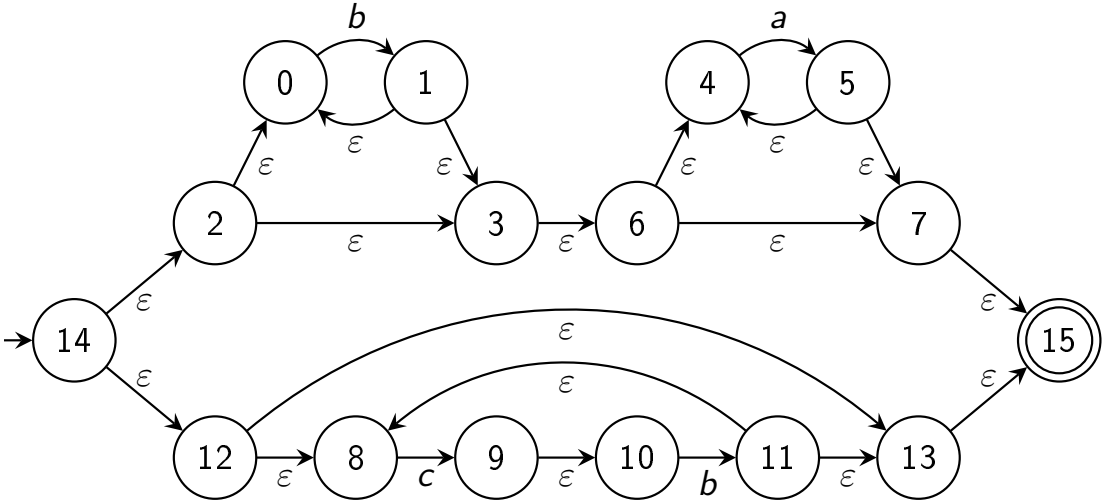
$E_2 = \{0, 1, 3, 4, 6, 7, 15\}$

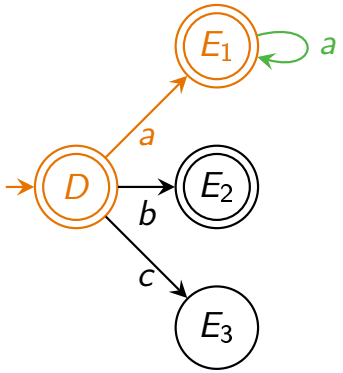
$E_3 = \{9, 10\}$

Etat actuel : D

- x = c
- transitions : 8c9
- $EpsilonFermeture(\{9\}) = \{9, 10\} = E_3$
- etat final : non

expr = b*a*|(cb)*



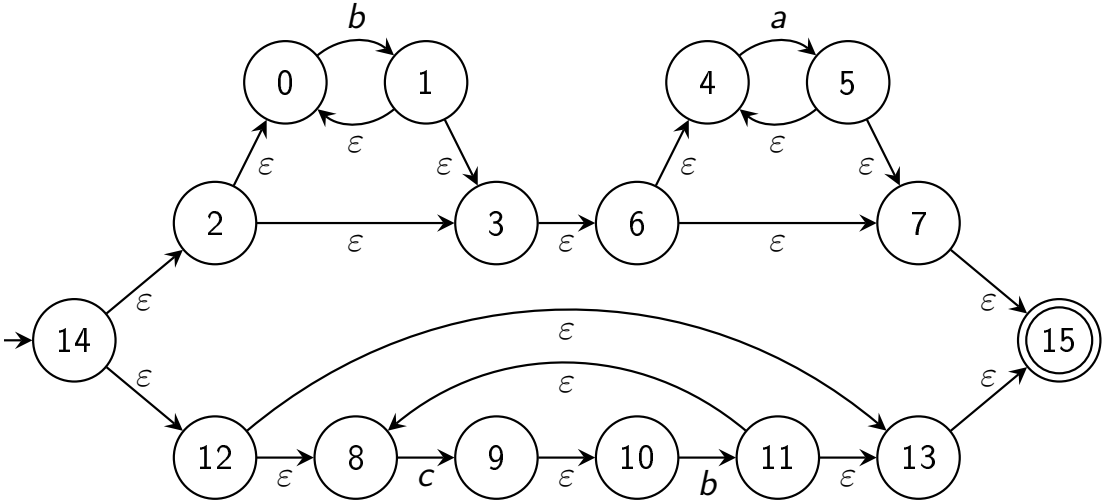


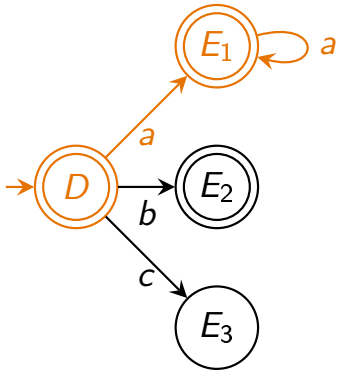
$D = \{0, 2, 3, 4, 6, 7, 8, 12, 13, 14, 15\}$
$E_1 = \{4, 5, 7, 15\}$
$E_2 = \{0, 1, 3, 4, 6, 7, 15\}$
$E_3 = \{9, 10\}$

Etat actuel : E_1

- x = a
- transitions : 4a5
- $EpsilonFermeture(\{5\}) = \{4, 5, 7, 15\} = E_1$
- etat final : oui

expr = b*a*|(cb)*





$D = \{0, 2, 3, 4, 6, 7, 8, 12, 13, 14, 15\}$

$E_1 = \{4, 5, 7, 15\}$

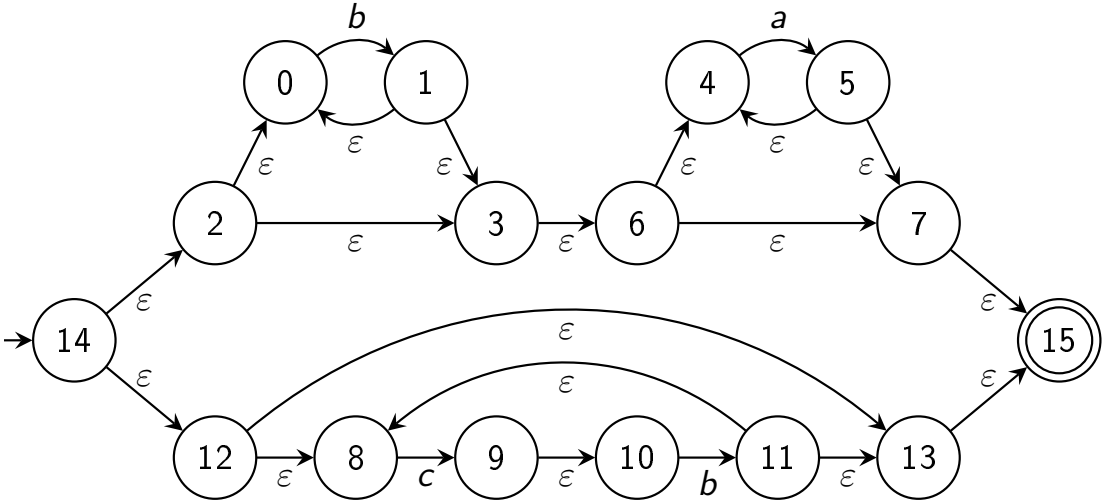
$E_2 = \{0, 1, 3, 4, 6, 7, 15\}$

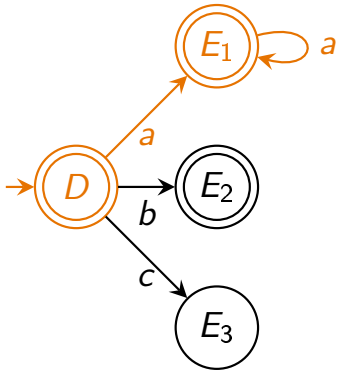
$E_3 = \{9, 10\}$

Etat actuel : E_1

- x = b
- transitions : aucune

expr = b*a*|(cb)*



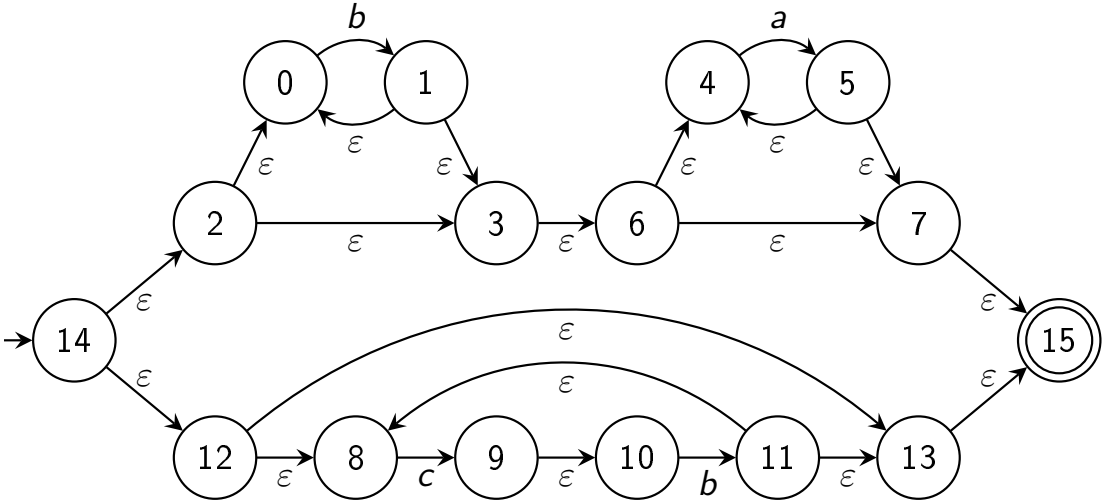


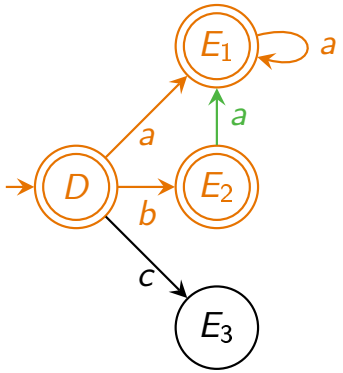
$$D = \{0, 2, 3, 4, 6, 7, 8, 12, 13, 14, 15\}$$
$$E_1 = \{4, 5, 7, 15\}$$
$$E_2 = \{0, 1, 3, 4, 6, 7, 15\}$$
$$E_3 = \{9, 10\}$$

Etat actuel : E_1

- x = c
- transitions : aucune

expr = b*a*|(cb)*



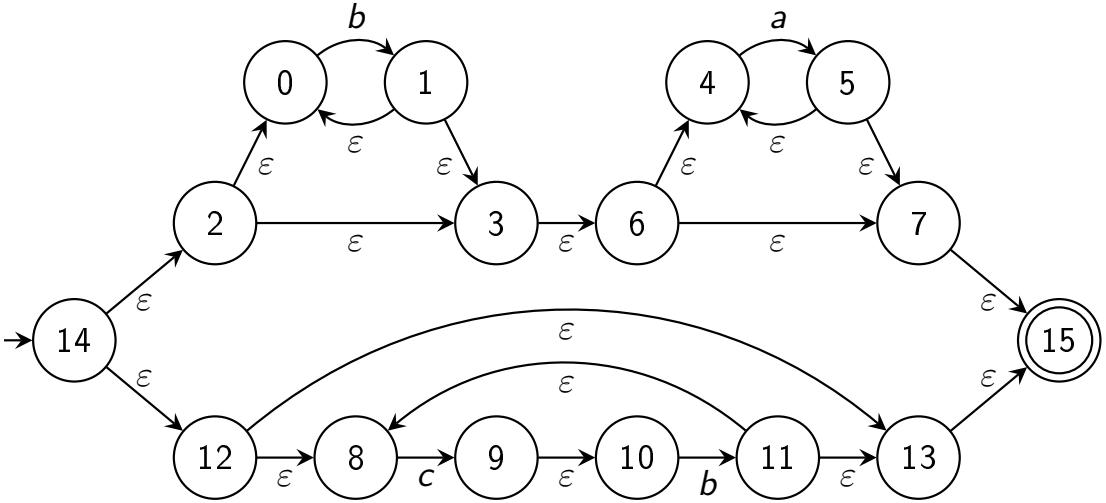


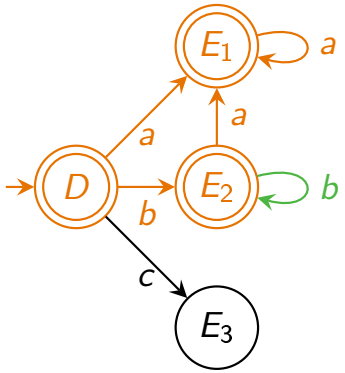
$D = \{0, 2, 3, 4, 6, 7, 8, 12, 13, 14, 15\}$
$E_1 = \{4, 5, 7, 15\}$
$E_2 = \{0, 1, 3, 4, 6, 7, 15\}$
$E_3 = \{9, 10\}$

Etat actuel : E_2

- x = a
- transitions : 4a5
- $EpsilonFermeture(\{5\}) = \{4, 5, 7, 15\} = E_1$
- etat final : oui

expr = b*a*|(cb)*



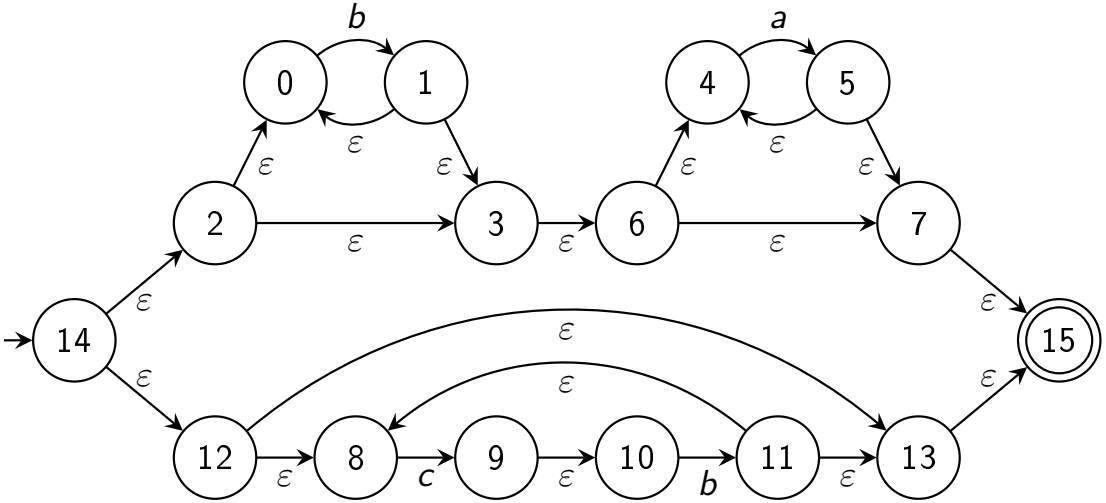


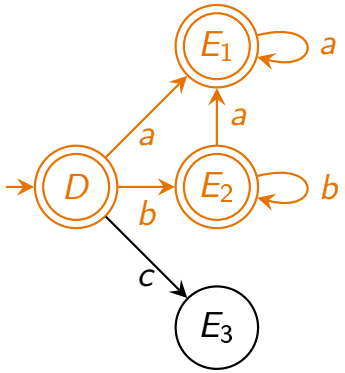
$D = \{0, 2, 3, 4, 6, 7, 8, 12, 13, 14, 15\}$
$E_1 = \{4, 5, 7, 15\}$
$E_2 = \{0, 1, 3, 4, 6, 7, 15\}$
$E_3 = \{9, 10\}$

Etat actuel : E_2

- x = b
- transitions : 0b1
- $EpsilonFermeture(\{1\}) = \{0, 1, 3, 4, 6, 7, 15\} = E_2$
- etat final : oui

expr = b*a*|(cb)*



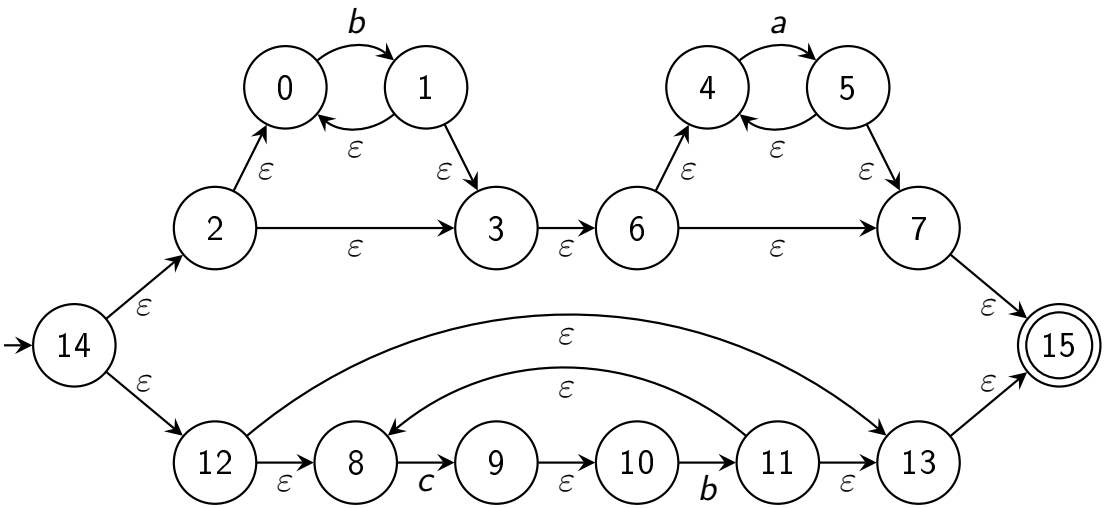


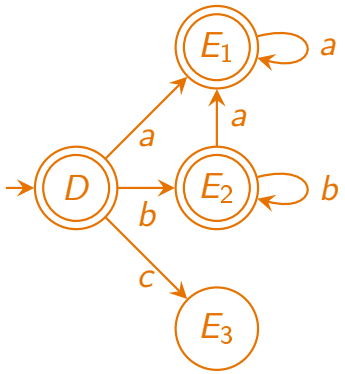
$$D = \{0, 2, 3, 4, 6, 7, 8, 12, 13, 14, 15\}$$
$$E_1 = \{4, 5, 7, 15\}$$
$$E_2 = \{0, 1, 3, 4, 6, 7, 15\}$$
$$E_3 = \{9, 10\}$$

Etat actuel : E_2

- x = c
- transitions : aucune

expr = b*a*|(cb)*



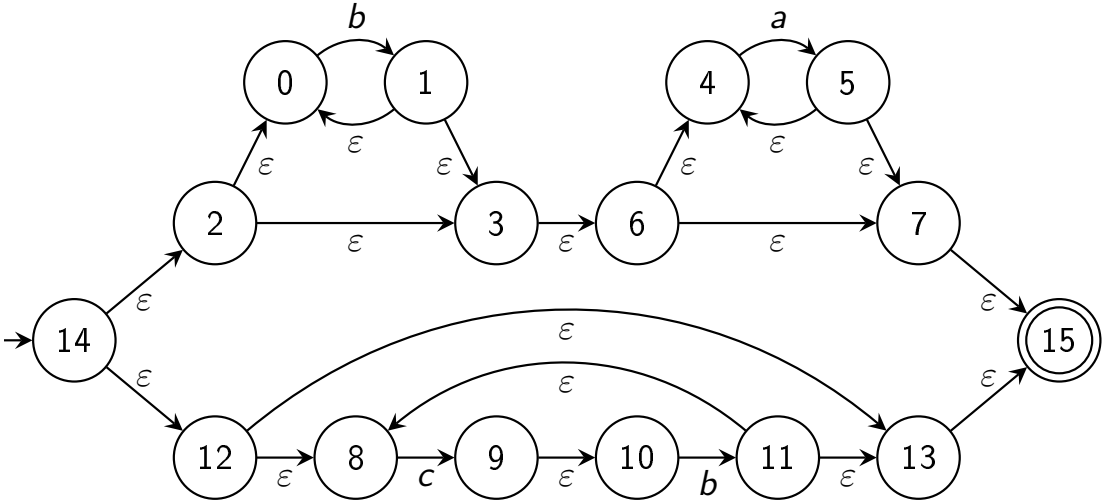


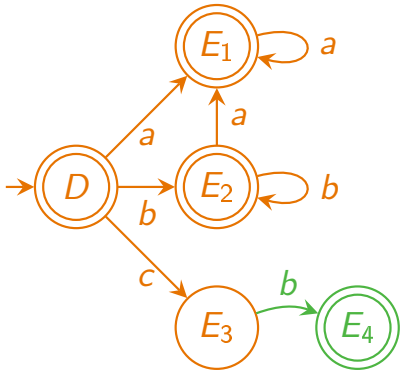
$$D = \{0, 2, 3, 4, 6, 7, 8, 12, 13, 14, 15\}$$
$$E_1 = \{4, 5, 7, 15\}$$
$$E_2 = \{0, 1, 3, 4, 6, 7, 15\}$$
$$E_3 = \{9, 10\}$$

Etat actuel : E_3

- x = a
- transitions : aucune

expr = b*a*|(cb)*



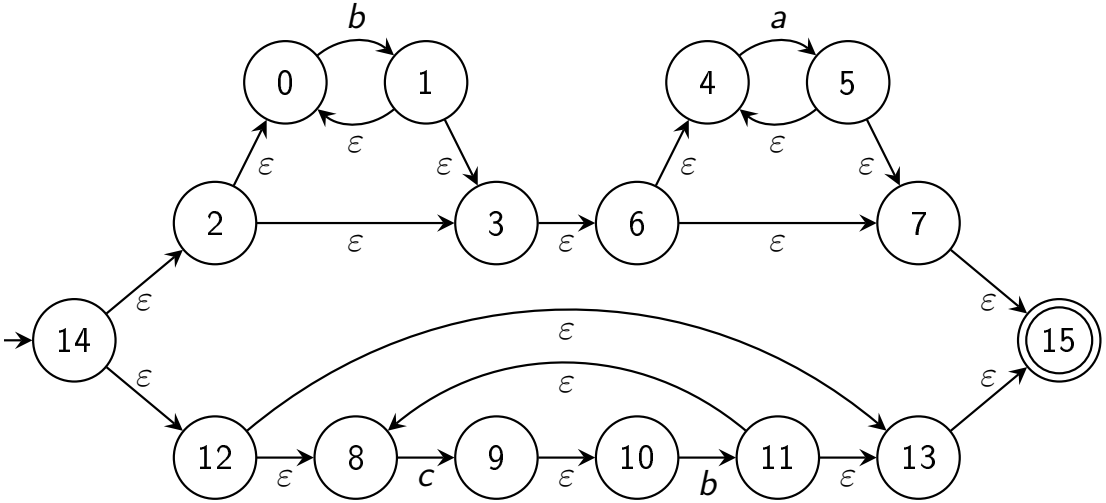


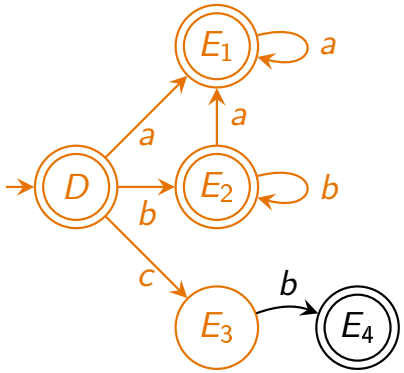
$D = \{0, 2, 3, 4, 6, 7, 8, 12, 13, 14, 15\}$
$E_1 = \{4, 5, 7, 15\}$
$E_2 = \{0, 1, 3, 4, 6, 7, 15\}$
$E_3 = \{9, 10\}$
$E_4 = \{8, 11, 13, 15\}$

Etat actuel : E_3

- x = b
- transitions : 10b11
- $EpsilonFermeture(\{11\}) = \{8, 11, 13, 15\} = E_4$
- etat final : oui

expr = b*a*|(cb)*



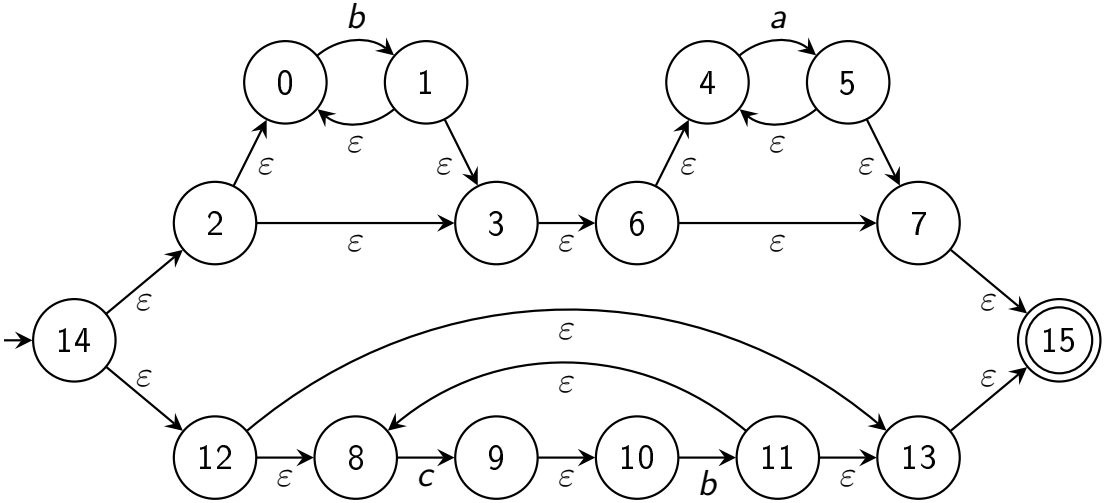


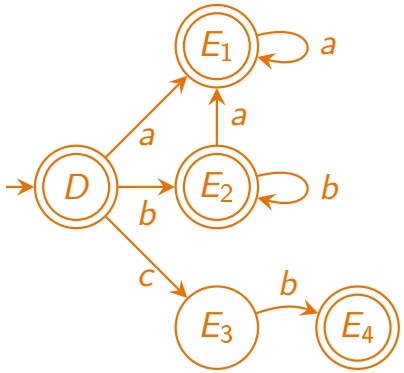
$$D = \{0, 2, 3, 4, 6, 7, 8, 12, 13, 14, 15\}$$
$$E_1 = \{4, 5, 7, 15\}$$
$$E_2 = \{0, 1, 3, 4, 6, 7, 15\}$$
$$E_3 = \{9, 10\}$$
$$E_4 = \{8, 11, 13, 15\}$$

Etat actuel : E_3

- x = c
- transitions : aucune

expr = b*a*|(cb)*



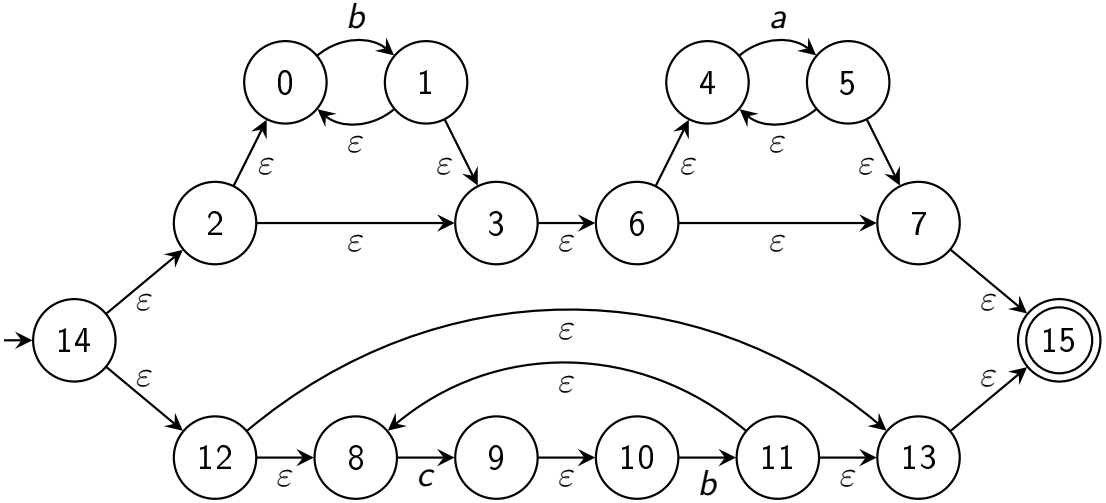


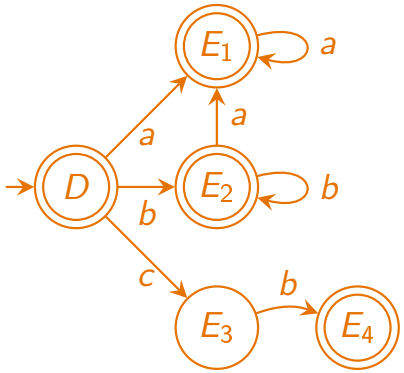
$$D = \{0, 2, 3, 4, 6, 7, 8, 12, 13, 14, 15\}$$
$$E_1 = \{4, 5, 7, 15\}$$
$$E_2 = \{0, 1, 3, 4, 6, 7, 15\}$$
$$E_3 = \{9, 10\}$$
$$E_4 = \{8, 11, 13, 15\}$$

Etat actuel : E_4

- x = a
- transitions : aucune

expr = b*a*|(cb)*



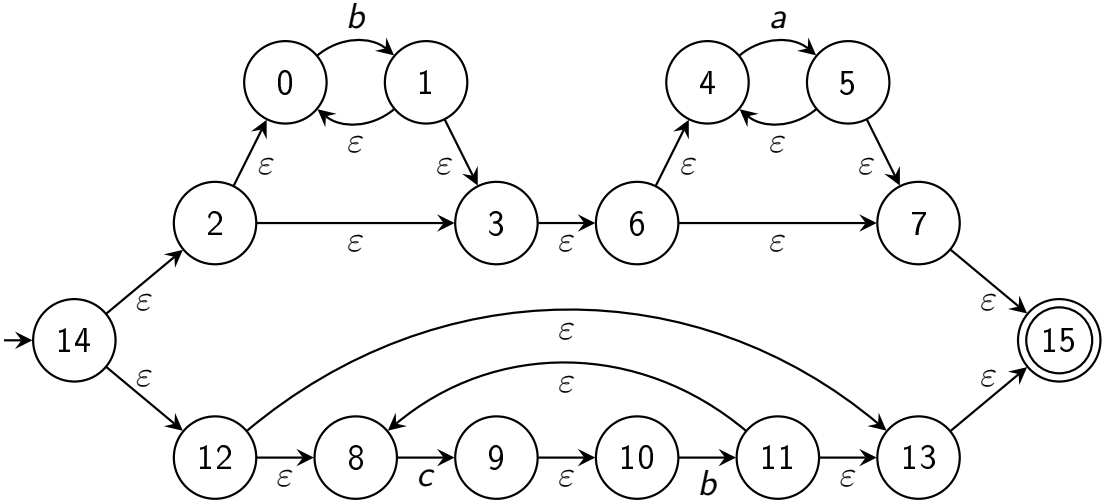


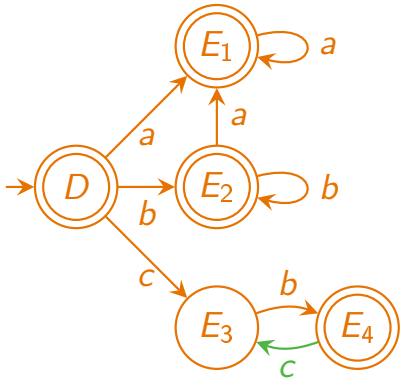
$D = \{0, 2, 3, 4, 6, 7, 8, 12, 13, 14, 15\}$
$E_1 = \{4, 5, 7, 15\}$
$E_2 = \{0, 1, 3, 4, 6, 7, 15\}$
$E_3 = \{9, 10\}$
$E_4 = \{8, 11, 13, 15\}$

Etat actuel : E_4

- x = b
- transitions : aucune

expr = b*a*|(cb)*



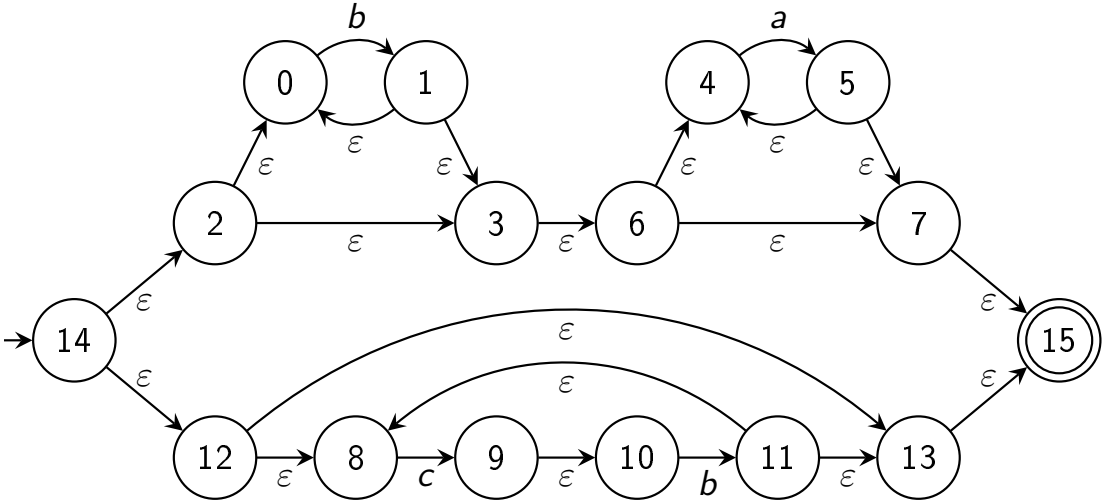


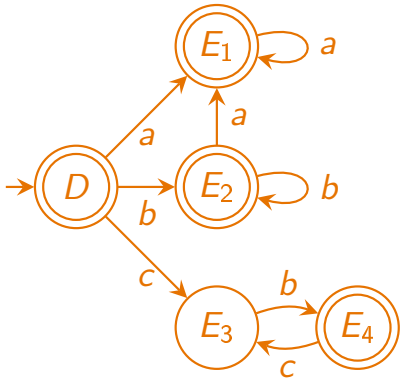
$$D = \{0, 2, 3, 4, 6, 7, 8, 12, 13, 14, 15\}$$
$$E_1 = \{4, 5, 7, 15\}$$
$$E_2 = \{0, 1, 3, 4, 6, 7, 15\}$$
$$E_3 = \{9, 10\}$$
$$E_4 = \{8, 11, 13, 15\}$$

Etat actuel : E_4

- x = c
- transitions : 8c9
- $EpsilonFermeture(\{9\}) = \{9, 10\} = E_3$
- etat final : non

expr = b*a*|(cb)*

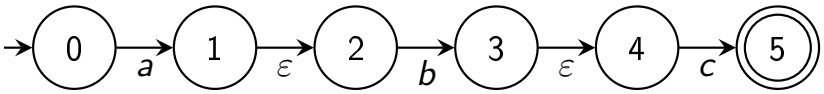


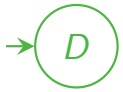


$$D = \{0, 2, 3, 4, 6, 7, 8, 12, 13, 14, 15\}$$
$$E_1 = \{4, 5, 7, 15\}$$
$$E_2 = \{0, 1, 3, 4, 6, 7, 15\}$$
$$E_3 = \{9, 10\}$$
$$E_4 = \{8, 11, 13, 15\}$$

AFD terminé, nombre d'états : 5

expr = abc



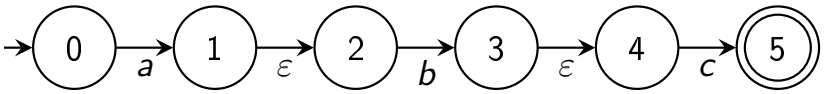


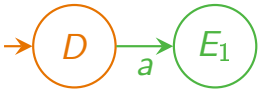
$D = \{0\}$

Calcul de l'état de départ :

$$D = EpsilonFermeture(\{0\}) = \{0\}$$

expr = abc





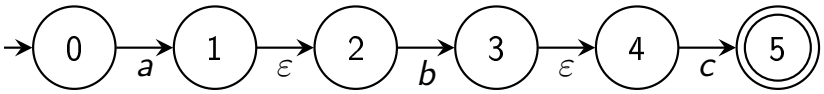
$D = \{0\}$

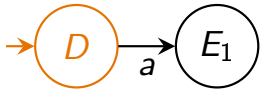
$E_1 = \{1, 2\}$

Etat actuel : D

- x = a
- transitions : 0a1
- $EpsilonFermeture(\{1\}) = \{1, 2\} = E_1$
- etat final : non

expr = abc





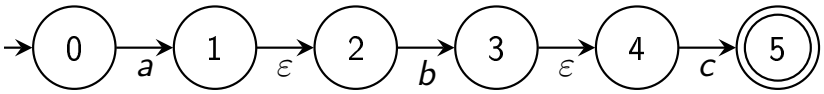
$D = \{0\}$

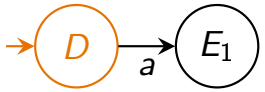
$E_1 = \{1, 2\}$

Etat actuel : D

- x = b
- transitions : aucune

expr = abc





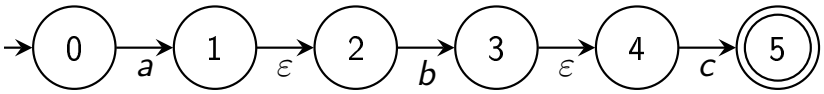
$D = \{0\}$

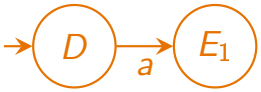
$E_1 = \{1, 2\}$

Etat actuel : D

- x = c
- transitions : aucune

expr = abc





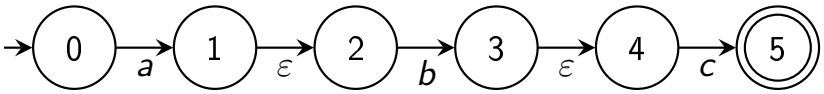
$D = \{0\}$

$E_1 = \{1, 2\}$

Etat actuel : E_1

- x = a
- transitions : aucune

expr = abc

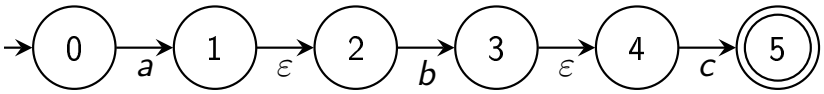


$D = \{0\}$
$E_1 = \{1, 2\}$
$E_2 = \{3, 4\}$

Etat actuel : E_1

- x = b
- transitions : 2b3
- $EpsilonFermeture(\{3\}) = \{3, 4\} = E_2$
- etat final : non

expr = abc

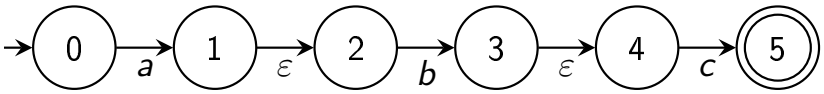


$D = \{0\}$
$E_1 = \{1, 2\}$
$E_2 = \{3, 4\}$

Etat actuel : E_1

- x = c
- transitions : aucune

expr = abc

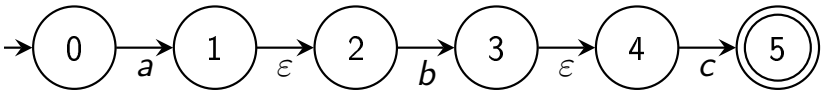


$D = \{0\}$
$E_1 = \{1, 2\}$
$E_2 = \{3, 4\}$

Etat actuel : E_2

- x = a
- transitions : aucune

expr = abc

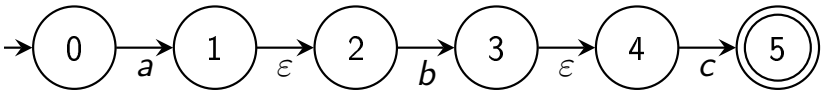


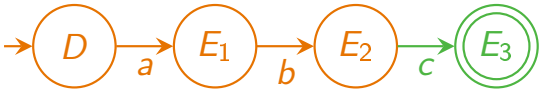
$D = \{0\}$
$E_1 = \{1, 2\}$
$E_2 = \{3, 4\}$

Etat actuel : E_2

- x = b
- transitions : aucune

expr = abc



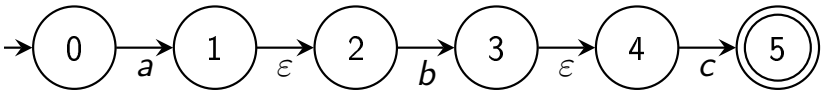


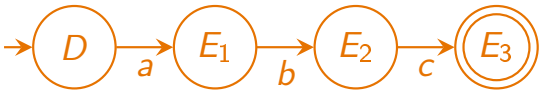
$D = \{0\}$
$E_1 = \{1, 2\}$
$E_2 = \{3, 4\}$
$E_3 = \{5\}$

Etat actuel : E_2

- x = c
- transitions : 4c5
- $EpsilonFermeture(\{5\}) = \{5\} = E_3$
- etat final : oui

expr = abc



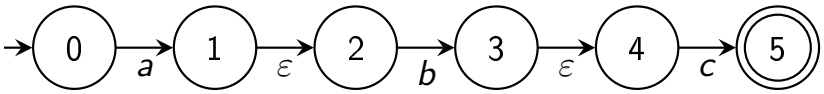


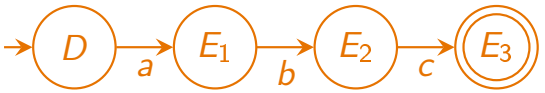
$D = \{0\}$
$E_1 = \{1, 2\}$
$E_2 = \{3, 4\}$
$E_3 = \{5\}$

Etat actuel : E_3

- x = a
- transitions : aucune

expr = abc



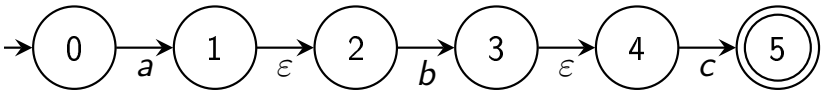


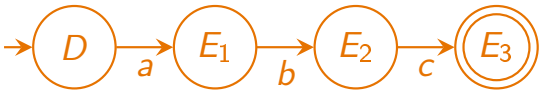
$D = \{0\}$
$E_1 = \{1, 2\}$
$E_2 = \{3, 4\}$
$E_3 = \{5\}$

Etat actuel : E_3

- x = b
- transitions : aucune

expr = abc



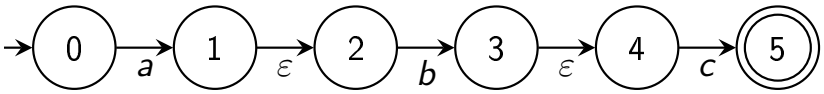


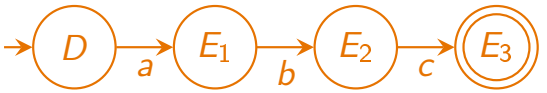
$D = \{0\}$
$E_1 = \{1, 2\}$
$E_2 = \{3, 4\}$
$E_3 = \{5\}$

Etat actuel : E_3

- x = c
- transitions : aucune

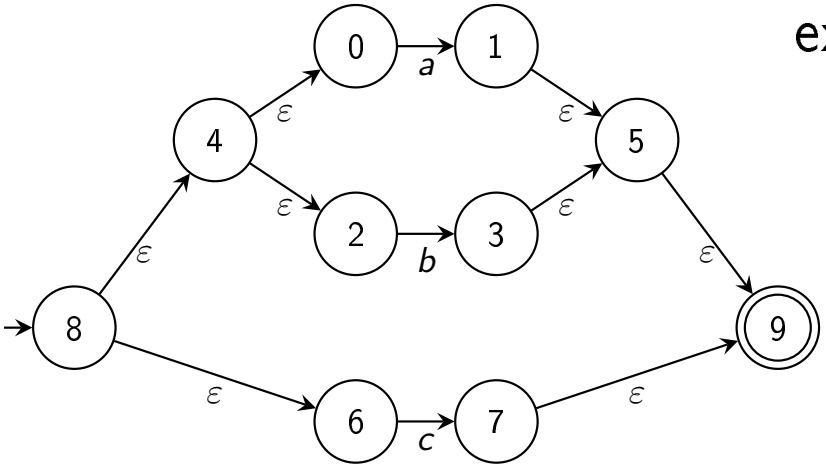
expr = abc



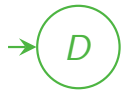


$D = \{0\}$
$E_1 = \{1, 2\}$
$E_2 = \{3, 4\}$
$E_3 = \{5\}$

AFD terminé, nombre d'états : 4



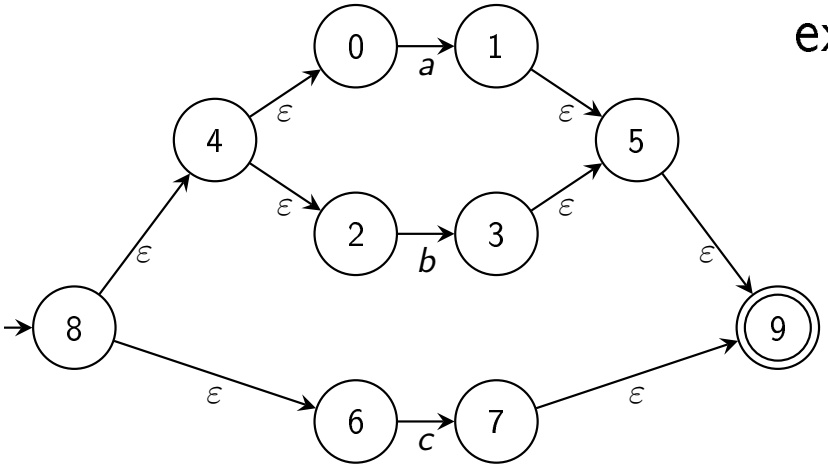
expr = (a|b)|c



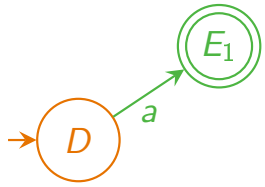
$D = \{0, 2, 4, 6, 8\}$

Calcul de l'état de départ :

$$D = EpsilonFermeture(\{8\}) = \{0, 2, 4, 6, 8\}$$



expr = (a|b)|c

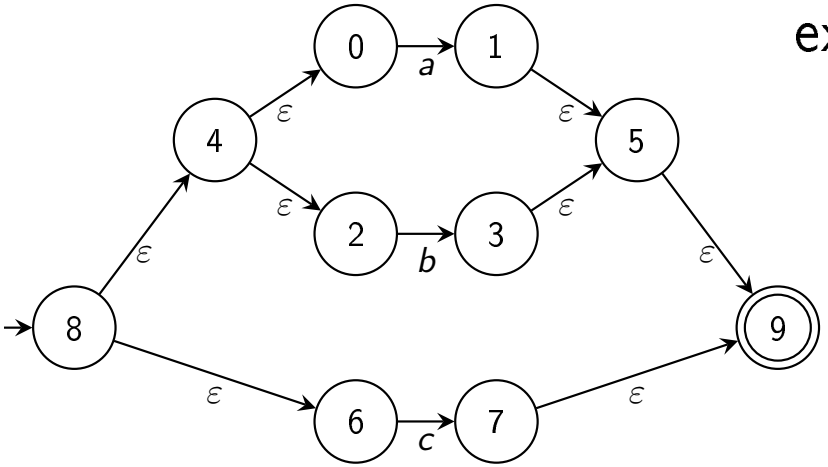


$D = \{0, 2, 4, 6, 8\}$

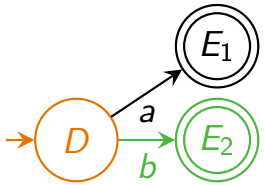
$E_1 = \{1, 5, 9\}$

Etat actuel : D

- x = a
- transitions : 0a1
- $EpsilonFermeture(\{1\}) = \{1, 5, 9\} = E_1$
- etat final : oui



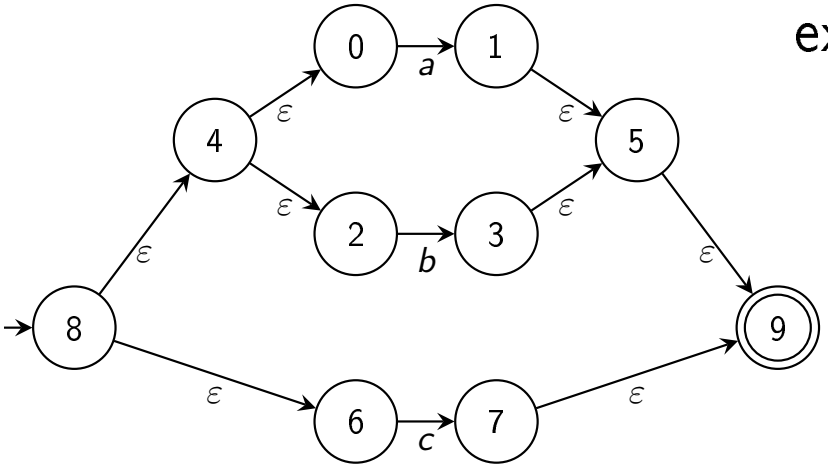
expr = (a|b)|c



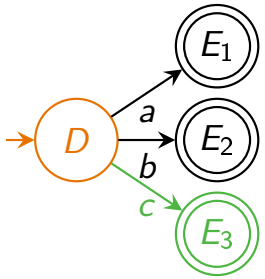
$D = \{0, 2, 4, 6, 8\}$
$E_1 = \{1, 5, 9\}$
$E_2 = \{3, 5, 9\}$

Etat actuel : D

- x = b
- transitions : 2b3
- $EpsilonFermeture(\{3\}) = \{3, 5, 9\} = E_2$
- etat final : oui



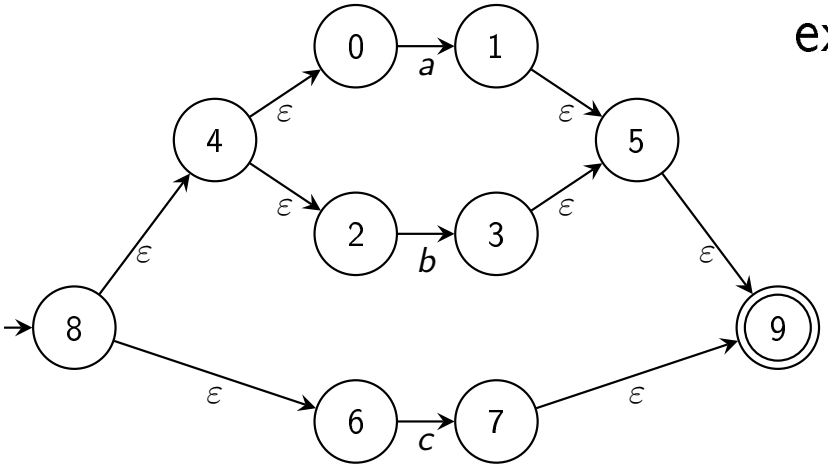
expr = (a|b)|c



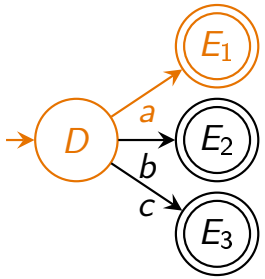
$D = \{0, 2, 4, 6, 8\}$
$E_1 = \{1, 5, 9\}$
$E_2 = \{3, 5, 9\}$
$E_3 = \{7, 9\}$

Etat actuel : D

- x = c
- transitions : 6c7
- $EpsilonFermeture(\{5\}) = \{7, 9\} = E_3$
- etat final : oui



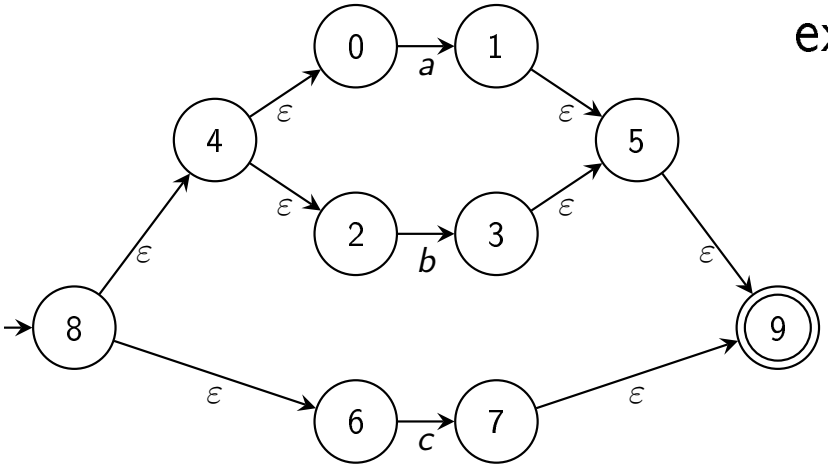
expr = (a|b)|c



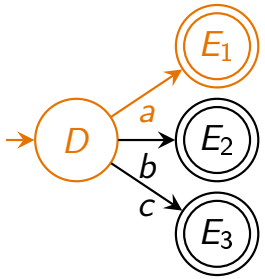
$D = \{0, 2, 4, 6, 8\}$
$E_1 = \{1, 5, 9\}$
$E_2 = \{3, 5, 9\}$
$E_3 = \{7, 9\}$

Etat actuel : E_1

- x = a
- transitions : aucune



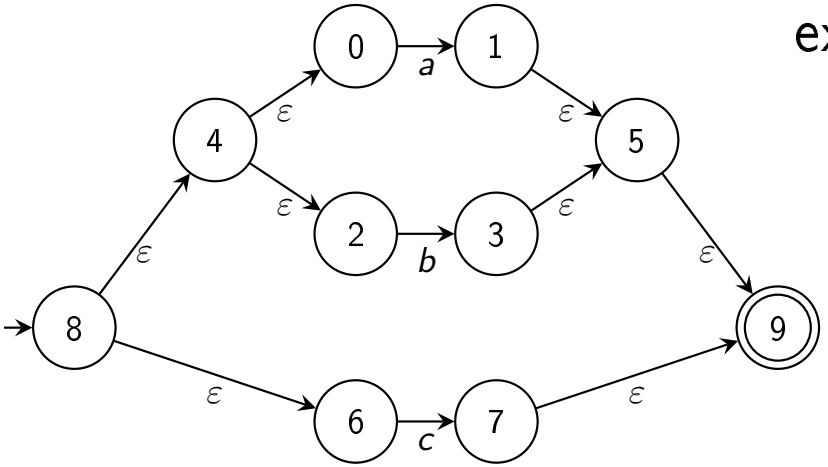
expr = (a|b)|c



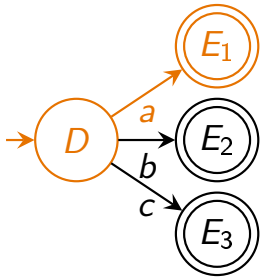
$D = \{0, 2, 4, 6, 8\}$
$E_1 = \{1, 5, 9\}$
$E_2 = \{3, 5, 9\}$
$E_3 = \{7, 9\}$

Etat actuel : E_1

- x = b
- transitions : aucune



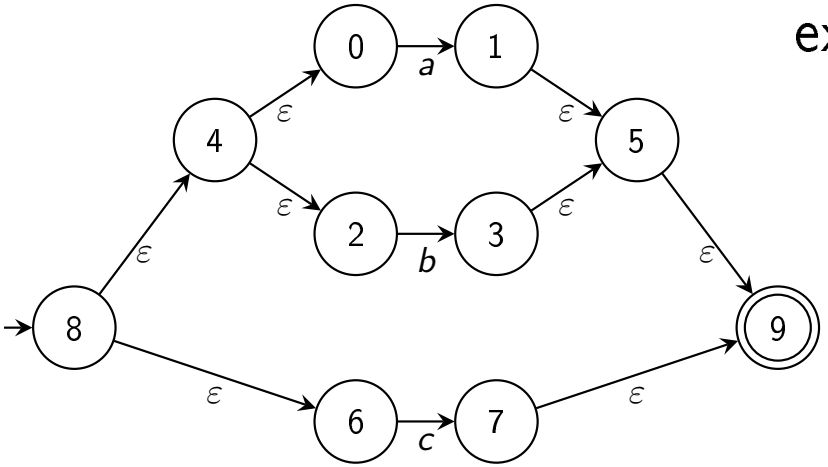
expr = (a|b)|c



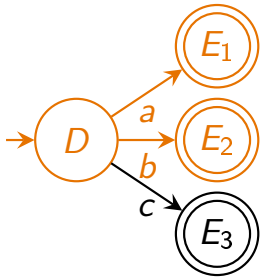
$D = \{0, 2, 4, 6, 8\}$
$E_1 = \{1, 5, 9\}$
$E_2 = \{3, 5, 9\}$
$E_3 = \{7, 9\}$

Etat actuel : E_1

- x = c
- transitions : aucune



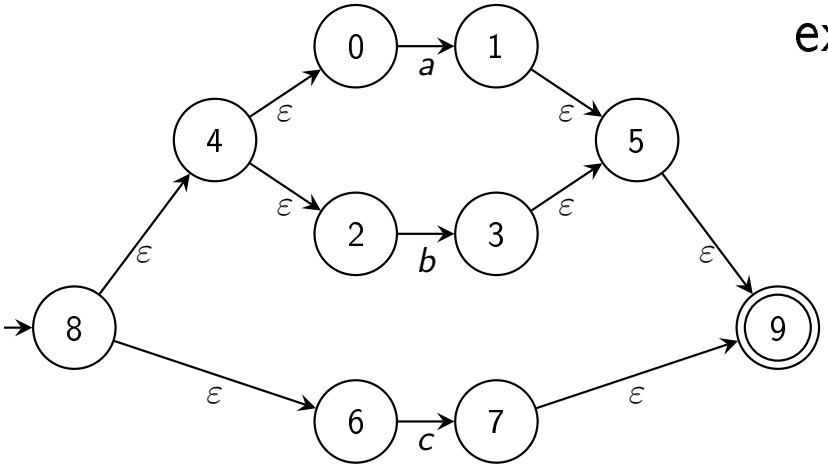
expr = (a|b)|c



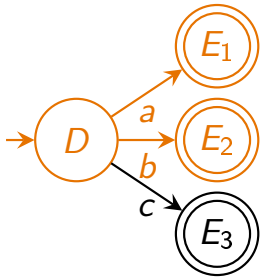
$D = \{0, 2, 4, 6, 8\}$
$E_1 = \{1, 5, 9\}$
$E_2 = \{3, 5, 9\}$
$E_3 = \{7, 9\}$

Etat actuel : E_2

- x = a
- transitions : aucune



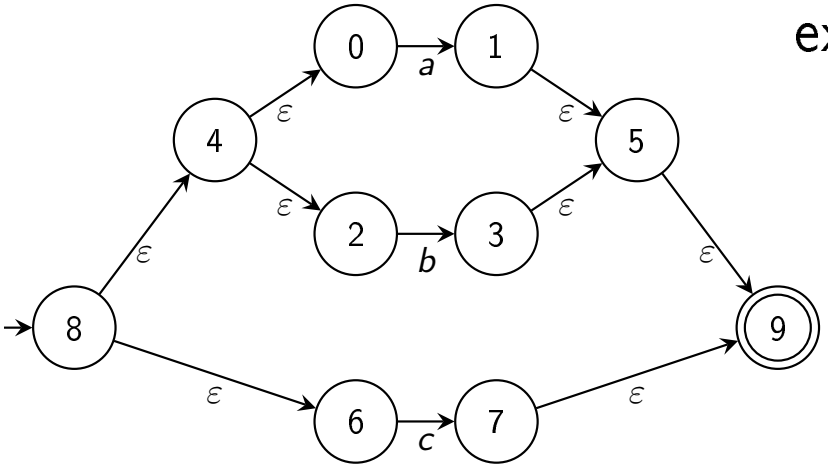
expr = (a|b)|c



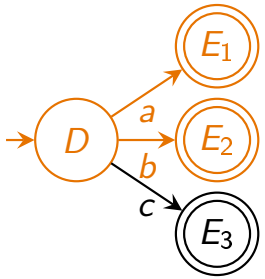
$D = \{0, 2, 4, 6, 8\}$
$E_1 = \{1, 5, 9\}$
$E_2 = \{3, 5, 9\}$
$E_3 = \{7, 9\}$

Etat actuel : E_2

- x = b
- transitions : aucune



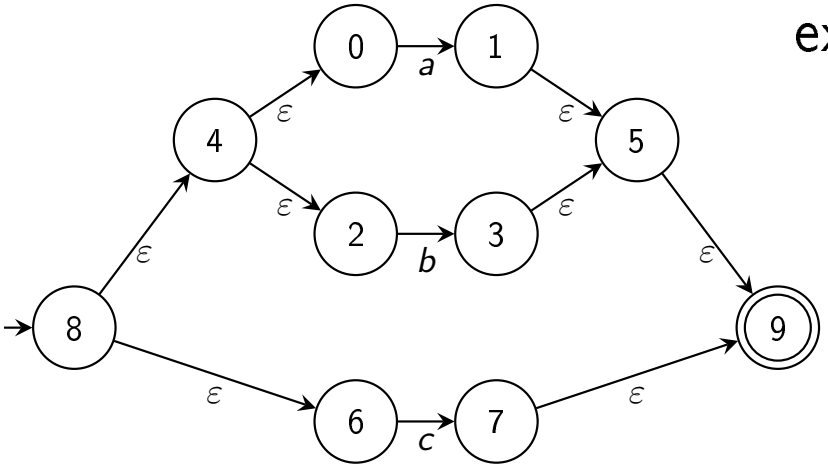
expr = (a|b)|c



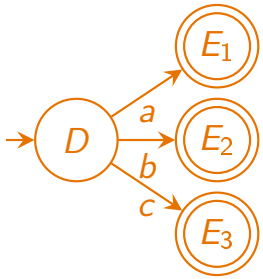
$D = \{0, 2, 4, 6, 8\}$
$E_1 = \{1, 5, 9\}$
$E_2 = \{3, 5, 9\}$
$E_3 = \{7, 9\}$

Etat actuel : E_2

- x = c
- transitions : aucune



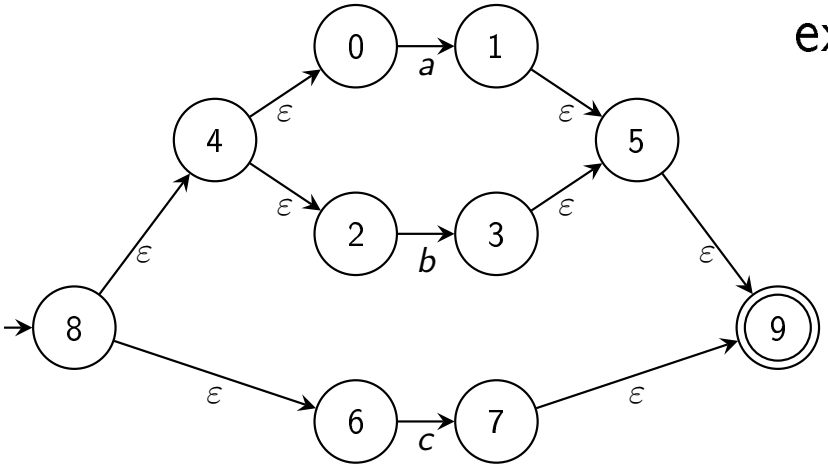
expr = (a|b)|c



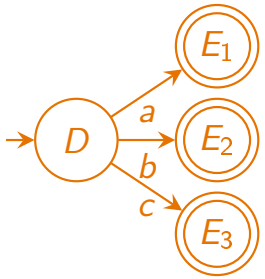
$D = \{0, 2, 4, 6, 8\}$
$E_1 = \{1, 5, 9\}$
$E_2 = \{3, 5, 9\}$
$E_3 = \{7, 9\}$

Etat actuel : E_3

- x = a
- transitions : aucune



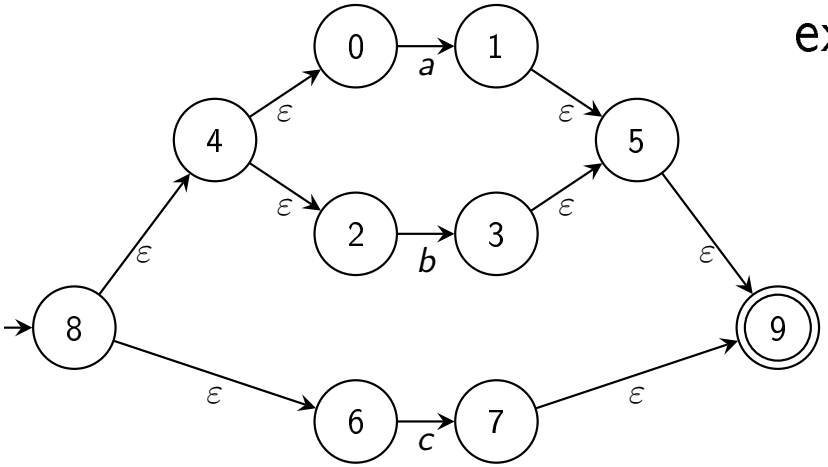
expr = (a|b)|c



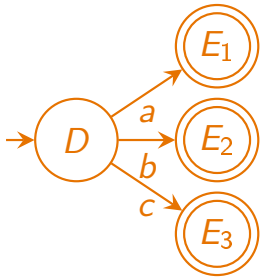
$D = \{0, 2, 4, 6, 8\}$
$E_1 = \{1, 5, 9\}$
$E_2 = \{3, 5, 9\}$
$E_3 = \{7, 9\}$

Etat actuel : E_3

- x = b
- transitions : aucune



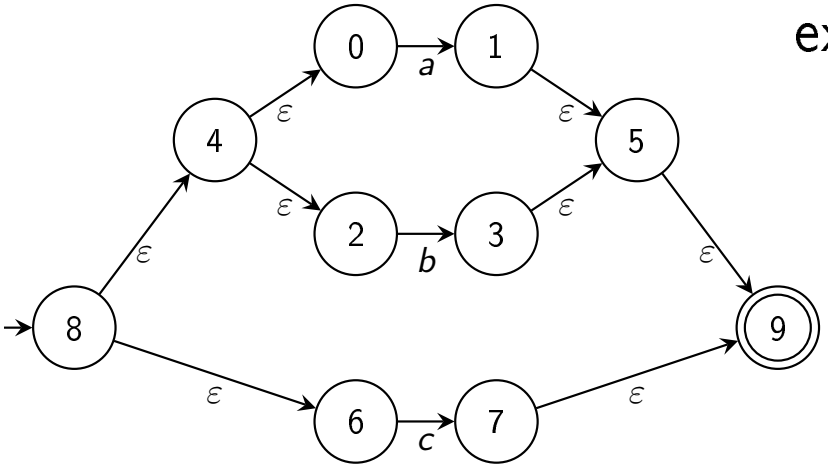
expr = (a|b)|c



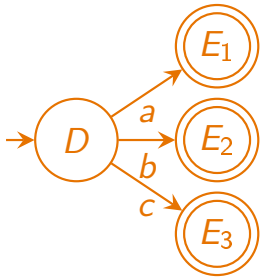
$D = \{0, 2, 4, 6, 8\}$
$E_1 = \{1, 5, 9\}$
$E_2 = \{3, 5, 9\}$
$E_3 = \{7, 9\}$

Etat actuel : E_3

- x = c
- transitions : aucune

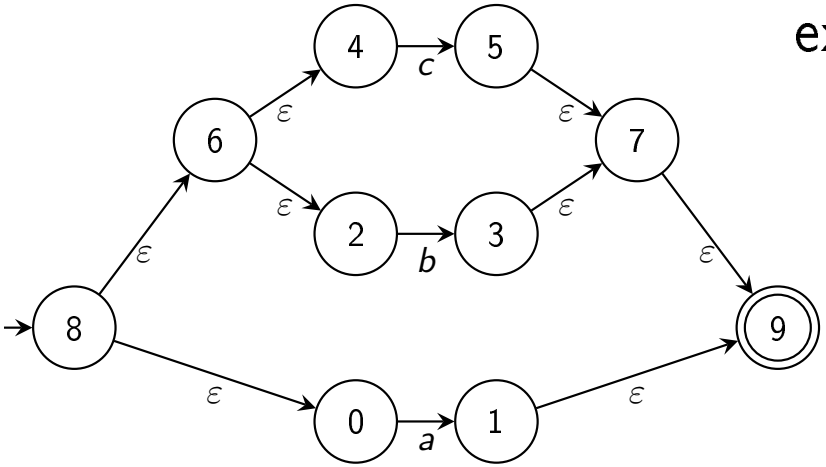


expr = (a|b)|c

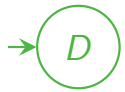


$D = \{0, 2, 4, 6, 8\}$
$E_1 = \{1, 5, 9\}$
$E_2 = \{3, 5, 9\}$
$E_3 = \{7, 9\}$

AFD terminé, nombre d'états : 4



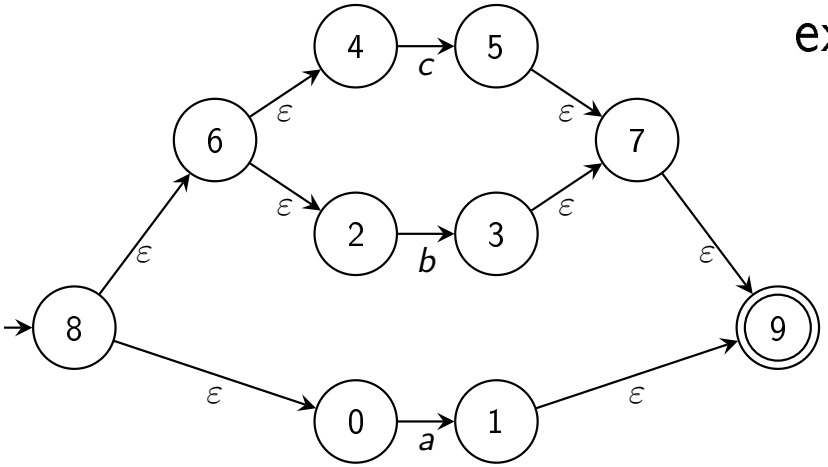
expr = a|(b|c)



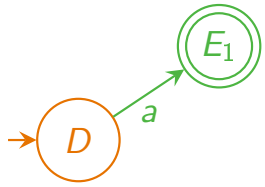
$D = \{0, 2, 4, 6, 8\}$

Calcul de l'état de départ :

$$D = EpsilonFermeture(\{8\}) = \{0,\ 2,\ 4,\ 6,\ 8\}$$



expr = a|(b|c)

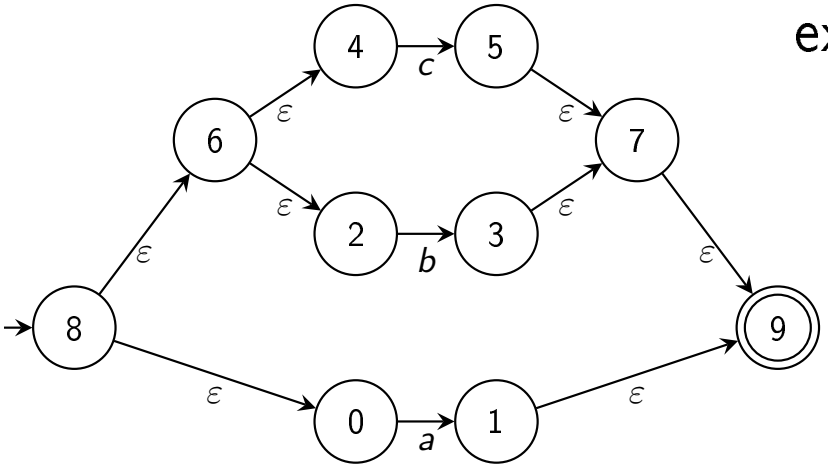


$D = \{0, 2, 4, 6, 8\}$

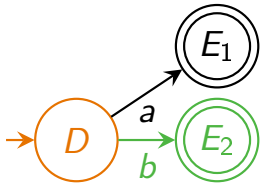
$E_1 = \{1, 9\}$

Etat actuel : D

- x = a
- transitions : 0a1
- $EpsilonFermeture(\{1\}) = \{1, 9\} = E_1$
- etat final : oui



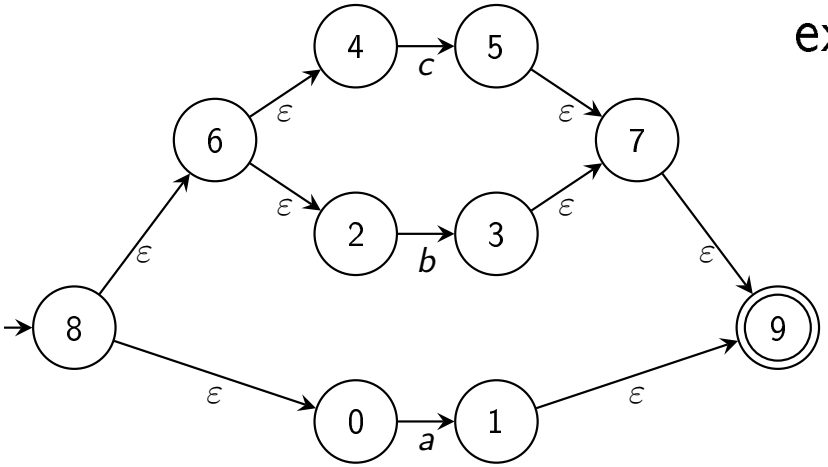
expr = a|(b|c)



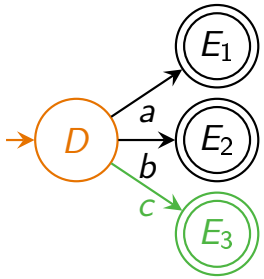
$D = \{0, 2, 4, 6, 8\}$
$E_1 = \{1, 9\}$
$E_2 = \{3, 7, 9\}$

Etat actuel : D

- x = b
- transitions : 2b3
- $EpsilonFermeture(\{3\}) = \{3, 7, 9\} = E_2$
- etat final : oui



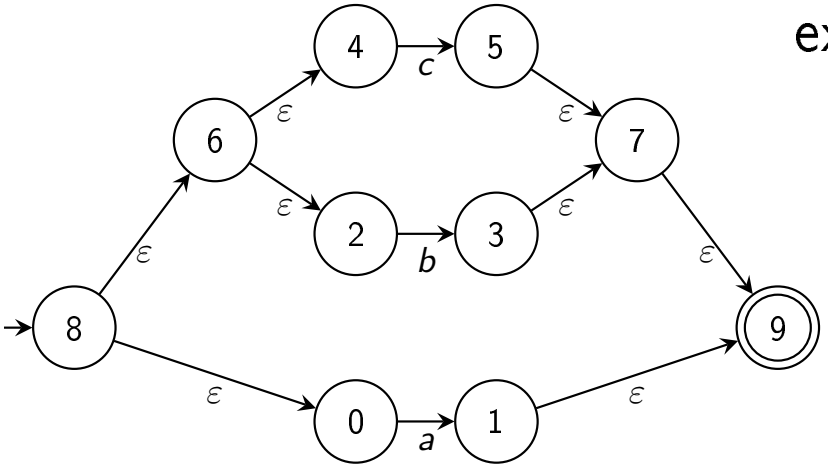
expr = a|(b|c)



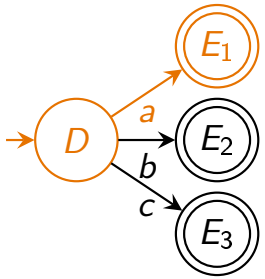
$D = \{0, 2, 4, 6, 8\}$
$E_1 = \{1, 9\}$
$E_2 = \{3, 7, 9\}$
$E_3 = \{5, 7, 9\}$

Etat actuel : D

- x = c
- transitions : 4c5
- $EpsilonFermeture(\{5\}) = \{5, 7, 9\} = E_3$
- etat final : oui



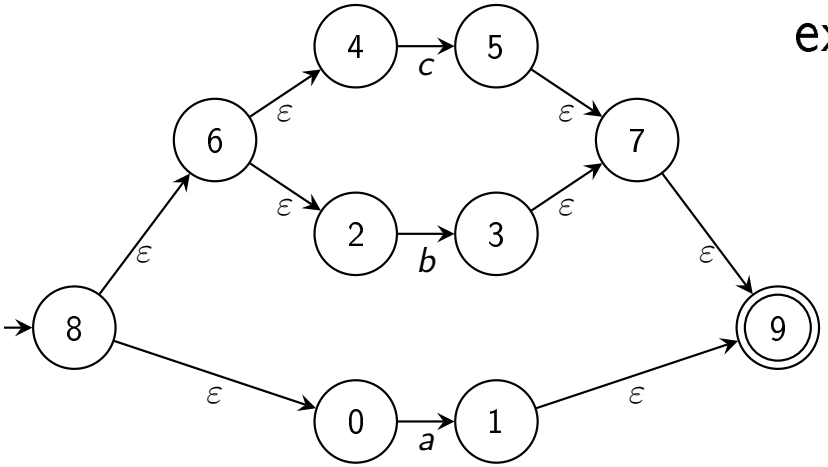
expr = a|(b|c)



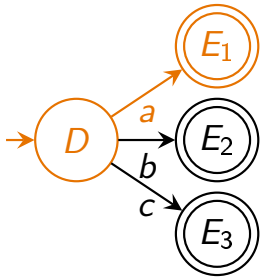
$D = \{0, 2, 4, 6, 8\}$
$E_1 = \{1, 9\}$
$E_2 = \{3, 7, 9\}$
$E_3 = \{5, 7, 9\}$

Etat actuel : E_1

- x = a
- transitions : aucune



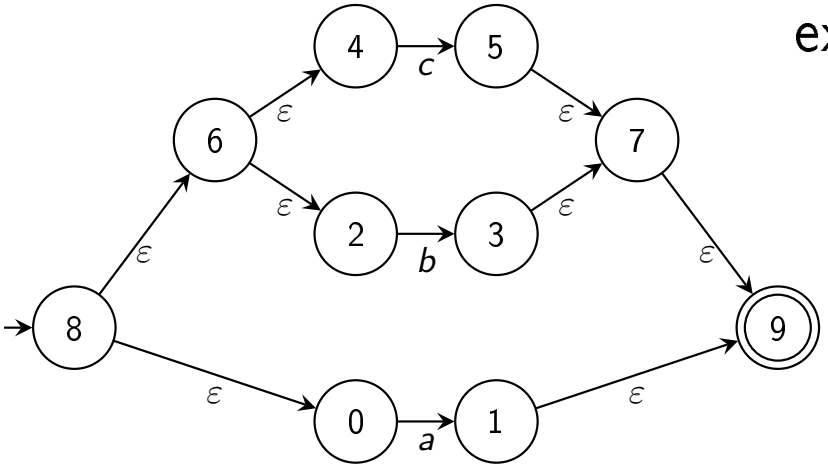
expr = a|(b|c)



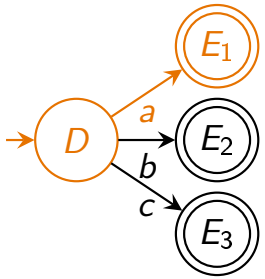
$D = \{0, 2, 4, 6, 8\}$
$E_1 = \{1, 9\}$
$E_2 = \{3, 7, 9\}$
$E_3 = \{5, 7, 9\}$

Etat actuel : E_1

- x = b
- transitions : aucune



expr = a|(b|c)



$D = \{0, 2, 4, 6, 8\}$
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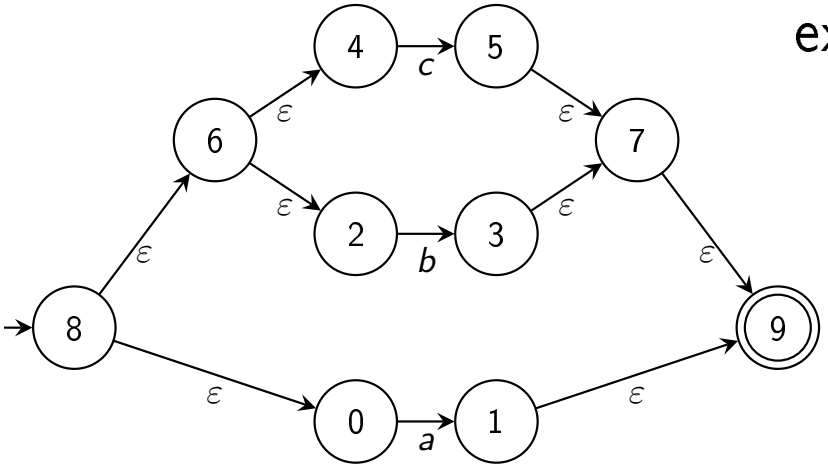
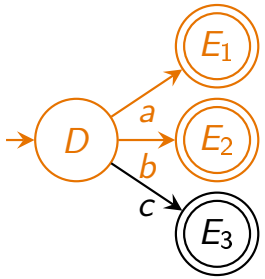
$E_3 = \{5, 7, 9\}$

Etat actuel : E_1

- x = c
- transitions : aucune



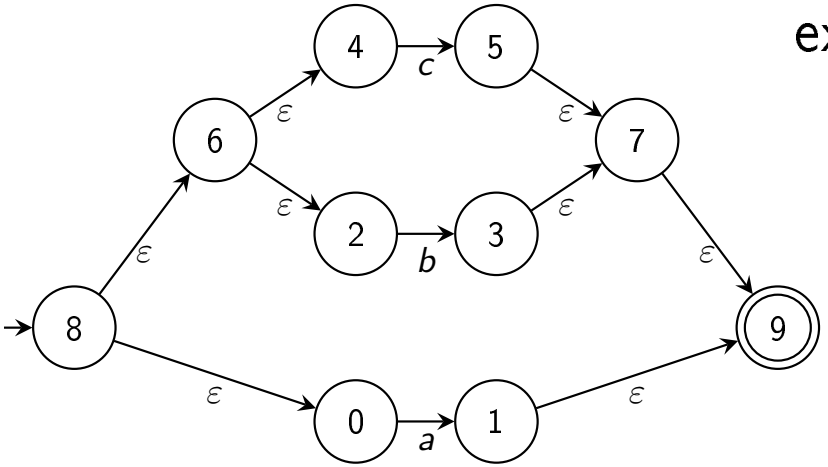
expr = a|(b|c)



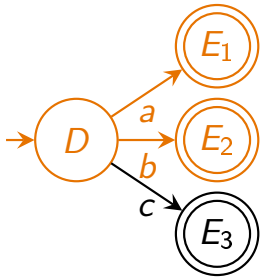
$D = \{0, 2, 4, 6, 8\}$
$E_1 = \{1, 9\}$
$E_2 = \{3, 7, 9\}$
$E_3 = \{5, 7, 9\}$

Etat actuel : E_2

- x = a
- transitions : aucune



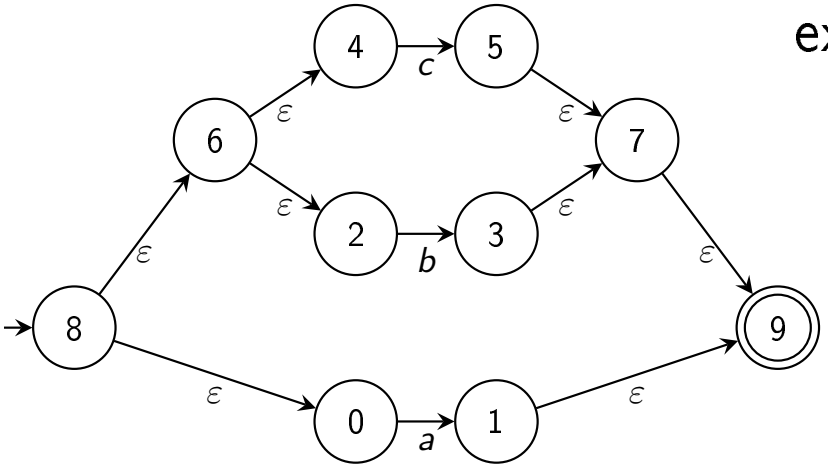
expr = a|(b|c)



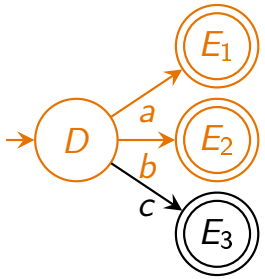
$D = \{0, 2, 4, 6, 8\}$
$E_1 = \{1, 9\}$
$E_2 = \{3, 7, 9\}$
$E_3 = \{5, 7, 9\}$

Etat actuel : E_2

- x = b
- transitions : aucune



expr = a|(b|c)



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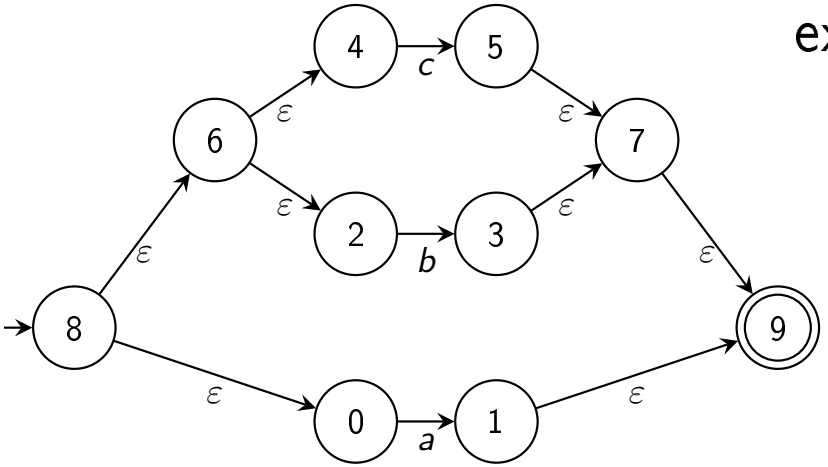
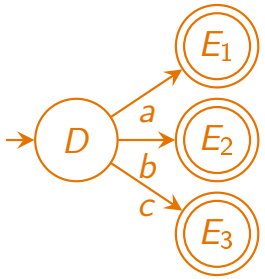
$E_3 = \{5, 7, 9\}$

Etat actuel : E_2

- x = c
- transitions : aucune



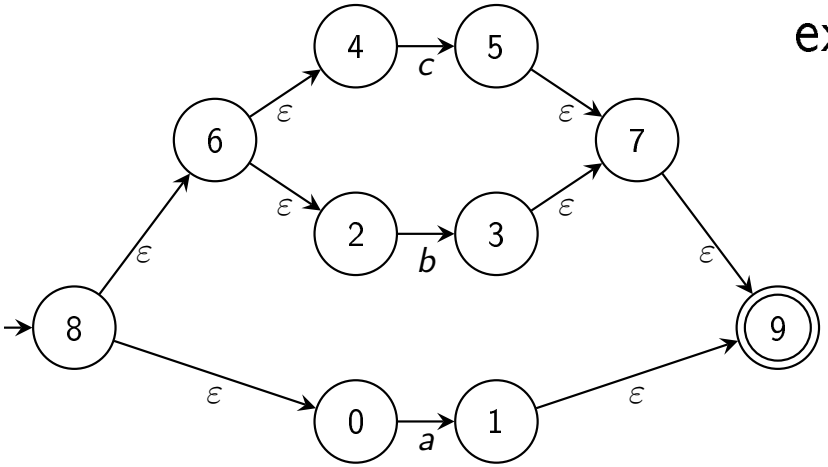
expr = a|(b|c)



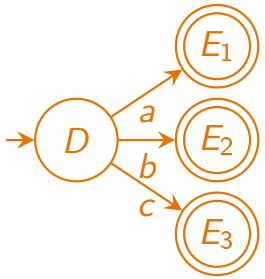
$D = \{0, 2, 4, 6, 8\}$
$E_1 = \{1, 9\}$
$E_2 = \{3, 7, 9\}$
$E_3 = \{5, 7, 9\}$

Etat actuel : E_3

- x = a
- transitions : aucune



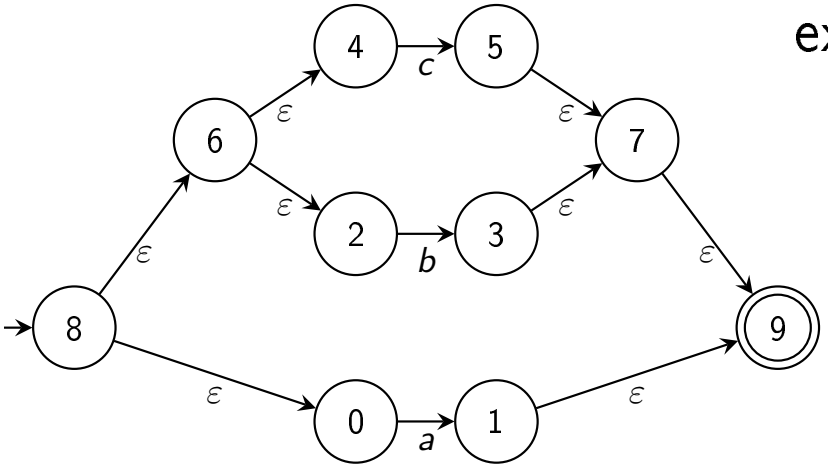
expr = a|(b|c)



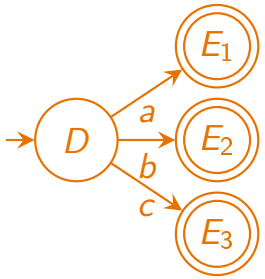
$D = \{0, 2, 4, 6, 8\}$
$E_1 = \{1, 9\}$
$E_2 = \{3, 7, 9\}$
$E_3 = \{5, 7, 9\}$

Etat actuel : E_3

- x = b
- transitions : aucune



expr = a|(b|c)



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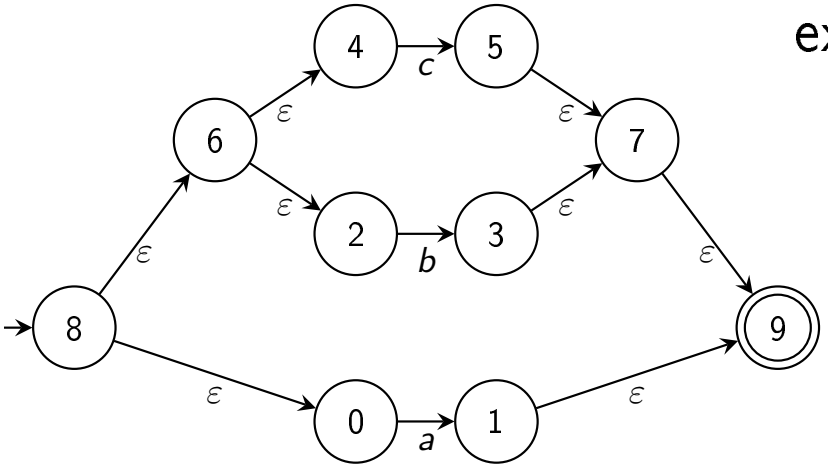
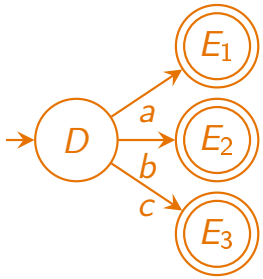
$E_3 = \{5, 7, 9\}$

Etat actuel : E_3

- x = c
- transitions : aucune



expr = a|(b|c)



$D = \{0, 2, 4, 6, 8\}$
$E_1 = \{1, 9\}$
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$E_3 = \{5, 7, 9\}$

AFD terminé, nombre d'états : 4

Les AFD des expressions régulières (a|b)|c et a|(b|c) sont strictement identiques. Cela montre que quels que soient a, b et c, la règle | est associative. Il est donc possible de supprimer les parenthèses sans ambiguité.

$$(a|b)|c = a|(b|c) = a|b|c$$