HAI6011 - Exercices de révisions

Benoît Huftier

2022

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Construction d'un AFN d'expression régulière

Enoncé

Donner l'automate fini non déterministe (AFN) des expressions régulières suivantes grâce à l'algorithme de Thompson.

- ab
- a b
- a

- ab c
- **o** ab*|c
- b*a*|(cb)*

- (a|b)|c
- a | (b|c)
- (ab)c
- a(bc)

Avec les AFN des expressions régulières (ab)c et a(bc), que peut-on en déduire sur la concaténation ?

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$$expr = ab$$

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$$expr = ab$$

arbre

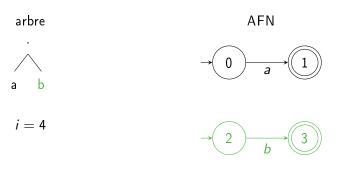


$$i = 0$$

$$expr = ab$$

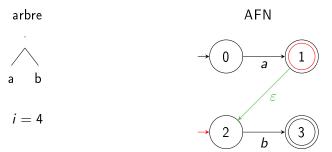


$$expr = ab$$



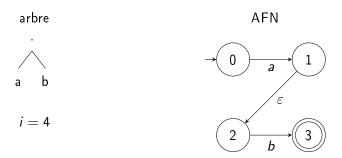
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$$expr = ab$$



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$$expr = ab$$



AFN final à 4 états

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$$\mathsf{expr} = \mathsf{b*}$$

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$$expr = b^*$$

arbre

- *
- b

$$i = 0$$

$$\mathsf{expr} = \mathsf{b*}$$

*

|

i = 2

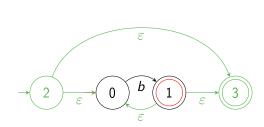
 AFN

$$\rightarrow 0$$
 b 1

$$expr = b^*$$



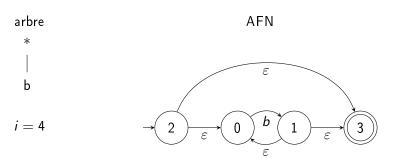
$$i = 4$$



AFN

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$$expr = b^*$$



AFN final à 4 états

$$expr = a|b$$

$$expr = a|b$$



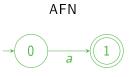


$$i = 0$$

$$expr = a|b$$



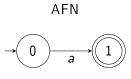
$$i = 2$$



$$expr = a|b$$



$$i = 4$$

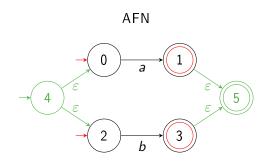




$$\mathsf{expr} = \mathsf{a}|\mathsf{b}$$

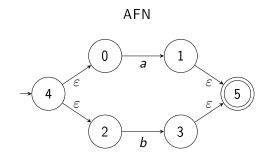


$$i = 6$$



$$expr = a|b$$





i = 6

AFN final à 6 états

$$expr = a$$

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$$expr = a$$

 arbre

a

$$i = 0$$

$$expr = a$$

 ${\it arbre}$

a

i = 2

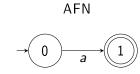
AFN



$$expr = a$$

arbre a

i = 2



AFN final à 2 états

$$expr = ab|c$$

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expr = ab|c

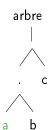






$$i = 0$$

$$expr = ab|c$$

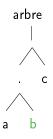


$$i = 2$$

AFN



expr = ab|c

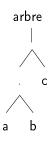


i = 4

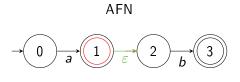


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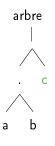
$$expr = ab|c$$



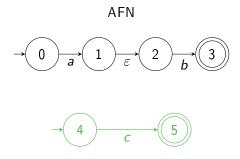
i = 4



$\mathsf{expr} = \mathsf{ab}|\mathsf{c}$

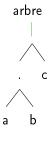


$$i = 6$$

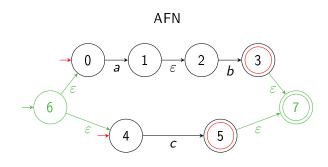


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expr = ab|c

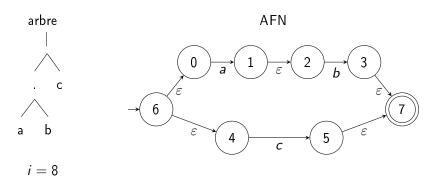






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expr = ab|c



AFN final à 8 états

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$$expr = ab*|c$$

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$$expr = ab*|c$$

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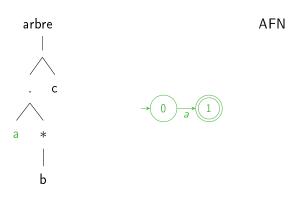


$$i = 0$$

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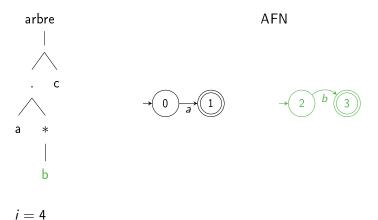
$$expr = ab*|c$$



i = 2

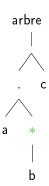
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$$expr = ab*|c$$

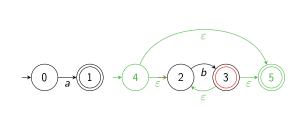




$$expr = ab*|c$$

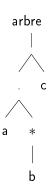


$$i = 6$$

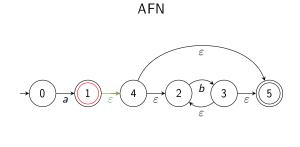


AFN

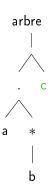
$$expr = ab*|c$$



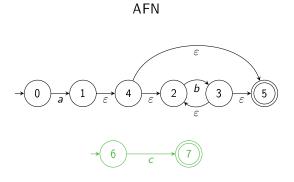
$$i = 6$$



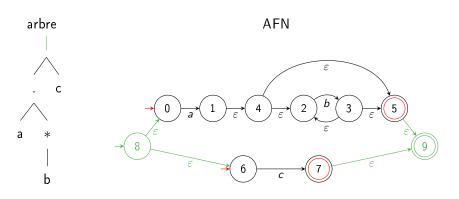
$$expr = ab*|c$$



$$i = 8$$



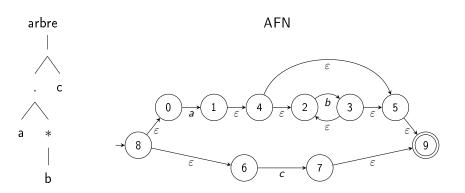
expr = ab*|c





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expr = ab*|c



AFN final à 10 états



i = 10

$$expr = ((a|b)|cc)*$$

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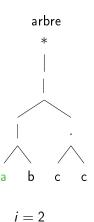
$$\mathsf{expr} = ((\mathsf{a}|\mathsf{b})|\mathsf{cc})^*$$

$\quad \text{arbre} \quad$



$$i = 0$$

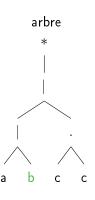
$$expr = ((a|b)|cc)*$$





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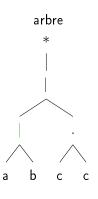
$$expr = ((a|b)|cc)*$$



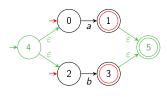
$$i = 4$$



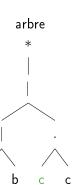
$$expr = ((a|b)|cc)*$$



$$i = 6$$

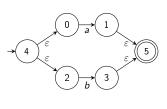


$$expr = ((a|b)|cc)*$$



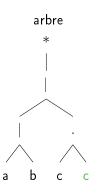
$$i = 8$$

а

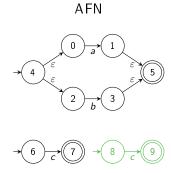




$$expr = ((a|b)|cc)*$$

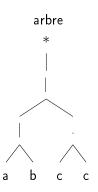


i = 10

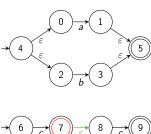


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$$expr = ((a|b)|cc)*$$

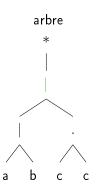


$$i = 10$$

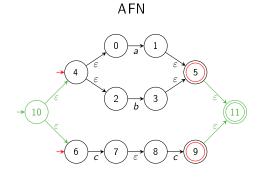




$$expr = ((a|b)|cc)*$$

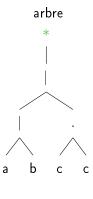


$$i = 12$$

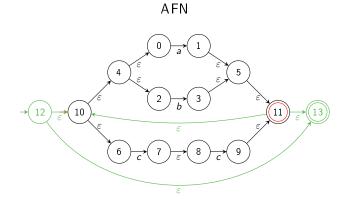


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$$expr = ((a|b)|cc)*$$

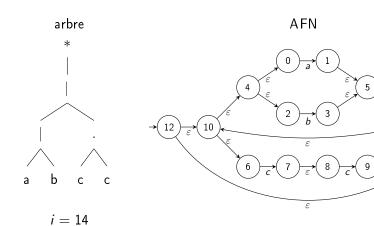


$$i = 14$$



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$$expr = ((a|b)|cc)*$$



AFN final à 14 états

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$$\mathsf{expr} = \mathsf{b*a*}|(\mathsf{cb})*$$

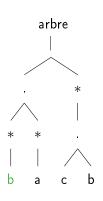
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$$\mathsf{expr} = \mathsf{b*a*}|(\mathsf{cb})^*$$



$$i = 0$$

$$expr = b*a*|(cb)*$$

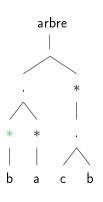


$$i = 2$$

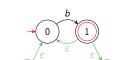


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$$expr = b*a*|(cb)*$$



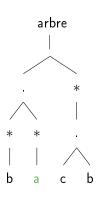
$$i = 4$$



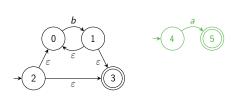


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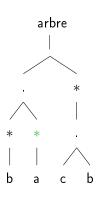
$$expr = b*a*|(cb)*$$



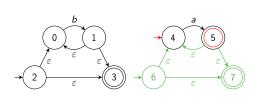
$$i = 6$$



$$expr = b*a*|(cb)*$$



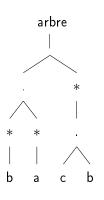
$$i = 8$$



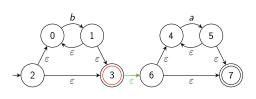
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$$expr = b*a*|(cb)*$$

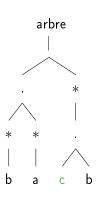


$$i = 8$$



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$$expr = b*a*|(cb)*$$



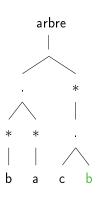
$$i = 10$$





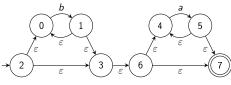
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$$expr = b*a*|(cb)*$$



$$i = 12$$

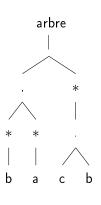




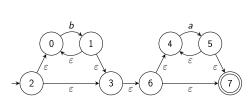
$$\rightarrow 8 \rightarrow 9 \rightarrow 10 \rightarrow 11$$

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$$expr = b*a*|(cb)*$$

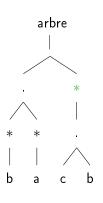


$$i = 12$$

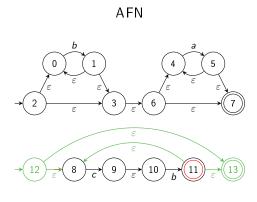




$$expr = b*a*|(cb)*$$

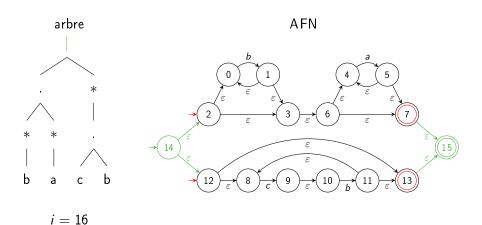


$$i = 14$$



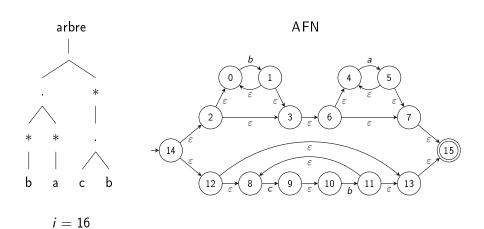
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$$expr = b*a*|(cb)*$$



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$$expr = b*a*|(cb)*$$



AFN final à 16 états

$$expr = (a|b)|c$$

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$$expr = (a|b)|c$$



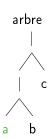




$$i = 0$$

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$$expr = (a|b)|c$$



$$i = 2$$

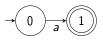
AFN



$$expr = (a|b)|c$$



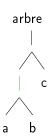
$$i = 4$$



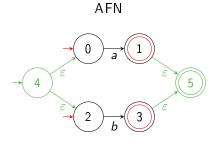


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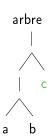
$$expr = (a|b)|c$$



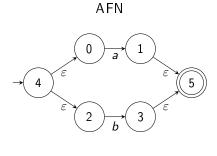
$$i = 6$$



$$expr = (a|b)|c$$

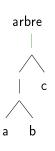


$$i = 8$$

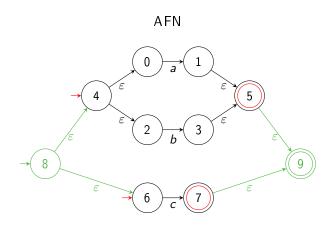


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$$expr = (a|b)|c$$

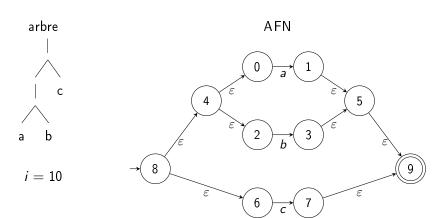


$$i = 10$$



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$$expr = (a|b)|c$$



AFN final à 10 états

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$$expr = a|(b|c)$$

$$expr = a|(b|c)$$



$$i = 0$$

$$expr = a|(b|c)$$

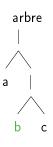


$$i = 2$$



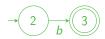


$$expr = a|(b|c)$$



$$i = 4$$

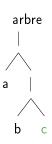




$$\rightarrow 0$$

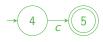
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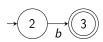
$$\mathsf{expr} = \mathsf{a} | (\mathsf{b} | \mathsf{c})$$



$$i = 6$$

AFN





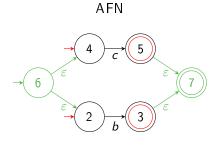
$$\rightarrow 0$$

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$$expr = a|(b|c)$$

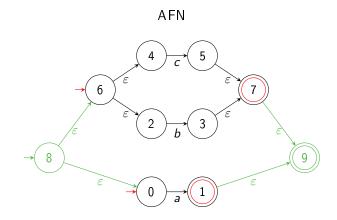


$$i = 8$$

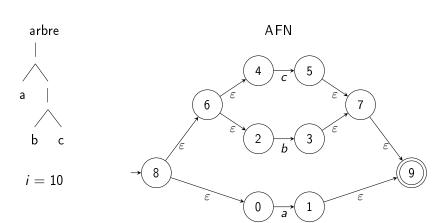


$$expr = a|(b|c)$$





$$expr = a|(b|c)$$



AFN final à 10 états

$$expr = (ab)c$$

$$expr = (ab)c$$



$$i = 0$$

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$$expr = (ab)c$$



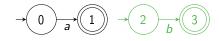
$$i = 2$$

$$expr = (ab)c$$





$$i = 4$$

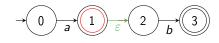


$$expr = (ab)c$$





$$i = 4$$



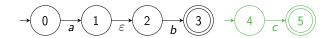
$$expr = (ab)c$$





$$i = 6$$

AFN



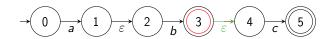
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$$expr = (ab)c$$





$$i = 6$$



$$expr = (ab)c$$

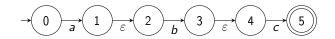






$$i = 6$$

AFN



AFN final à 6 états

$$expr = a(bc)$$

$$expr = a(bc)$$



$$i = 0$$

14 / 15

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$$expr = a(bc)$$

AFN

arbre



a .

$$i = 2$$

b

[□▶ 4**□**▶ 4토▶ 4토▶ 출 ∽9<은

$$expr = a(bc)$$



а



i = 4

AFN



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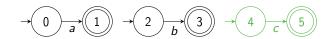
$$expr = a(bc)$$



a .



$$i = 6$$



$$expr = a(bc)$$

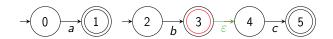


a



$$i = 6$$

AFN



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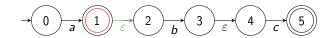
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$$expr = a(bc)$$



 \wedge

i = 6



$$expr = a(bc)$$



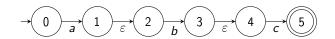






$$i = 6$$

AFN



AFN final à 6 états

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Les AFN des expressions régulières (ab)c et a(bc) sont strictement identiques. Cela montre que quels que soient a, b et c, la concaténation est associative. Il est donc possible de supprimer les parenthèses sans ambiguité.

$$(ab)c = a(bc) = abc$$