HAI6011 - Exercices de révisions

Benoît Huftier

2022

Construction d'un AFD à partir d'un AFN

Enoncé

Donner l'automate fini déterministe (AFD) de certaines des expressions régulières de l'exercice précédent.

ab

ab*|c

abc

b*

- (a|b)|c

a | b

b*a*|(cb)*

a | (b | c)

Avec les AFD des expressions régulières (a|b)|c et a|(b|c), que peut-on en déduire sur la règle | ?

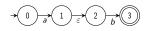
$$\underbrace{0 \quad a \quad 1 \quad e}_{a} \underbrace{2 \quad b}_{b} \underbrace{3}_{a}$$
 expr = ab

On rappelle que pour créer un AFD à partir d'un AFN, il faut créer des ε -fermetures d'ensemble d'états, en commençant par l'état de départ (ici l'état 0).

EpsilonFermeture(
$$\{0\}$$
) = $\{0\}$

En effet, l' ε -fermeture comprend tous les états de l'ensemble (ici uniquement 0) et comprend également tous les états qui peuvent être atteint avec une ε -transition, il n'y en a aucune ici.

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$$expr = ab$$



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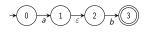
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Voici donc D, notre nouvel état de départ, on l'ajoute à notre AFD :

$$D = \{0\}$$

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3/12





3/12

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Voici donc D, notre nouvel état de départ, on l'ajoute à notre AFD :

$$D = \{0\}$$

On part maintenant de D, pour créer notre AFD.

$$expr = ab$$

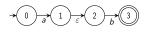
$$D = \{0\}$$

3 / 12

$$\mathsf{expr} = \mathsf{ab}$$
 $D = \{0\}$

On va parcourir tout l'alphabet de notre vocabulaire et regarder les transitions depuis D.





$$\mathsf{expr} = \mathsf{ab} \qquad \qquad \mathsf{D}$$

$$D = \{0\}$$

On va parcourir tout l'alphabet de notre vocabulaire et regarder les transitions depuis D.

Pour x = a, la seule transition de l'AFN est 0a1. On calcul donc l' ε -fermeture de 1.

EpsilonFermeture(
$$\{1\}$$
) = $E_1 = \{1, 2\}$

expr = ab
$$D = \{0\}$$

$$E_1 = \{1, 2\}$$

On va parcourir tout l'alphabet de notre vocabulaire et regarder les transitions depuis D.

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EpsilonFermeture(
$$\{1\}$$
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On ajoute ensuite l'état E_1 et la transition DaE_1 dans notre AFD.

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expr = ab
$$D = \{0\}$$

$$E_1 = \{1, 2\}$$

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Pour x = a, la seule transition de l'AFN est 0a1. On calcul donc l' ε -fermeture de 1.

EpsilonFermeture(
$$\{1\}$$
) = $E_1 = \{1,2\}$

On ajoute ensuite l'état E_1 et la transition DaE_1 dans notre AFD. Pour x = b, il n'y a aucune transition depuis l'état 0.

expr = ab
$$D = \{0\}$$

$$E_1 = \{1, 2\}$$

On va parcourir tout l'alphabet de notre vocabulaire et regarder les transitions depuis D.

Pour x = a, la seule transition de l'AFN est 0a1. On calcul donc l' ε -fermeture de 1.

EpsilonFermeture(
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On ajoute ensuite l'état E_1 et la transition DaE_1 dans notre AFD.

Pour x = b, il n'y a aucune transition depuis l'état 0.

On marque maintenant D et on regarde s'il reste des états non marqués, c'est le cas donc on continue.

$$expr = ab$$

$$D \rightarrow E_1$$

$$D = \{0\} \\ E_1 = \{1, 2\}$$

3 / 12

$$\rightarrow \boxed{0} \rightarrow \boxed{1} \rightarrow \boxed{2} \rightarrow \boxed{3}$$

$$expr = ab$$

$$D \rightarrow E_1$$

$$D = \{0\}$$

 $E_1 = \{1, 2\}$

On va parcourir tout l'alphabet de notre vocabulaire et regarder les transitions depuis E_1 .



$$expr = ab$$

$$D \rightarrow E_1$$

$$D = \{0\} \\ E_1 = \{1, 2\}$$

On va parcourir tout l'alphabet de notre vocabulaire et regarder les transitions depuis E_1 .

Pour x = a, il n'y a aucune transition.

expr = ab
$$D = \{0\}$$

$$E_1 = \{1, 2\}$$

On va parcourir tout l'alphabet de notre vocabulaire et regarder les transitions depuis E_1 .

Pour x = a, il n'y a aucune transition.

Pour x = b, il y a la transition 2b3, on calcule donc l' ε -fermeture de 3.

EpsilonFermeture(
$$\{3\}$$
) = E_2 = $\{3\}$

$$expr = ab$$

$$D \rightarrow E_1 \rightarrow E_2$$

$$D = \{0\}$$

 $E_1 = \{1, 2\}$
 $E_2 = \{3\}$

On va parcourir tout l'alphabet de notre vocabulaire et regarder les transitions depuis E_1 .

Pour x = a, il n'y a aucune transition.

Pour x = b, il y a la transition 2b3, on calcule donc l' ε -fermeture de 3.

EpsilonFermeture(
$$\{3\}$$
) = $E_2 = \{3\}$

On ajoute ensuite l'état E_2 et la transition E_1bE_2 dans notre AFD. Comme $3 \in E_2$ et que 3 est un état final de l'AFN alors E_2 est également final.

3 / 12

$$0 \xrightarrow{a} 1 \xrightarrow{\varepsilon} 2 \xrightarrow{b} 3$$

$$D \xrightarrow{a} E_1 \xrightarrow{b} E_2$$

$$D = \{0\}$$

 $E_1 = \{1, 2\}$
 $E_2 = \{3\}$

On va parcourir tout l'alphabet de notre vocabulaire et regarder les transitions depuis E_1 .

Pour x = a, il n'y a aucune transition.

Pour x = b, il y a la transition 2b3, on calcule donc l' ε -fermeture de 3.

EpsilonFermeture(
$$\{3\}$$
) = E_2 = $\{3\}$

On ajoute ensuite l'état E_2 et la transition E_1bE_2 dans notre AFD. Comme $3 \in E_2$ et que 3 est un état final de l'AFN alors E_2 est également final. On marque maintenant E_1 et on regarde s'il reste des états non marqués, c'est le cas donc on continue.

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$$expr = ab$$

$$D \xrightarrow{a} E_1 \xrightarrow{b} E_2$$

$$D = \{0\}$$

 $E_1 = \{1, 2\}$
 $E_2 = \{3\}$

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$$expr = ab$$

$$D \rightarrow E_1 \rightarrow E_2$$

$$D = \{0\}$$

 $E_1 = \{1, 2\}$
 $E_2 = \{3\}$

Il n'y a aucune transition depuis l'état 3 donc on marque directement l'état E_2 .

$$\rightarrow \boxed{0} \rightarrow \boxed{1} \rightarrow \boxed{2} \rightarrow \boxed{3}$$

$$expr = ab$$

$$D \rightarrow E_1 \rightarrow E_2$$

$$D = \{0\}$$

 $E_1 = \{1, 2\}$
 $E_2 = \{3\}$

Il n'y a aucune transition depuis l'état 3 donc on marque directement l'état E_2 .

Tous les états sont maintenant marqués, l'AFD est terminé.

$$expr = ab$$

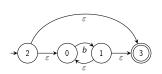
$$D \xrightarrow{a} E_1 \xrightarrow{b} E_2$$

$$D = \{0\}$$

 $E_1 = \{1, 2\}$
 $E_2 = \{3\}$

Il n'y a aucune transition depuis l'état 3 donc on marque directement l'état E_2 .

Notez que la correction a été expliquée pour cet exemple mais ne le sera pas pour les prochains.



$$expr = b*$$

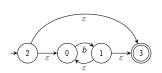
$$D = \{0, 2, 3\}$$

Calcul de l'état de départ :

$$D = EpsilonFermeture(\{2\}) = \{0, 2, 3\}$$



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 4 / 12



$$expr = b^*$$



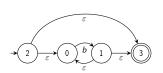
$$D = \{0, 2, 3\}$$

 $E_1 = \{0, 1, 3\}$

Etat actuel: D

- $\bullet x = b$
- transitions: 0b1
- EpsilonFermeture($\{1\}$) = $\{0, 1, 3\} = E_1$
- etat final : oui





$$expr = b*$$

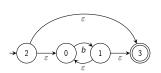


$$D = \{0, 2, 3\}$$

$$E_1 = \{0, 1, 3\}$$

Etat actuel : E_1

- $\bullet x = b$
- transitions: 0b1
- EpsilonFermeture($\{1\}$) = $\{0, 1, 3\} = E_1$
- etat final : oui



$$expr = b*$$

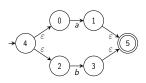


$$D = \{0, 2, 3\}$$

 $E_1 = \{0, 1, 3\}$

AFD terminé, nombre d'états : 2





$$expr = a|b$$



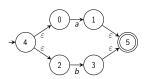
$$D = \{0, 2, 4\}$$

Calcul de l'état de départ :

$$D = EpsilonFermeture(\{4\}) = \{0, 2, 4\}$$



5 / 12





$$D = \{0, 2, 4\}$$

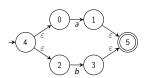
 $E_1 = \{1, 5\}$

Etat actuel: D

• transitions: 0a1

• EpsilonFermeture(
$$\{1\}$$
) = $\{1, 5\}$ = E_1

• etat final : oui



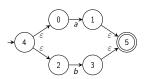


$$D = \{0, 2, 4\}$$

 $E_1 = \{1, 5\}$
 $E_2 = \{3, 5\}$

Etat actuel: D

- $\bullet x = b$
- transitions: 2b3
- EpsilonFermeture($\{3\}$) = $\{3, 5\}$ = E_2
- etat final : oui



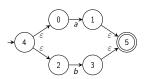


$$D = \{0, 2, 4\}$$

 $E_1 = \{1, 5\}$
 $E_2 = \{3, 5\}$

Etat actuel : E_1

- x = a
- transitions : aucune



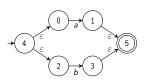


$$D = \{0, 2, 4\}$$

 $E_1 = \{1, 5\}$
 $E_2 = \{3, 5\}$

Etat actuel : E_1

- $\bullet x = b$
- transitions : aucune



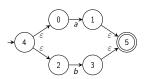


$$D = \{0, 2, 4\}$$

 $E_1 = \{1, 5\}$
 $E_2 = \{3, 5\}$

Etat actuel : E_2

- \bullet x = a
- transitions : aucune



$$expr = a|b$$

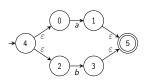


$$D = \{0, 2, 4\}$$

 $E_1 = \{1, 5\}$
 $E_2 = \{3, 5\}$

Etat actuel : E_2

- $\bullet x = b$
- transitions : aucune





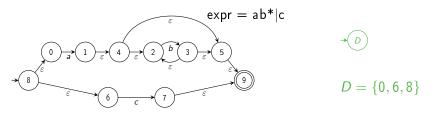
$$D = \{0, 2, 4\}$$

 $E_1 = \{1, 5\}$

$$E_2 = \{3,5\}$$

AFD terminé, nombre d'états : 3

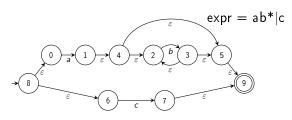




Calcul de l'état de départ :

$$D = EpsilonFermeture(\{8\}) = \{0, 6, 8\}$$

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$$D = \{0, 6, 8\}$$

 $E_1 = \{1, 2, 4, 5, 9\}$

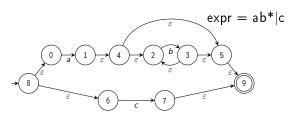
Etat actuel: D

 \bullet x = a

transitions: 0a1

• EpsilonFermeture($\{1\}$) = $\{1, 2, 4, 5, 9\}$ = E_1

• etat final : oui



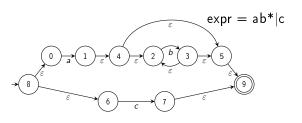


$$D = \{0, 6, 8\}$$

$$E_1 = \{1, 2, 4, 5, 9\}$$

Etat actuel: D

- $\bullet x = b$
- transitions : aucune



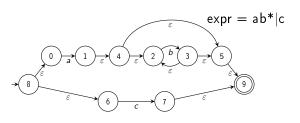


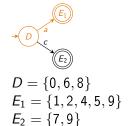
$$D = \{0, 6, 8\}$$

$$E_1 = \{1, 2, 4, 5, 9\}$$

$$E_2 = \{7, 9\}$$

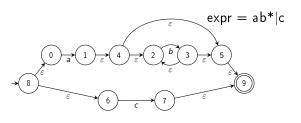
- $\bullet x = c$
- transitions: 6c7
- EpsilonFermeture($\{7\}$) = $\{7, 9\}$ = E_2
- etat final : oui





- \bullet x = a
- transitions : aucune

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$$D = \{0,6,8\}$$

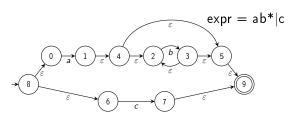
$$E_1 = \{1,2,4,5,9\}$$

$$E_2 = \{7,9\}$$

$$E_3 = \{2,3,5,9\}$$

- $\bullet x = b$
- transitions: 2b3
- EpsilonFermeture($\{3\}$) = $\{2, 3, 5, 9\}$ = E_3
- etat final : oui







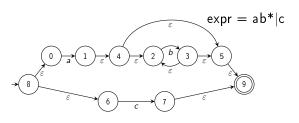
$$D = \{0, 6, 8\}$$

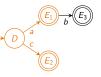
$$E_1 = \{1, 2, 4, 5, 9\}$$

$$E_2 = \{7, 9\}$$

$$E_3 = \{2, 3, 5, 9\}$$

- $\bullet x = c$
- transitions : aucune





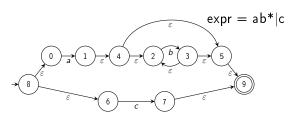
$$D = \{0, 6, 8\}$$

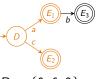
$$E_1 = \{1, 2, 4, 5, 9\}$$

$$E_2 = \{7, 9\}$$

$$E_3 = \{2, 3, 5, 9\}$$

- x = a
- transitions: aucune





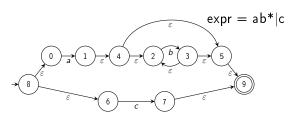
$$D = \{0, 6, 8\}$$

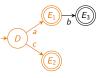
$$E_1 = \{1, 2, 4, 5, 9\}$$

$$E_2 = \{7, 9\}$$

$$E_3 = \{2, 3, 5, 9\}$$

- $\bullet x = b$
- transitions: aucune





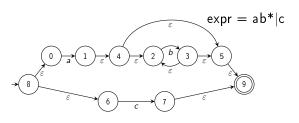
$$D = \{0, 6, 8\}$$

$$E_1 = \{1, 2, 4, 5, 9\}$$

$$E_2 = \{7, 9\}$$

$$E_3 = \{2, 3, 5, 9\}$$

- $\bullet x = c$
- transitions: aucune





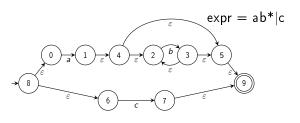
$$D = \{0,6,8\}$$

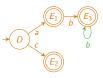
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$$E_3 = \{2,3,5,9\}$$

- x = a
- transitions: aucune





$$D = \{0,6,8\}$$

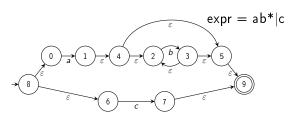
$$E_1 = \{1,2,4,5,9\}$$

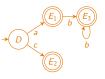
$$E_2 = \{7,9\}$$

$$E_3 = \{2,3,5,9\}$$

- $\bullet x = b$
- transitions: 2b3
- EpsilonFermeture($\{3\}$) = $\{2, 3, 5, 9\}$ = E_3
- etat final : oui







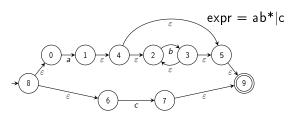
$$D = \{0, 6, 8\}$$

$$E_1 = \{1, 2, 4, 5, 9\}$$

$$E_2 = \{7, 9\}$$

$$E_3 = \{2, 3, 5, 9\}$$

- $\bullet x = c$
- transitions: aucune





$$D = \{0, 6, 8\}$$

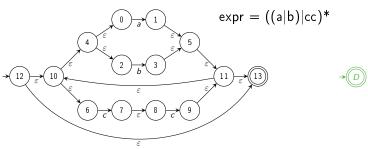
$$E_1 = \{1, 2, 4, 5, 9\}$$

$$E_2 = \{7, 9\}$$

$$E_3 = \{2, 3, 5, 9\}$$

AFD terminé, nombre d'états : 4



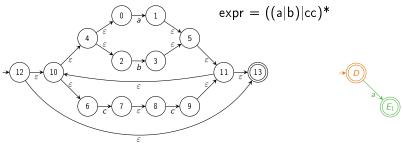


$$D = \{0, 2, 4, 6, 10, 12, 13\}$$

Calcul de l'état de départ :

$$D = EpsilonFermeture(\{12\}) = \{0, 2, 4, 6, 10, 12, 13\}$$

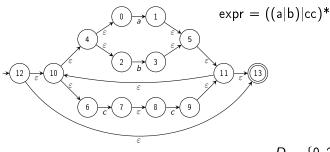
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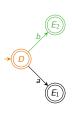


$$D = \{0, 2, 4, 6, 10, 12, 13\}$$

$$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$$

- \bullet x = a
- transitions: 0a1
- EpsilonFermeture($\{1\}$) = $\{0, 1, 2, 4, 6, 5, 10, 11, 13\} = E_1$
- etat final : oui



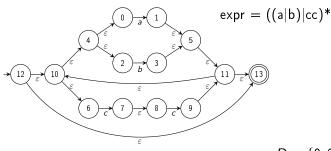


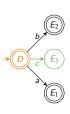
$$D = \{0, 2, 4, 6, 10, 12, 13\}$$

$$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$$

$$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$$

- $\bullet x = b$
- transitions: 2b3
- EpsilonFermeture($\{3\}$) = $\{0, 2, 3, 4, 6, 5, 10, 11, 13\} = E_2$
- etat final : oui





$$\bullet x = c$$

• transitions: 6c7

• EpsilonFermeture(
$$\{7\}$$
) = $\{7, 8\} = E_3$

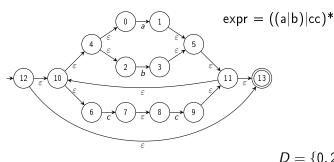
etat final : non

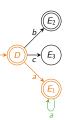
$$D = \{0, 2, 4, 6, 10, 12, 13\}$$

$$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$$

$$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$$

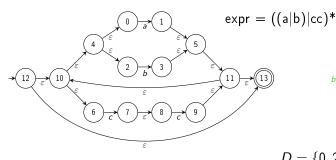
$$E_3 = \{7, 8\}$$

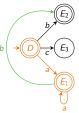




- x = a
- transitions: 0a1
- tiunsitions : our
- EpsilonFermeture($\{1\}$) = $\{0, 1, 2, 4, 6, 5, 10, 11, 13\} = E_1$
- etat final : oui

 $D = \{0, 2, 4, 6, 10, 12, 13\}$ $E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$ $E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$ $E_3 = \{7, 8\}$





$$\bullet x = b$$

transitions: 2b3

• EpsilonFermeture($\{3\}$) = $\{0, 2, 3, 4, 6, 5, 10, 11, 13\} = E_2$

etat final : oui

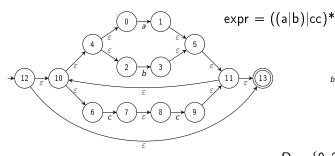
$$D = \{0, 2, 4, 6, 10, 12, 13\}$$

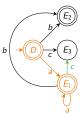
$$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$$

$$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$$

$$E_3 = \{7, 8\}$$

7 / 12





$$\bullet x = c$$

transitions: 6c7

• EpsilonFermeture(
$$\{7\}$$
) = $\{7, 8\} = E_3$

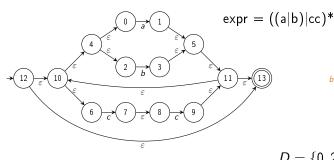
• etat final : non

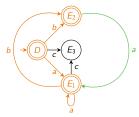
$$D = \{0, 2, 4, 6, 10, 12, 13\}$$

$$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$$

$$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$$

$$E_3 = \{7, 8\}$$





transitions: 0a1

• EpsilonFermeture($\{1\}$) = $\{0, 1, 2, 4, 6, 5, 10, 11, 13\} = E_1$

etat final : oui

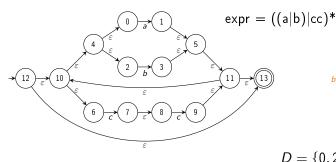
$$D = \{0, 2, 4, 6, 10, 12, 13\}$$

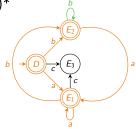
$$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$$

$$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$$

$$E_3 = \{7, 8\}$$

7 / 12





$$\bullet x = b$$

transitions: 2b3

• EpsilonFermeture($\{3\}$) = $\{0, 2, 3, 4, 6, 5, 10, 11, 13\} = E_2$

etat final : oui

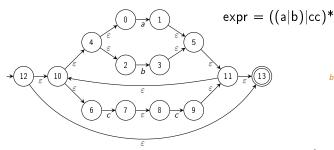
$$D = \{0, 2, 4, 6, 10, 12, 13\}$$

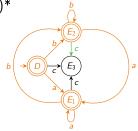
$$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$$

$$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$$

$$E_3 = \{7, 8\}$$

7 / 12





$$\bullet x = c$$

transitions: 6c7

• EpsilonFermeture(
$$\{7\}$$
) = $\{7, 8\} = E_3$

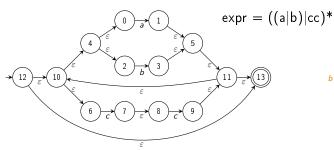
• etat final : non

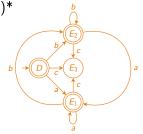
$$D = \{0, 2, 4, 6, 10, 12, 13\}$$

$$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$$

$$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$$

$$E_3 = \{7, 8\}$$





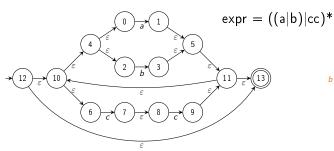
- x = a
- transitions : aucune

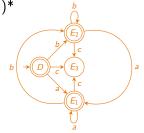
$$D = \{0, 2, 4, 6, 10, 12, 13\}$$

$$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$$

$$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$$

$$E_3 = \{7, 8\}$$





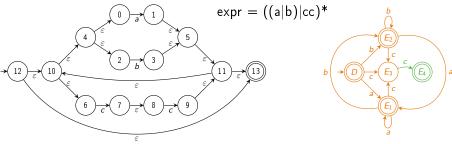
- $\bullet x = b$
- transitions : aucune

$$D = \{0, 2, 4, 6, 10, 12, 13\}$$

$$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$$

$$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$$

$$E_3 = \{7, 8\}$$



$$\bullet x = c$$

• EpsilonFermeture($\{9\}$) = $\{0, 2, 4, 6, 9, 10, 11, 13\} = E_4$

etat final : non

$$D = \{0, 2, 4, 6, 10, 12, 13\}$$

$$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$$

$$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$$

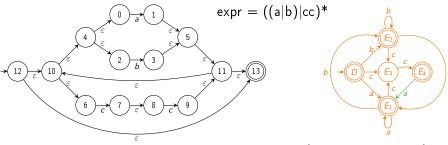
$$E_3 = \{7, 8\}$$

$$E_4 = \{0, 2, 4, 6, 9, 10, 11, 13\}$$

$$E_3 = \{7, 8\}$$

 $E_4 = \{0, 2, 4, 6, 9, 10, 11, 13\}$

$$E_4 = \{0, 2, 4, 6, 9, 10, 11, 13\}$$

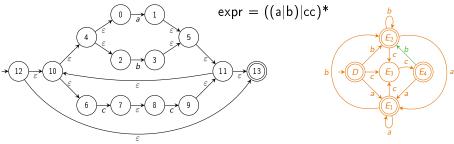


$$\bullet$$
 x = a

• EpsilonFermeture($\{1\}$) = $\{0, 1, 2, 4, 6, 5, 10, 11, 13\} = E_1$

etat final : oui

 $D = \{0, 2, 4, 6, 10, 12, 13\}$ $E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$ $E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$ $E_3 = \{7, 8\}$ $E_4 = \{0, 2, 4, 6, 9, 10, 11, 13\}$



- $\bullet x = b$
- transitions: 2b3
- EpsilonFermeture($\{3\}$) = $\{0, 2, 3, 4, 6, 5, 10, 11, 13\} = E_2$
- etat final : oui

$$D = \{0, 2, 4, 6, 10, 12, 13\}$$

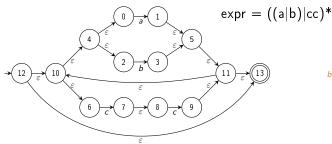
$$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$$

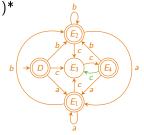
$$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$$

$$E_3 = \{7, 8\}$$

$$E_4 = \{0, 2, 4, 6, 9, 10, 11, 13\}$$

$$E_4 = \{0, 2, 4, 6, 9, 10, 11, 13\}$$





$$\bullet x = c$$

- transitions: 6c7
- EpsilonFermeture($\{7\}$) = $\{7, 8\} = E_3$
- etat final : non

$$D = \{0, 2, 4, 6, 10, 12, 13\}$$

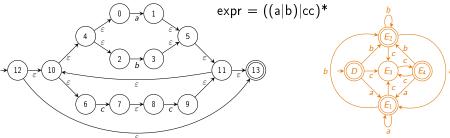
$$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$$

$$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$$

$$E_3 = \{7, 8\}$$

$$E_4 = \{0, 2, 4, 6, 9, 10, 11, 13\}$$

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AFD terminé, nombre d'états : 5

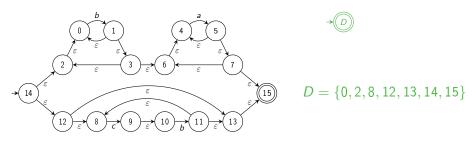
$$D = \{0, 2, 4, 6, 10, 12, 13\}$$

$$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$$

$$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$$

$$E_3 = \{7, 8\}$$

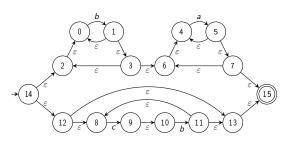
$$E_4 = \{0, 2, 4, 6, 9, 10, 11, 13\}$$



Calcul de l'état de départ :

$$D = EpsilonFermeture(\{14\}) = \{0, 2, 8, 12, 13, 14, 15\}$$

4□ > 4□ > 4 = > 4 = > = 90

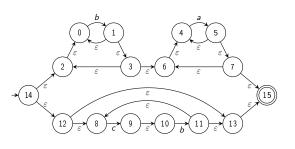


$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

Etat actuel: D

- \bullet x = a
- transitions : aucune

 Benoît Huftier
 HA|601| - révisions
 2022
 8 / 12



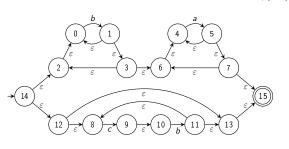


$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

Etat actuel: D

- $\bullet x = b$
- transitions: 0b1
- EpsilonFermeture($\{1\}$) = $\{0, 1, 2, 3, 4, 6\} = E_1$
- etat final : non





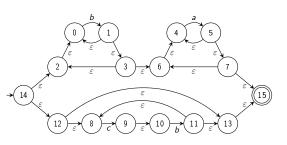
$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

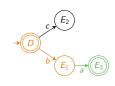
$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

$$E_2 = \{9, 10\}$$

Etat actuel: D

- \bullet x = c
- transitions: 8c9
- EpsilonFermeture($\{9\}$) = $\{9, 10\}$ = E_2
- etat final : non





$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

$$E_2 = \{9, 10\}$$

$$E_3 = \{4, 5, 6, 7, 15\}$$

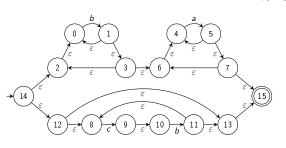
Etat actuel : E_1

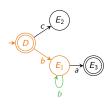
x = a

transitions: 4a5

• EpsilonFermeture($\{5\}$) = $\{4, 5, 6, 7, 15\}$ = E_3

etat final : oui





$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

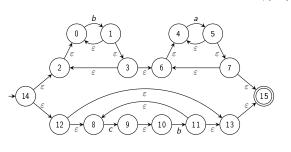
$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

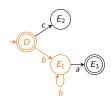
$$E_2 = \{9, 10\}$$

$$E_3 = \{4, 5, 6, 7, 15\}$$

Etat actuel : E_1

- $\bullet x = b$
- transitions: 0b1
- EpsilonFermeture($\{1\}$) = $\{0, 1, 2, 3, 4, 6\}$ = E_1
- etat final : non





$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

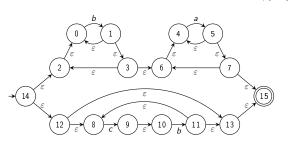
$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

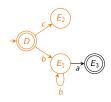
$$E_2 = \{9, 10\}$$

$$E_3 = \{4, 5, 6, 7, 15\}$$

Etat actuel : E_1

- $\bullet x = c$
- transitions : aucune





$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

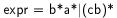
$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

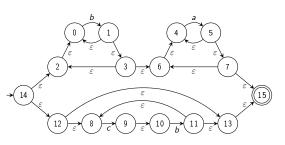
$$E_2 = \{9, 10\}$$

$$E_3 = \{4, 5, 6, 7, 15\}$$

Etat actuel : E_2

- \bullet x = a
- transitions: aucune





$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

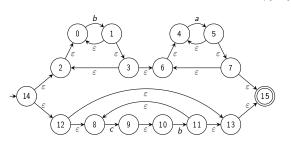
$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

$$E_2 = \{9, 10\}$$

$$E_3 = \{4, 5, 6, 7, 15\}$$

$$E_4 = \{8, 10, 11, 13, 15\}$$

- $\bullet x = b$
- transitions: 10b11
- EpsilonFermeture($\{11\}$) = $\{8, 10, 11, 13, 15\}$ = E_4
- etat final : oui



$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

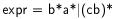
$$E_2 = \{9, 10\}$$

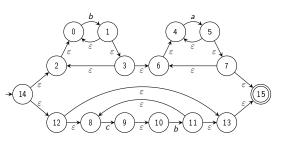
$$E_3 = \{4, 5, 6, 7, 15\}$$

$$E_4 = \{8, 10, 11, 13, 15\}$$

Etat actuel : E_2

- \bullet x = c
- transitions: aucune





$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

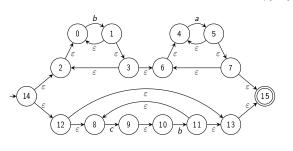
$$E_2 = \{9, 10\}$$

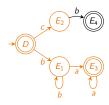
$$E_3 = \{4, 5, 6, 7, 15\}$$

$$E_4 = \{8, 10, 11, 13, 15\}$$

Etat actuel : E_3

- x = a
- transitions: 4c5
- EpsilonFermeture($\{5\}$) = $\{4, 5, 6, 7, 15\}$ = E_3
- etat final : oui





$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

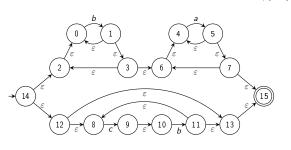
$$E_2 = \{9, 10\}$$

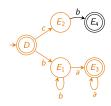
$$E_3 = \{4, 5, 6, 7, 15\}$$

$$E_4 = \{8, 10, 11, 13, 15\}$$

Etat actuel : E_3

- $\bullet x = b$
- transitions : aucune





$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

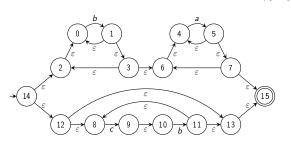
$$E_2 = \{9, 10\}$$

$$E_3 = \{4, 5, 6, 7, 15\}$$

$$E_4 = \{8, 10, 11, 13, 15\}$$

Etat actuel : E_3

- \bullet x = c
- transitions: aucune



$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

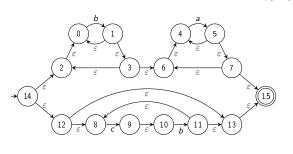
$$E_2 = \{9, 10\}$$

$$E_3 = \{4, 5, 6, 7, 15\}$$

$$E_4 = \{8, 10, 11, 13, 15\}$$

Etat actuel: E_4

- x = a
- transitions: aucune



$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

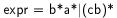
$$E_2 = \{9, 10\}$$

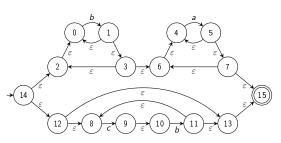
$$E_3 = \{4, 5, 6, 7, 15\}$$

$$E_4 = \{8, 10, 11, 13, 15\}$$

Etat actuel: E_4

- $\bullet x = b$
- transitions : aucune





$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

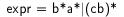
$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

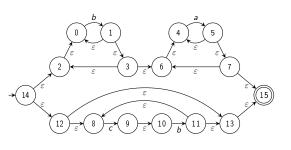
$$E_2 = \{9, 10\}$$

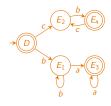
$$E_3 = \{4, 5, 6, 7, 15\}$$

$$E_4 = \{8, 10, 11, 13, 15\}$$

- \bullet x = c
- transitions: 8c9
- EpsilonFermeture($\{9\}$) = $\{9, 10\}$ = E_2
- etat final : non







$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

$$E_2 = \{9, 10\}$$

$$E_3 = \{4, 5, 6, 7, 15\}$$

$$E_4 = \{8, 10, 11, 13, 15\}$$

AFD terminé, nombre d'états : 5

$$expr = abc$$

$$D = \{0\}$$

Calcul de l'état de départ :

$$D = EpsilonFermeture(\{0\}) = \{0\}$$

$$expr = abc$$

$$D \rightarrow E_1$$

$$D = \{0\}$$

 $E_1 = \{1, 2\}$

- x = a
- transitions: 0a1
- EpsilonFermeture($\{1\}$) = $\{1, 2\}$ = E_1
- etat final : non

$$expr = abc$$



$$D = \{0\} \\ E_1 = \{1, 2\}$$

- $\bullet x = b$
- transitions : aucune

$$expr = abc$$

$$D \rightarrow E_1$$

$$D = \{0\} \\ E_1 = \{1, 2\}$$

- $\bullet x = c$
- transitions : aucune

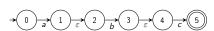
$$expr = abc$$



$$D = \{0\} \\ E_1 = \{1, 2\}$$

- \bullet x = a
- transitions : aucune

$$expr = abc$$



$$D = \{0\}$$

$$E_1 = \{1, 2\}$$

- $\bullet x = b$
- transitions: 2b3
- EpsilonFermeture($\{3\}$) = $\{3, 4\}$ = E_2
- etat final : non

$$expr = abc$$



$$D = \{0\}$$

$$E_1 = \{1, 2\}$$

$$E_2 = \{3, 4\}$$

- $\bullet x = c$
- transitions : aucune

$$expr = abc$$



$$D = \{0\}$$

 $E_1 = \{1, 2\}$
 $E_2 = \{3, 4\}$

- \bullet x = a
- transitions : aucune

$$expr = abc$$

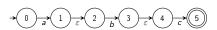


$$D = \{0\}$$

 $E_1 = \{1, 2\}$
 $E_2 = \{3, 4\}$

- $\bullet x = b$
- transitions : aucune

$$expr = abc$$



- \bullet x = c
- transitions: 4c5
- *EpsilonFermeture*($\{5\}$) = $\{5\}$ = E_3
- etat final : oui

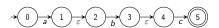
$$D \xrightarrow{a} E_1 \xrightarrow{b} E_2 \xrightarrow{c} E_3$$

$$D = \{0\}$$

 $E_1 = \{1, 2\}$
 $E_2 = \{3, 4\}$

 $E_3 = \{5\}$

$$expr = abc$$



$$D = \{0\}$$

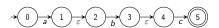
$$E_1 = \{1, 2\}$$

$$E_2 = \{3, 4\}$$

$$E_3 = \{5\}$$

- \bullet x = a
- transitions : aucune

$$expr = abc$$



$$D = \{0\}$$

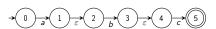
$$E_1 = \{1, 2\}$$

$$E_2 = \{3, 4\}$$

$$E_3 = \{5\}$$

- $\bullet x = b$
- transitions: aucune

$$expr = abc$$



$$D = \{0\}$$

$$E_1 = \{1, 2\}$$

$$E_2 = \{3, 4\}$$

$$E_3 = \{5\}$$

- $\bullet x = c$
- transitions : aucune

$$expr = abc$$





$$0 \xrightarrow{a} 1 \xrightarrow{\varepsilon} 2 \xrightarrow{b} 3 \xrightarrow{\varepsilon} 4 \xrightarrow{c} 5$$

$$D = \{0\}$$

$$E_1 = \{1, 2\}$$

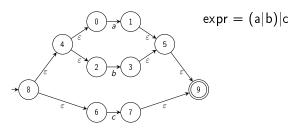
$$E_2 = \{3, 4\}$$

$$E_3 = \{5\}$$

AFD terminé, nombre d'états : 4

9 / 12

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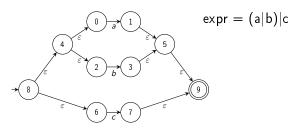


$$D = \{0, 2, 4, 6, 8\}$$

Calcul de l'état de départ :

$$D = EpsilonFermeture(\{8\}) = \{0, 2, 4, 6, 8\}$$

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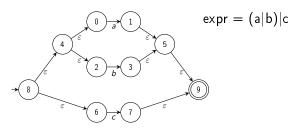




$$D = \{0, 2, 4, 6, 8\}$$

$$E_1 = \{1, 5, 9\}$$

- x = a
- transitions: 0a1
- EpsilonFermeture($\{1\}$) = $\{1, 5, 9\}$ = E_1
- etat final : oui



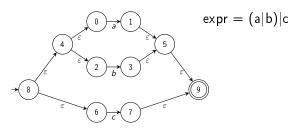


$$D = \{0, 2, 4, 6, 8\}$$

$$E_1 = \{1, 5, 9\}$$

$$E_2 = \{3, 5, 9\}$$

- $\bullet x = b$
- transitions: 2b3
- EpsilonFermeture($\{3\}$) = $\{3, 5, 9\}$ = E_2
- etat final : oui





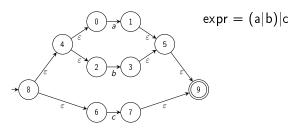
$$D = \{0, 2, 4, 6, 8\}$$

$$E_1 = \{1, 5, 9\}$$

$$E_2 = \{3, 5, 9\}$$

$$E_3 = \{7, 9\}$$

- $\bullet x = c$
- transitions: 6c7
- EpsilonFermeture($\{5\}$) = $\{7, 9\}$ = E_3
- etat final : oui





$$D = \{0, 2, 4, 6, 8\}$$

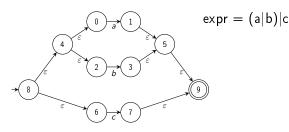
$$E_1 = \{1, 5, 9\}$$

$$E_2 = \{3, 5, 9\}$$

$$E_3 = \{7, 9\}$$

- \bullet x = a
- transitions: aucune

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 2022
 10 / 12





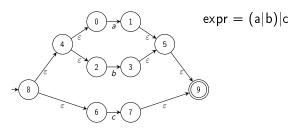
$$D = \{0, 2, 4, 6, 8\}$$

$$E_1 = \{1, 5, 9\}$$

$$E_2 = \{3, 5, 9\}$$

$$E_3 = \{7, 9\}$$

- $\bullet x = b$
- transitions: aucune





$$D = \{0, 2, 4, 6, 8\}$$

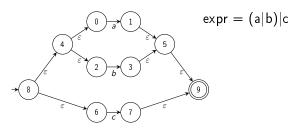
$$E_1 = \{1, 5, 9\}$$

$$E_2 = \{3, 5, 9\}$$

$$E_3 = \{7, 9\}$$

- $\bullet x = c$
- transitions: aucune

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 2022
 10 / 12





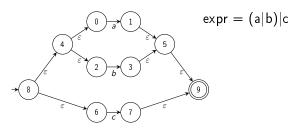
$$D = \{0, 2, 4, 6, 8\}$$

$$E_1 = \{1, 5, 9\}$$

$$E_2 = \{3, 5, 9\}$$

$$E_3 = \{7, 9\}$$

- \bullet x = a
- transitions : aucune





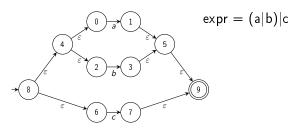
$$D = \{0, 2, 4, 6, 8\}$$

$$E_1 = \{1, 5, 9\}$$

$$E_2 = \{3, 5, 9\}$$

$$E_3 = \{7, 9\}$$

- $\bullet x = b$
- transitions: aucune





$$D = \{0, 2, 4, 6, 8\}$$

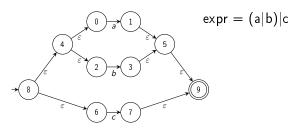
$$E_1 = \{1, 5, 9\}$$

$$E_2 = \{3, 5, 9\}$$

$$E_3 = \{7, 9\}$$

- $\bullet x = c$
- transitions: aucune

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 2022
 10 / 12





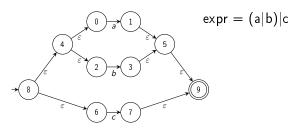
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$$E_3 = \{7, 9\}$$

- \bullet x = a
- transitions: aucune





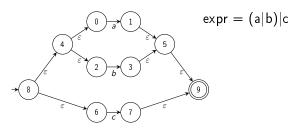
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- $\bullet x = b$
- transitions: aucune





$$D = \{0, 2, 4, 6, 8\}$$

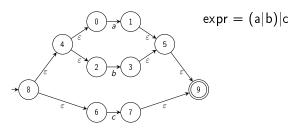
$$E_1 = \{1, 5, 9\}$$

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- $\bullet x = c$
- transitions: aucune

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 2022
 10 / 12





$$D = \{0, 2, 4, 6, 8\}$$

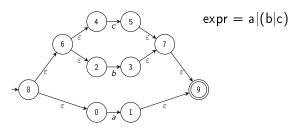
$$E_1 = \{1, 5, 9\}$$

$$E_2 = \{3, 5, 9\}$$

$$E_3 = \{7, 9\}$$

AFD terminé, nombre d'états : 4



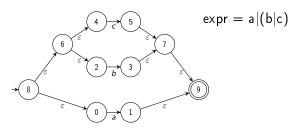




$$D = \{0, 2, 4, 6, 8\}$$

Calcul de l'état de départ :

$$D = EpsilonFermeture(\{8\}) = \{0, 2, 4, 6, 8\}$$



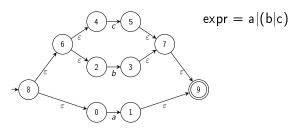


$$D = \{0, 2, 4, 6, 8\}$$

$$E_1 = \{1, 9\}$$

Etat actuel: D

- x = a
- transitions: 0a1
- EpsilonFermeture($\{1\}$) = $\{1, 9\}$ = E_1
- etat final : oui





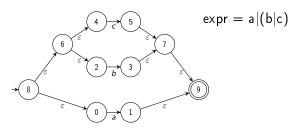
$$D = \{0, 2, 4, 6, 8\}$$

$$E_1 = \{1, 9\}$$

$$E_2 = \{3, 7, 9\}$$

Etat actuel: D

- $\bullet x = b$
- transitions: 2b3
- EpsilonFermeture($\{3\}$) = $\{3, 7, 9\}$ = E_2
- etat final : oui





$$D = \{0, 2, 4, 6, 8\}$$

$$E_1 = \{1, 9\}$$

$$E_2 = \{3, 7, 9\}$$

$$E_3 = \{5, 7, 9\}$$

Etat actuel: D

- $\bullet x = c$
- transitions: 4c5
- EpsilonFermeture($\{5\}$) = $\{5, 7, 9\}$ = E_3
- etat final : oui





$$D = \{0, 2, 4, 6, 8\}$$

$$E_1 = \{1, 9\}$$

$$E_2 = \{3, 7, 9\}$$

$$E_3 = \{5, 7, 9\}$$

- x = a
- transitions : aucune

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 2022
 11 / 12





$$D = \{0, 2, 4, 6, 8\}$$

$$E_1 = \{1, 9\}$$

$$E_2 = \{3, 7, 9\}$$

$$E_3 = \{5, 7, 9\}$$

- $\bullet x = b$
- transitions: aucune





$$D = \{0, 2, 4, 6, 8\}$$

$$E_1 = \{1, 9\}$$

$$E_2 = \{3, 7, 9\}$$

$$E_3 = \{5, 7, 9\}$$

- $\bullet x = c$
- transitions : aucune

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 2022
 11 / 12





$$D = \{0, 2, 4, 6, 8\}$$

$$E_1 = \{1, 9\}$$

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$$E_3 = \{5, 7, 9\}$$

- x = a
- transitions : aucune





$$D = \{0, 2, 4, 6, 8\}$$

$$E_1 = \{1, 9\}$$

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$$E_3 = \{5, 7, 9\}$$

- $\bullet x = b$
- transitions: aucune





$$D = \{0, 2, 4, 6, 8\}$$

$$E_1 = \{1, 9\}$$

$$E_2 = \{3, 7, 9\}$$

$$E_3 = \{5, 7, 9\}$$

- \bullet x = c
- transitions: aucune



$$\begin{array}{c}
E_1 \\
D \\
b \\
E_2
\end{array}$$

$$D = \{0, 2, 4, 6, 8\}$$

$$E_1 = \{1, 9\}$$

$$E_2 = \{3, 7, 9\}$$

$$E_3 = \{5, 7, 9\}$$

- x = a
- transitions: aucune





$$D = \{0, 2, 4, 6, 8\}$$

$$E_1 = \{1, 9\}$$

$$E_2 = \{3, 7, 9\}$$

$$E_3 = \{5, 7, 9\}$$

- $\bullet x = b$
- transitions: aucune





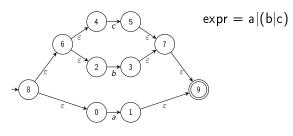
$$D = \{0, 2, 4, 6, 8\}$$

$$E_1 = \{1, 9\}$$

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- $\bullet x = c$
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$$D = \{0, 2, 4, 6, 8\}$$

$$E_1 = \{1, 9\}$$

$$E_2 = \{3, 7, 9\}$$

$$E_3 = \{5, 7, 9\}$$

AFD terminé, nombre d'états : 4



Les AFD des expressions régulières (a|b)|c et a|(b|c) sont strictement identiques. Cela montre que quels que soient a, b et c, la règle | est associative. Il est donc possible de supprimer les parenthèses sans ambiguité.

$$(a|b)|c = a|(b|c) = a|b|c$$