

HA601I - Exercices de révisions

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2022

Construction d'un AFD à partir d'un AFN

Enoncé

Donner l'automate fini déterministe (AFD) de certaines des expressions régulières de l'exercice précédent.

1 ab

2 b^*

3 $a|b$

4 $ab^*|c$

5 $((a|b)|cc)^*$

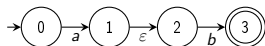
6 $b^*a^*(cb)^*$

7 abc

8 $(a|b)|c$

9 $a|(b|c)$

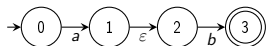
Avec les AFD des expressions régulières $(a|b)|c$ et $a|(b|c)$, que peut-on en déduire sur la règle $|$?



On rappelle que pour créer un AFD à partir d'un AFN, il faut créer des ε -fermetures d'ensemble d'états, en commençant par l'état de départ (ici l'état 0).

$$\textit{EpsilonFermeture}(\{0\}) = \{0\}$$

En effet, l' ε -fermeture comprend tous les états de l'ensemble (ici uniquement 0) et comprend également tous les états qui peuvent être atteint avec une ε -transition, il n'y en a aucune ici.



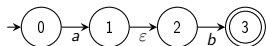
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Voici donc D, notre nouvel état de départ, on l'ajoute à notre AFD :

$$D = \{0\}$$



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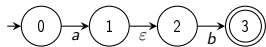
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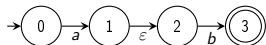
$$D = \{0\}$$

On part maintenant de D, pour créer notre AFD.



$$D = \{0\}$$

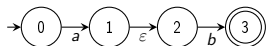
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On va parcourir tout l'alphabet de notre vocabulaire et regarder les transitions depuis D .



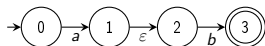
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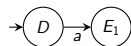
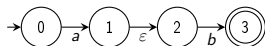
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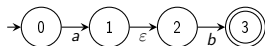
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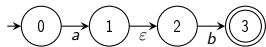
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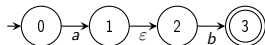
On marque maintenant D et on regarde s'il reste des états non marqués, c'est le cas donc on continue.



$$D = \{0\}$$

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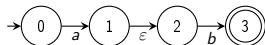


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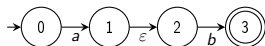
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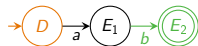
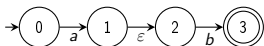
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Pour $x = a$, il n'y a aucune transition.

Pour $x = b$, il y a la transition $2b3$, on calcule donc l' ε -fermeture de 3.

$$\text{EpsilonFermeture}(\{3\}) = E_2 = \{3\}$$



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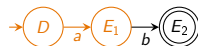
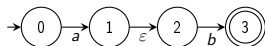
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On ajoute ensuite l'état E_2 et la transition $E_1 b E_2$ dans notre AFD. Comme $3 \in E_2$ et que 3 est un état final de l'AFN alors E_2 est également final.



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$$E_1 = \{1, 2\}$$

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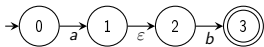
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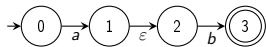


$$D = \{0\}$$

$$E_1 = \{1, 2\}$$

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Etat actuel : $E_2 = \{3\}$.



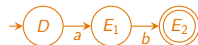
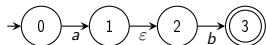
$$D = \{0\}$$

$$E_1 = \{1, 2\}$$

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Etat actuel : $E_2 = \{3\}$.

Il n'y a aucune transition depuis l'état 3 donc on marque directement l'état E_2 .



$$D = \{0\}$$

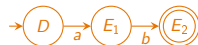
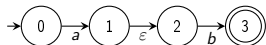
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Tous les états sont maintenant marqués, l'AFD est terminé.



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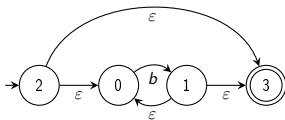
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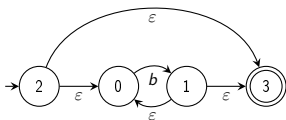
Notez que la correction a été expliquée pour cet exemple mais ne le sera pas pour les prochains.



$$D = \{0, 2, 3\}$$

Calcul de l'état de départ :

$$D = \text{EpsilonFermeture}(\{2\}) = \{0, 2, 3\}$$

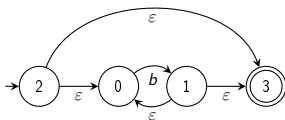


$$D = \{0, 2, 3\}$$

$$E_1 = \{0, 1, 3\}$$

Etat actuel : D

- $x = b$
- transitions : 0b1
- $EpsilonFermeture(\{1\}) = \{0, 1, 3\} = E_1$
- etat final : oui

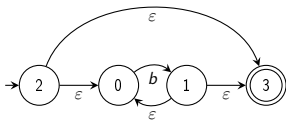


$$D = \{0, 2, 3\}$$

$$E_1 = \{0, 1, 3\}$$

Etat actuel : E_1

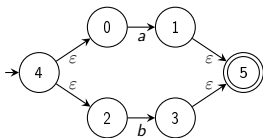
- $x = b$
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- etat final : oui



$$D = \{0, 2, 3\}$$

$$E_1 = \{0, 1, 3\}$$

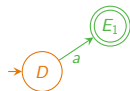
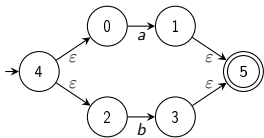
AFD terminé, nombre d'états : 2



$$D = \{0, 2, 4\}$$

Calcul de l'état de départ :

$$D = \text{EpsilonFermeture}(\{4\}) = \{0, 2, 4\}$$

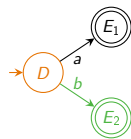
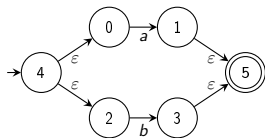


$$D = \{0, 2, 4\}$$

$$E_1 = \{1, 5\}$$

Etat actuel : D

- $x = a$
- transitions : $0a1$
- $EpsilonFermeture(\{1\}) = \{1, 5\} = E_1$
- etat final : oui



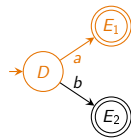
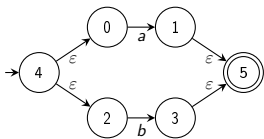
$$D = \{0, 2, 4\}$$

$$E_1 = \{1, 5\}$$

$$E_2 = \{3, 5\}$$

Etat actuel : D

- $x = b$
- transitions : 2b3
- $EpsilonFermeture(\{3\}) = \{3, 5\} = E_2$
- etat final : oui



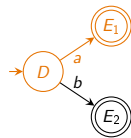
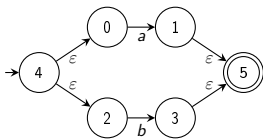
$$D = \{0, 2, 4\}$$

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Etat actuel : E_1

- $x = a$
- transitions : aucune



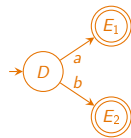
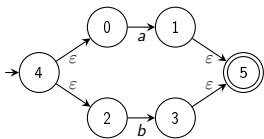
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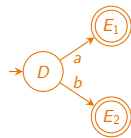
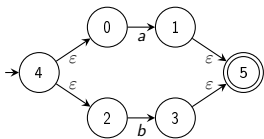
$$D = \{0, 2, 4\}$$

$$E_1 = \{1, 5\}$$

$$E_2 = \{3, 5\}$$

Etat actuel : E_2

- $x = a$
- transitions : aucune



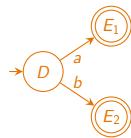
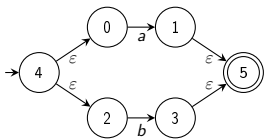
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Etat actuel : E_2

- $x = b$
- transitions : aucune

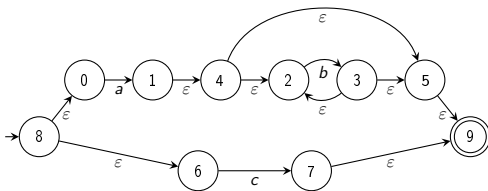


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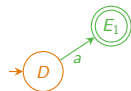
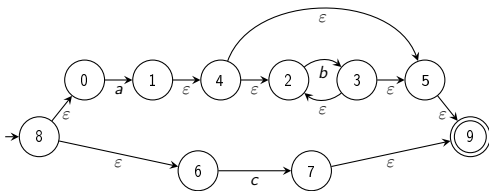
AFD terminé, nombre d'états : 3



$$D = \{0, 6, 8\}$$

Calcul de l'état de départ :

$$D = \text{EpsilonFermeture}(\{8\}) = \{0, 6, 8\}$$

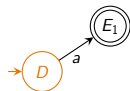
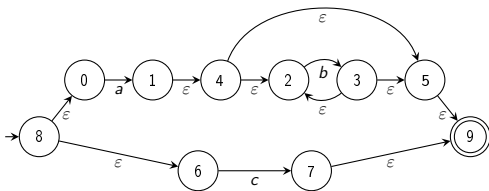


$$D = \{0, 6, 8\}$$

$$E_1 = \{1, 2, 4, 5, 9\}$$

Etat actuel : D

- $x = a$
- transitions : $0a1$
- $EpsilonFermeture(\{1\}) = \{1, 2, 4, 5, 9\} = E_1$
- etat final : oui

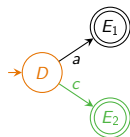
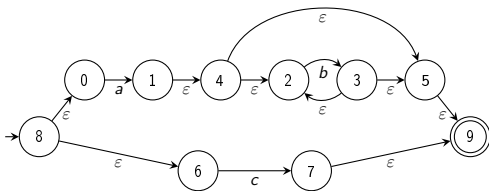


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Etat actuel : D

- $x = b$
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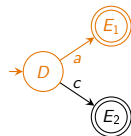
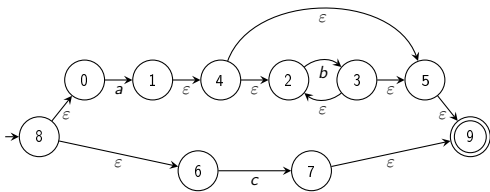
$$D = \{0, 6, 8\}$$

$$E_1 = \{1, 2, 4, 5, 9\}$$

$$E_2 = \{7, 9\}$$

Etat actuel : D

- $x = c$
- transitions : $6c7$
- $Epsilon\text{Fermeture}(\{7\}) = \{7, 9\} = E_2$
- etat final : oui



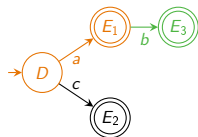
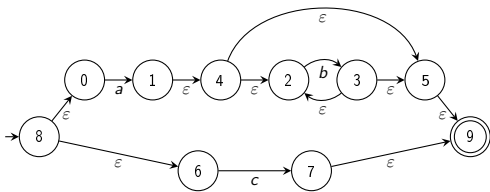
$$D = \{0, 6, 8\}$$

$$E_1 = \{1, 2, 4, 5, 9\}$$

$$E_2 = \{7, 9\}$$

Etat actuel : E_1

- $x = a$
- transitions : aucune



$$D = \{0, 6, 8\}$$

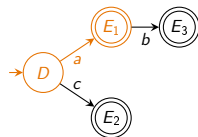
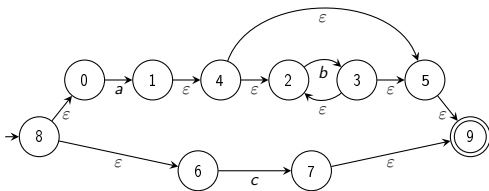
$$E_1 = \{1, 2, 4, 5, 9\}$$

$$E_2 = \{7, 9\}$$

$$E_3 = \{2, 3, 5, 9\}$$

Etat actuel : E_1

- $x = b$
- transitions : 2b3
- $EpsilonFermeture(\{3\}) = \{2, 3, 5, 9\} = E_3$
- etat final : oui



$$D = \{0, 6, 8\}$$

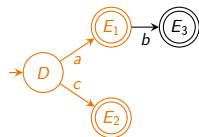
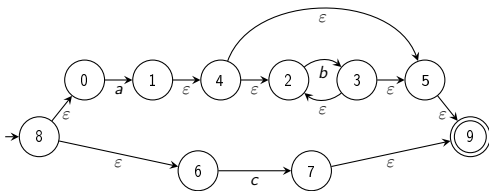
$$E_1 = \{1, 2, 4, 5, 9\}$$

$$E_2 = \{7, 9\}$$

$$E_3 = \{2, 3, 5, 9\}$$

Etat actuel : E_1

- $x = c$
- transitions : aucune



$$D = \{0, 6, 8\}$$

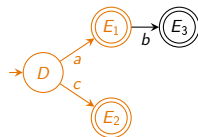
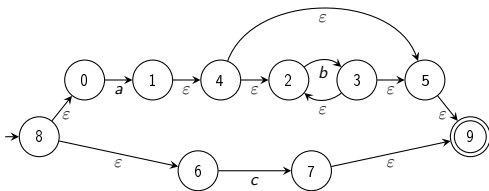
$$E_1 = \{1, 2, 4, 5, 9\}$$

$$E_2 = \{7, 9\}$$

$$E_3 = \{2, 3, 5, 9\}$$

Etat actuel : E_2

- $x = a$
- transitions : aucune



$$D = \{0, 6, 8\}$$

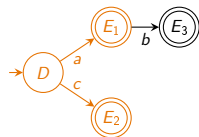
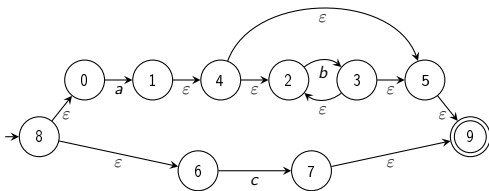
$$E_1 = \{1, 2, 4, 5, 9\}$$

$$E_2 = \{7, 9\}$$

$$E_3 = \{2, 3, 5, 9\}$$

Etat actuel : E_2

- $x = b$
- transitions : aucune



$$D = \{0, 6, 8\}$$

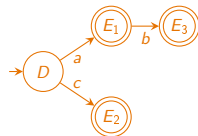
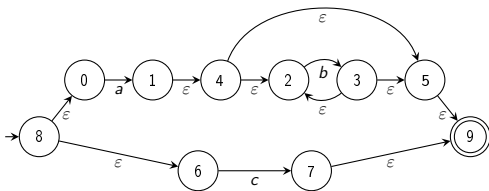
$$E_1 = \{1, 2, 4, 5, 9\}$$

$$E_2 = \{7, 9\}$$

$$E_3 = \{2, 3, 5, 9\}$$

Etat actuel : E_2

- $x = c$
- transitions : aucune



$$D = \{0, 6, 8\}$$

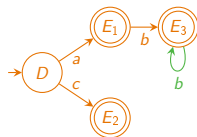
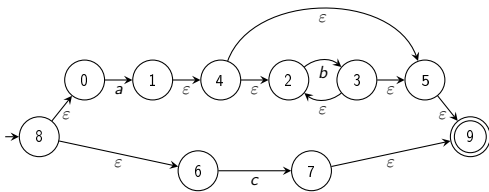
$$E_1 = \{1, 2, 4, 5, 9\}$$

$$E_2 = \{7, 9\}$$

$$E_3 = \{2, 3, 5, 9\}$$

Etat actuel : E_3

- $x = a$
- transitions : aucune



$$D = \{0, 6, 8\}$$

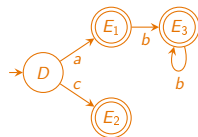
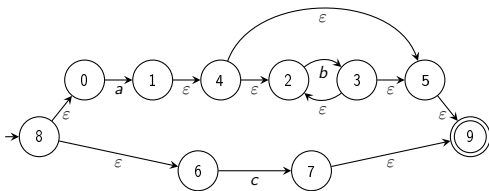
$$E_1 = \{1, 2, 4, 5, 9\}$$

$$E_2 = \{7, 9\}$$

$$E_3 = \{2, 3, 5, 9\}$$

Etat actuel : E_3

- $x = b$
- transitions : 2b3
- $EpsilonFermeture(\{3\}) = \{2, 3, 5, 9\} = E_3$
- etat final : oui



$$D = \{0, 6, 8\}$$

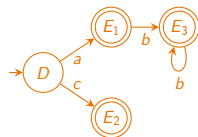
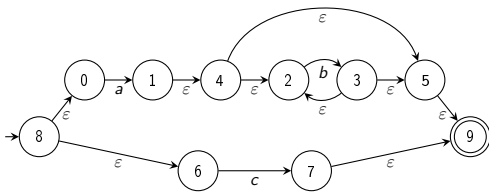
$$E_1 = \{1, 2, 4, 5, 9\}$$

$$E_2 = \{7, 9\}$$

$$E_3 = \{2, 3, 5, 9\}$$

Etat actuel : E_3

- $x = c$
- transitions : aucune



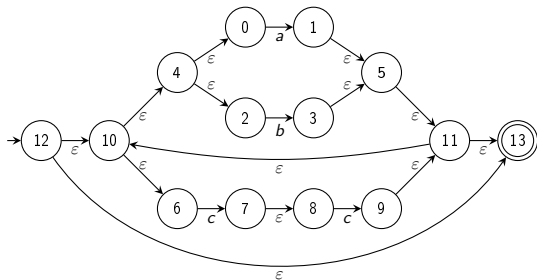
$$D = \{0, 6, 8\}$$

$$E_1 = \{1, 2, 4, 5, 9\}$$

$$E_2 = \{7, 9\}$$

$$E_3 = \{2, 3, 5, 9\}$$

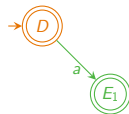
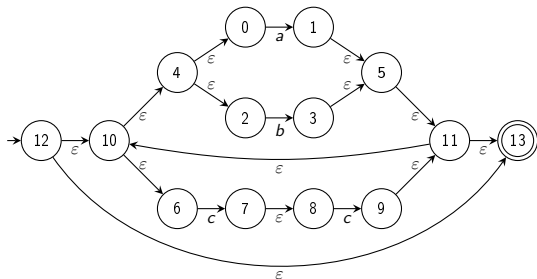
AFD terminé, nombre d'états : 4



$$D = \{0, 2, 4, 6, 10, 12, 13\}$$

Calcul de l'état de départ :

$$D = \text{EpsilonFermeture}(\{12\}) = \{0, 2, 4, 6, 10, 12, 13\}$$

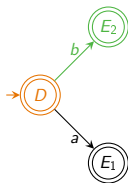
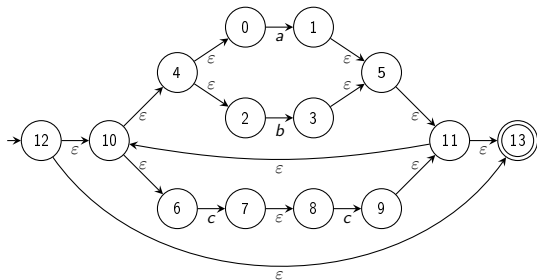


Etat actuel : D

$$D = \{0, 2, 4, 6, 10, 12, 13\}$$

$$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$$

- $x = a$
- transitions : $0a1$
- $EpsilonFermeture(\{1\}) = \{0, 1, 2, 4, 6, 5, 10, 11, 13\} = E_1$
- etat final : oui



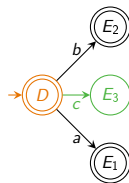
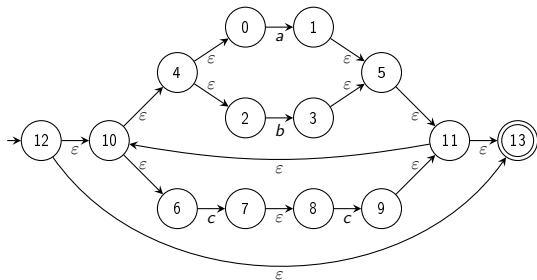
Etat actuel : D

$$D = \{0, 2, 4, 6, 10, 12, 13\}$$

$$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$$

$$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$$

- $x = b$
- transitions : 2b3
- $EpsilonFermeture(\{3\}) = \{0, 2, 3, 4, 6, 5, 10, 11, 13\} = E_2$
- etat final : oui



Etat actuel : D

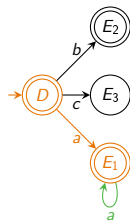
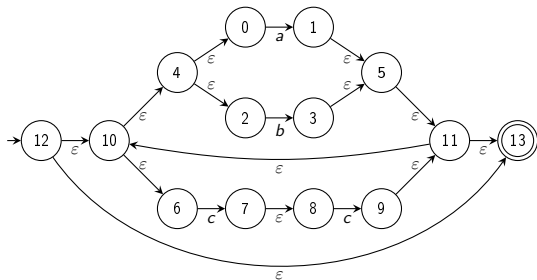
$$D = \{0, 2, 4, 6, 10, 12, 13\}$$

$$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$$

$$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$$

$$E_3 = \{7, 8\}$$

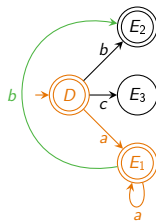
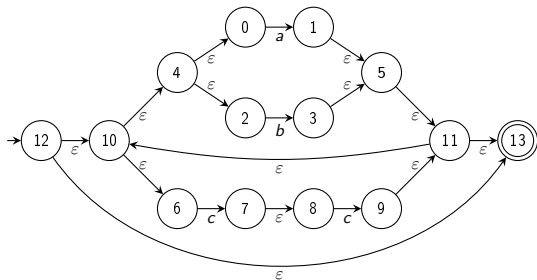
- $x = c$
- transitions : $6c7$
- $EpsilonFermeture(\{7\}) = \{7, 8\} = E_3$
- etat final : non



Etat actuel : E_1

- $x = a$
- transitions : 0a1
- $EpsilonFermeture(\{1\}) = \{0, 1, 2, 4, 6, 5, 10, 11, 13\} = E_1$
- etat final : oui

$$\begin{aligned}
 D &= \{0, 2, 4, 6, 10, 12, 13\} \\
 E_1 &= \{0, 1, 2, 4, 5, 6, 10, 11, 13\} \\
 E_2 &= \{0, 2, 3, 4, 5, 6, 10, 11, 13\} \\
 E_3 &= \{7, 8\}
 \end{aligned}$$



Etat actuel : E_1

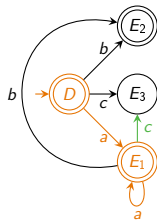
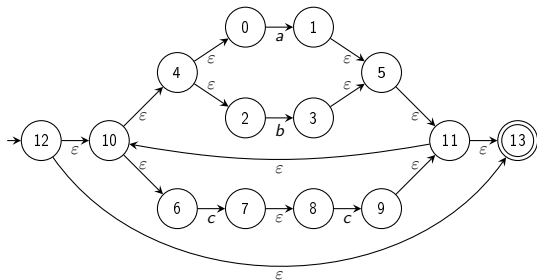
- $x = b$
- transitions : 2b3
- $EpsilonFermeture(\{3\}) = \{0, 2, 3, 4, 6, 5, 10, 11, 13\} = E_2$
- etat final : oui

$$D = \{0, 2, 4, 6, 10, 12, 13\}$$

$$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$$

$$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$$

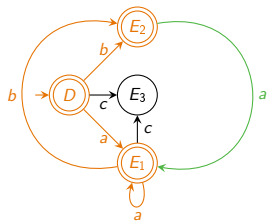
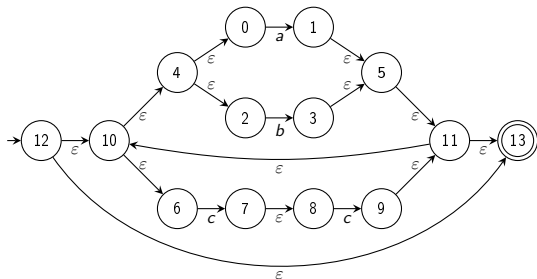
$$E_3 = \{7, 8\}$$



Etat actuel : E_1

- $x = c$
- transitions : 6c7
- $EpsilonFermeture(\{7\}) = \{7, 8\} = E_3$
- etat final : non

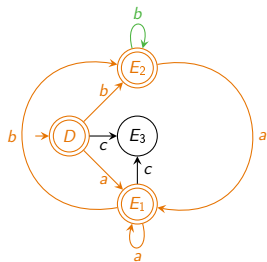
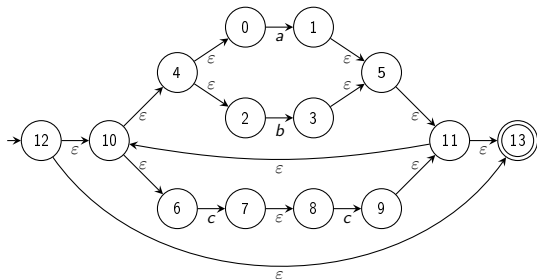
$$\begin{aligned}
 D &= \{0, 2, 4, 6, 10, 12, 13\} \\
 E_1 &= \{0, 1, 2, 4, 5, 6, 10, 11, 13\} \\
 E_2 &= \{0, 2, 3, 4, 5, 6, 10, 11, 13\} \\
 E_3 &= \{7, 8\}
 \end{aligned}$$



Etat actuel : E_2

$$\begin{aligned}
 D &= \{0, 2, 4, 6, 10, 12, 13\} \\
 E_1 &= \{0, 1, 2, 4, 5, 6, 10, 11, 13\} \\
 E_2 &= \{0, 2, 3, 4, 5, 6, 10, 11, 13\} \\
 E_3 &= \{7, 8\}
 \end{aligned}$$

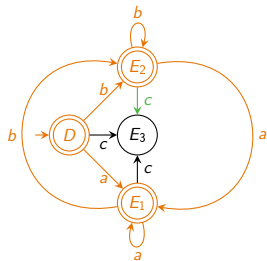
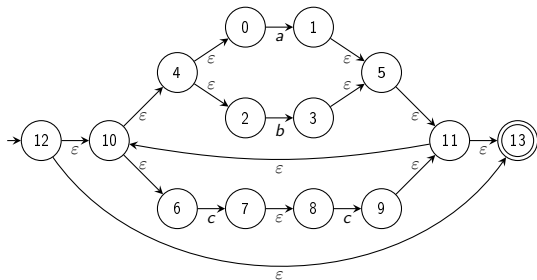
- $x = a$
- transitions : 0a1
- $EpsilonFermeture(\{1\}) = \{0, 1, 2, 4, 6, 5, 10, 11, 13\} = E_1$
- etat final : oui



Etat actuel : E_2

- $x = b$
- transitions : 2b3
- $EpsilonFermeture(\{3\}) = \{0, 2, 3, 4, 6, 5, 10, 11, 13\} = E_2$
- etat final : oui

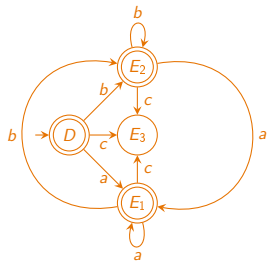
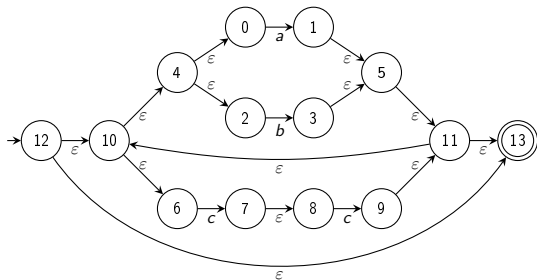
$$\begin{aligned}
 D &= \{0, 2, 4, 6, 10, 12, 13\} \\
 E_1 &= \{0, 1, 2, 4, 5, 6, 10, 11, 13\} \\
 E_2 &= \{0, 2, 3, 4, 5, 6, 10, 11, 13\} \\
 E_3 &= \{7, 8\}
 \end{aligned}$$



Etat actuel : E_2

- $x = c$
- transitions : 6c7
- $EpsilonFermeture(\{7\}) = \{7, 8\} = E_3$
- etat final : non

$$\begin{aligned}
 D &= \{0, 2, 4, 6, 10, 12, 13\} \\
 E_1 &= \{0, 1, 2, 4, 5, 6, 10, 11, 13\} \\
 E_2 &= \{0, 2, 3, 4, 5, 6, 10, 11, 13\} \\
 E_3 &= \{7, 8\}
 \end{aligned}$$



Etat actuel : E_3

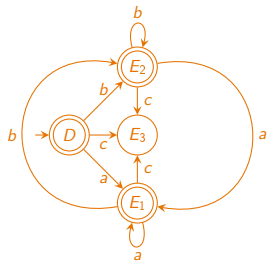
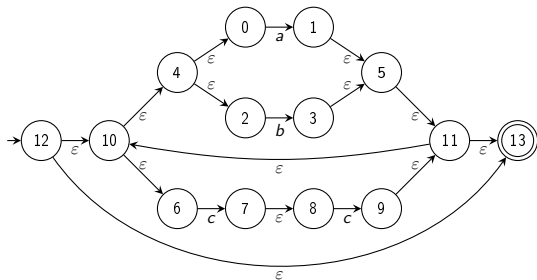
$$D = \{0, 2, 4, 6, 10, 12, 13\}$$

$$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$$

$$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$$

$$E_3 = \{7, 8\}$$

- $x = a$
- transitions : aucune



Etat actuel : E_3

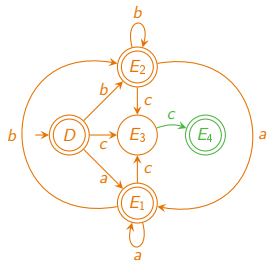
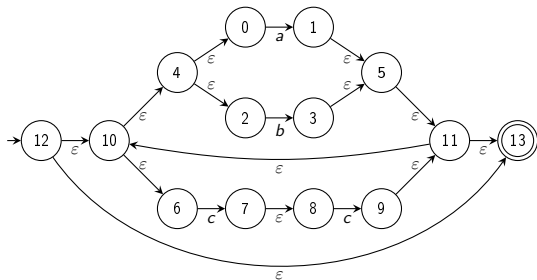
- $x = b$
- transitions : aucune

$$D = \{0, 2, 4, 6, 10, 12, 13\}$$

$$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$$

$$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$$

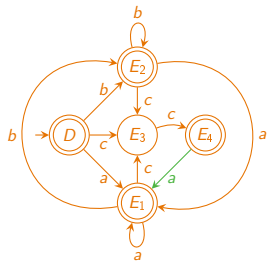
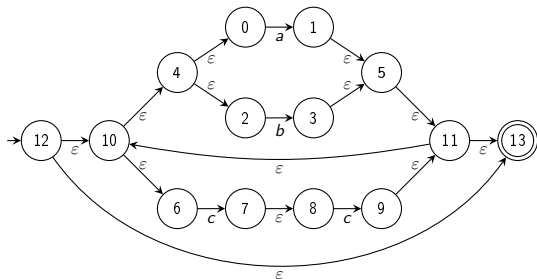
$$E_3 = \{7, 8\}$$



Etat actuel : E_3

- $x = c$
- transitions : 8c9
- $EpsilonFermeture(\{9\}) = \{0, 2, 4, 6, 9, 10, 11, 13\} = E_4$
- etat final : non

$$\begin{aligned}
 D &= \{0, 2, 4, 6, 10, 12, 13\} \\
 E_1 &= \{0, 1, 2, 4, 5, 6, 10, 11, 13\} \\
 E_2 &= \{0, 2, 3, 4, 5, 6, 10, 11, 13\} \\
 E_3 &= \{7, 8\} \\
 E_4 &= \{0, 2, 4, 6, 9, 10, 11, 13\}
 \end{aligned}$$



Etat actuel : E_4

- $x = a$
- transitions : 0a1
- $EpsilonFermeture(\{1\}) = \{0, 1, 2, 4, 6, 5, 10, 11, 13\} = E_1$
- etat final : oui

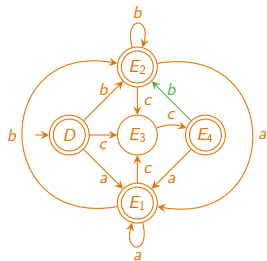
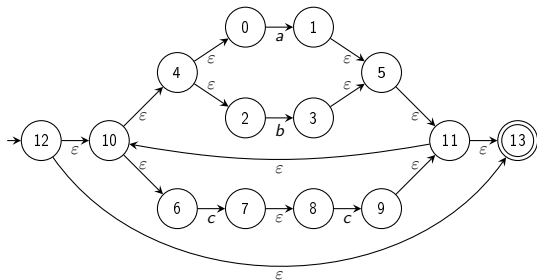
$$D = \{0, 2, 4, 6, 10, 12, 13\}$$

$$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$$

$$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$$

$$E_3 = \{7, 8\}$$

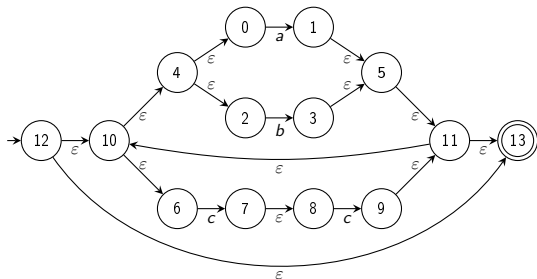
$$E_4 = \{0, 2, 4, 6, 9, 10, 11, 13\}$$



Etat actuel : E_4

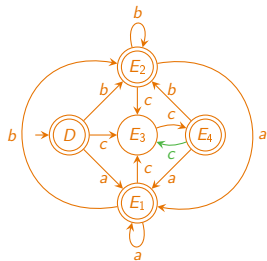
- $x = b$
- transitions : 2b3
- $EpsilonFermeture(\{3\}) = \{0, 2, 3, 4, 6, 5, 10, 11, 13\} = E_2$
- etat final : oui

$$\begin{aligned}
 D &= \{0, 2, 4, 6, 10, 12, 13\} \\
 E_1 &= \{0, 1, 2, 4, 5, 6, 10, 11, 13\} \\
 E_2 &= \{0, 2, 3, 4, 5, 6, 10, 11, 13\} \\
 E_3 &= \{7, 8\} \\
 E_4 &= \{0, 2, 4, 6, 9, 10, 11, 13\}
 \end{aligned}$$



Etat actuel : E_4

- $x = c$
- transitions : $6c7$
- $EpsilonFermeture(\{7\}) = \{7, 8\} = E_3$
- etat final : non



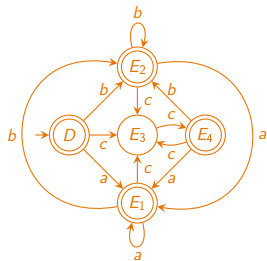
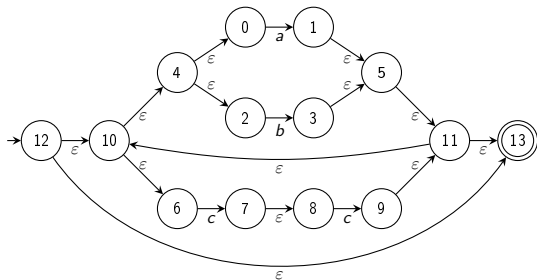
$$D = \{0, 2, 4, 6, 10, 12, 13\}$$

$$E_1 = \{0, 1, 2, 4, 5, 6, 10, 11, 13\}$$

$$E_2 = \{0, 2, 3, 4, 5, 6, 10, 11, 13\}$$

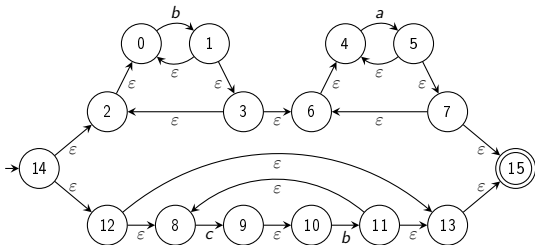
$$E_3 = \{7, 8\}$$

$$E_4 = \{0, 2, 4, 6, 9, 10, 11, 13\}$$



AFD terminé, nombre d'états : 5

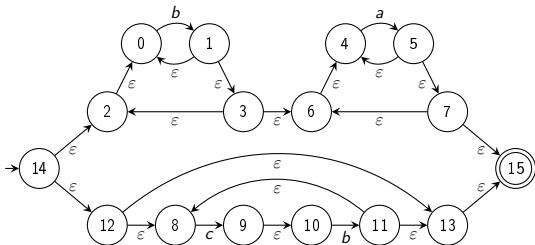
$$\begin{aligned}
 D &= \{0, 2, 4, 6, 10, 12, 13\} \\
 E_1 &= \{0, 1, 2, 4, 5, 6, 10, 11, 13\} \\
 E_2 &= \{0, 2, 3, 4, 5, 6, 10, 11, 13\} \\
 E_3 &= \{7, 8\} \\
 E_4 &= \{0, 2, 4, 6, 9, 10, 11, 13\}
 \end{aligned}$$



$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

Calcul de l'état de départ :

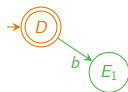
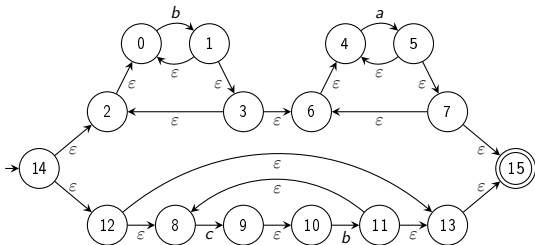
$$D = \text{EpsilonFermeture}(\{14\}) = \{0, 2, 8, 12, 13, 14, 15\}$$



$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

Etat actuel : D

- $x = a$
- transitions : aucune

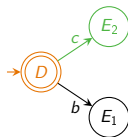
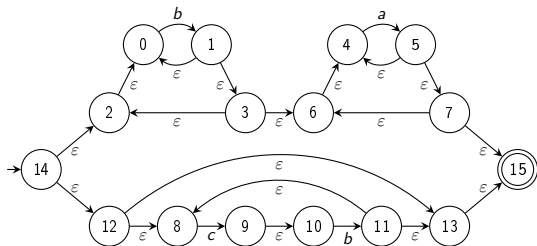


$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

Etat actuel : D

- $x = b$
- transitions : 0b1
- $EpsilonFermeture(\{1\}) = \{0, 1, 2, 3, 4, 6\} = E_1$
- etat final : non



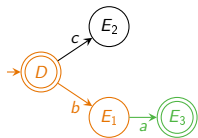
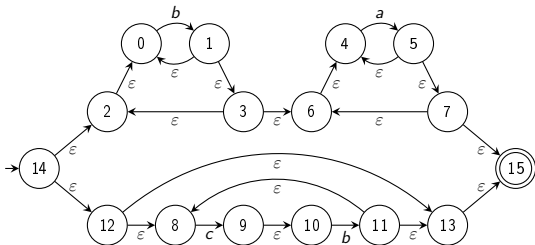
$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

$$E_2 = \{9, 10\}$$

Etat actuel : D

- $x = c$
- transitions : $8c9$
- $EpsilonFermeture(\{9\}) = \{9, 10\} = E_2$
- etat final : non



Etat actuel : E_1

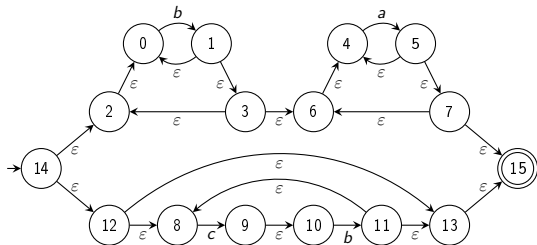
$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

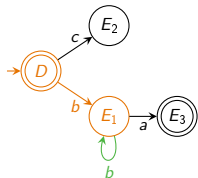
$$E_2 = \{9, 10\}$$

$$E_3 = \{4, 5, 6, 7, 15\}$$

- $x = a$
- transitions : 4a5
- $EpsilonFermeture(\{5\}) = \{4, 5, 6, 7, 15\} = E_3$
- etat final : oui



Etat actuel : E_1



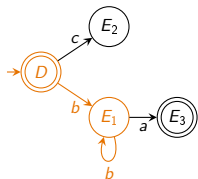
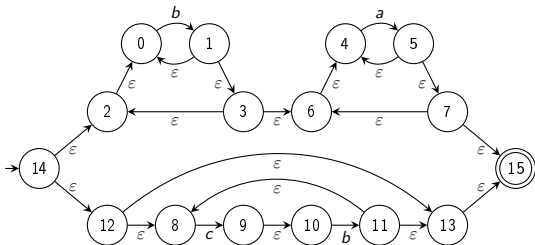
$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

$$E_2 = \{9, 10\}$$

$$E_3 = \{4, 5, 6, 7, 15\}$$

- $x = b$
- transitions : 0b1
- $EpsilonFermeture(\{1\}) = \{0, 1, 2, 3, 4, 6\} = E_1$
- etat final : non



$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

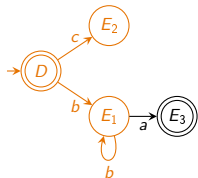
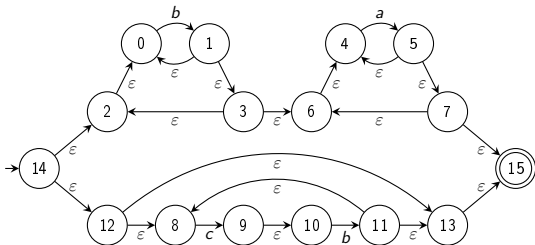
$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

$$E_2 = \{9, 10\}$$

$$E_3 = \{4, 5, 6, 7, 15\}$$

Etat actuel : E_1

- $x = c$
- transitions : aucune



$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

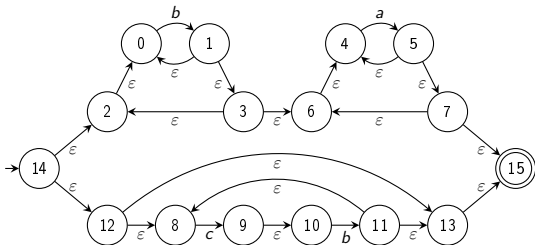
$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

$$E_2 = \{9, 10\}$$

$$E_3 = \{4, 5, 6, 7, 15\}$$

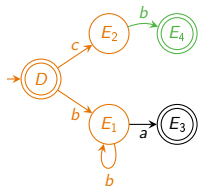
Etat actuel : E_2

- $x = a$
- transitions : aucune



Etat actuel : E_2

- $x = b$
- transitions : 10b11
- $EpsilonFermeture(\{11\}) = \{8, 10, 11, 13, 15\} = E_4$
- etat final : oui



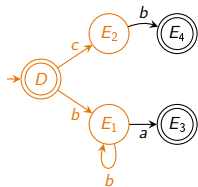
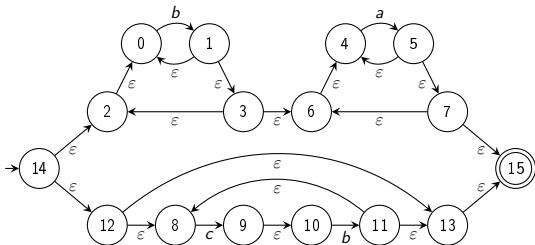
$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

$$E_2 = \{9, 10\}$$

$$E_3 = \{4, 5, 6, 7, 15\}$$

$$E_4 = \{8, 10, 11, 13, 15\}$$



Etat actuel : E_2

- $x = c$
- transitions : aucune

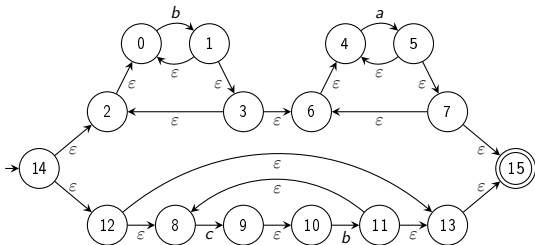
$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

$$E_2 = \{9, 10\}$$

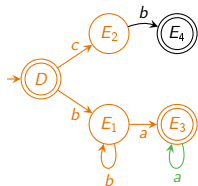
$$E_3 = \{4, 5, 6, 7, 15\}$$

$$E_4 = \{8, 10, 11, 13, 15\}$$



Etat actuel : E_3

- $x = a$
- transitions : 4c5
- $EpsilonFermeture(\{5\}) = \{4, 5, 6, 7, 15\} = E_3$
- etat final : oui



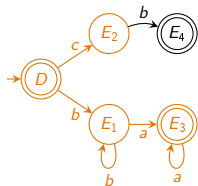
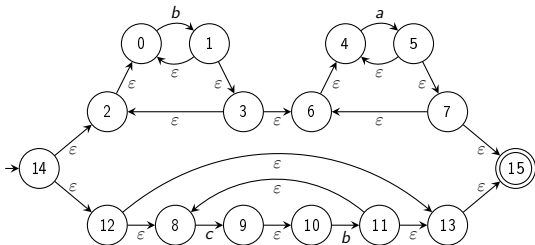
$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

$$E_2 = \{9, 10\}$$

$$E_3 = \{4, 5, 6, 7, 15\}$$

$$E_4 = \{8, 10, 11, 13, 15\}$$



Etat actuel : E_3

- $x = b$
- transitions : aucune

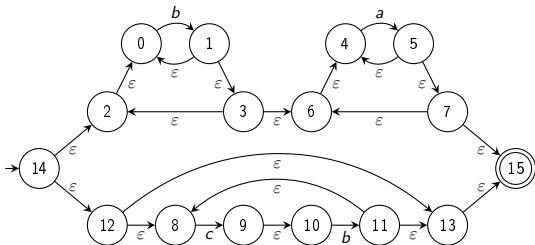
$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

$$E_2 = \{9, 10\}$$

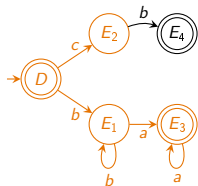
$$E_3 = \{4, 5, 6, 7, 15\}$$

$$E_4 = \{8, 10, 11, 13, 15\}$$



Etat actuel : E_3

- $x = c$
- transitions : aucune



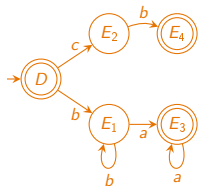
$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

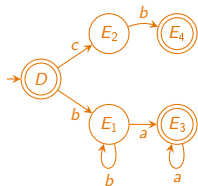
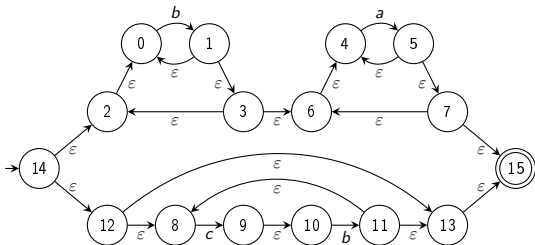
$$E_2 = \{9, 10\}$$

$$E_3 = \{4, 5, 6, 7, 15\}$$

$$E_4 = \{8, 10, 11, 13, 15\}$$


$$\begin{aligned} D &= \{0, 2, 8, 12, 13, 14, 15\} \\ E_1 &= \{0, 1, 2, 3, 4, 6\} \\ E_2 &= \{9, 10\} \\ E_3 &= \{4, 5, 6, 7, 15\} \\ E_4 &= \{8, 10, 11, 13, 15\} \end{aligned}$$

- A set of small navigation icons typically found in Beamer presentations, including symbols for back, forward, search, and other slide controls.



Etat actuel : E_4

- $x = b$
- transitions : aucune

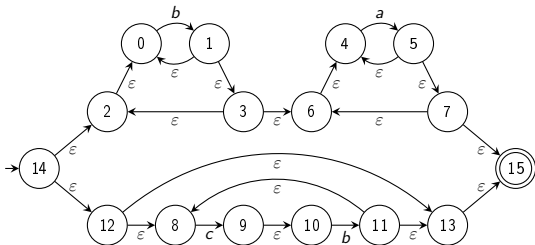
$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

$$E_2 = \{9, 10\}$$

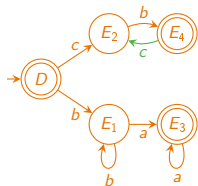
$$E_3 = \{4, 5, 6, 7, 15\}$$

$$E_4 = \{8, 10, 11, 13, 15\}$$



Etat actuel : E_4

- $x = c$
- transitions : 8c9
- $EpsilonFermeture(\{9\}) = \{9, 10\} = E_2$
- etat final : non



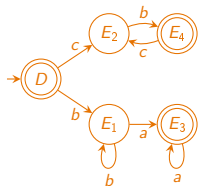
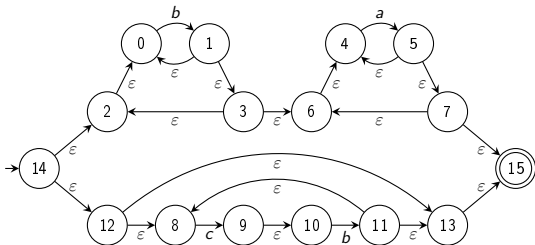
$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

$$E_2 = \{9, 10\}$$

$$E_3 = \{4, 5, 6, 7, 15\}$$

$$E_4 = \{8, 10, 11, 13, 15\}$$



$$D = \{0, 2, 8, 12, 13, 14, 15\}$$

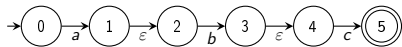
$$E_1 = \{0, 1, 2, 3, 4, 6\}$$

$$E_2 = \{9, 10\}$$

$$E_3 = \{4, 5, 6, 7, 15\}$$

$$E_4 = \{8, 10, 11, 13, 15\}$$

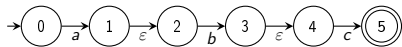
AFD terminé, nombre d'états : 5



$$D = \{0\}$$

Calcul de l'état de départ :

$$D = \textit{EpsilonFermeture}(\{0\}) = \{0\}$$

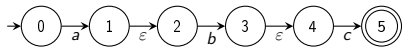


$$D = \{0\}$$

$$E_1 = \{1, 2\}$$

Etat actuel : D

- $x = a$
- transitions : $0a1$
- $EpsilonFermeture(\{1\}) = \{1, 2\} = E_1$
- etat final : non

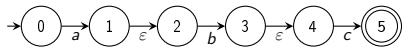


$$D = \{0\}$$

$$E_1 = \{1, 2\}$$

Etat actuel : D

- $x = b$
- transitions : aucune

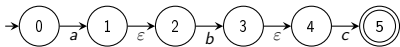


$$D = \{0\}$$

$$E_1 = \{1, 2\}$$

Etat actuel : D

- $x = c$
- transitions : aucune

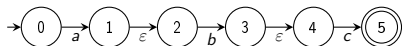


$$D = \{0\}$$

$$E_1 = \{1, 2\}$$

Etat actuel : E_1

- $x = a$
- transitions : aucune



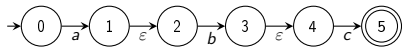
$$D = \{0\}$$

$$E_1 = \{1, 2\}$$

$$E_2 = \{3, 4\}$$

Etat actuel : E_1

- $x = b$
- transitions : 2b3
- $EpsilonFermeture(\{3\}) = \{3, 4\} = E_2$
- etat final : non



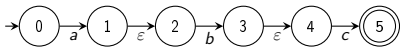
$$D = \{0\}$$

$$E_1 = \{1, 2\}$$

$$E_2 = \{3, 4\}$$

Etat actuel : E_1

- $x = c$
- transitions : aucune



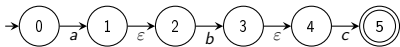
$$D = \{0\}$$

$$E_1 = \{1, 2\}$$

$$E_2 = \{3, 4\}$$

Etat actuel : E_2

- $x = a$
- transitions : aucune



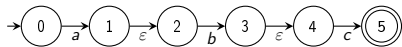
$$D = \{0\}$$

$$E_1 = \{1, 2\}$$

$$E_2 = \{3, 4\}$$

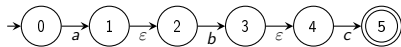
Etat actuel : E_2

- $x = b$
- transitions : aucune



Etat actuel : E_2

- $x = c$
- transitions : 4c5
- $EpsilonFermeture(\{5\}) = \{5\} = E_3$
- etat final : oui



$$D = \{0\}$$

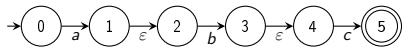
$$E_1 = \{1, 2\}$$

$$E_2 = \{3, 4\}$$

$$E_3 = \{5\}$$

Etat actuel : E_3

- $x = a$
- transitions : aucune



$$D = \{0\}$$

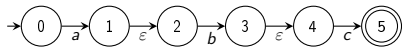
$$E_1 = \{1, 2\}$$

$$E_2 = \{3, 4\}$$

$$E_3 = \{5\}$$

Etat actuel : E_3

- $x = b$
- transitions : aucune



$$D = \{0\}$$

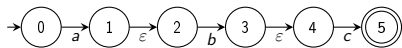
$$E_1 = \{1, 2\}$$

$$E_2 = \{3, 4\}$$

$$E_3 = \{5\}$$

Etat actuel : E_3

- $x = c$
- transitions : aucune



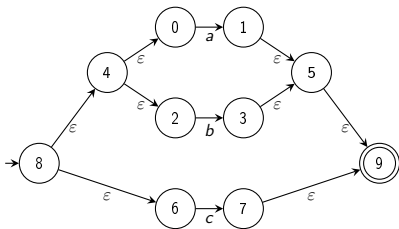
$$D = \{0\}$$

$$E_1 = \{1, 2\}$$

$$E_2 = \{3, 4\}$$

$$E_3 = \{5\}$$

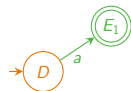
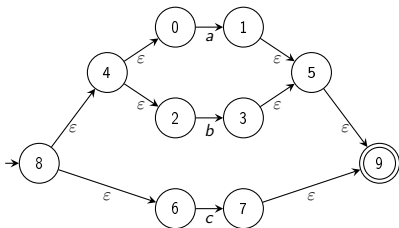
AFD terminé, nombre d'états : 4



$$D = \{0, 2, 4, 6, 8\}$$

Calcul de l'état de départ :

$$D = \text{EpsilonFermeture}(\{8\}) = \{0, 2, 4, 6, 8\}$$

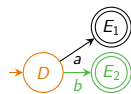
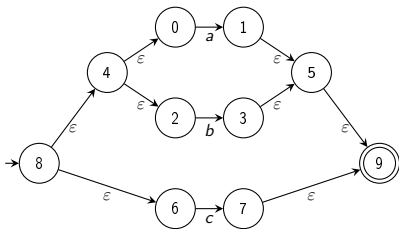


$$D = \{0, 2, 4, 6, 8\}$$

$$E_1 = \{1, 5, 9\}$$

Etat actuel : D

- $x = a$
- transitions : 0a1
- $EpsilonFermeture(\{1\}) = \{1, 5, 9\} = E_1$
- etat final : oui



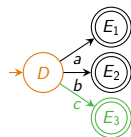
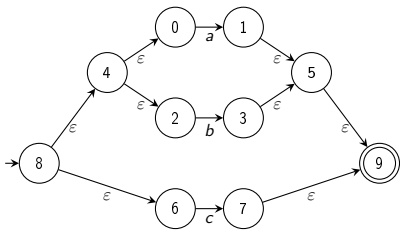
$$D = \{0, 2, 4, 6, 8\}$$

$$E_1 = \{1, 5, 9\}$$

$$E_2 = \{3, 5, 9\}$$

Etat actuel : D

- $x = b$
- transitions : 2b3
- $Epsilon\text{Fermeture}(\{3\}) = \{3, 5, 9\} = E_2$
- etat final : oui



$$D = \{0, 2, 4, 6, 8\}$$

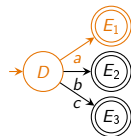
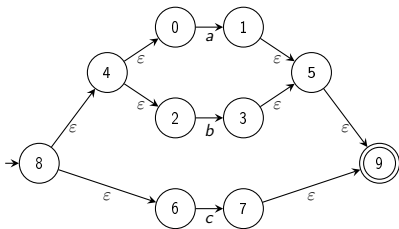
$$E_1 = \{1, 5, 9\}$$

$$E_2 = \{3, 5, 9\}$$

$$E_3 = \{7, 9\}$$

Etat actuel : D

- $x = c$
- transitions : $6c7$
- $EpsilonFermeture(\{5\}) = \{7, 9\} = E_3$
- etat final : oui



$$D = \{0, 2, 4, 6, 8\}$$

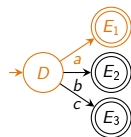
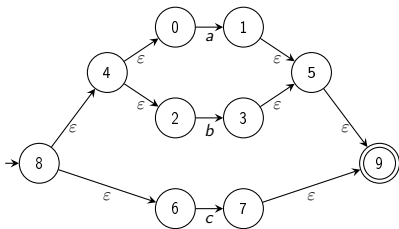
$$E_1 = \{1, 5, 9\}$$

$$E_2 = \{3, 5, 9\}$$

$$E_3 = \{7, 9\}$$

Etat actuel : E_1

- $x = a$
- transitions : aucune



$$D = \{0, 2, 4, 6, 8\}$$

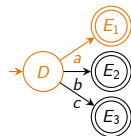
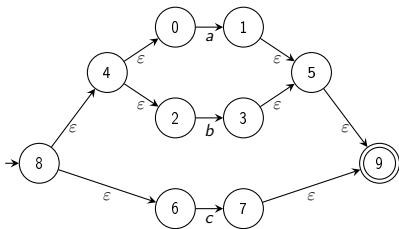
$$E_1 = \{1, 5, 9\}$$

$$E_2 = \{3, 5, 9\}$$

$$E_3 = \{7, 9\}$$

Etat actuel : E_1

- $x = b$
- transitions : aucune



$$D = \{0, 2, 4, 6, 8\}$$

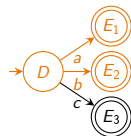
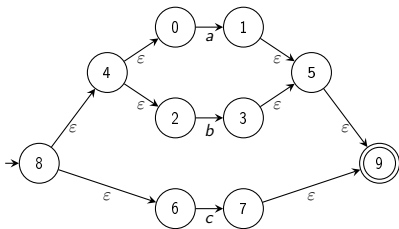
$$E_1 = \{1, 5, 9\}$$

$$E_2 = \{3, 5, 9\}$$

$$E_3 = \{7, 9\}$$

Etat actuel : E_1

- $x = c$
- transitions : aucune



$$D = \{0, 2, 4, 6, 8\}$$

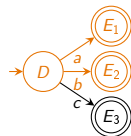
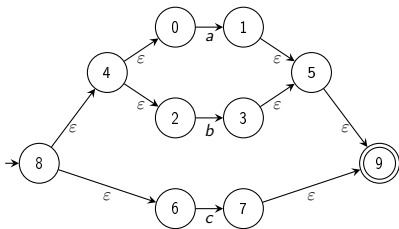
$$E_1 = \{1, 5, 9\}$$

$$E_2 = \{3, 5, 9\}$$

$$E_3 = \{7, 9\}$$

Etat actuel : E_2

- $x = a$
- transitions : aucune



$$D = \{0, 2, 4, 6, 8\}$$

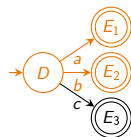
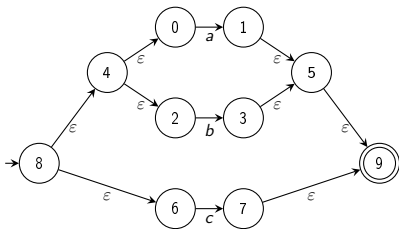
$$E_1 = \{1, 5, 9\}$$

$$E_2 = \{3, 5, 9\}$$

$$E_3 = \{7, 9\}$$

Etat actuel : E_2

- $x = b$
- transitions : aucune



$$D = \{0, 2, 4, 6, 8\}$$

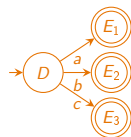
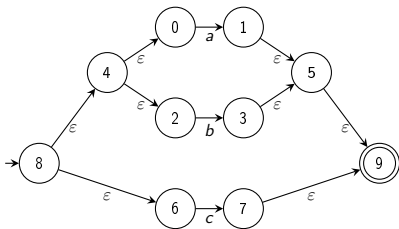
$$E_1 = \{1, 5, 9\}$$

$$E_2 = \{3, 5, 9\}$$

$$E_3 = \{7, 9\}$$

Etat actuel : E_2

- $x = c$
- transitions : aucune



$$D = \{0, 2, 4, 6, 8\}$$

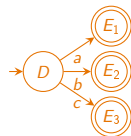
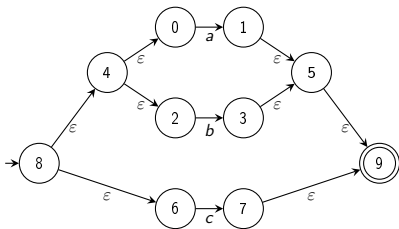
$$E_1 = \{1, 5, 9\}$$

$$E_2 = \{3, 5, 9\}$$

$$E_3 = \{7, 9\}$$

Etat actuel : E_3

- $x = a$
- transitions : aucune



$$D = \{0, 2, 4, 6, 8\}$$

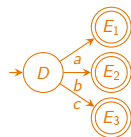
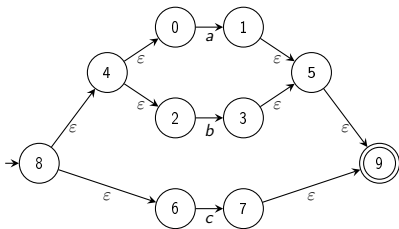
$$E_1 = \{1, 5, 9\}$$

$$E_2 = \{3, 5, 9\}$$

$$E_3 = \{7, 9\}$$

Etat actuel : E_3

- $x = b$
- transitions : aucune



$$D = \{0, 2, 4, 6, 8\}$$

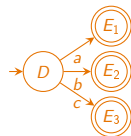
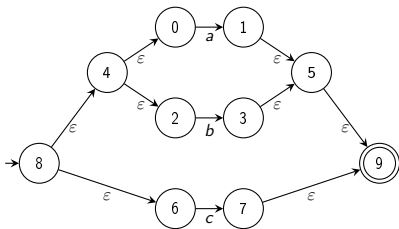
$$E_1 = \{1, 5, 9\}$$

$$E_2 = \{3, 5, 9\}$$

$$E_3 = \{7, 9\}$$

Etat actuel : E_3

- $x = c$
- transitions : aucune



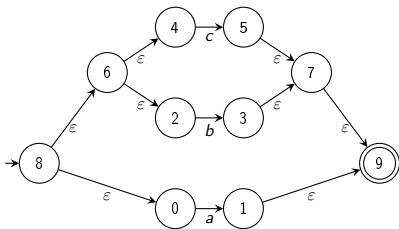
$$D = \{0, 2, 4, 6, 8\}$$

$$E_1 = \{1, 5, 9\}$$

$$E_2 = \{3, 5, 9\}$$

$$E_3 = \{7, 9\}$$

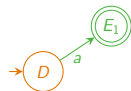
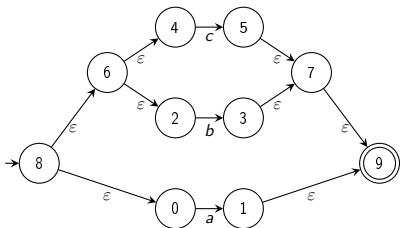
AFD terminé, nombre d'états : 4



$$D = \{0, 2, 4, 6, 8\}$$

Calcul de l'état de départ :

$$D = \text{EpsilonFermeture}(\{8\}) = \{0, 2, 4, 6, 8\}$$

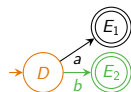
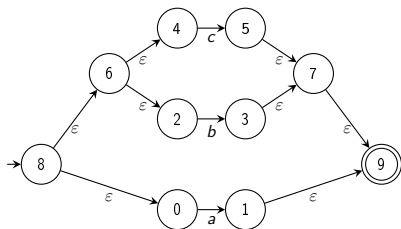


$$D = \{0, 2, 4, 6, 8\}$$

$$E_1 = \{1, 9\}$$

Etat actuel : D

- $x = a$
- transitions : 0a1
- $EpsilonFermeture(\{1\}) = \{1, 9\} = E_1$
- etat final : oui



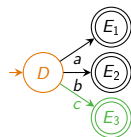
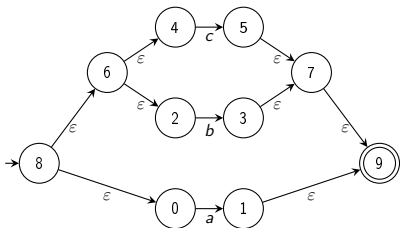
$$D = \{0, 2, 4, 6, 8\}$$

$$E_1 = \{1, 9\}$$

$$E_2 = \{3, 7, 9\}$$

Etat actuel : D

- $x = b$
- transitions : 2b3
- $EpsilonFermeture(\{3\}) = \{3, 7, 9\} = E_2$
- etat final : oui



$$D = \{0, 2, 4, 6, 8\}$$

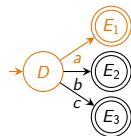
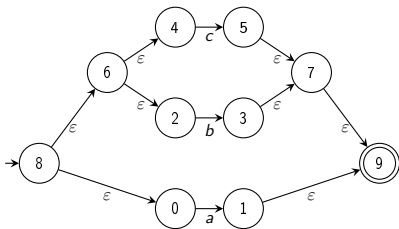
$$E_1 = \{1, 9\}$$

$$E_2 = \{3, 7, 9\}$$

$$E_3 = \{5, 7, 9\}$$

Etat actuel : D

- $x = c$
- transitions : 4c5
- $EpsilonFermeture(\{5\}) = \{5, 7, 9\} = E_3$
- etat final : oui



$$D = \{0, 2, 4, 6, 8\}$$

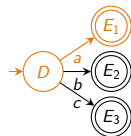
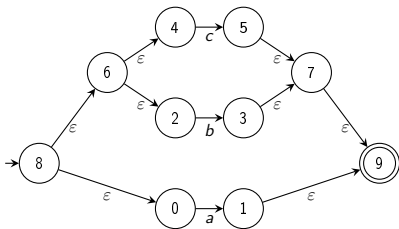
$$E_1 = \{1, 9\}$$

$$E_2 = \{3, 7, 9\}$$

$$E_3 = \{5, 7, 9\}$$

Etat actuel : E_1

- $x = a$
- transitions : aucune



$$D = \{0, 2, 4, 6, 8\}$$

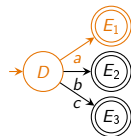
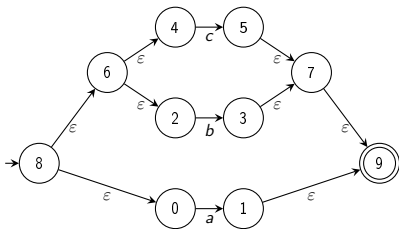
$$E_1 = \{1, 9\}$$

$$E_2 = \{3, 7, 9\}$$

$$E_3 = \{5, 7, 9\}$$

Etat actuel : E_1

- $x = b$
- transitions : aucune



$$D = \{0, 2, 4, 6, 8\}$$

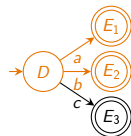
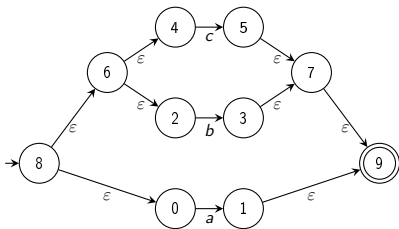
$$E_1 = \{1, 9\}$$

$$E_2 = \{3, 7, 9\}$$

$$E_3 = \{5, 7, 9\}$$

Etat actuel : E_1

- $x = c$
- transitions : aucune



$$D = \{0, 2, 4, 6, 8\}$$

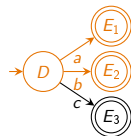
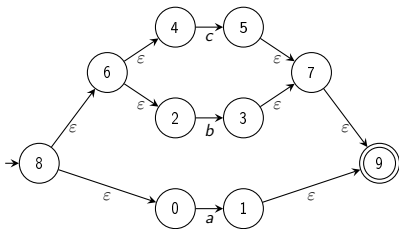
$$E_1 = \{1, 9\}$$

$$E_2 = \{3, 7, 9\}$$

$$E_3 = \{5, 7, 9\}$$

Etat actuel : E_2

- $x = a$
- transitions : aucune



$$D = \{0, 2, 4, 6, 8\}$$

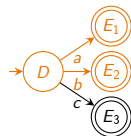
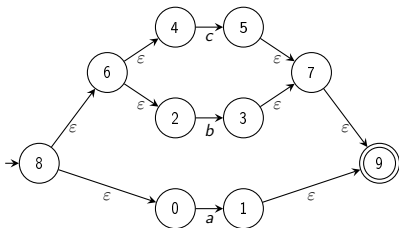
$$E_1 = \{1, 9\}$$

$$E_2 = \{3, 7, 9\}$$

$$E_3 = \{5, 7, 9\}$$

Etat actuel : E_2

- $x = b$
- transitions : aucune



$$D = \{0, 2, 4, 6, 8\}$$

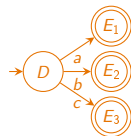
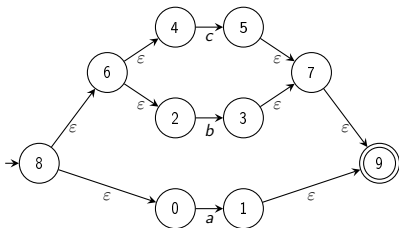
$$E_1 = \{1, 9\}$$

$$E_2 = \{3, 7, 9\}$$

$$E_3 = \{5, 7, 9\}$$

Etat actuel : E_2

- $x = c$
- transitions : aucune



$$D = \{0, 2, 4, 6, 8\}$$

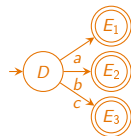
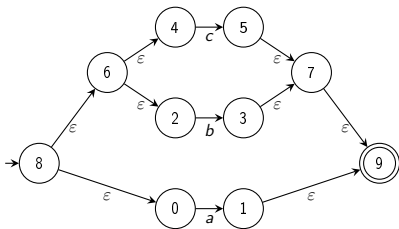
$$E_1 = \{1, 9\}$$

$$E_2 = \{3, 7, 9\}$$

$$E_3 = \{5, 7, 9\}$$

Etat actuel : E_3

- $x = a$
- transitions : aucune



$$D = \{0, 2, 4, 6, 8\}$$

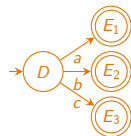
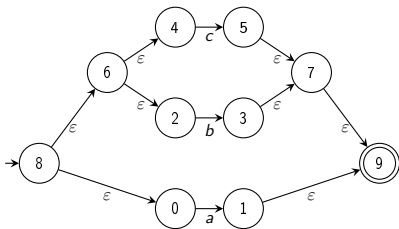
$$E_1 = \{1, 9\}$$

$$E_2 = \{3, 7, 9\}$$

$$E_3 = \{5, 7, 9\}$$

Etat actuel : E_3

- $x = b$
- transitions : aucune



$$D = \{0, 2, 4, 6, 8\}$$

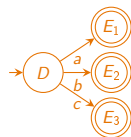
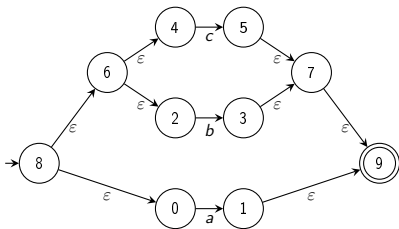
$$E_1 = \{1, 9\}$$

$$E_2 = \{3, 7, 9\}$$

$$E_3 = \{5, 7, 9\}$$

Etat actuel : E_3

- $x = c$
- transitions : aucune



$$D = \{0, 2, 4, 6, 8\}$$

$$E_1 = \{1, 9\}$$

$$E_2 = \{3, 7, 9\}$$

$$E_3 = \{5, 7, 9\}$$

AFD terminé, nombre d'états : 4

Les AFD des expressions régulières $(a|b)|c$ et $a|(b|c)$ sont strictement identiques. Cela montre que quels que soient a , b et c , la règle $|$ est associative. Il est donc possible de supprimer les parenthèses sans ambiguïté.

$$(a|b)|c = a|(b|c) = a|b|c$$