HAI601I - Exercices de révisions

Benoît Huftier

2022

Construction d'un AFN d'expression régulière

Enoncé

Donner l'automate fini non déterministe (AFN) des expressions régulières suivantes grâce à l'algorithme de Thompson.

1. ab
2. b*
3. a|b
4. a
5. ab|c
6. ab*|c
7. ((a|b)|cc)*
8. b*a*|(cb)*
9. (a|b)|c
10. a|(b|c)
11. (ab)c
12. a(bc)

Avec les AFN des expressions régulières (ab)c et a(bc), que peut-on en déduire sur la concaténation ?

$$\text{expr} = \text{ab}$$

La première chose à faire c'est toujours de construire l'arbre de dérivation correspondant à l'expression régulière. On se base ensuite sur cet arbre pour créer notre AFD. On initialise $i = 0$ et on lance l'algorithme sur l'arbre.

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arbre

$i = 0$

$$\text{expr} = \text{ab}$$

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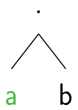
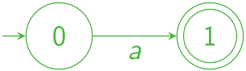
arbre



$i = 2$

AFN



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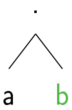
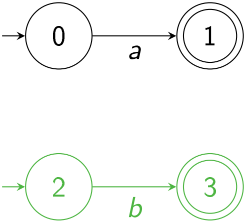
arbre



$i = 4$

AFN



$$\text{expr} = \text{ab}$$

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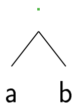
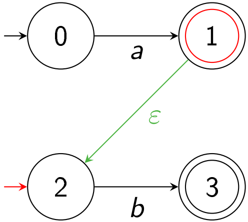
arbre



$i = 4$

AFN



expr = ab

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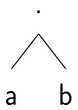
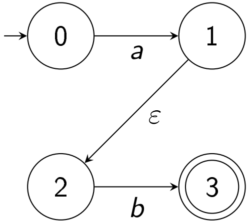
arbre



$i = 4$

AFN



AFN final à 4 états

expr = b*

$$\text{expr} = \text{b*}$$

arbre

$*$

$|$

b

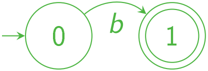
$i = 0$

$$\text{expr} = b*$$

arbre

$i = 2$

AFN

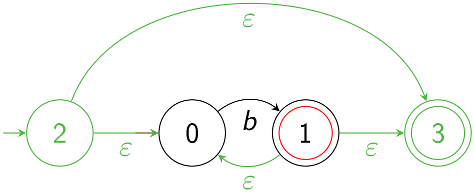


expr = b*

arbre

$i = 4$

AFN



expr = b*

arbre

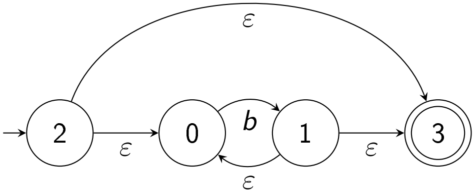
$*$

|

b

$i = 4$

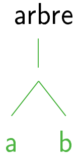
AFN



AFN final à 4 états

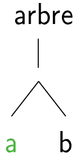
$$\text{expr} = a \mid b$$

$$\text{expr} = \text{a}|\text{b}$$



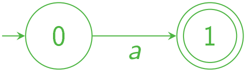
$$i = 0$$

expr = a|b



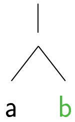
$i = 2$

AFN



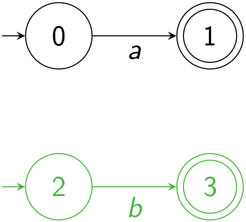
$$\text{expr} = \text{a}|\text{b}$$

arbre



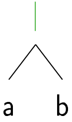
$i = 4$

AFN



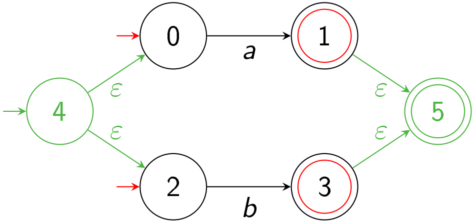
$$\text{expr} = \text{a}|\text{b}$$

arbre



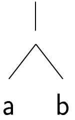
$i = 6$

AFN



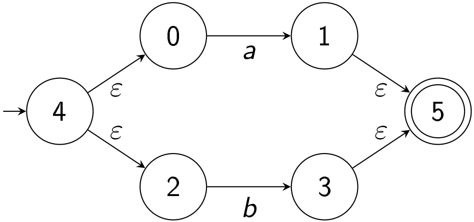
$\text{expr} = a|b$

arbre



$i = 6$

AFN



AFN final à 6 états

expr = a

expr = a

arbre

a

$i = 0$

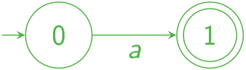
expr = a

arbre

a

$i = 2$

AFN



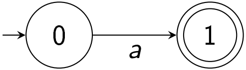
expr = a

arbre

a

$i = 2$

AFN

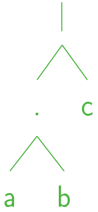


AFN final à 2 états

$$\text{expr} = \text{ab}|\text{c}$$

$$\text{expr} = \text{ab}|\text{c}$$

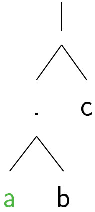
arbre



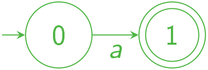
$i = 0$

expr = ab|c

arbre



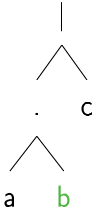
AFN



$i = 2$

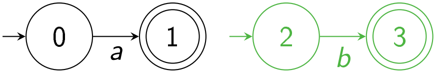
$$\text{expr} = \text{ab|c}$$

arbre



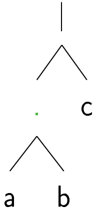
$i = 4$

AFN



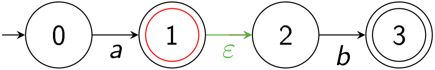
$$\text{expr} = \text{ab|c}$$

arbre



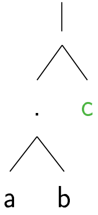
$i = 4$

AFN



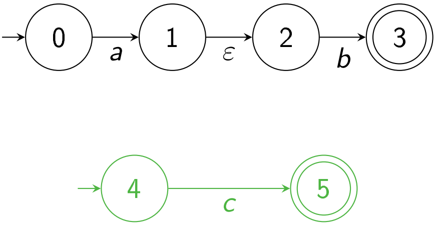
$$\text{expr} = \text{ab}|\text{c}$$

arbre



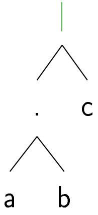
$i = 6$

AFN



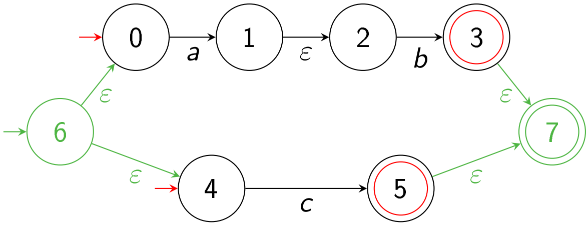
$$\text{expr} = ab|c$$

arbre



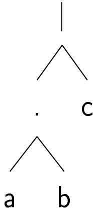
$i = 8$

AFN



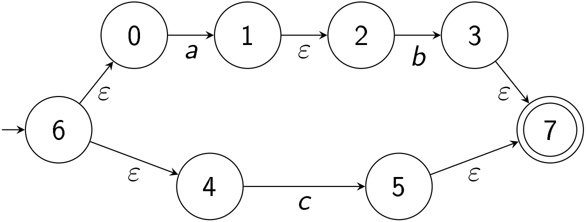
$$\text{expr} = ab|c$$

arbre



$i = 8$

AFN

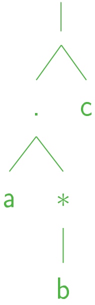


AFN final à 8 états

expr = ab*|c

$$\text{expr} = ab^*|c$$

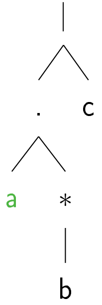
arbre



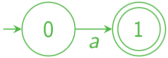
$i = 0$

$$\text{expr} = \text{ab}^*|\text{c}$$

arbre



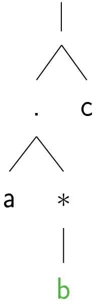
AFN



$i = 2$

expr = ab*|c

arbre

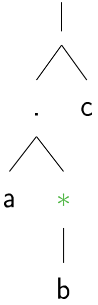


AFN

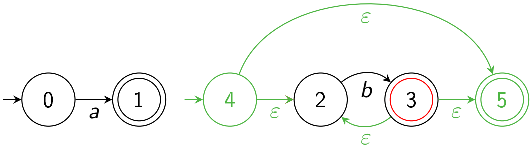
$i = 4$

$$\text{expr} = \text{ab*|c}$$

arbre

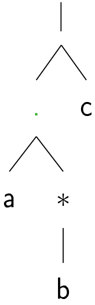


AFN

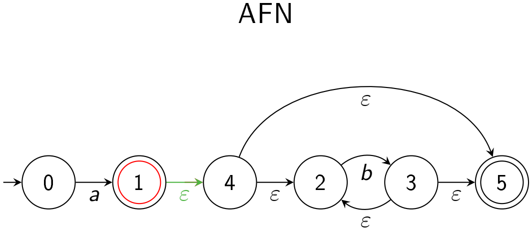


$i = 6$

expr = ab*|c

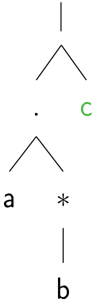


$i = 6$

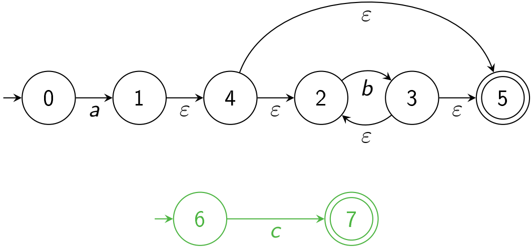


expr = ab*|c

arbre

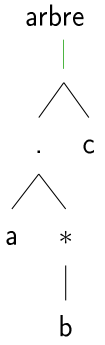


AFN

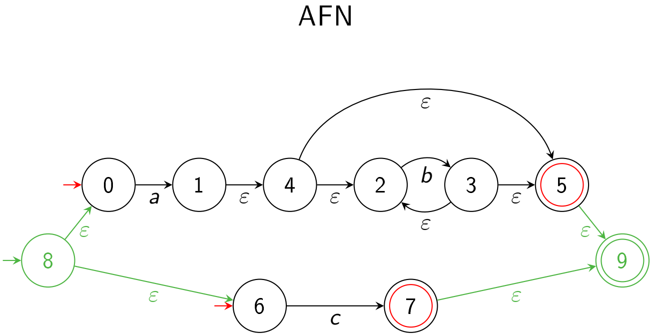


$i = 8$

$\text{expr} = ab^*|c$

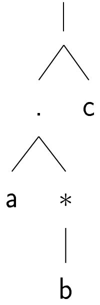


$i = 10$



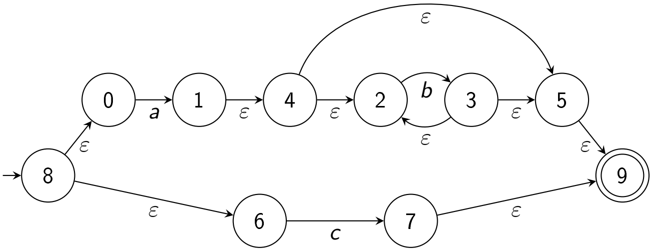
expr = ab*|c

arbre



$i = 10$

AFN

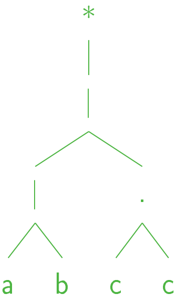


AFN final à 10 états

$$expr = ((a|b)|cc)^*$$

$$\text{expr} = ((a|b)|cc)*$$

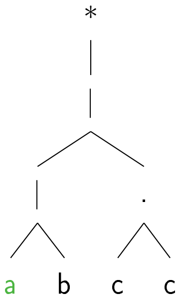
arbre



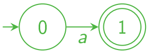
$i = 0$

expr = ((a|b)|cc)*

arbre



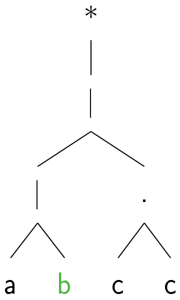
AFN



$i = 2$

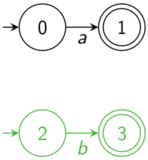
$$\text{expr} = ((a|b)|cc)^*$$

arbre



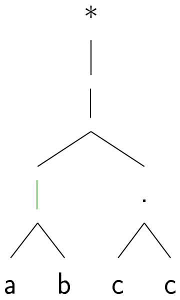
$i = 4$

AFN



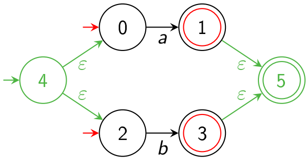
expr = ((a|b)|cc)*

arbre



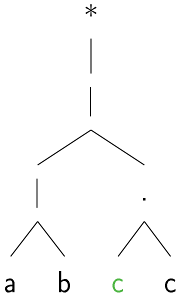
$i = 6$

AFN



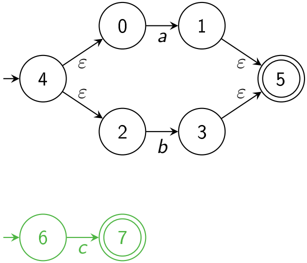
expr = ((a|b)|cc)*

arbre

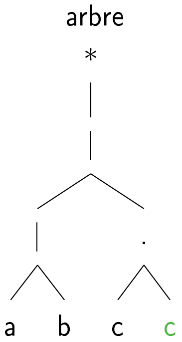


$i = 8$

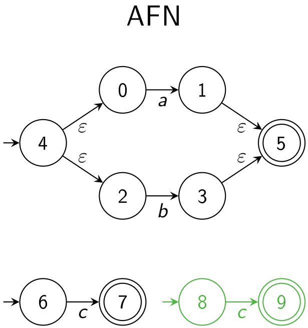
AFN



expr = ((a|b)|cc)*

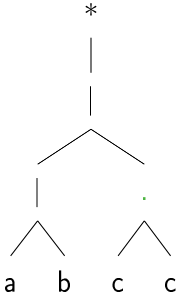


$i = 10$

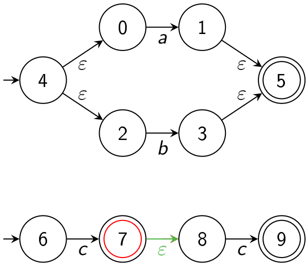


expr = ((a|b)|cc)*

arbre



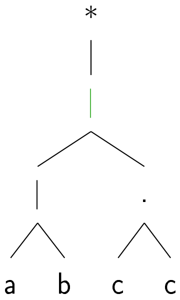
AFN



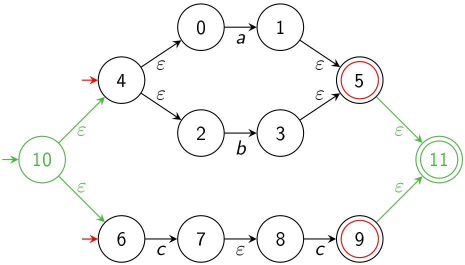
$i = 10$

expr = ((a|b)|cc)*

arbre



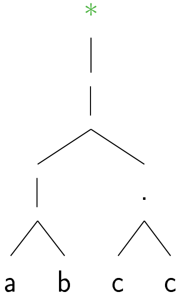
AFN



$i = 12$

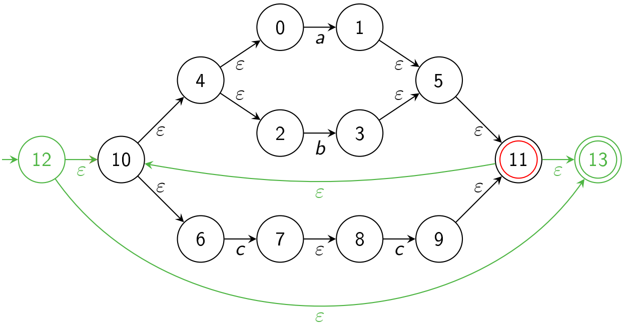
$$\text{expr} = ((a|b)|cc)^*$$

arbre



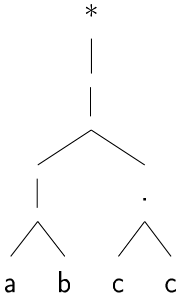
$$i = 14$$

AFN



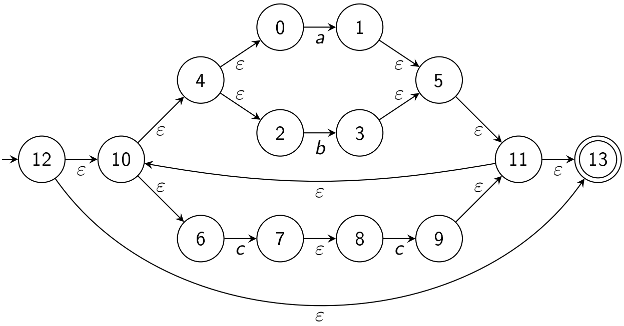
expr = ((a|b)|cc)*

arbre



$i = 14$

AFN

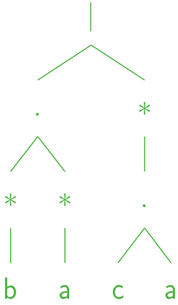


AFN final à 14 états

$$\text{expr} = b^*a^*|(cb)^*$$

expr = b*a*|(cb)*

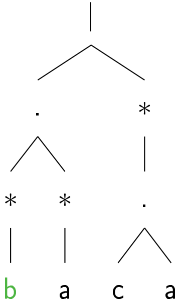
arbre



$i = 0$

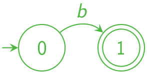
$$\text{expr} = b^*a^*|(cb)^*$$

arbre

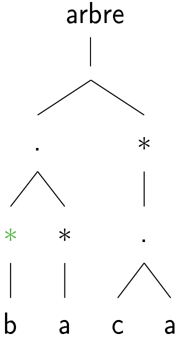


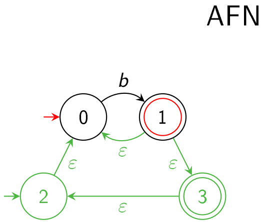
$i = 2$

AFN



expr = b*a*|(cb)*

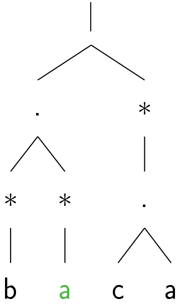




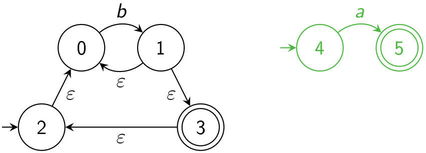
$i = 4$

expr = b*a*|(cb)*

arbre



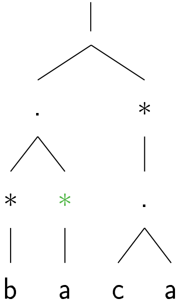
AFN



$i = 6$

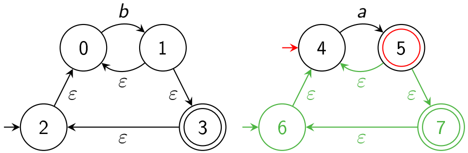
$$\text{expr} = b{*}a{*}|(cb){*}$$

arbre



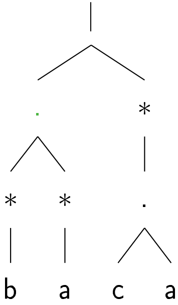
$i = 8$

AFN

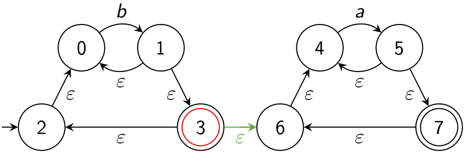


$$\text{expr} = \text{b*a*}|\text{(cb)*}$$

arbre



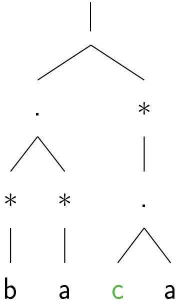
AFN



$i = 8$

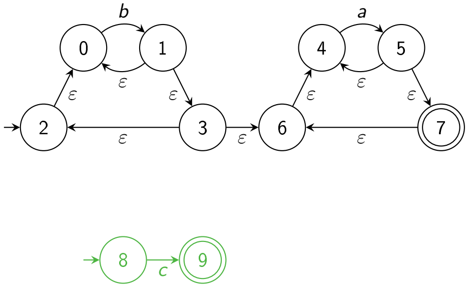
expr = b*a*|(cb)*

arbre

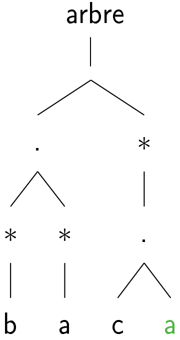


$i = 10$

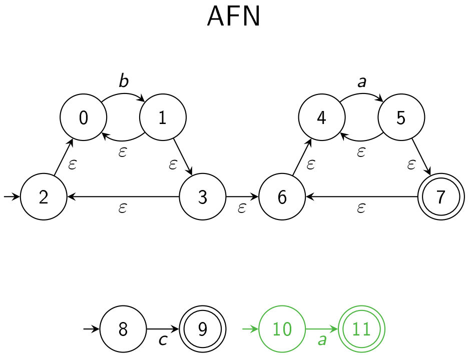
AFN



expr = b*a*|(cb)*

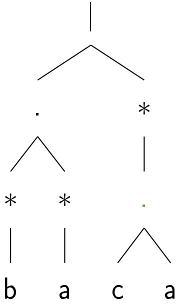


$i = 12$



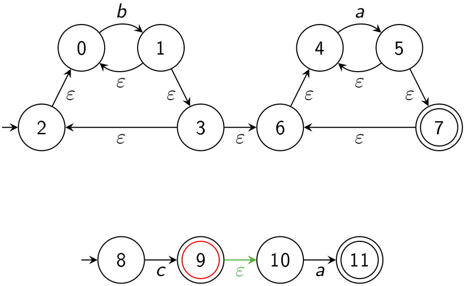
$$\text{expr} = b{*}a{*}|(cb){*}$$

arbre



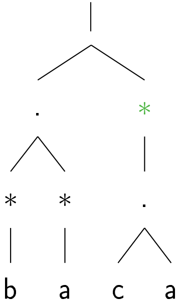
$i = 12$

AFN



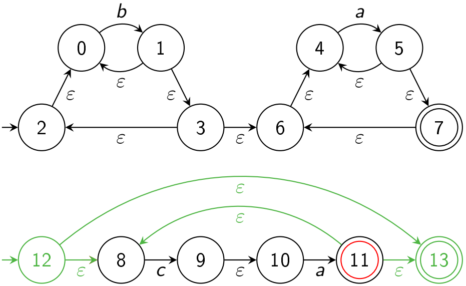
$$\text{expr} = b^*a^*|(cb)^*$$

arbre



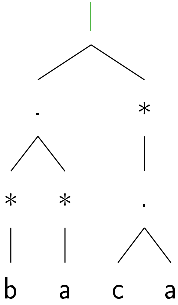
$i = 14$

AFN



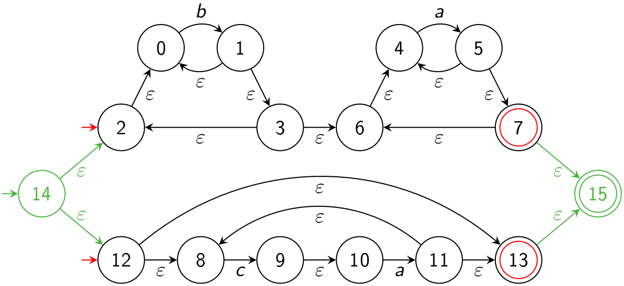
expr = b*a*|(cb)*

arbre



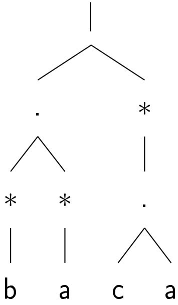
$i = 16$

AFN

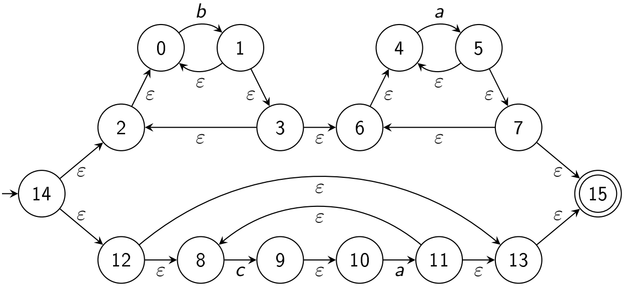


expr = b*a*|(cb)*

arbre



AFN

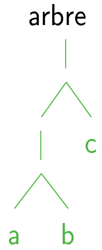


$i = 16$

AFN final à 16 états

$$\text{expr} = (a|b)|c$$

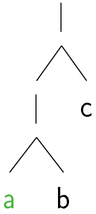
$$\text{expr} = (a|b)|c$$



$i = 0$

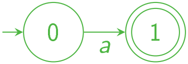
$$\text{expr} = (a|b)|c$$

arbre

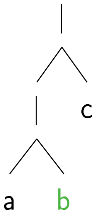


$i = 2$

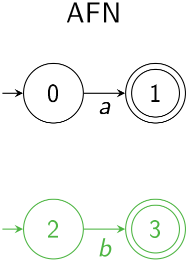
AFN



expr = (a|b)|c

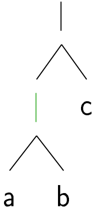


$i = 4$



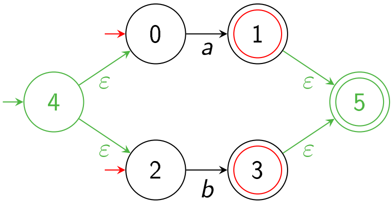
expr = (a|b)|c

arbre



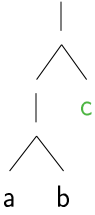
$i = 6$

AFN



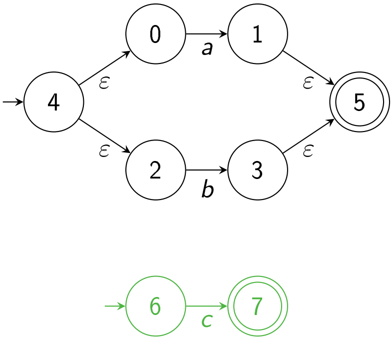
$$\text{expr} = (a|b)|c$$

arbre



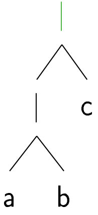
$i = 8$

AFN



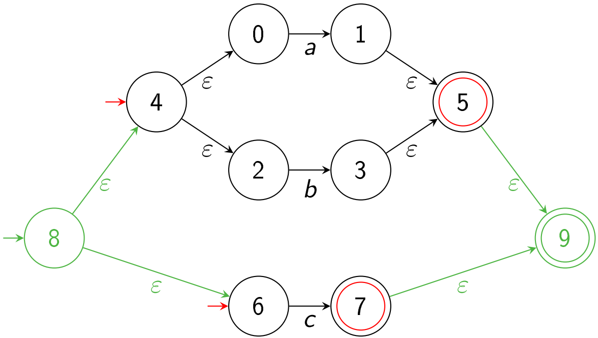
$$\text{expr} = (a|b)|c$$

arbre



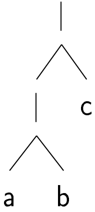
$i = 10$

AFN



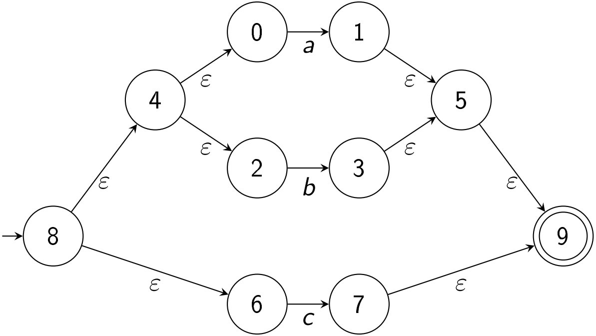
$\text{expr} = (a|b)|c$

arbre



$i = 10$

AFN

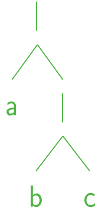


AFN final à 10 états

$$\mathrm{expr} = (a|b)|c$$

$$\text{expr} = (a|b)|c$$

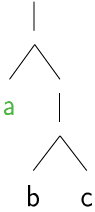
arbre



$i = 0$

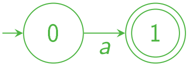
$$\text{expr} = (a|b)|c$$

arbre



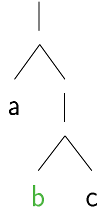
$i = 2$

AFN



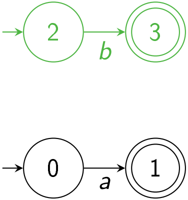
$$\text{expr} = (a|b)|c$$

arbre



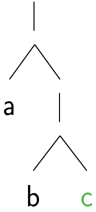
$i = 4$

AFN



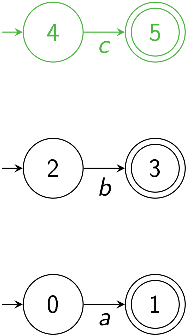
$$\text{expr} = (\text{a}|\text{b})|\text{c}$$

arbre



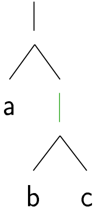
$i = 6$

AFN



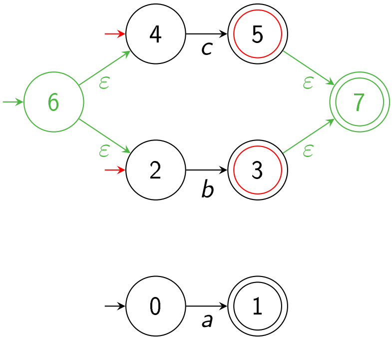
$$\text{expr} = (\text{a}|\text{b})|\text{c}$$

arbre



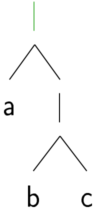
$i = 8$

AFN



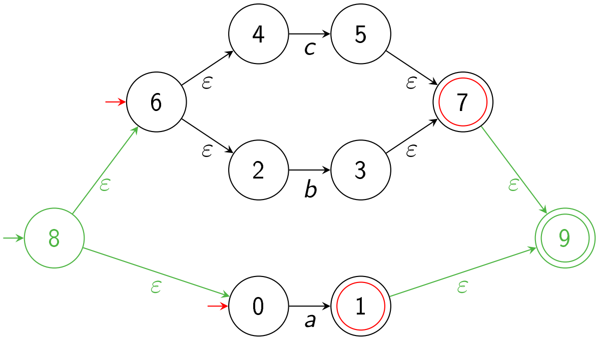
$$\text{expr} = (a|b)|c$$

arbre



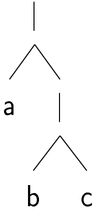
$i = 10$

AFN



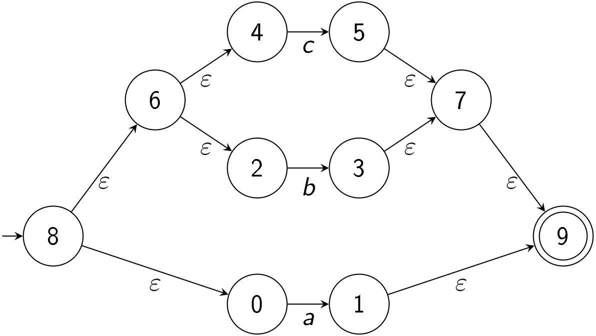
expr = (a|b)|c

arbre



$i = 10$

AFN

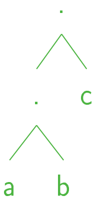


AFN final à 10 états

$$\text{expr} = (ab)c$$

$$\text{expr} = (ab)c$$

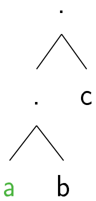
arbre



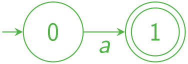
$i = 0$

expr $= (ab)c$

arbre



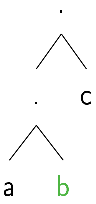
AFN



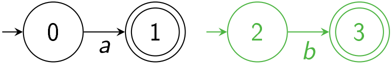
$i = 2$

expr = (ab)c

arbre



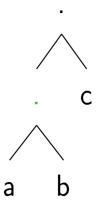
AFN



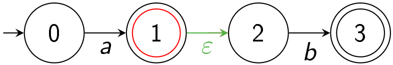
$i = 4$

expr = (ab)c

arbre



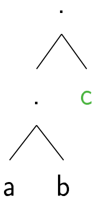
AFN



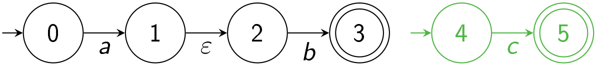
$i = 4$

$$\text{expr} = (\text{ab})\text{c}$$

arbre



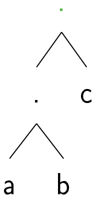
AFN



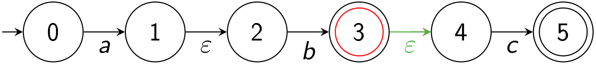
$i = 6$

expr = (ab)c

arbre



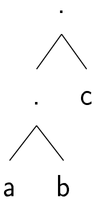
AFN



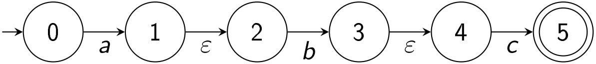
$i = 6$

expr = (ab)c

arbre



AFN



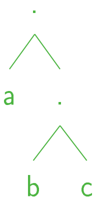
$i = 6$

AFN final à 6 états

$$expr = a(bc)$$

$$\text{expr} = \text{a(bc)}$$

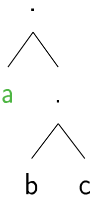
arbre



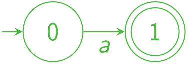
$i = 0$

expr = a(bc)

arbre



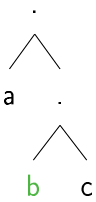
AFN



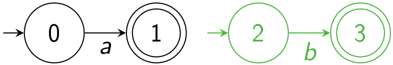
$i = 2$

$$\text{expr} = \text{a(bc)}$$

arbre



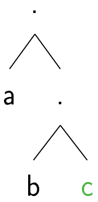
AFN



$i = 4$

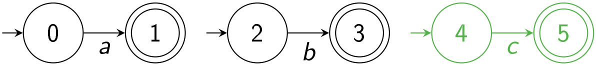
$$\text{expr} = \text{a(bc)}$$

arbre



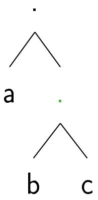
$i = 6$

AFN

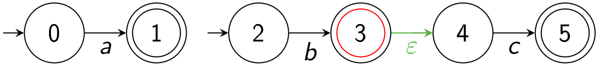


$$\text{expr} = a(bc)$$

arbre



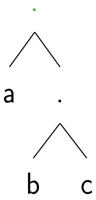
AFN



$i = 6$

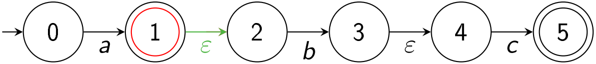
expr = a(bc)

arbre



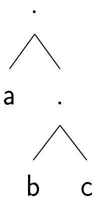
$i = 6$

AFN



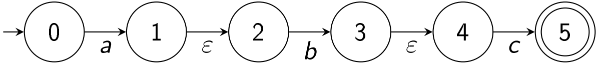
expr = a(bc)

arbre



$i = 6$

AFN



AFN final à 6 états

Les AFN des expressions régulières (ab)c et a(bc) sont strictement identiques. Cela montre que quels que soient a, b et c, la concaténation est associative, il est donc possible de supprimer les parenthèses sans ambiguité.

$$(ab)c = a(bc) = abc$$