

# HA601I - Exercices de révisions

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2022

## Enoncé

Donner l'automate fini non déterministe (AFN) des expressions régulières suivantes grâce à l'algorithme de Thompson.

❶  $ab$

❷  $b^*$

❸  $a|b$

❹  $a$

❺  $ab|c$

❻  $ab^*|c$

❼  $((a|b)|cc)^*$

❽  $b^*a^*(cb)^*$

❾  $(a|b)|c$

❿  $a|(b|c)$

⓫  $(ab)c$

⓬  $a(bc)$

Avec les AFN des expressions régulières  $(ab)c$  et  $a(bc)$ , que peut-on en déduire sur la concaténation ?

$$\text{expr} = ab$$

La première chose à faire c'est toujours de construire l'arbre de dérivation correspondant à l'expression régulière. On se base ensuite sur cet arbre pour créer notre AFD. On initialise  $i = 0$  et on lance l'algorithme sur l'arbre.

$$\text{expr} = ab$$

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arbre



$$i = 0$$

$$\text{expr} = ab$$

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arbre



$$i = 2$$

AFN



$$\text{expr} = ab$$

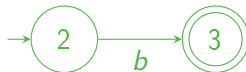
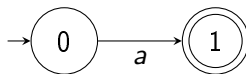
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arbre



$$i = 4$$

AFN



$$\text{expr} = ab$$

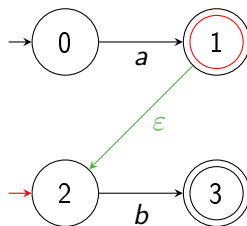
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arbre



$$i = 4$$

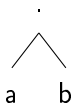
AFN



expr = ab

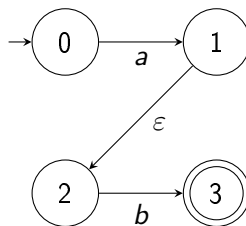
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arbre



$i = 4$

AFN



AFN final à 4 états



$\text{expr} = b^*$

$\text{expr} = b^*$

arbre

\*

|

b

$i = 0$

$\text{expr} = b^*$

arbre

\*

|

*b*

$i = 2$

AFN



$\text{expr} = b^*$

arbre

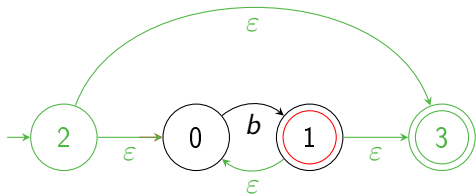
\*

|

b

$i = 4$

AFN



$\text{expr} = b^*$

arbre

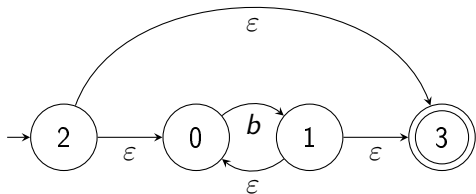
\*

|

b

$i = 4$

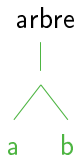
AFN



AFN final à 4 états

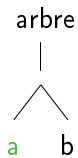
$\text{expr} = a|b$

$\text{expr} = a|b$

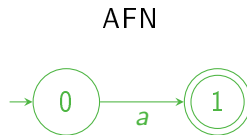


$i = 0$

$\text{expr} = a|b$

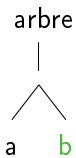


$i = 2$



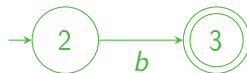
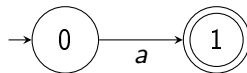


$\text{expr} = a|b$



$i = 4$

AFN



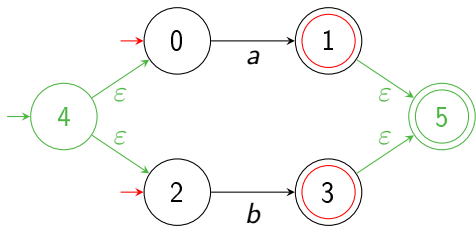
$\text{expr} = a|b$

arbre

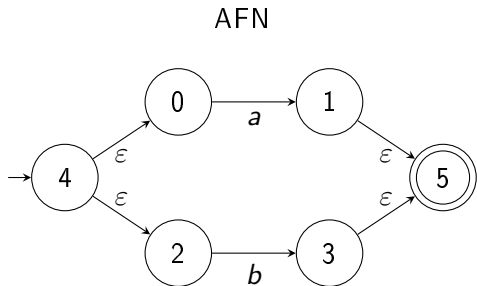
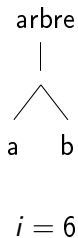


$i = 6$

AFN



$\text{expr} = a|b$



AFN final à 6 états

`expr = a`

$\text{expr} = a$

arbre

*a*

$i = 0$

expr = a

arbre

a

$i = 2$

AFN



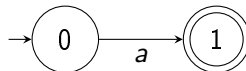
expr = a

arbre

a

$i = 2$

AFN



AFN final à 2 états

$\text{expr} = ab|c$



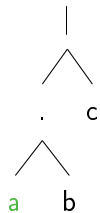
$\text{expr} = ab|c$



$i = 0$

$\text{expr} = ab|c$

arbre



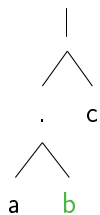
$i = 2$

AFN



$\text{expr} = ab|c$

arbre



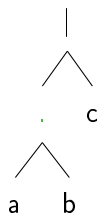
$i = 4$

AFN



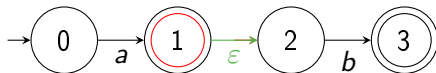
$\text{expr} = ab|c$

arbre



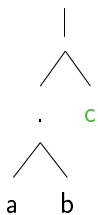
$i = 4$

AFN



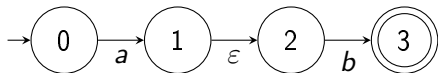
$\text{expr} = ab|c$

arbre



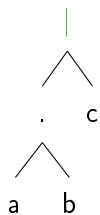
$i = 6$

AFN



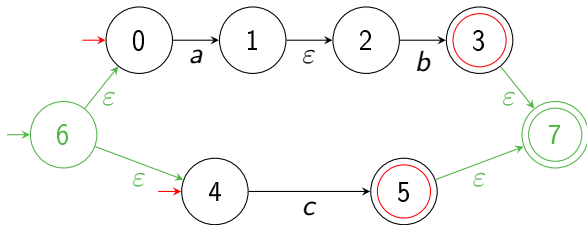
$\text{expr} = ab|c$

arbre

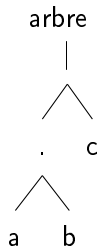


$i = 8$

AFN

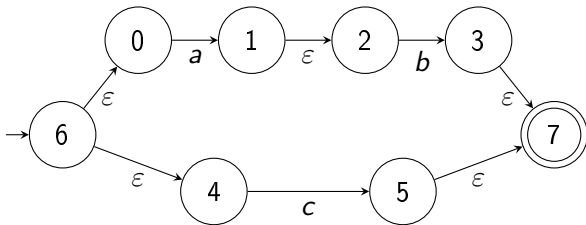


$\text{expr} = ab|c$



$i = 8$

AFN



AFN final à 8 états

$\text{expr} = ab^*|c$



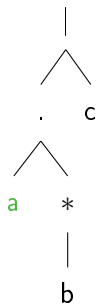
$\text{expr} = ab^*|c$



$i = 0$

$$\text{expr} = ab^*|c$$

arbre



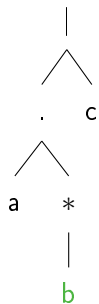
AFN



$i = 2$

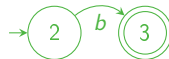
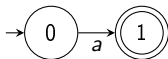
$$\text{expr} = ab^*|c$$

arbre



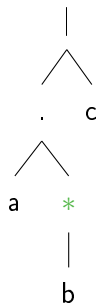
$$i = 4$$

AFN



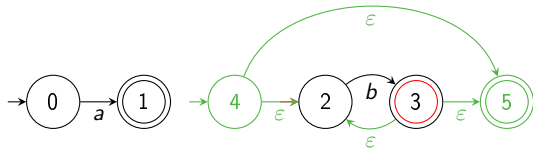
$$\text{expr} = ab^*|c$$

arbre



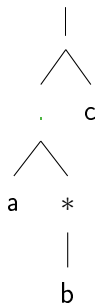
$$i = 6$$

AFN



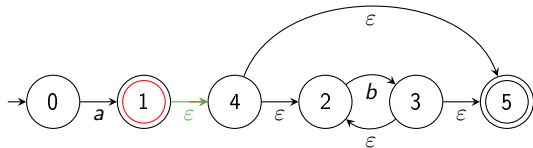
$$\text{expr} = ab^*|c$$

arbre



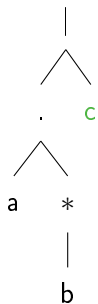
$$i = 6$$

AFN

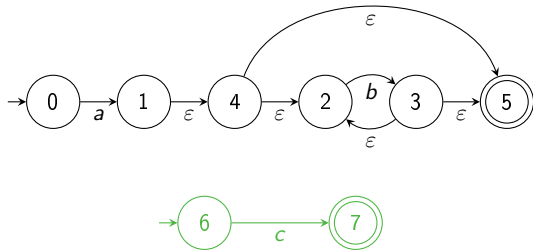


$$\text{expr} = ab^*|c$$

arbre



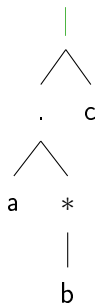
AFN



$$i = 8$$

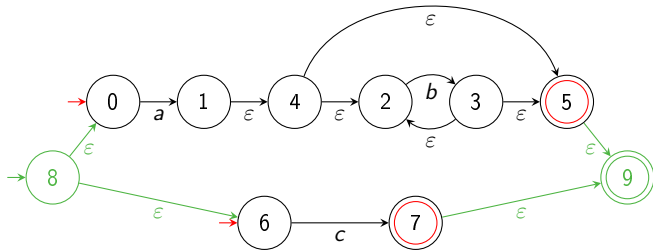
$\text{expr} = ab^*|c$

arbre



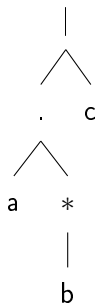
$i = 10$

AFN



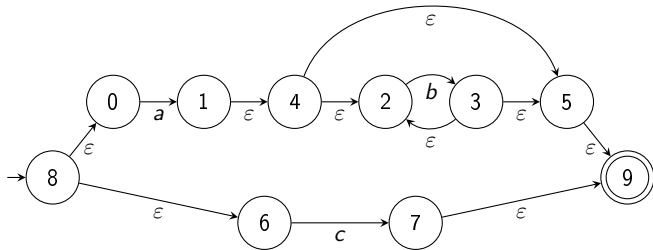
$\text{expr} = ab^*|c$

arbre



$i = 10$

AFN

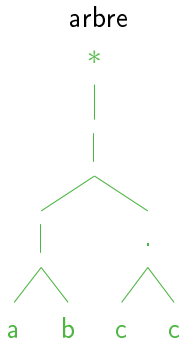


AFN final à 10 états



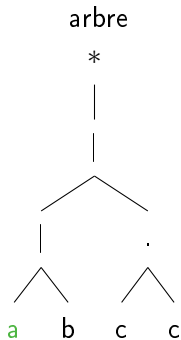
$$\text{expr} = ((a|b)|cc)^*$$

$$\text{expr} = ((a|b)|cc)^*$$



$$i = 0$$

$$\text{expr} = ((a|b)|cc)^*$$

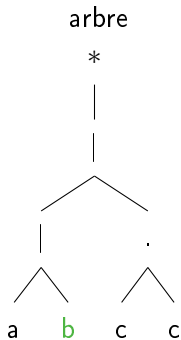


$$i = 2$$

AFN

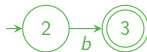
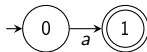


$$\text{expr} = ((a|b)|cc)^*$$



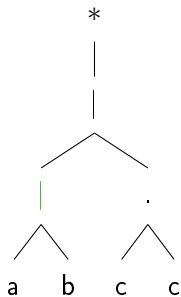
$$i = 4$$

AFN



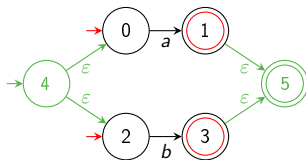
$$\text{expr} = ((a|b)|cc)^*$$

arbre



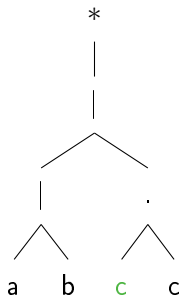
$$i = 6$$

AFN



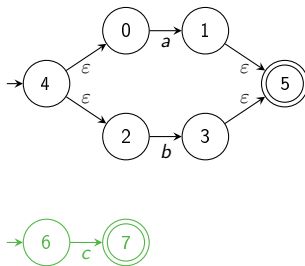
$$\text{expr} = ((a|b)|cc)^*$$

arbre



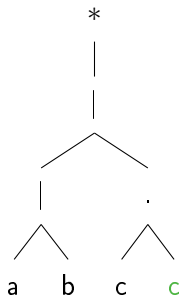
$i = 8$

AFN



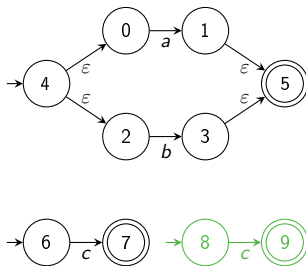
$$\text{expr} = ((a|b)|cc)^*$$

arbre



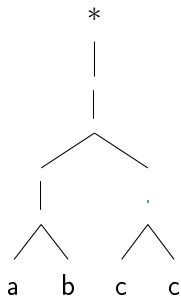
$i = 10$

AFN



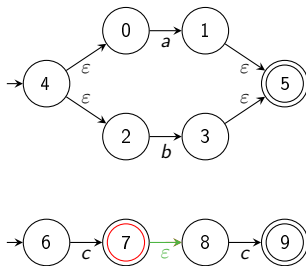
$$\text{expr} = ((a|b)|cc)^*$$

arbre



$i = 10$

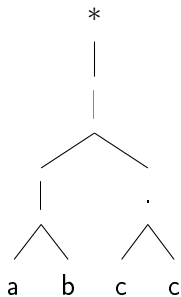
AFN





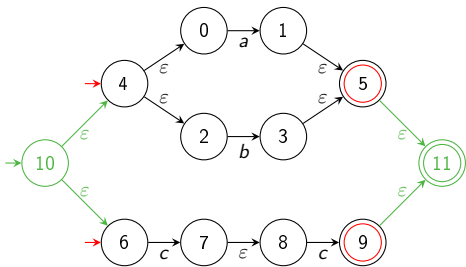
$$\text{expr} = ((a|b)|cc)^*$$

arbre



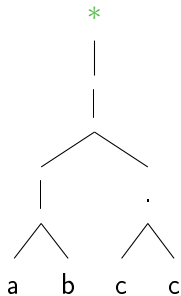
$i = 12$

AFN



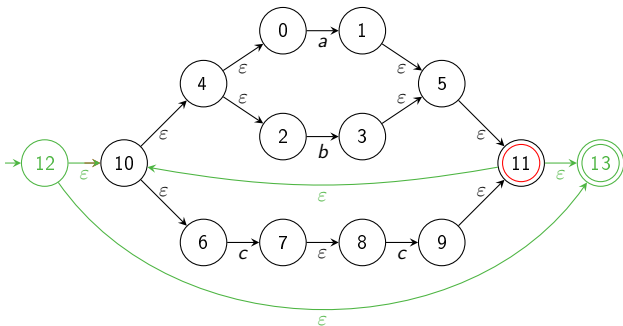
$$\text{expr} = ((a|b)|cc)^*$$

arbre



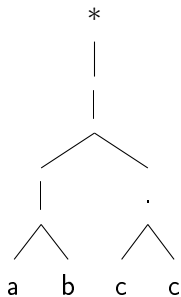
$i = 14$

AFN



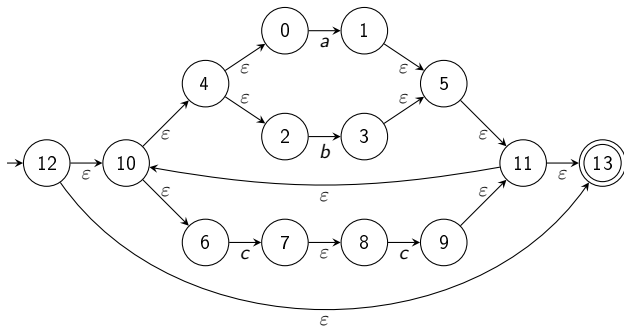
$$\text{expr} = ((a|b)|cc)^*$$

arbre



$i = 14$

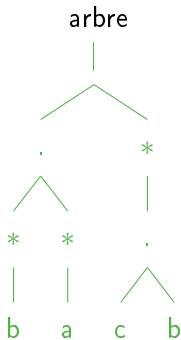
AFN



AFN final à 14 états

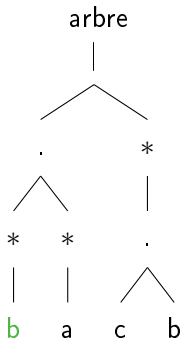
$\text{expr} = b*a*|(cb)^*$

$$\text{expr} = b * a * (cb)^*$$

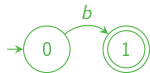


$$i = 0$$

$$\text{expr} = b^*a^*(cb)^*$$

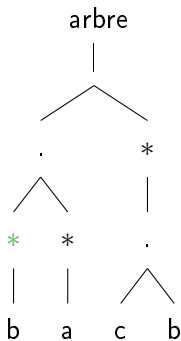


AFN



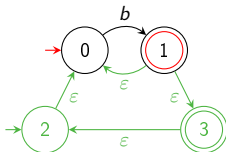
$$i = 2$$

$$\text{expr} = b^*a^*(cb)^*$$

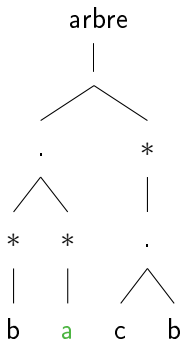


$$i = 4$$

AFN

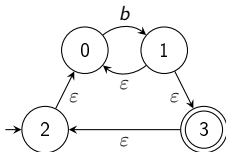


$$\text{expr} = b^*a^*(cb)^*$$



$$i = 6$$

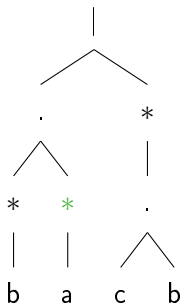
AFN





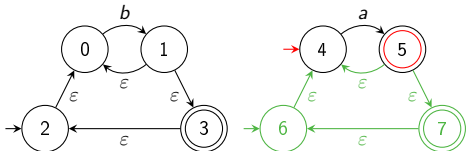
$$\text{expr} = b^*a^*(cb)^*$$

arbre

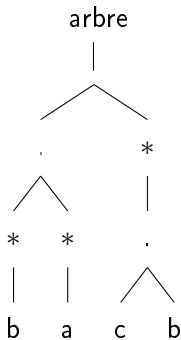


$$i = 8$$

AFN

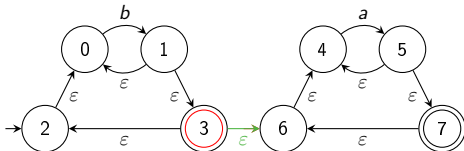


$\text{expr} = b^*a^*(cb)^*$

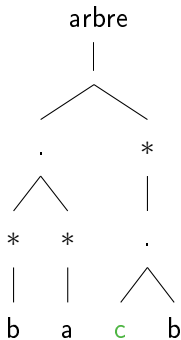


$i = 8$

AFN

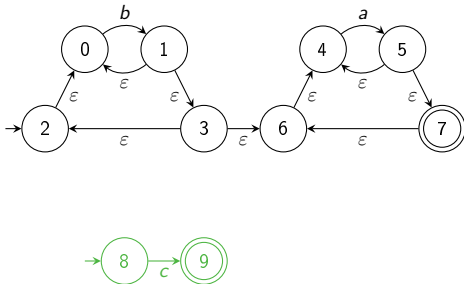


$$\text{expr} = b^*a^*(cb)^*$$

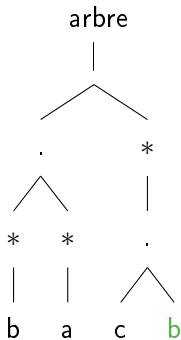


$$i = 10$$

AFN

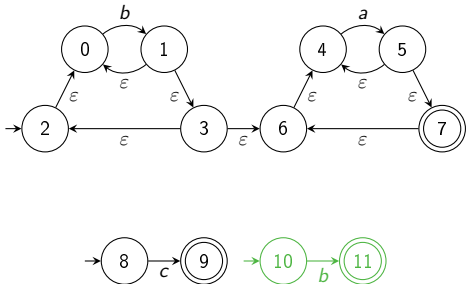


$$\text{expr} = b^*a^*(cb)^*$$

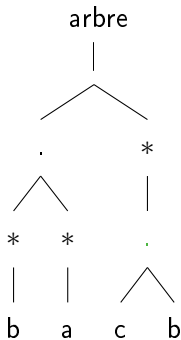


$i = 12$

AFN

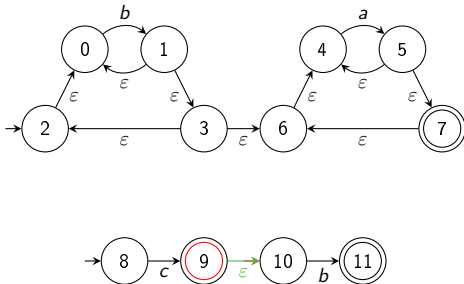


expr = b\*a\*|(cb)\*

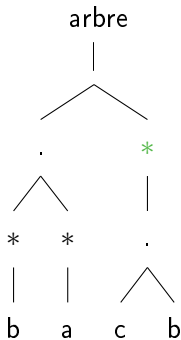


$i = 12$

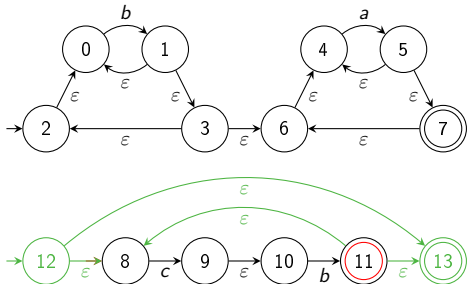
AFN



$\text{expr} = b^*a^*(cb)^*$

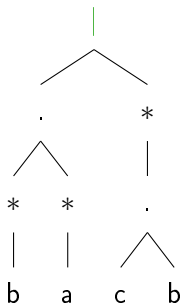


AFN



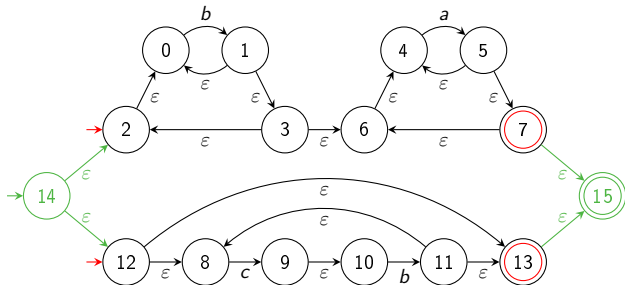
$$\text{expr} = b^*a^*(cb)^*$$

arbre



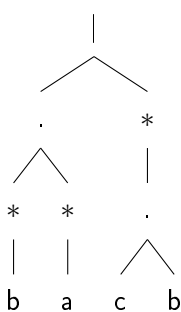
$i = 16$

AFN



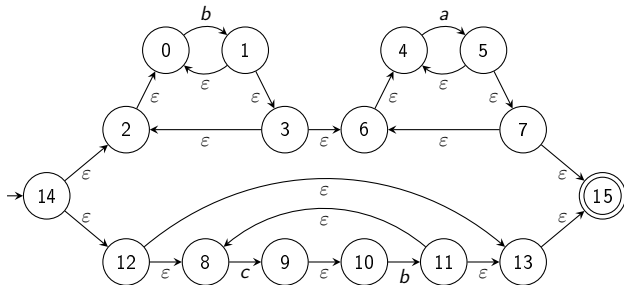
$$\text{expr} = b^*a^*(cb)^*$$

arbre



$i = 16$

AFN



AFN final à 16 états



$$\text{expr} = (a|b)|c$$

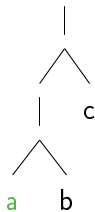
$\text{expr} = (a|b)|c$



$i = 0$

$\text{expr} = (a|b)|c$

arbre

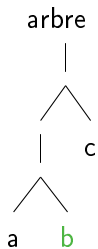


$i = 2$

AFN

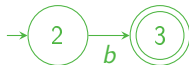
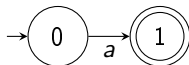


$$\text{expr} = (a|b)|c$$

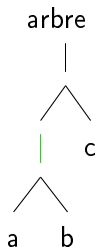


$$i = 4$$

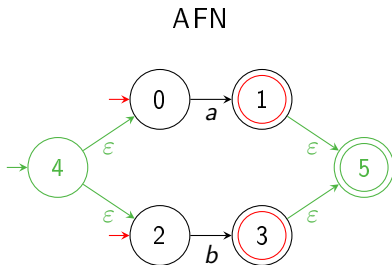
AFN



$\text{expr} = (a|b)|c$

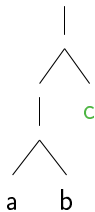


$i = 6$



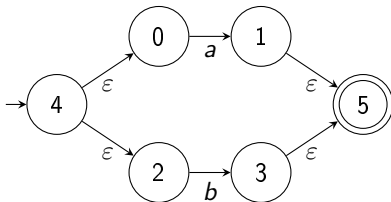
$$\text{expr} = (a|b)|c$$

arbre



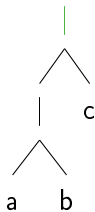
$$i = 8$$

AFN



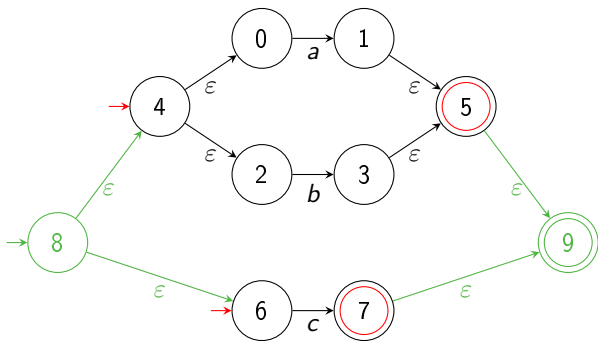
$$\text{expr} = (a|b)|c$$

arbre



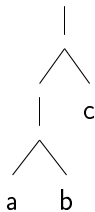
$i = 10$

AFN



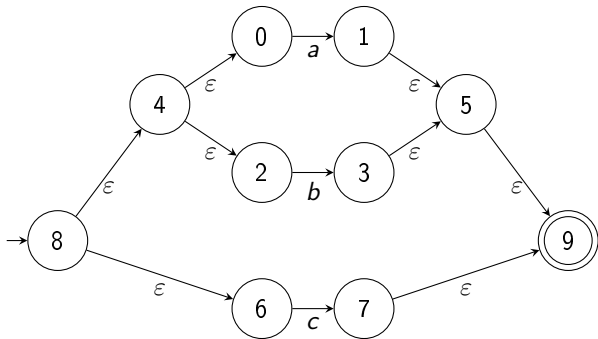
$\text{expr} = (a|b)|c$

arbre



$i = 10$

AFN



AFN final à 10 états



$$\text{expr} = a|(b|c)$$

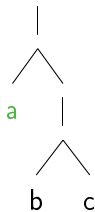
$\text{expr} = a|(b|c)$



$i = 0$

$\text{expr} = a|(b|c)$

arbre



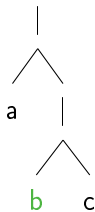
$i = 2$

AFN



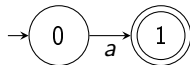
$\text{expr} = a|(b|c)$

arbre



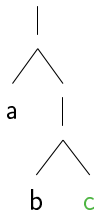
$i = 4$

AFN



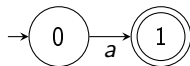
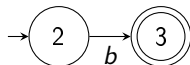
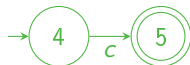
$\text{expr} = a|(b|c)$

arbre



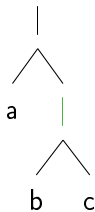
$i = 6$

AFN



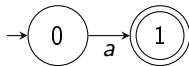
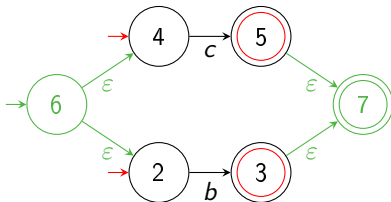
$\text{expr} = a|(b|c)$

arbre



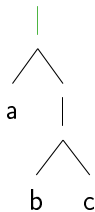
$i = 8$

AFN



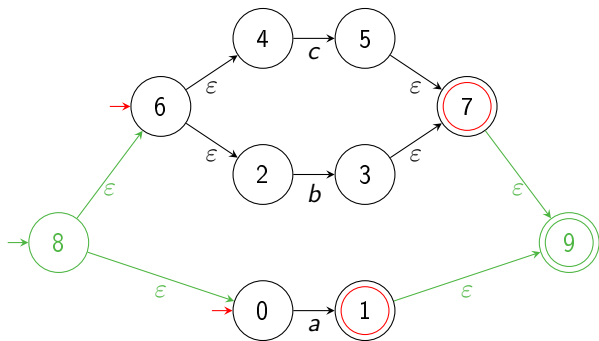
$\text{expr} = a|(b|c)$

arbre



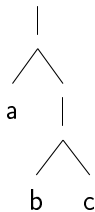
$i = 10$

AFN



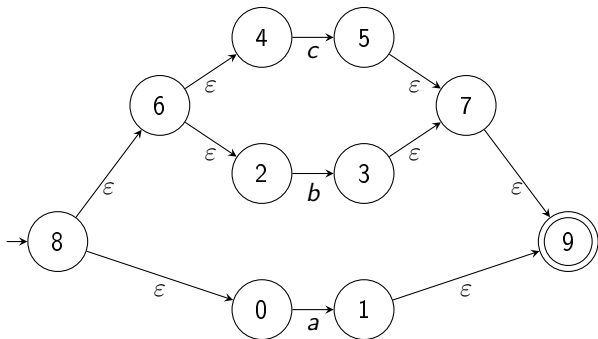
$\text{expr} = a|(b|c)$

arbre



$i = 10$

AFN



AFN final à 10 états



$\text{expr} = (ab)c$

$\text{expr} = (ab)c$

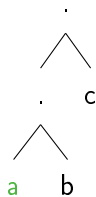
arbre



$i = 0$

$\text{expr} = (ab)c$

arbre



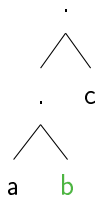
AFN



$i = 2$

$\text{expr} = (ab)c$

arbre



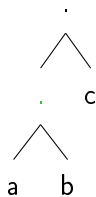
$i = 4$

AFN



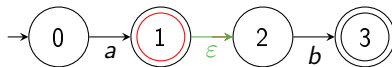
$\text{expr} = (ab)c$

arbre



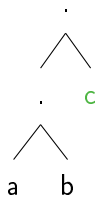
$i = 4$

AFN



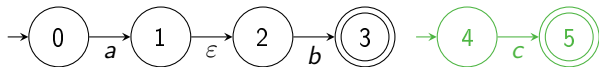
$$\text{expr} = (ab)c$$

arbre



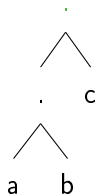
$$i = 6$$

AFN



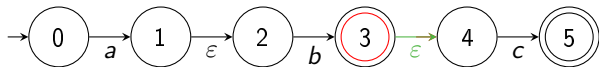
$$\text{expr} = (ab)c$$

arbre



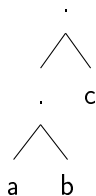
$$i = 6$$

AFN



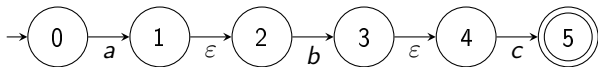
$$\text{expr} = (ab)c$$

arbre



$$i = 6$$

AFN



AFN final à 6 états



$\text{expr} = a(bc)$

$\text{expr} = a(bc)$

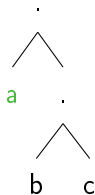
arbre



$i = 0$

$\text{expr} = a(bc)$

arbre



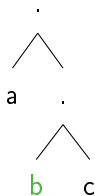
AFN



$i = 2$

$\text{expr} = a(bc)$

arbre



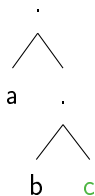
$i = 4$

AFN



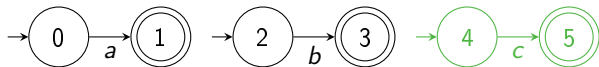
$\text{expr} = a(bc)$

arbre



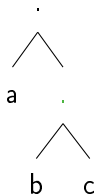
$i = 6$

AFN



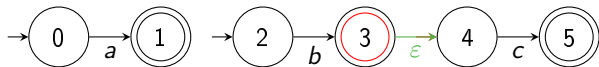
$\text{expr} = a(bc)$

arbre



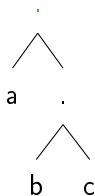
$i = 6$

AFN



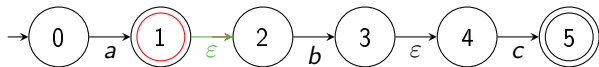
$\text{expr} = a(bc)$

arbre



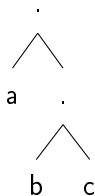
$i = 6$

AFN



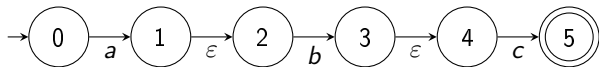
$\text{expr} = a(bc)$

arbre



$i = 6$

AFN



AFN final à 6 états



Les AFN des expressions régulières  $(ab)c$  et  $a(bc)$  sont strictement identiques. Cela montre que quels que soient  $a$ ,  $b$  et  $c$ , la concaténation est associative. Il est donc possible de supprimer les parenthèses sans ambiguïté.

$$(ab)c = a(bc) = abc$$