

HA601I - Exercices de révisions

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2022

Enoncé

Donner l'automate fini déterministe minimale (AFDM) de certaines des expressions régulières de l'exercice précédent.

1 ab

2 b^*

3 $a|b$

4 $ab^*|c$

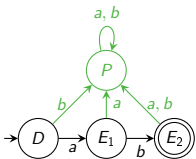
5 $((a|b)|cc)^*$

6 $b^*a^*(cb)^*$

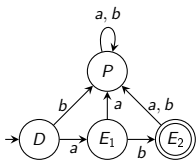
7 abc

8 $a|b|c$

$\text{expr} = ab$



Ajout du puits.

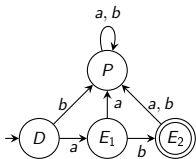


expr = ab

$N = \{D, E_1, P\}$

$F = \{E_2\}$

Première séparation entre états finaux et non finaux : $N = \{D, E_1, P\}$,
 $F = \{E_2\}$.



expr = ab

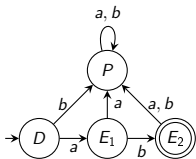
$N = \{D, E_1, P\}$

$F = \{E_2\}$

$N_1 = \{D, P\}$

$N_2 = \{E_1\}$

Séparation de N par b : $N_1 = \{D, P\}$ et $N_2 = \{E_1\}$.



$\text{expr} = ab$

$$F = \{E_2\}$$

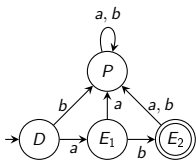
$$N_1 = \{D, P\}$$

$$N_2 = \{E_1\}$$

$$N_{11} = \{D\}$$

$$N_{12} = \{P\}$$

Séparation de N_1 par a : $N_{11} = \{D\}$ et $N_{12} = \{P\}$.



$\text{expr} = ab$

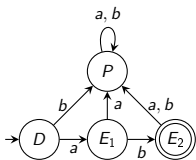
$$F = \{E_2\}$$

$$N_2 = \{E_1\}$$

$$N_{11} = \{D\}$$

$$N_{12} = \{P\}$$

On supprime le puits.



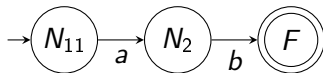
expr = ab

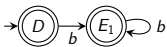
$$F = \{E_2\}$$

$$N_2 = \{E_1\}$$

$$N_{11} = \{D\}$$

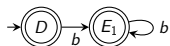
Et voici l'AFDM final :





$\text{expr} = b^*$

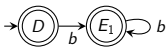
Pas besoin de puits l'AFD est déjà complet.



$\text{expr} = b^*$

$F = \{D, E_1\}$

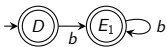
Première séparation entre état finaux et non finaux : $F = \{D, E_1\}$.



$$\text{expr} = b^*$$

$$F = \{D, E_1\}$$

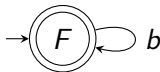
Impossible de séparer F par b , l'automate est donc minimisé.



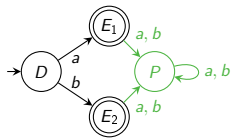
$\text{expr} = b^*$

$F = \{D, E_1\}$

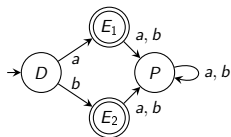
Et voici l'AFDM final :



$\text{expr} = a|b$



Ajout du puits.

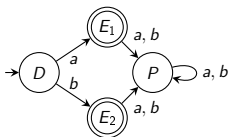


$\text{expr} = a|b$

$N = \{D, P\}$

$F = \{E_1, E_2\}$

Première séparation entre états finaux et non finaux : $N = \{D, P\}$,
 $F = \{E_1, E_2\}$.



$\text{expr} = a|b$

$N = \{D, P\}$

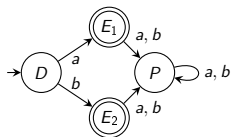
$N = \{D\}$

$F = \{E_1, E_2\}$

Impossible de séparer N par a ou b .

Il en va de même pour F .

On supprime juste P et l'automate est minimisé.

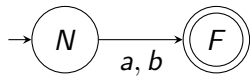


$\text{expr} = a|b$

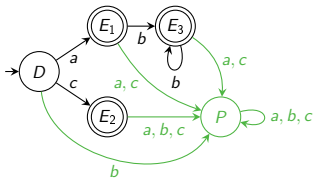
$N = \{D\}$

$F = \{E_1, E_2\}$

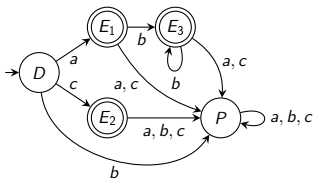
Et voici l'AFDM final :



$\text{expr} = ab^*|c$



Ajout du puits.

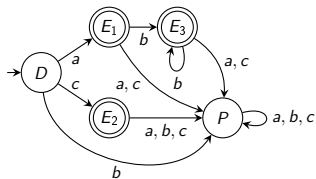


$\text{expr} = ab^*|c$

$N = \{D, P\}$

$F = \{E_1, E_2, E_3\}$

Première séparation entre états finaux et non finaux : $N = \{D, P\}$,
 $F = \{E_1, E_2, E_3\}$.



$\text{expr} = ab^*|c$

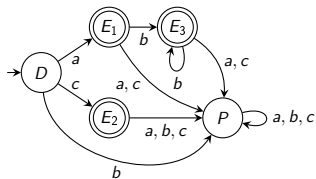
$N = \{D, P\}$

$F = \{E_1, E_2, E_3\}$

$F_1 = \{E_1, E_3\}$

$F_2 = \{E_2\}$

Séparation de F par b : $F_1 = \{E_1, E_3\}$ et $F_2 = \{E_2\}$.



$\text{expr} = ab^*|c$

$N = \{D, P\}$

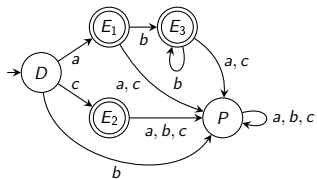
$F_1 = \{E_1, E_3\}$

$F_2 = \{E_2\}$

$N_1 = \{D\}$

$N_2 = \{P\}$

Séparation de N par c : $N_1 = \{D\}$ et $N_2 = \{P\}$.



$\text{expr} = ab^*|c$

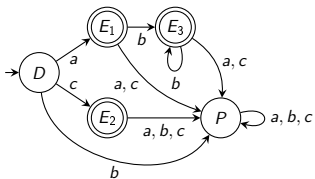
$F_1 = \{E_1, E_3\}$

$F_2 = \{E_2\}$

$N_1 = \{D\}$

$N_2 = \{P\}$

On supprime le puits.



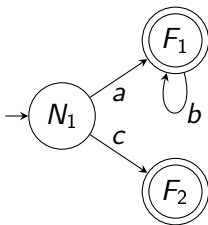
$\text{expr} = ab^*|c$

$F_1 = \{E_1, E_3\}$

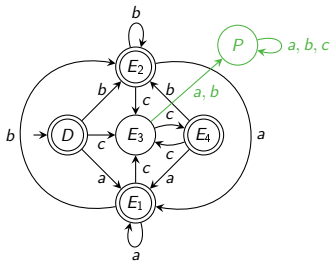
$F_2 = \{E_2\}$

$N_1 = \{D\}$

Et voici l'AFDM final :



$$\text{expr} = ((a|b)|cc)^*$$

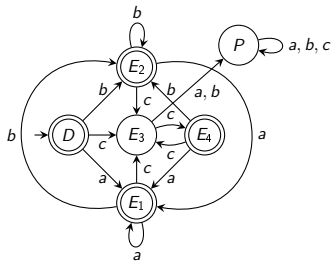


Ajout du puits.

$\text{expr} = ((a|b)|cc)^*$

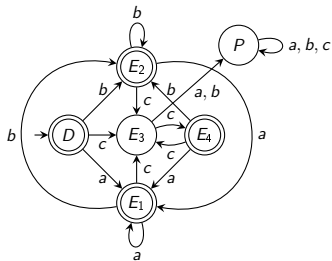
$N = \{E_3, P\}$

$F = \{D, E_1, E_2, E_4\}$



Première séparation entre états finaux et non finaux : $N = \{E_3, P\}$,
 $F = \{D, E_1, E_2, E_4\}$.

$\text{expr} = ((a|b)|cc)^*$



$N = \{E_3, P\}$

$F = \{D, E_1, E_2, E_4\}$

$N_1 = \{E_3\}$

$N_2 = \{P\}$

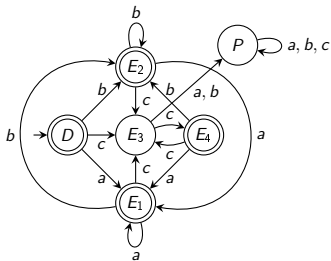
Séparation de N par c : $N_1 = \{E_3\}$ et $N_2 = \{P\}$.

$\text{expr} = ((a|b)|cc)^*$

$F = \{D, E_1, E_2, E_4\}$

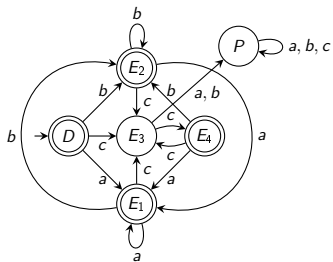
$N_1 = \{E_3\}$

$N_2 = \{P\}$



Impossible de séparer F , tous ses éléments ramènent vers F par a et b et vers N_1 par c .

$\text{expr} = ((a|b)|cc)^*$



$F = \{D, E_1, E_2, E_4\}$

$N_1 = \{E_3\}$

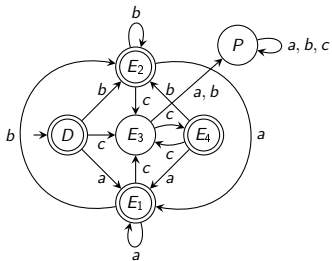
$N_2 = \{P\}$

On supprime le puits.

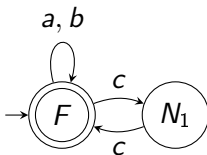
$\text{expr} = ((a|b)|cc)^*$

$F = \{D, E_1, E_2, E_4\}$

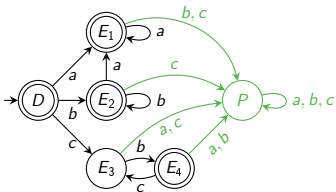
$N_1 = \{E_3\}$



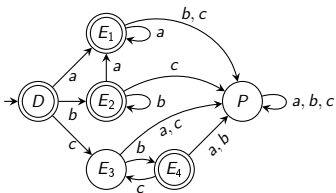
Et voici l'AFDM final :



$$\text{expr} = b^*a^*(cb)^*$$



Ajout du puits.

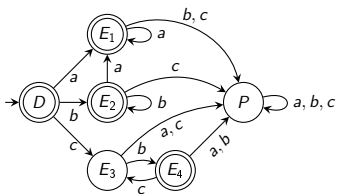


$\text{expr} = b^*a^*(cb)^*$

$N = \{E_3, P\}$

$F = \{D, E_1, E_2, E_4\}$

Première séparation entre états finaux et non finaux : $N = \{E_3, P\}$,
 $F = \{D, E_1, E_2, E_4\}$.



$\text{expr} = b^*a^*(cb)^*$

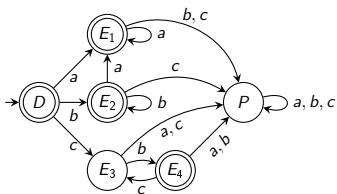
$N = \{E_3, P\}$

$F = \{D, E_1, E_2, E_4\}$

$N_1 = \{E_3\}$

$N_2 = \{P\}$

Séparation de N par b : $N_1 = \{E_3\}$ et $N_2 = \{P\}$.



$\text{expr} = b^*a^*(cb)^*$

$F = \{D, E_1, E_2, E_4\}$

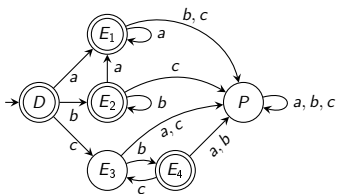
$N_1 = \{E_3\}$

$N_2 = \{P\}$

$F_1 = \{D, E_1, E_2\}$

$F_2 = \{E_4\}$

Séparation de F par a : $F_1 = \{D, E_1, E_2\}$ et $F_2 = \{E_4\}$.



$\text{expr} = b^*a^*(cb)^*$

$N_1 = \{E_3\}$

$N_2 = \{P\}$

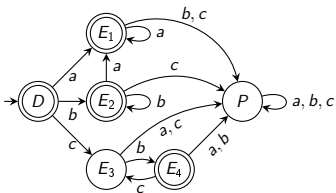
$F_1 = \{D, E_1, E_2\}$

$F_2 = \{E_4\}$

$F_{11} = \{D, E_2\}$

$F_{12} = \{E_1\}$

Séparation de F_1 par b : $F_{11} = \{D, E_2\}$ et $F_{12} = \{E_1\}$.



$\text{expr} = b^*a^*(cb)^*$

$N_1 = \{E_3\}$

$N_2 = \{P\}$

$F_2 = \{E_4\}$

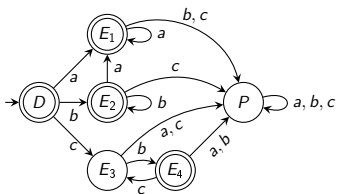
$F_{11} = \{D, E_2\}$

$F_{12} = \{E_1\}$

$F_{111} = \{D\}$

$F_{112} = \{E_2\}$

Séparation de F_{11} par c : $F_{111} = \{D\}$ et $F_{112} = \{E_2\}$.



$\text{expr} = b^*a^*(cb)^*$

$N_1 = \{E_3\}$

$N_2 = \{P\}$

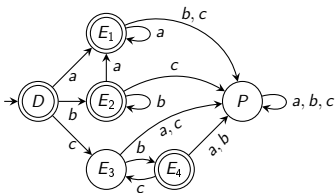
$F_2 = \{E_4\}$

$F_{12} = \{E_1\}$

$F_{111} = \{D\}$

$F_{112} = \{E_2\}$

On supprime le puits.



$\text{expr} = b^*a^*(cb)^*$

$N_1 = \{E_3\}$

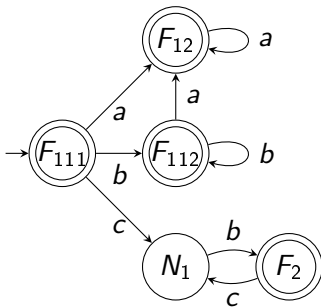
$F_2 = \{E_4\}$

$F_{12} = \{E_1\}$

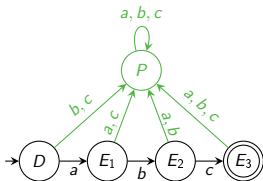
$F_{111} = \{D\}$

$F_{112} = \{E_2\}$

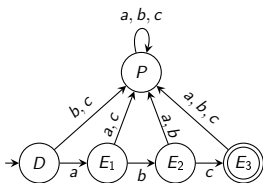
Et voici l'AFDM final :



$\text{expr} = abc$



Ajout du puits.

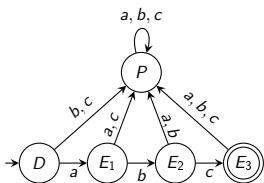


expr = abc

$N = \{D, E_1, E_2, P\}$

$F = \{E_3\}$

Première séparation entre états finaux et non finaux : $N = \{D, E_1, E_2, P\}$,
 $F = \{E_3\}$.



expr = abc

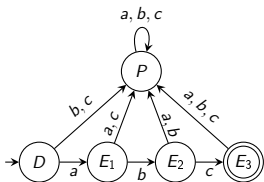
$N = \{D, E_1, E_2, P\}$

$F = \{E_3\}$

$N_1 = \{E_2\}$

$N_2 = \{D, E_1, P\}$

Séparation de N par c : $N_1 = \{E_2\}$ et $N_2 = \{D, E_1, P\}$.



expr = abc

$$F = \{E_3\}$$

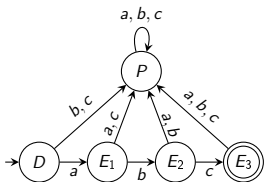
$$N_1 = \{E_2\}$$

$$N_2 = \{D, E_1, P\}$$

$$N_{21} = \{E_1\}$$

$$N_{22} = \{D, P\}$$

Séparation de N_2 par b : $N_{21} = \{E_1\}$ et $N_{22} = \{D, P\}$.



expr = abc

$$F = \{E_3\}$$

$$N_1 = \{E_2\}$$

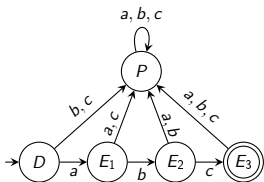
$$N_{21} = \{E_1\}$$

$$N_{22} = \{D, P\}$$

$$N_{221} = \{D\}$$

$$N_{222} = \{P\}$$

Séparation de N_{22} par a : $N_{221} = \{D\}$ et $N_{222} = \{P\}$.



$\text{expr} = abc$

$$F = \{E_3\}$$

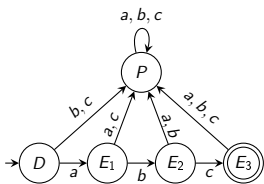
$$N_1 = \{E_2\}$$

$$N_{21} = \{E_1\}$$

$$N_{221} = \{D\}$$

$$N_{222} = \{P\}$$

On supprime le puits.



expr = abc

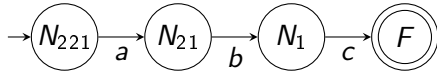
$$F = \{E_3\}$$

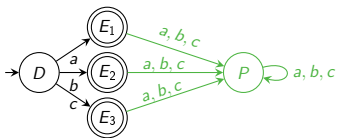
$$N_1 = \{E_2\}$$

$$N_{21} = \{E_1\}$$

$$N_{221} = \{D\}$$

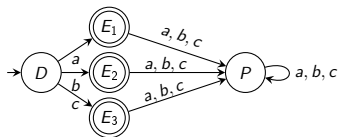
Et voici l'AFDM final :





$\text{expr} = a|b|c$

Ajout du puits.

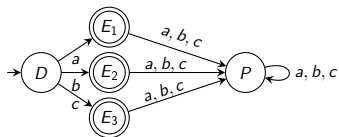


expr = $a|b|c$

$N = \{D, P\}$

$F = \{E_1, E_2, E_3\}$

Première séparation entre états finaux et non finaux : $N = \{D, E_1, E_2, P\}$,
 $F = \{E_3\}$.



expr = $a|b|c$

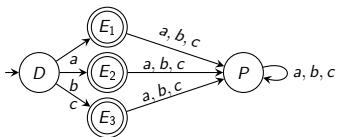
$N = \{D, P\}$

$F = \{E_1, E_2, E_3\}$

$N_1 = \{D\}$

$N_2 = \{P\}$

Séparation de N par a : $N_1 = \{D\}$ et $N_2 = \{P\}$.



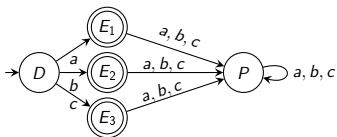
$\text{expr} = a|b|c$

$F = \{E_1, E_2, E_3\}$

$N_1 = \{D\}$

$N_2 = \{P\}$

Impossible de séparer F car toutes les transitions amènent vers le puits.



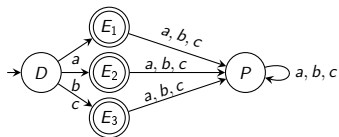
$\text{expr} = a|b|c$

$F = \{E_1, E_2, E_3\}$

$N_1 = \{D\}$

$N_2 = \{P\}$

On supprime le puits.



$\text{expr} = a|b|c$

$F = \{E_1, E_2, E_3\}$

$N_1 = \{D\}$

Et voici l'AFDM final :

