# HAI601I - Exercices de révisions

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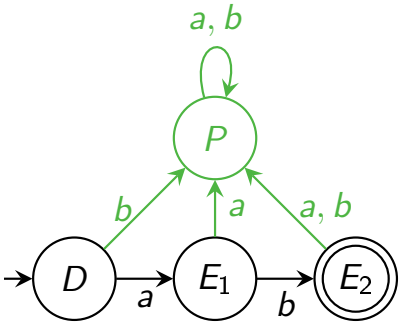
2022

# Construction d'un AFD à partir d'un AFN

## Enoncé

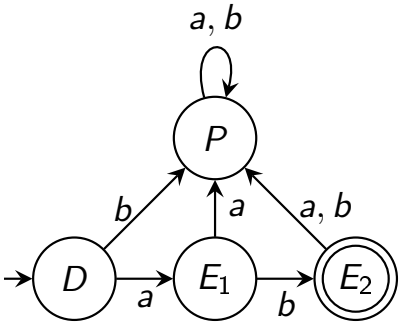
Donner l'automate fini déterministe minimale (AFDM) de certaines des expressions régulières de l'exercice précédent.

1. ab
2. b*
3. a|b
4. ab*|c
5. ((a|b)|cc)*
6. b*a*|(cb)*
7. abc
8. a|b|c




expr = ab

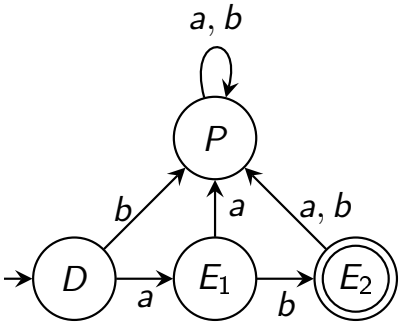
Ajout du puits.




expr = ab

$N = \{D, E_1, P\}$
$F = \{E_2\}$

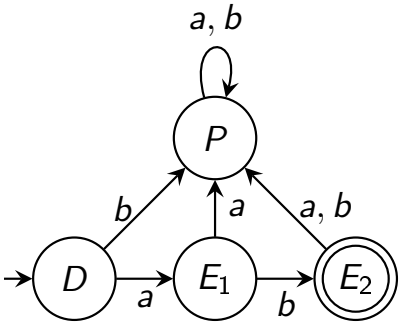
Première séparation entre états finaux et non finaux : $N = \{D, E_1, P\}$, $F = \{E_2\}$.




expr = ab

$N = \{D, E_1, P\}$
$F = \{E_2\}$
$N_1 = \{D, P\}$
$N_2 = \{E_1\}$

Séparation de $N$ par $b$ : $N_1 = \{D, P\}$ et $N_2 = \{E_1\}$.




expr = ab

$F = \{E_2\}$

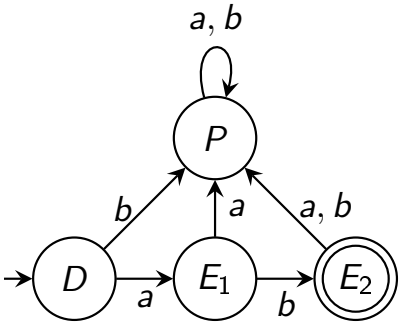
$N_1 = \{D, P\}$

$N_2 = \{E_1\}$

$N_{11} = \{D\}$

$N_{12} = \{P\}$

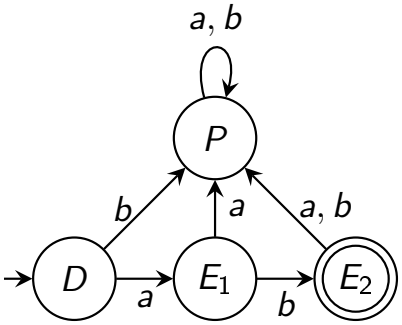
Séparation de $N_1$ par $a$ : $N_{11} = \{D\}$ et $N_{12} = \{P\}$.




expr = ab

$F = \{E_2\}$
$N_2 = \{E_1\}$
$N_{11} = \{D\}$
$N_{12} = \{P\}$

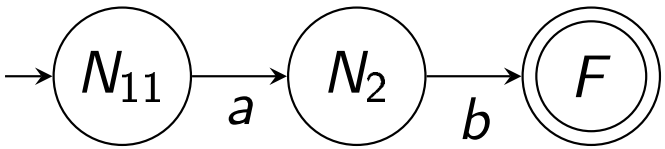
On supprime le puits.

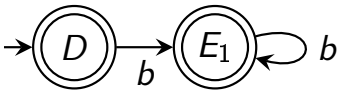



expr = ab

$F = \{E_2\}$
$N_2 = \{E_1\}$
$N_{11} = \{D\}$

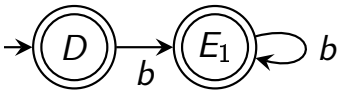
Et voici l'AFDM final :






expr = b*

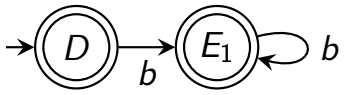
Pas besoin de puits l'AFD est déja complet.




expr = b*

$F = \{D, E_1\}$

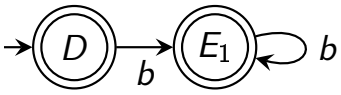
Première séparation entre état finaux et non finaux : $F = \{D, E_1\}$.




expr = b*

$F = \{D, E_1\}$

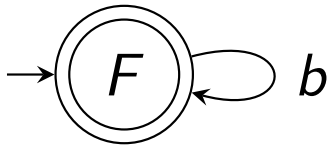
Impossible de séparer $F$ par $b$, l'automate est donc minimisé.

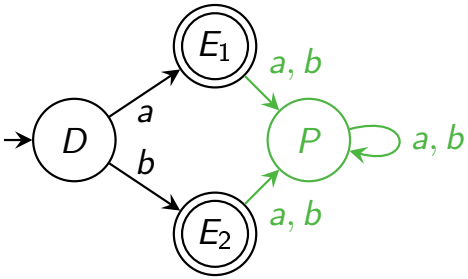



expr = b*

$F = \{D, E_1\}$

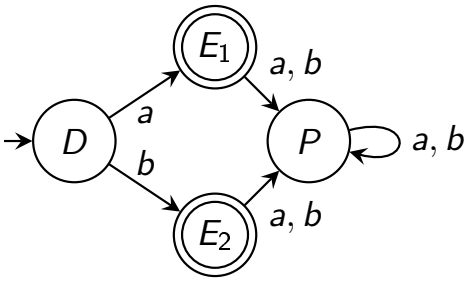
Et voici l'AFDM final :






expr = a|b

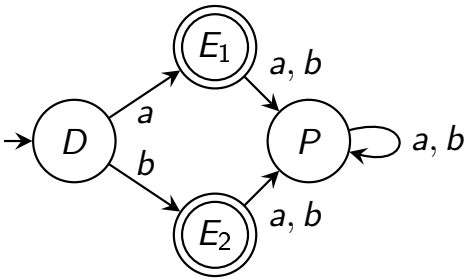
Ajout du puits.




expr = a|b

$N = \{D, P\}$
$F = \{E_1, E_2\}$

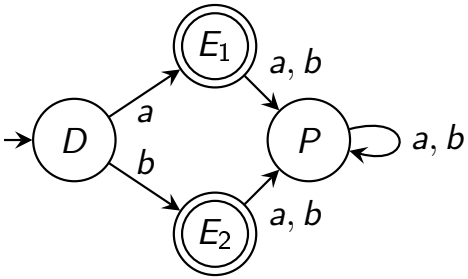
Première séparation entre états finaux et non finaux : $N = \{D, P\}$, $F = \{E_1, E_2\}$.




expr = a|b

$N = \{D, P\}$
$N = \{D\}$
$F = \{E_1, E_2\}$

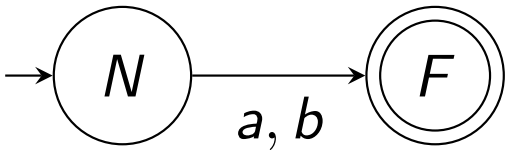
Impossible de séparer $N$ par $a$ ou $b$.
Il en va de même pour $F$.
On supprime juste $P$ et l'automate est minimisé.

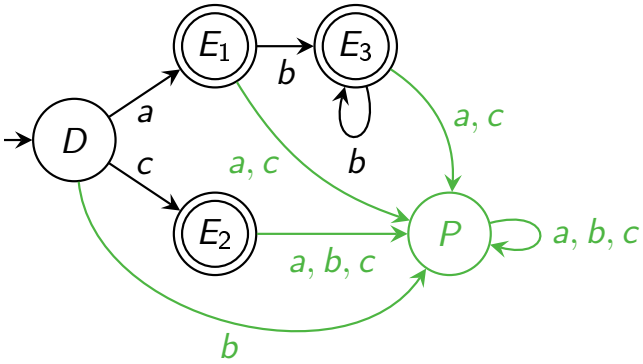



expr = a|b

$N = \{D\}$
$F = \{E_1, E_2\}$

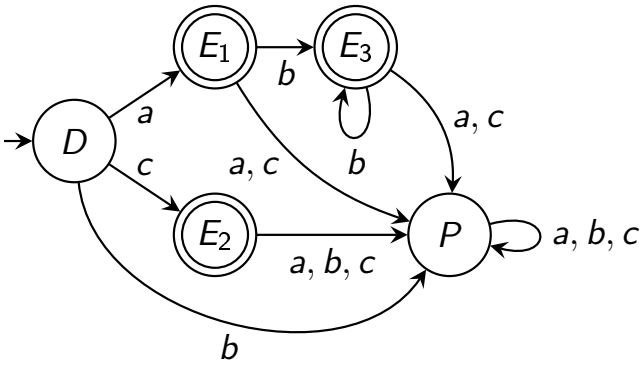
Et voici l'AFDM final :






expr = ab*|c

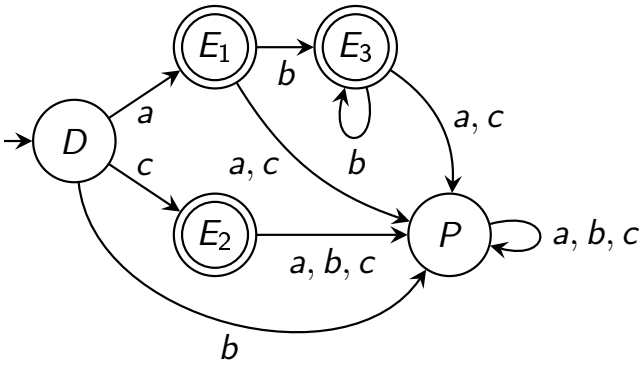
Ajout du puits.




expr = ab*|c

$N = \{D, P\}$
$F = \{E_1, E_2, E_3\}$

Première séparation entre états finaux et non finaux : $N = \{D, P\}$, $F = \{E_1, E_2, E_3\}$.




expr = ab*|c

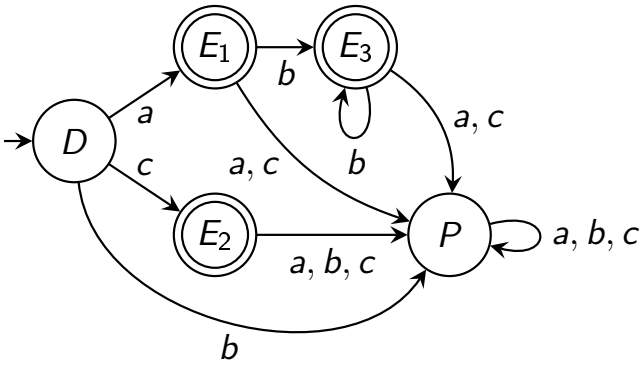
$N = \{D, P\}$

$F = \{E_1, E_2, E_3\}$

$F_1 = \{E_1, E_3\}$

$F_2 = \{E_2\}$

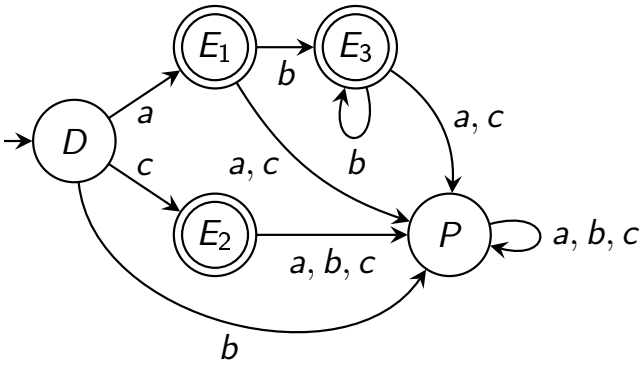
Séparation de $F$ par $b$ : $F_1 = \{E_1, E_3\}$ et $F_2 = \{E_2\}$.




expr = ab*|c

$N = \{D, P\}$
$F_1 = \{E_1, E_3\}$
$F_2 = \{E_2\}$
$N_1 = \{D\}$
$N_2 = \{P\}$

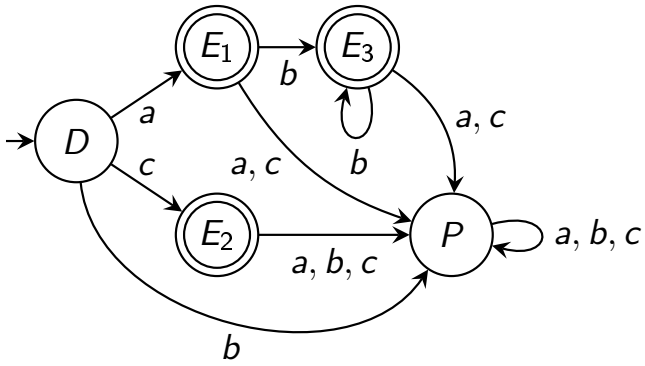
Séparation de $N$ par $c$ : $N_1 = \{D\}$ et $N_2 = \{P\}$.




expr = ab*|c

$F_1 = \{E_1, E_3\}$
$F_2 = \{E_2\}$
$N_1 = \{D\}$
$N_2 = \{P\}$

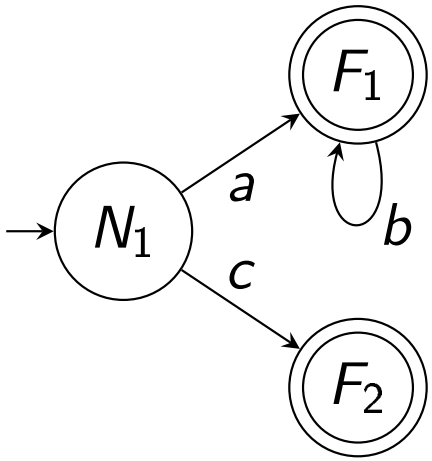
On supprime le puits.

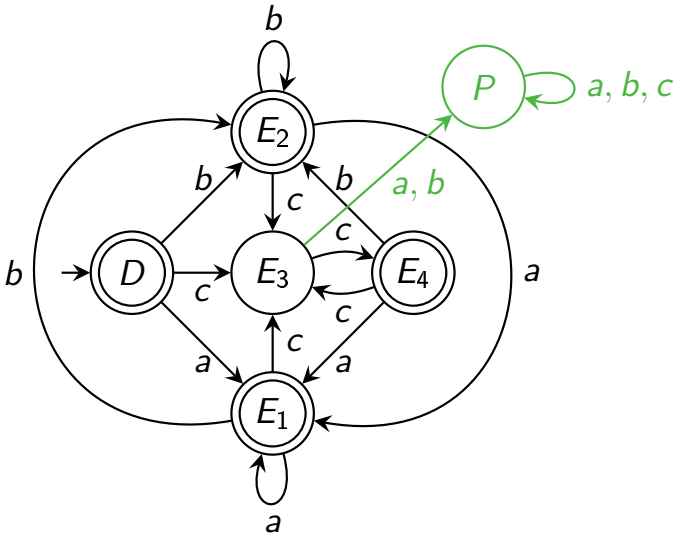



expr = ab*|c

$F_1 = \{E_1, E_3\}$
$F_2 = \{E_2\}$
$N_1 = \{D\}$

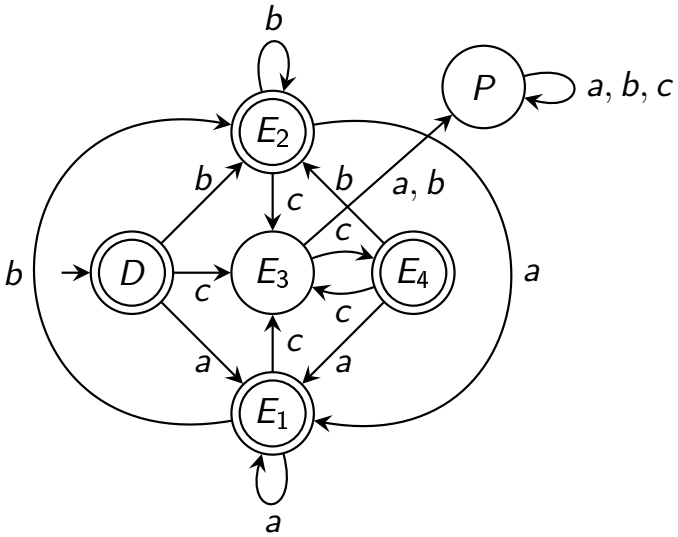
Et voici l'AFDM final :






$$\text{expr} = ((a|b)|cc)^*$$

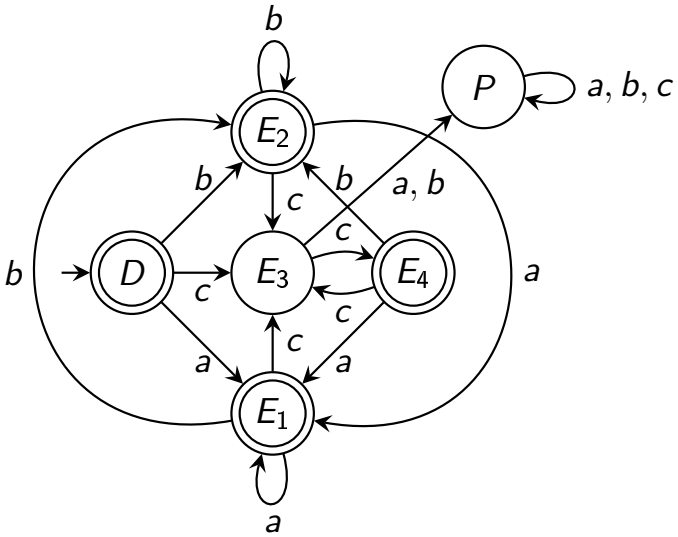
Ajout du puits.




expr = ((a|b)|cc)*

$N = \{E_3, P\}$
$F = \{D, E_1, E_2, E_4\}$

Première séparation entre états finaux et non finaux : $N = \{E_3, P\}$, $F = \{D, E_1, E_2, E_4\}$.




expr = ((a|b)|cc)*

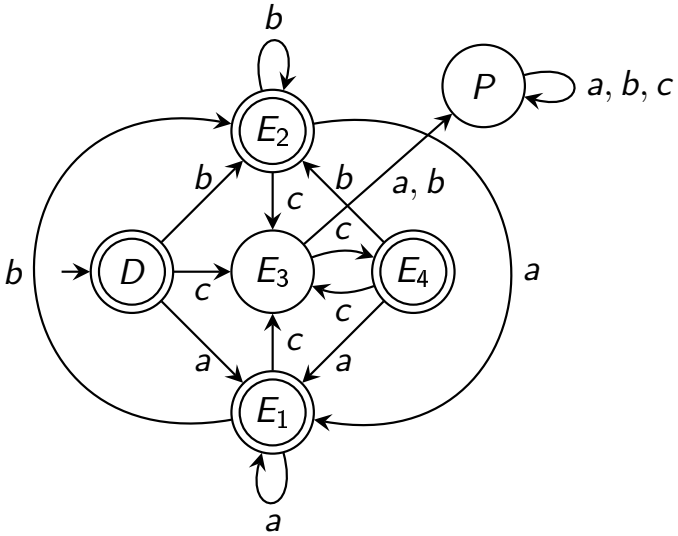
$N = \{E_3, P\}$

$F = \{D, E_1, E_2, E_4\}$

$N_1 = \{E_3\}$

$N_2 = \{P\}$

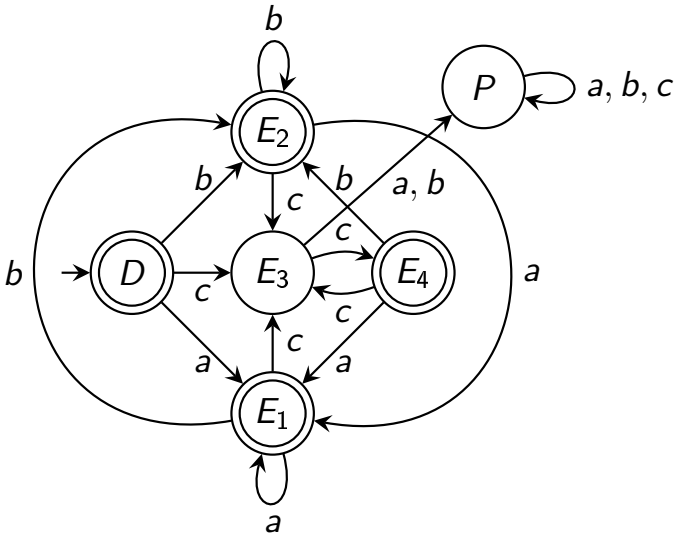
Séparation de $N$ par $c$ : $N_1 = \{E_3\}$ et $N_2 = \{P\}$.




expr = ((a|b)|cc)*

$F = \{D, E_1, E_2, E_4\}$
$N_1 = \{E_3\}$
$N_2 = \{P\}$

Impossible de séparer $F$, tous ses éléments ramènent vers $F$ par $a$ et $b$ et vers $N_1$ par $c$.



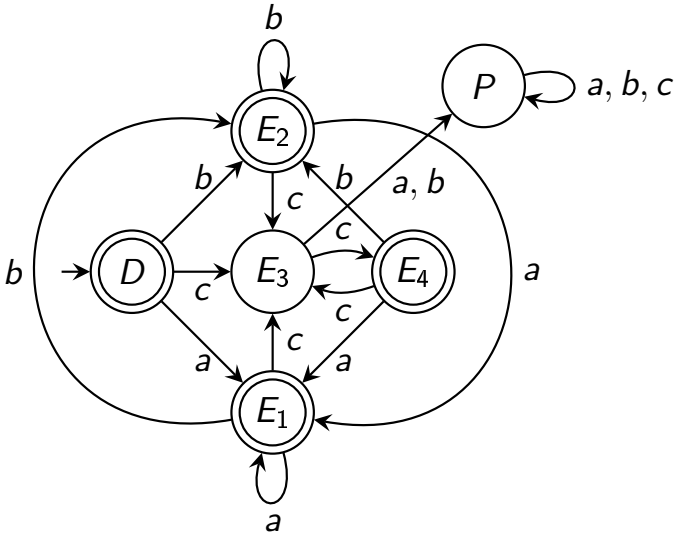

$\text{expr} = ((a|b)|cc)^*$

$F = \{D, E_1, E_2, E_4\}$

$N_1 = \{E_3\}$

$N_2 = \{P\}$

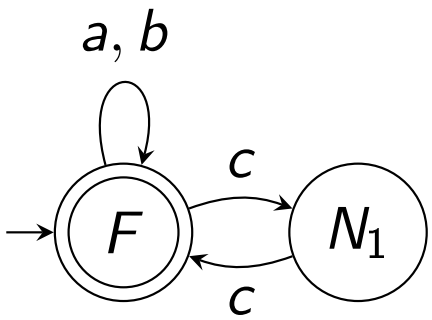
On supprime le puits.

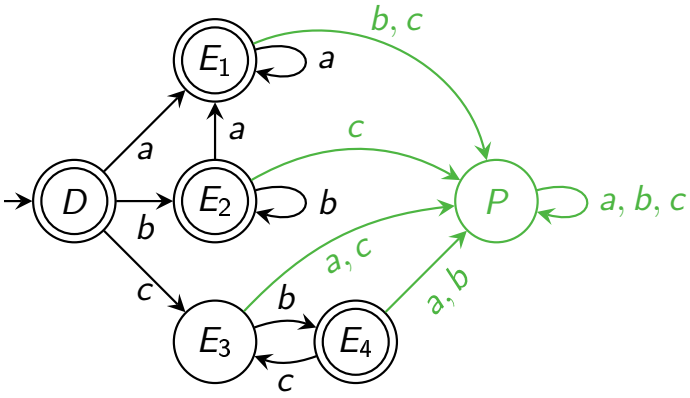



expr = ((a|b)|cc)*

$F = \{D, E_1, E_2, E_4\}$
$N_1 = \{E_3\}$

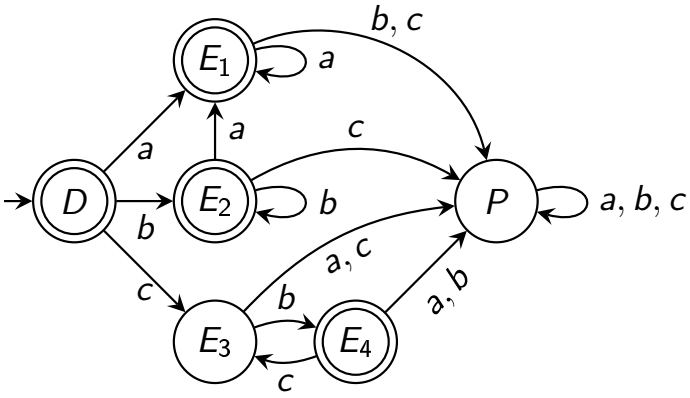
Et voici l'AFDM final :






expr = b*a*|(cb)*

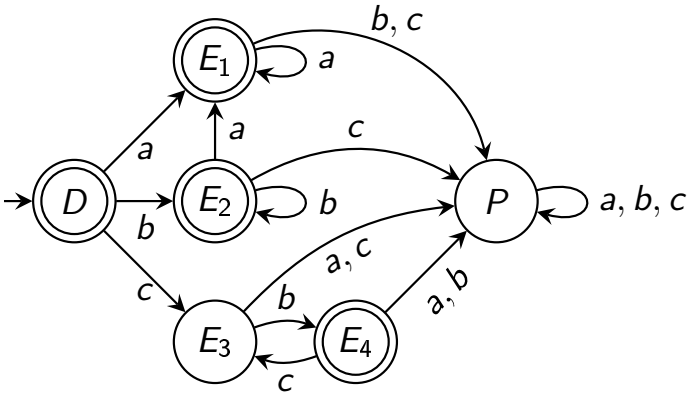
Ajout du puits.




expr = b*a*|(cb)*

$N = \{E_3, P\}$
$F = \{D, E_1, E_2, E_4\}$

Première séparation entre états finaux et non finaux : $N = \{E_3, P\}$, $F = \{D, E_1, E_2, E_4\}$.




expr = b*a*|(cb)*

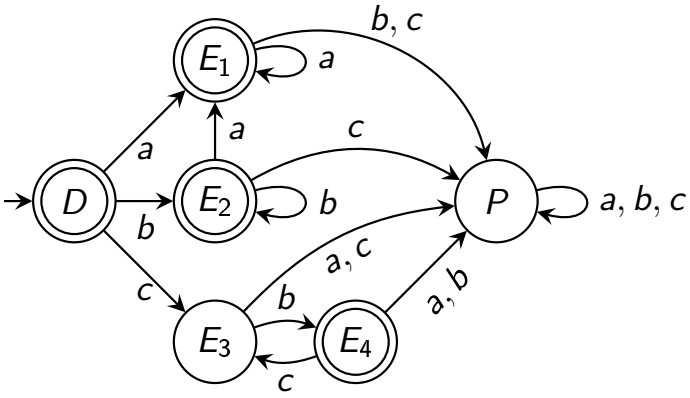
$N = \{E_3, P\}$

$F = \{D, E_1, E_2, E_4\}$

$N_1 = \{E_3\}$

$N_2 = \{P\}$

Séparation de $N$ par $b$ : $N_1 = \{E_3\}$ et $N_2 = \{P\}$.




expr = b*a*|(cb)*

$F = \{D, E_1, E_2, E_4\}$

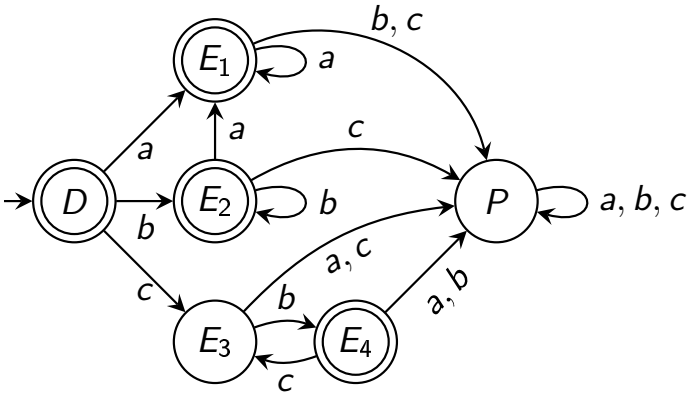
$N_1 = \{E_3\}$

$N_2 = \{P\}$

$F_1 = \{D, E_1, E_2\}$

$F_2 = \{E_4\}$

Séparation de $F$ par $a$ : $F_1 = \{D, E_1, E_2\}$ et $F_2 = \{E_4\}$.




expr = b*a*|(cb)*

$N_1 = \{E_3\}$

$N_2 = \{P\}$

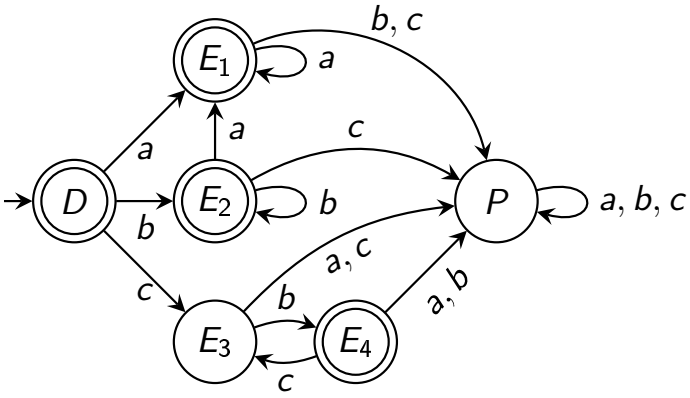
$F_1 = \{D, E_1, E_2\}$

$F_2 = \{E_4\}$

$F_{11} = \{D, E_2\}$

$F_{12} = \{E_1\}$

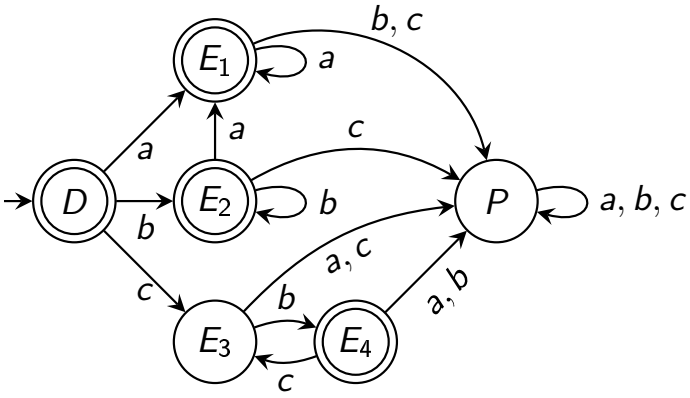
Séparation de $F_1$ par $b$ : $F_{11} = \{D, E_2\}$ et $F_{12} = \{E_1\}$.




expr = b*a*|(cb)*

$N_1 = \{E_3\}$
$N_2 = \{P\}$
$F_2 = \{E_4\}$
$F_{11} = \{D, E_2\}$
$F_{12} = \{E_1\}$
$F_{111} = \{D\}$
$F_{112} = \{E_2\}$

Séparation de $F_{11}$ par $c$ : $F_{111} = \{D\}$ et $F_{112} = \{E_2\}$.




expr = b*a*|(cb)*

$N_1 = \{E_3\}$

$N_2 = \{P\}$

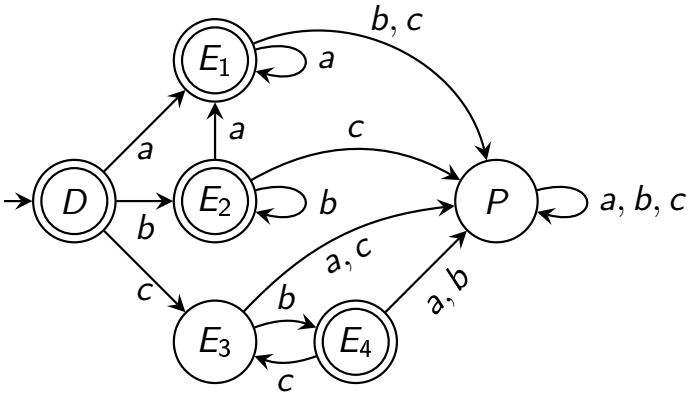
$F_2 = \{E_4\}$

$F_{12} = \{E_1\}$

$F_{111} = \{D\}$

$F_{112} = \{E_2\}$

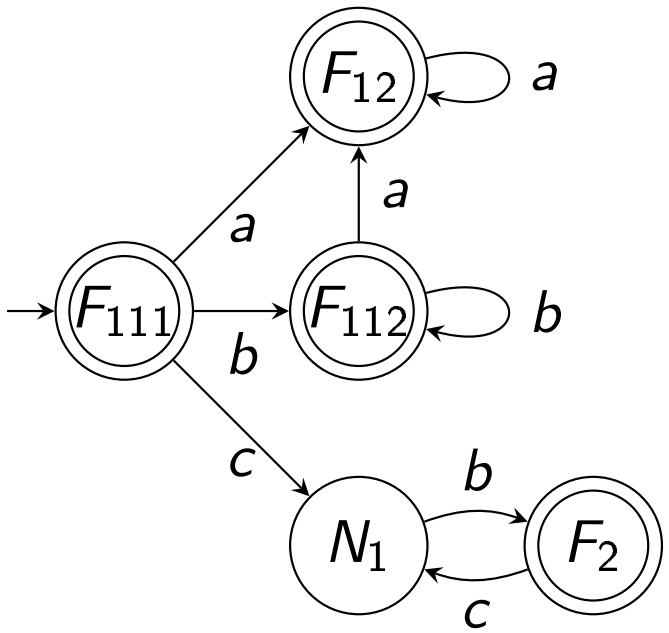
On supprime le puits.

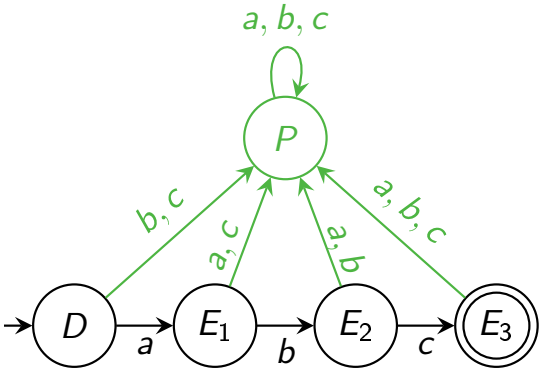



expr = b*a*|(cb)*

$N_1 = \{E_3\}$
$F_2 = \{E_4\}$
$F_{12} = \{E_1\}$
$F_{111} = \{D\}$
$F_{112} = \{E_2\}$

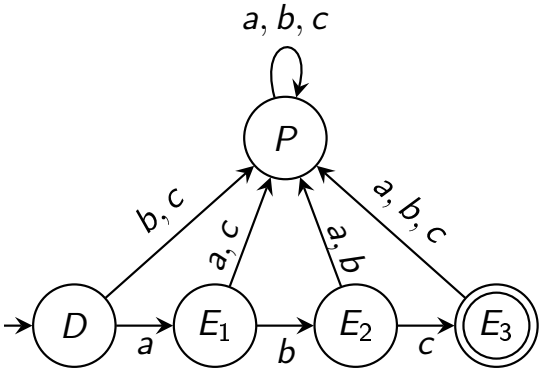
Et voici l'AFDM final :






expr = abc

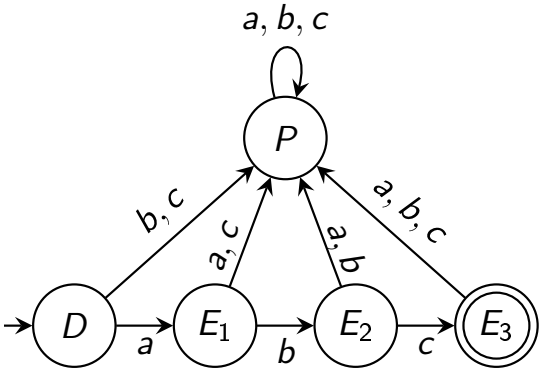
Ajout du puits.




expr = abc

$N = \{D, E_1, E_2, P\}$
$F = \{E_3\}$

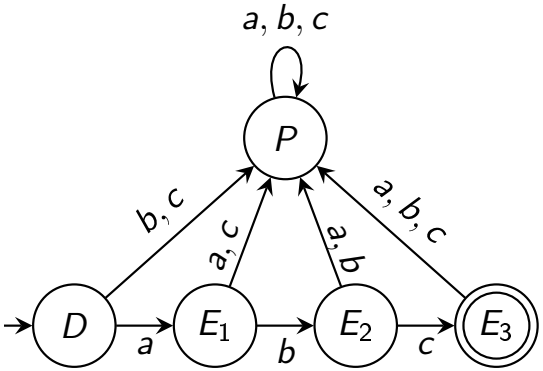
Première séparation entre états finaux et non finaux : $N = \{D, E_1, E_2, P\}$, $F = \{E_3\}$.




expr = abc

$N = \{D, E_1, E_2, P\}$
$F = \{E_3\}$
$N_1 = \{E_2\}$
$N_2 = \{D, E_1, P\}$

Séparation de $N$ par $c$ : $N_1 = \{E_2\}$ et $N_2 = \{D, E_1, P\}$.




expr = abc

$F = \{E_3\}$

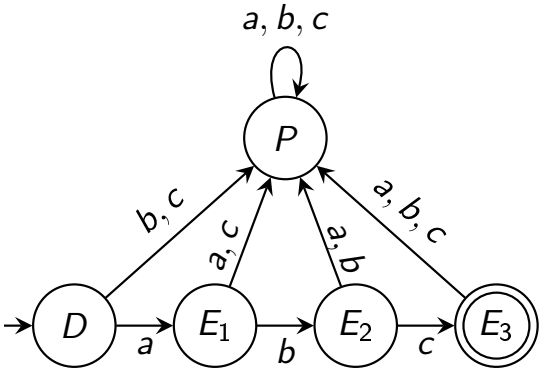
$N_1 = \{E_2\}$

$N_2 = \{D, E_1, P\}$

$N_{21} = \{E_1\}$

$N_{22} = \{D, P\}$

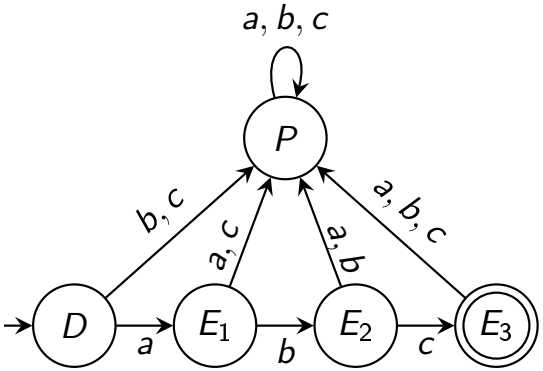
Séparation de $N_2$ par $b$ : $N_{21} = \{E_1\}$ et $N_{22} = \{D, P\}$.




expr = abc

$F = \{E_3\}$
$N_1 = \{E_2\}$
$N_{21} = \{E_1\}$
$N_{22} = \{D, P\}$
$N_{221} = \{D\}$
$N_{222} = \{P\}$

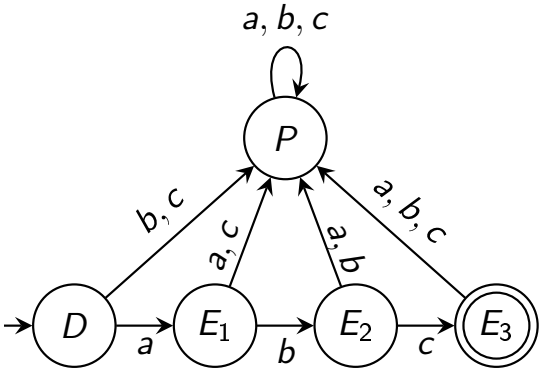
Séparation de $N_{22}$ par $a$ : $N_{221} = \{D\}$ et $N_{222} = \{P\}$.




expr = abc

$F = \{E_3\}$
$N_1 = \{E_2\}$
$N_{21} = \{E_1\}$
$N_{221} = \{D\}$
$N_{222} = \{P\}$

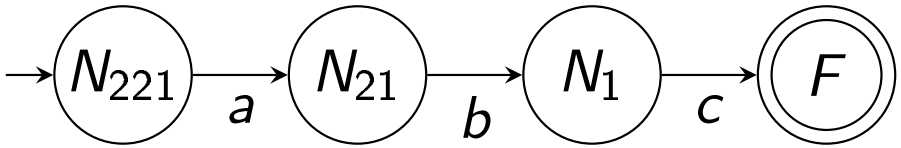
On supprime le puits.

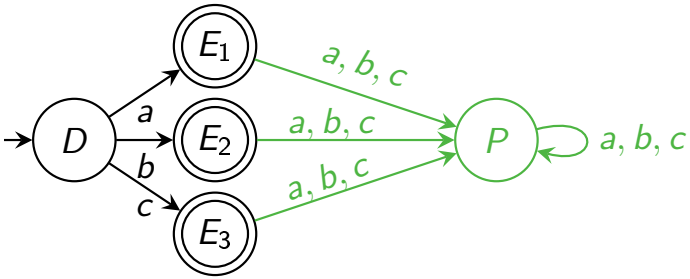



expr = abc

$F = \{E_3\}$
$N_1 = \{E_2\}$
$N_{21} = \{E_1\}$
$N_{221} = \{D\}$

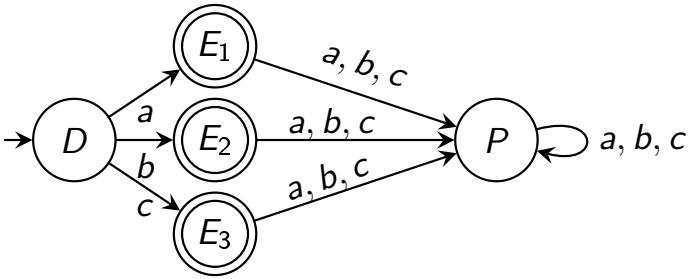
Et voici l'AFDM final :






expr = a|b|c

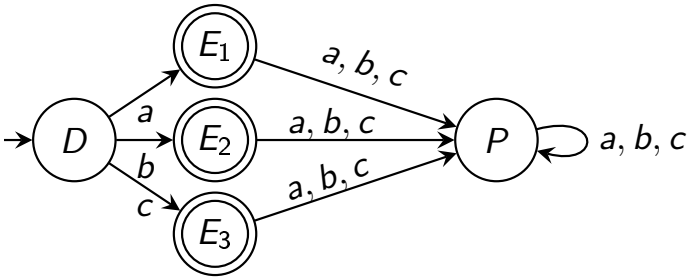
Ajout du puits.




expr = a|b|c

$N = \{D, P\}$
$F = \{E_1, E_2, E_3\}$

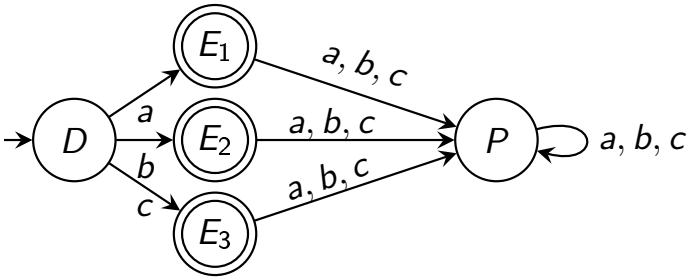
Première séparation entre états finaux et non finaux : $N = \{D, E_1, E_2, P\}$, $F = \{E_3\}$.




expr = a|b|c

$N = \{D, P\}$
$F = \{E_1, E_2, E_3\}$
$N_1 = \{D\}$
$N_2 = \{P\}$

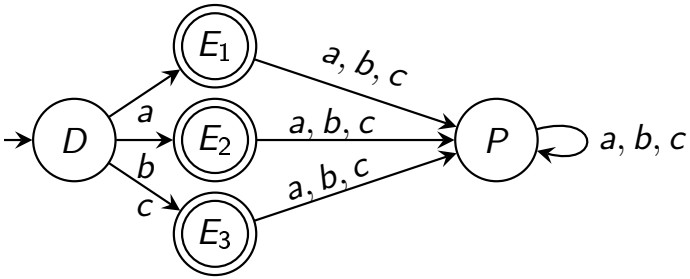
Séparation de $N$ par $a$ : $N_1 = \{D\}$ et $N_2 = \{P\}$.




$\text{expr} = \text{a}|\text{b}|\text{c}$

$F = \{E_1, E_2, E_3\}$
$N_1 = \{D\}$
$N_2 = \{P\}$

Impossible de séparer $F$ car toutes les transitions amènent vers le puits.



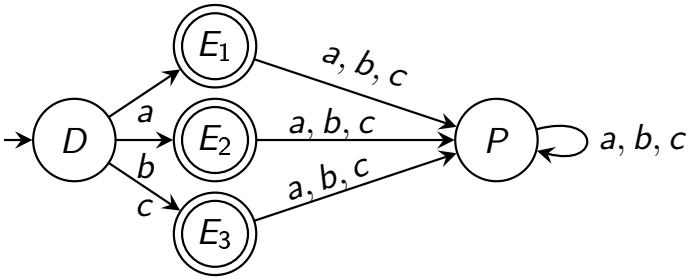

$\text{expr} = \text{a}|\text{b}|\text{c}$

$F = \{E_1, E_2, E_3\}$

$N_1 = \{D\}$

$N_2 = \{P\}$

On supprime le puits.




expr = a|b|c

$F = \{E_1, E_2, E_3\}$
$N_1 = \{D\}$

Et voici l'AFDM final :