

# HA1601I - Exercices de révisions

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## Enoncé

Donner l'automate fini déterministe minimale (AFDM) de certaines des expressions régulières de l'exercice précédent.

1  $ab$

2  $b^*$

3  $a|b$

4  $ab^*|c$

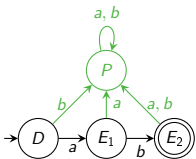
5  $((a|b)|cc)^*$

6  $b^*a^*(cb)^*$

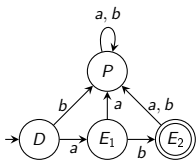
7  $abc$

8  $a|b|c$

$\text{expr} = ab$



Ajout du puits.

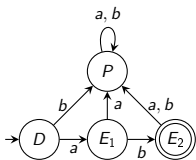


expr = ab

$N = \{D, E_1, P\}$

$F = \{E_2\}$

Première séparation entre états finaux et non finaux :  $N = \{D, E_1, P\}$ ,  
 $F = \{E_2\}$ .



expr = ab

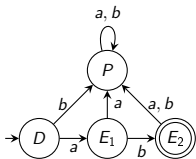
$N = \{D, E_1, P\}$

$F = \{E_2\}$

$N_1 = \{D, P\}$

$N_2 = \{E_1\}$

Séparation de  $N$  par  $b$  :  $N_1 = \{D, P\}$  et  $N_2 = \{E_1\}$ .



expr = ab

$$F = \{E_2\}$$

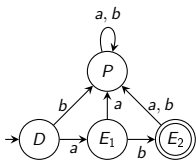
$$N_1 = \{D, P\}$$

$$N_2 = \{E_1\}$$

$$N_{11} = \{D\}$$

$$N_{12} = \{P\}$$

Séparation de  $N_1$  par  $a$  :  $N_{11} = \{D\}$  et  $N_{12} = \{P\}$ .



$\text{expr} = ab$

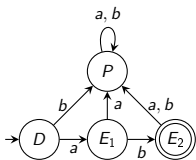
$$F = \{E_2\}$$

$$N_2 = \{E_1\}$$

$$N_{11} = \{D\}$$

$$N_{12} = \{P\}$$

On supprime le puits.



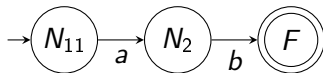
expr = ab

$$F = \{E_2\}$$

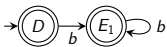
$$N_2 = \{E_1\}$$

$$N_{11} = \{D\}$$

Et voici l'AFDM final :

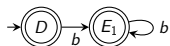






$\text{expr} = b^*$

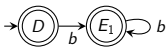
Pas besoin de puits l'AFD est déjà complet.



$\text{expr} = b^*$

$F = \{D, E_1\}$

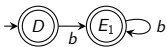
Première séparation entre état finaux et non finaux :  $F = \{D, E_1\}$ .



$$\text{expr} = b^*$$

$$F = \{D, E_1\}$$

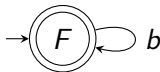
Impossible de séparer  $F$  par  $b$ , l'automate est donc minimisé.



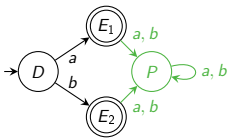
$\text{expr} = b^*$

$F = \{D, E_1\}$

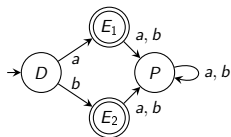
Et voici l'AFDM final :



$\text{expr} = a|b$



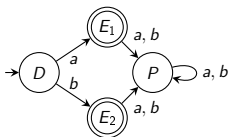
Ajout du puits.



$\text{expr} = a|b$

$N = \{D, P\}$   
 $F = \{E_1, E_2\}$

Première séparation entre états finaux et non finaux :  $N = \{D, P\}$ ,  
 $F = \{E_1, E_2\}$ .



$\text{expr} = a|b$

$N = \{D, P\}$

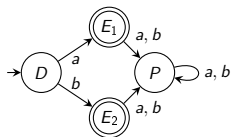
$N = \{D\}$

$F = \{E_1, E_2\}$

Impossible de séparer  $N$  par  $a$  ou  $b$ .

Il en va de même pour  $F$ .

On supprime juste  $P$  et l'automate est minimisé.

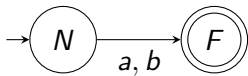


$\text{expr} = a|b$

$N = \{D\}$

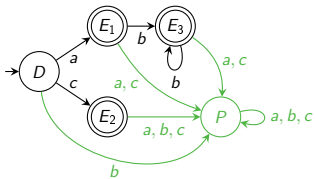
$F = \{E_1, E_2\}$

Et voici l'AFDM final :

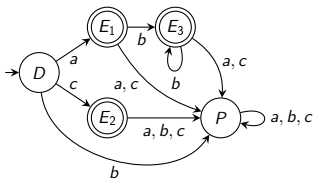




$\text{expr} = ab^*|c$



Ajout du puits.

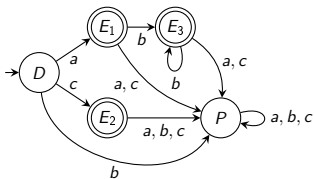


$\text{expr} = ab^*|c$

$N = \{D, P\}$

$F = \{E_1, E_2, E_3\}$

Première séparation entre états finaux et non finaux :  $N = \{D, P\}$ ,  
 $F = \{E_1, E_2, E_3\}$ .



$\text{expr} = ab^*|c$

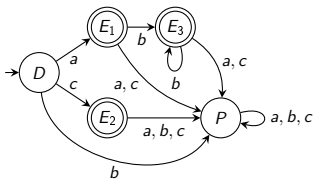
$N = \{D, P\}$

$F = \{E_1, E_2, E_3\}$

$F_1 = \{E_1, E_3\}$

$F_2 = \{E_2\}$

Séparation de  $F$  par  $b$  :  $F_1 = \{E_1, E_3\}$  et  $F_2 = \{E_2\}$ .



$\text{expr} = ab^*|c$

$N = \{D, P\}$

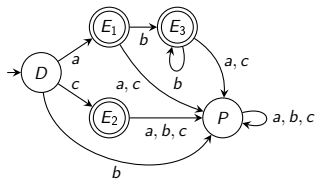
$F_1 = \{E_1, E_3\}$

$F_2 = \{E_2\}$

$N_1 = \{D\}$

$N_2 = \{P\}$

Séparation de  $N$  par  $c$  :  $N_1 = \{D\}$  et  $N_2 = \{P\}$ .



$\text{expr} = ab^*|c$

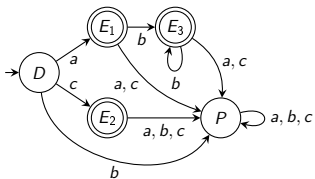
$F_1 = \{E_1, E_3\}$

$F_2 = \{E_2\}$

$N_1 = \{D\}$

$N_2 = \{P\}$

On supprime le puits.



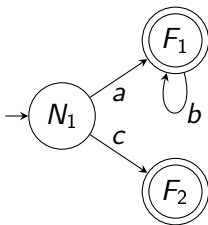
$\text{expr} = ab^*|c$

$F_1 = \{E_1, E_3\}$

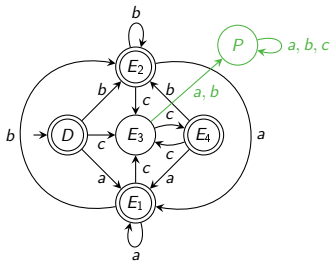
$F_2 = \{E_2\}$

$N_1 = \{D\}$

Et voici l'AFDM final :



$$\text{expr} = ((a|b)|cc)^*$$

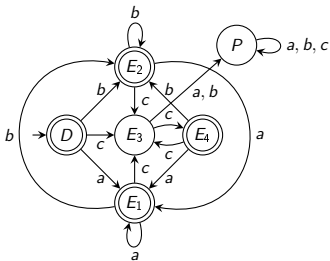


Ajout du puits.

$\text{expr} = ((a|b)|cc)^*$

$N = \{E_3, P\}$

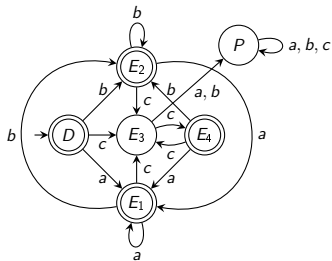
$F = \{D, E_1, E_2, E_4\}$



Première séparation entre états finaux et non finaux :  $N = \{E_3, P\}$ ,  
 $F = \{D, E_1, E_2, E_4\}$ .



$\text{expr} = ((a|b)|cc)^*$



$N = \{E_3, P\}$

$F = \{D, E_1, E_2, E_4\}$

$N_1 = \{E_3\}$

$N_2 = \{P\}$

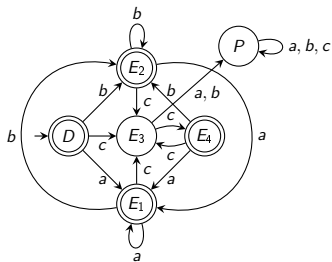
Séparation de  $N$  par  $c$  :  $N_1 = \{E_3\}$  et  $N_2 = \{P\}$ .

$\text{expr} = ((a|b)|cc)^*$

$F = \{D, E_1, E_2, E_4\}$

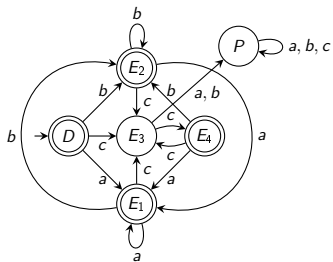
$N_1 = \{E_3\}$

$N_2 = \{P\}$



Impossible de séparer  $F$ , tous ses éléments ramènent vers  $F$  par  $a$  et  $b$  et vers  $N_1$  par  $c$ .

$\text{expr} = ((a|b)|cc)^*$



$F = \{D, E_1, E_2, E_4\}$

$N_1 = \{E_3\}$

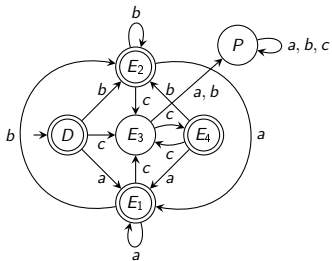
$N_2 = \{P\}$

On supprime le puits.

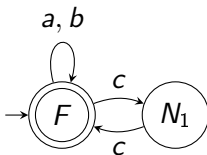
$\text{expr} = ((a|b)|cc)^*$

$F = \{D, E_1, E_2, E_4\}$

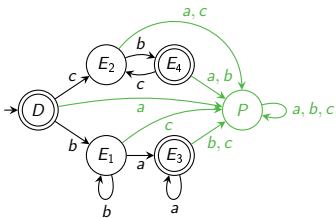
$N_1 = \{E_3\}$



Et voici l'AFDM final :



$\text{expr} = b^*a^*(cb)^*$

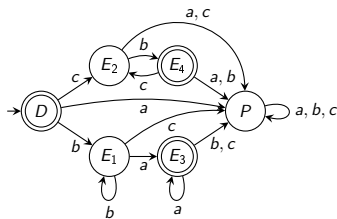


Ajout du puits.

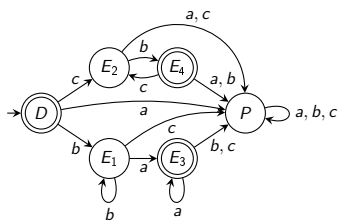
expr =  $b^*a^*(cb)^*$

$N = \{E_1, E_2, P\}$

$F = \{D, E_3, E_4\}$



Première séparation entre états finaux et non finaux :  $N = \{E_1, E_2, P\}$ ,  
 $F = \{D, E_3, E_4\}$ .



$\text{expr} = b^*a^*(cb)^*$

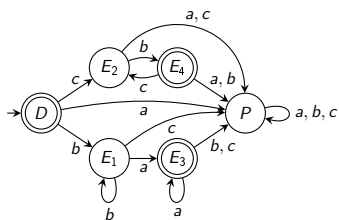
$N = \{E_1, E_2, P\}$

$F = \{D, E_3, E_4\}$

$N_1 = \{E_1\}$

$N_2 = \{E_2, P\}$

Séparation de  $N$  par  $a$  :  $N_1 = \{E_1\}$  et  $N_2 = \{E_2, P\}$ .



$\text{expr} = b^*a^*(cb)^*$

$F = \{D, E_3, E_4\}$

$N_1 = \{E_1\}$

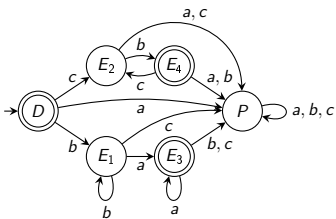
$N_2 = \{E_2, P\}$

$N_{21} = \{E_2\}$

$N_{22} = \{P\}$

Séparation de  $N_2$  par  $b$  :  $N_{21} = \{E_2\}$  et  $N_{22} = \{P\}$ .





$\text{expr} = b^*a^*(cb)^*$

$F = \{D, E_3, E_4\}$

$N_1 = \{E_1\}$

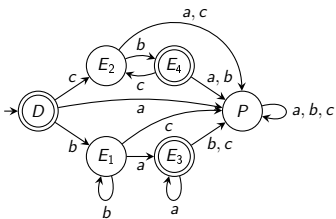
$N_{21} = \{E_2\}$

$N_{22} = \{P\}$

$F_1 = \{D, E_4\}$

$F_2 = \{E_3\}$

Séparation de  $F$  par  $a$  :  $F_1 = \{D, E_4\}$  et  $F_2 = \{E_3\}$ .



$\text{expr} = b^*a^*(cb)^*$

$N_1 = \{E_1\}$

$N_{21} = \{E_2\}$

$N_{22} = \{P\}$

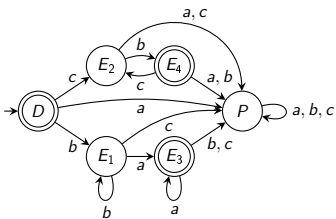
$F_1 = \{D, E_4\}$

$F_2 = \{E_3\}$

$F_{11} = \{D\}$

$F_{12} = \{E_4\}$

Séparation de  $F_1$  par  $b$  :  $F_{11} = \{D\}$  et  $F_{12} = \{E_4\}$ .



$\text{expr} = b^*a^*(cb)^*$

$N_1 = \{E_1\}$

$N_{21} = \{E_2\}$

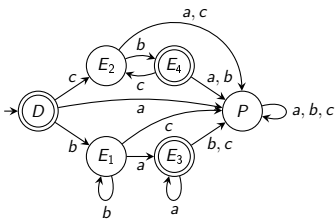
$N_{22} = \{P\}$

$F_2 = \{E_3\}$

$F_{11} = \{D\}$

$F_{12} = \{E_4\}$

On supprime le puits.



$\text{expr} = b^*a^*(cb)^*$

$N_1 = \{E_1\}$

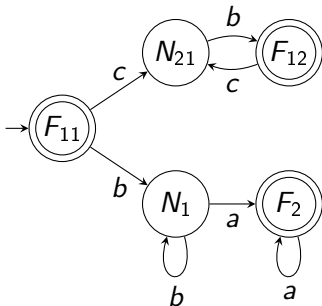
$N_{21} = \{E_2\}$

$F_2 = \{E_3\}$

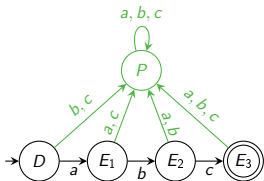
$F_{11} = \{D\}$

$F_{12} = \{E_4\}$

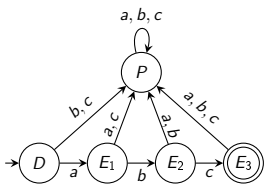
Et voici l'AFDM final :



$\text{expr} = abc$



Ajout du puits.

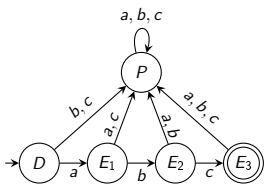


expr = abc

$N = \{D, E_1, E_2, P\}$

$F = \{E_3\}$

Première séparation entre états finaux et non finaux :  $N = \{D, E_1, E_2, P\}$ ,  
 $F = \{E_3\}$ .



expr = abc

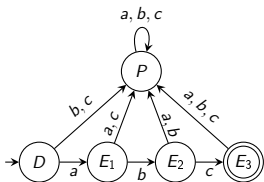
$N = \{D, E_1, E_2, P\}$

$F = \{E_3\}$

$N_1 = \{E_2\}$

$N_2 = \{D, E_1, P\}$

Séparation de  $N$  par  $c$  :  $N_1 = \{E_2\}$  et  $N_2 = \{D, E_1, P\}$ .



expr = abc

$$F = \{E_3\}$$

$$N_1 = \{E_2\}$$

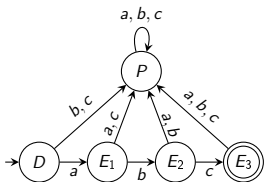
$$N_2 = \{D, E_1, P\}$$

$$N_{21} = \{E_1\}$$

$$N_{22} = \{D, P\}$$

Séparation de  $N_2$  par  $b$  :  $N_{21} = \{E_1\}$  et  $N_{22} = \{D, P\}$ .





expr = abc

$$F = \{E_3\}$$

$$N_1 = \{E_2\}$$

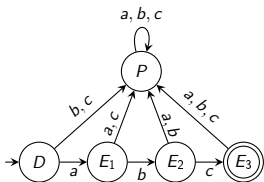
$$N_{21} = \{E_1\}$$

$$N_{22} = \{D, P\}$$

$$N_{221} = \{D\}$$

$$N_{222} = \{P\}$$

Séparation de  $N_{22}$  par  $a$  :  $N_{221} = \{D\}$  et  $N_{222} = \{P\}$ .



$\text{expr} = abc$

$$F = \{E_3\}$$

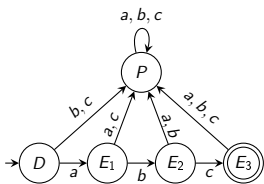
$$N_1 = \{E_2\}$$

$$N_{21} = \{E_1\}$$

$$N_{221} = \{D\}$$

$$N_{222} = \{P\}$$

On supprime le puits.



expr = abc

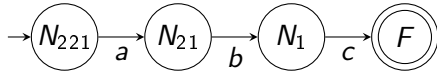
$$F = \{E_3\}$$

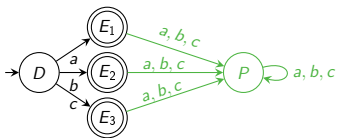
$$N_1 = \{E_2\}$$

$$N_{21} = \{E_1\}$$

$$N_{221} = \{D\}$$

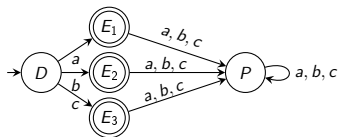
Et voici l'AFDM final :





$\text{expr} = a|b|c$

Ajout du puits.

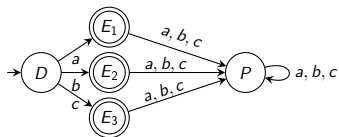


$\text{expr} = a|b|c$

$N = \{D, P\}$

$F = \{E_1, E_2, E_3\}$

Première séparation entre états finaux et non finaux :  $N = \{D, E_1, E_2, P\}$ ,  
 $F = \{E_3\}$ .



$\text{expr} = a|b|c$

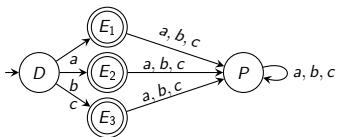
$N = \{D, P\}$

$F = \{E_1, E_2, E_3\}$

$N_1 = \{D\}$

$N_2 = \{P\}$

Séparation de  $N$  par  $a$  :  $N_1 = \{D\}$  et  $N_2 = \{P\}$ .



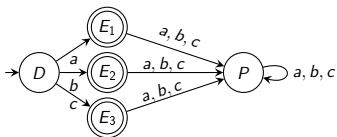
$\text{expr} = a|b|c$

$F = \{E_1, E_2, E_3\}$

$N_1 = \{D\}$

$N_2 = \{P\}$

Impossible de séparer  $F$  car toutes les transitions amènent vers le puits.



$\text{expr} = a|b|c$

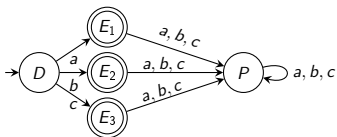
$F = \{E_1, E_2, E_3\}$

$N_1 = \{D\}$

$N_2 = \{P\}$

On supprime le puits.





$\text{expr} = a|b|c$

$F = \{E_1, E_2, E_3\}$

$N_1 = \{D\}$

Et voici l'AFDM final :

