

HA601I - Exercices de révisions

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2022

Enoncé

Donner l'automate fini non déterministe (AFN) des expressions régulières suivantes grâce à l'algorithme de Thompson.

❶ ab

❷ b^*

❸ $a|b$

❹ a

❺ $ab|c$

❻ $ab^*|c$

❼ $((a|b)|cc)^*$

❽ $b^*a^*(cb)^*$

❾ $(a|b)|c$

❿ $a|(b|c)$

⓫ $(ab)c$

⓬ $a(bc)$

Avec les AFN des expressions régulières $(ab)c$ et $a(bc)$, que peut-on en déduire sur la concaténation ?

$$\text{expr} = ab$$

La première chose à faire c'est toujours de construire l'arbre de dérivation correspondant à l'expression régulière. On se base ensuite sur cet arbre pour créer notre AFD. On initialise $i = 0$ et on lance l'algorithme sur l'arbre.

$$\text{expr} = ab$$

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arbre



$$i = 0$$

$$\text{expr} = ab$$

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arbre



$$i = 2$$

AFN



$$\text{expr} = ab$$

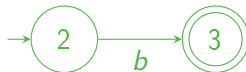
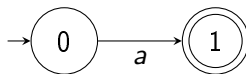
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arbre



$$i = 4$$

AFN



$$\text{expr} = ab$$

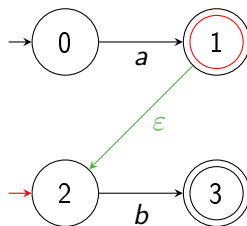
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arbre



$$i = 4$$

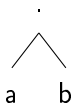
AFN



$\text{expr} = ab$

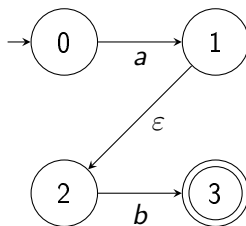
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arbre



$i = 4$

AFN



AFN final à 4 états

$\text{expr} = b^*$

$\text{expr} = b^*$

arbre

*

|

b

$i = 0$

$\text{expr} = b^*$

arbre

*

|

b

$i = 2$

AFN



$\text{expr} = b^*$

arbre

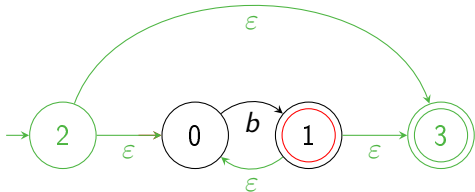
*

|

b

$i = 4$

AFN



$\text{expr} = b^*$

arbre

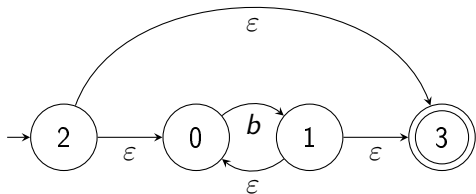
*

|

b

$i = 4$

AFN



AFN final à 4 états

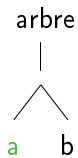
$\text{expr} = a|b$

$\text{expr} = a|b$



$i = 0$

$\text{expr} = a|b$

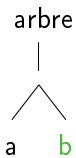


$i = 2$

AFN

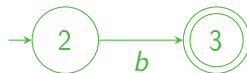
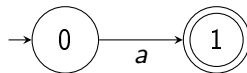


$\text{expr} = a|b$



$i = 4$

AFN



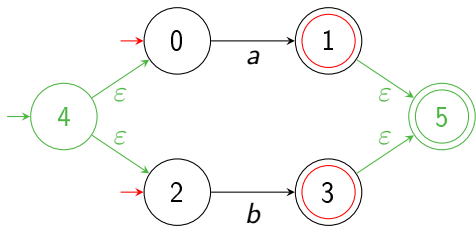
$\text{expr} = a|b$

arbre



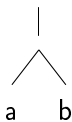
$i = 6$

AFN



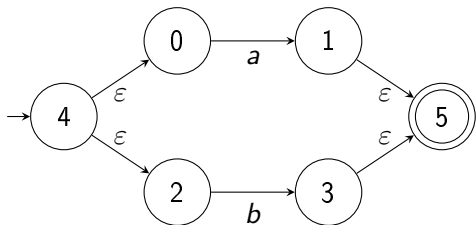
$\text{expr} = a|b$

arbre



$i = 6$

AFN



AFN final à 6 états

`expr = a`

$\text{expr} = a$

arbre

a

$i = 0$

expr = a

arbre

a

$i = 2$

AFN



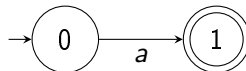
expr = a

arbre

a

$i = 2$

AFN



AFN final à 2 états

$\text{expr} = ab|c$

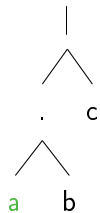
$\text{expr} = ab|c$



$i = 0$

$\text{expr} = ab|c$

arbre



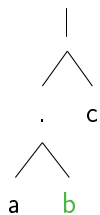
$i = 2$

AFN



$\text{expr} = ab|c$

arbre



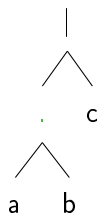
$i = 4$

AFN



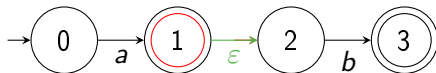
$\text{expr} = ab|c$

arbre



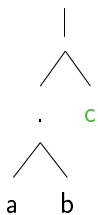
$i = 4$

AFN



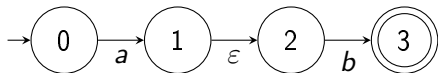
$\text{expr} = ab|c$

arbre



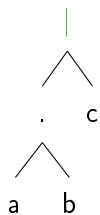
$i = 6$

AFN



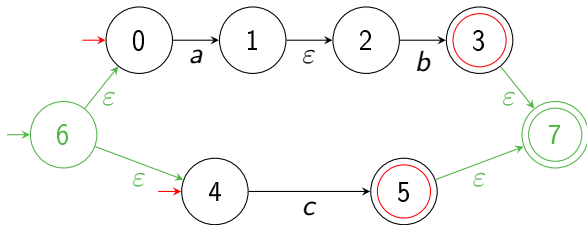
$\text{expr} = ab|c$

arbre

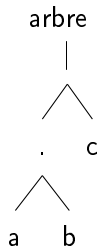


$i = 8$

AFN

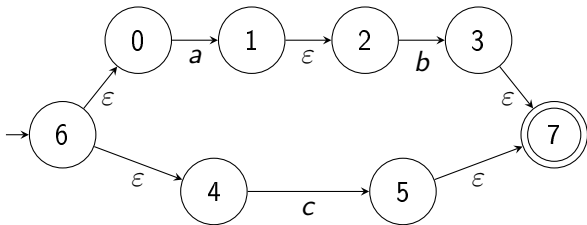


$\text{expr} = ab|c$



$i = 8$

AFN



AFN final à 8 états

$\text{expr} = ab^*|c$

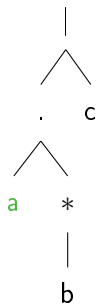
$\text{expr} = ab^*|c$



$i = 0$

$$\text{expr} = ab^*|c$$

arbre



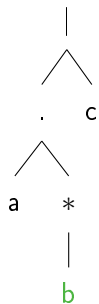
AFN



$i = 2$

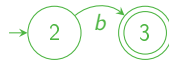
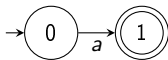
$$\text{expr} = ab^*|c$$

arbre



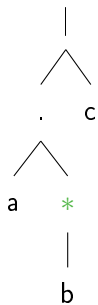
$$i = 4$$

AFN



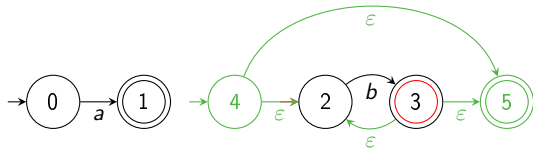
$$\text{expr} = ab^*|c$$

arbre



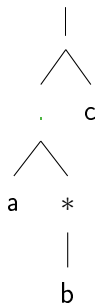
$$i = 6$$

AFN



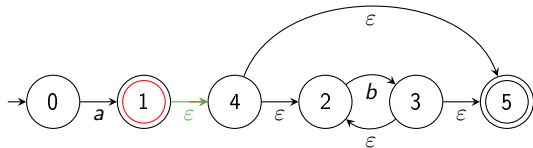
$$\text{expr} = ab^*|c$$

arbre



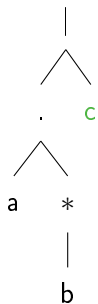
$$i = 6$$

AFN



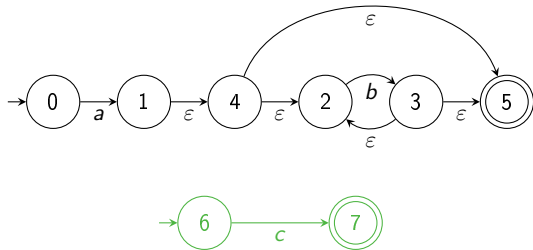
$$\text{expr} = ab^*|c$$

arbre



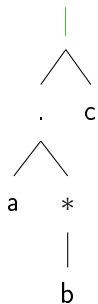
$i = 8$

AFN

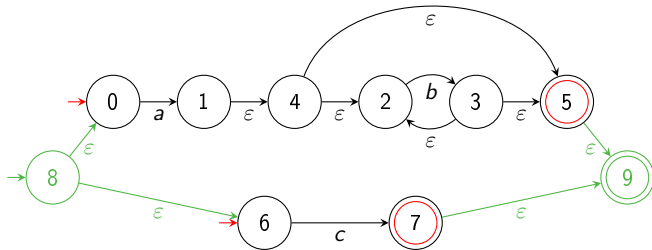


$\text{expr} = ab^*|c$

arbre



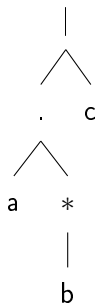
AFN



$i = 10$

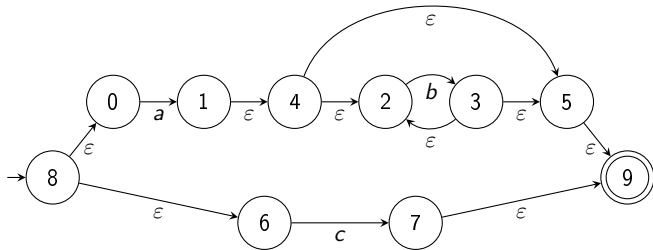
$\text{expr} = ab^*|c$

arbre



$i = 10$

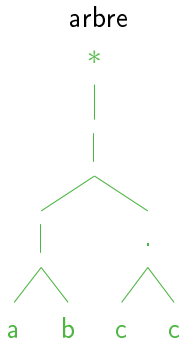
AFN



AFN final à 10 états

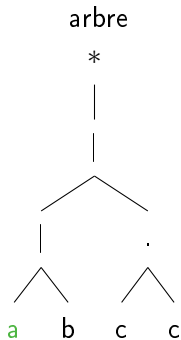
$$\text{expr} = ((a|b)|cc)^*$$

$$\text{expr} = ((a|b)|cc)^*$$



$$i = 0$$

$$\text{expr} = ((a|b)|cc)^*$$

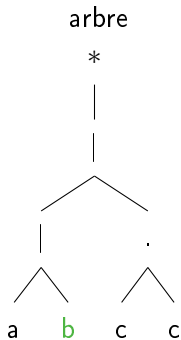


$$i = 2$$

AFN

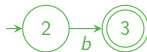
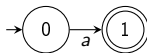


$$\text{expr} = ((a|b)|cc)^*$$



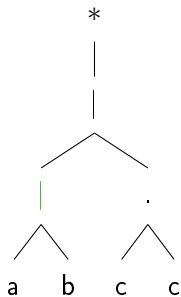
$i = 4$

AFN



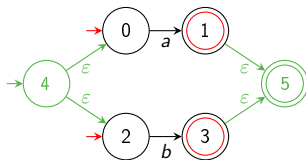
$$\text{expr} = ((a|b)|cc)^*$$

arbre



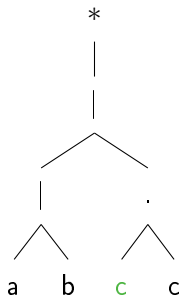
$$i = 6$$

AFN



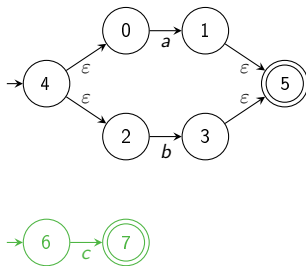
$$\text{expr} = ((a|b)|cc)^*$$

arbre



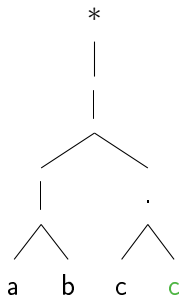
$i = 8$

AFN



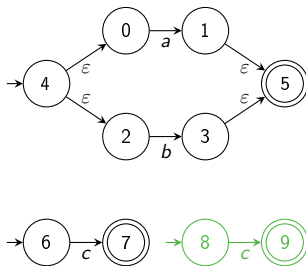
$$\text{expr} = ((a|b)|cc)^*$$

arbre



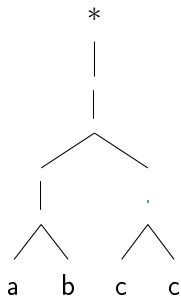
$i = 10$

AFN



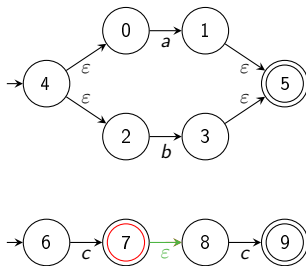
$$\text{expr} = ((a|b)|cc)^*$$

arbre



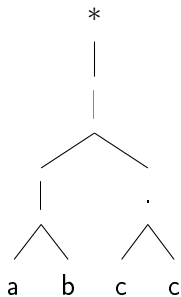
$i = 10$

AFN



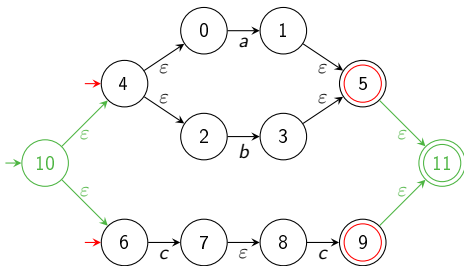
$$\text{expr} = ((a|b)|cc)^*$$

arbre



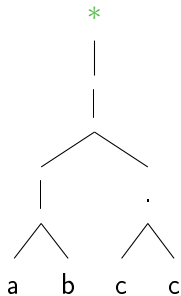
$i = 12$

AFN



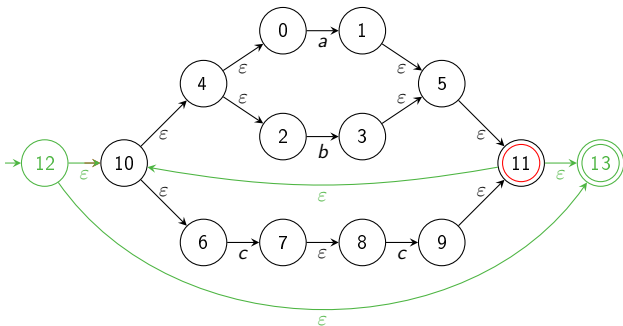
$$\text{expr} = ((a|b)|cc)^*$$

arbre



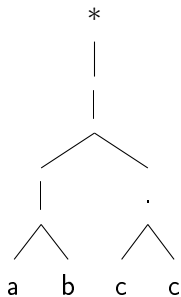
$i = 14$

AFN



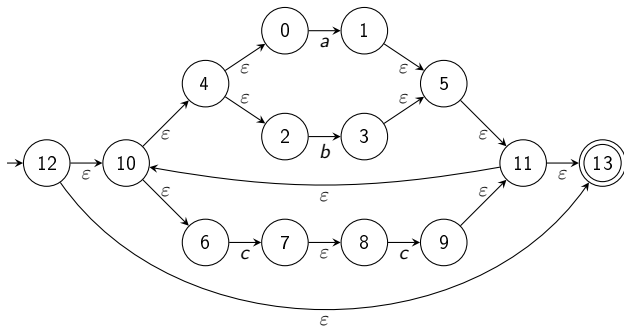
$$\text{expr} = ((a|b)|cc)^*$$

arbre



$i = 14$

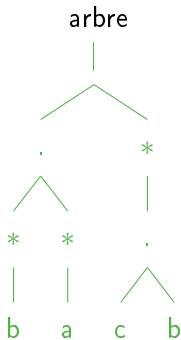
AFN



AFN final à 14 états

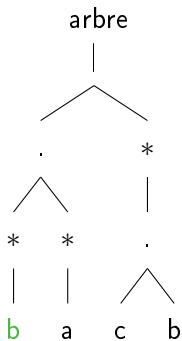
$\text{expr} = b*a*|(cb)^*$

$$\text{expr} = b * a * (cb)^*$$



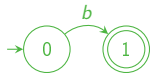
$$i = 0$$

$$\text{expr} = b^*a^*(cb)^*$$

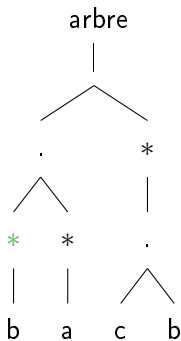


$$i = 2$$

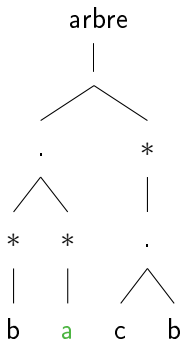
AFN



$$\text{expr} = b^*a^*(cb)^*$$

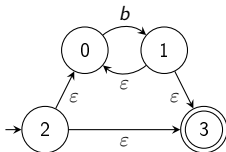


$$\text{expr} = b^*a^*(cb)^*$$

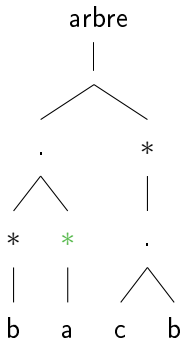


$$i = 6$$

AFN

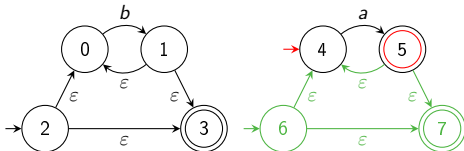


$$\text{expr} = b^*a^*(cb)^*$$

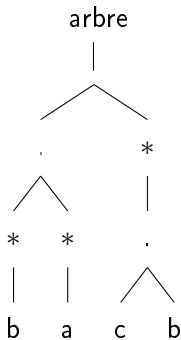


$$i = 8$$

AFN

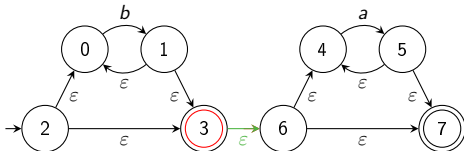


$$\text{expr} = b^*a^*(cb)^*$$

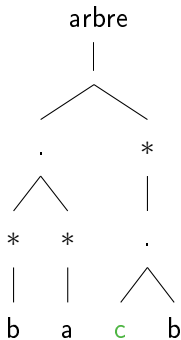


$$i = 8$$

AFN

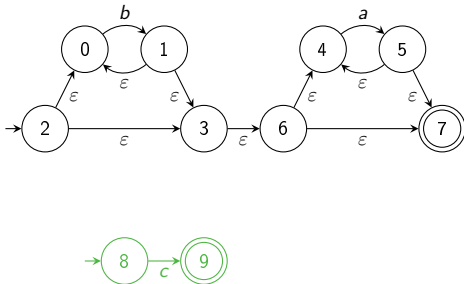


$$\text{expr} = b^*a^*(cb)^*$$

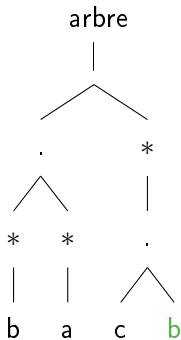


$$i = 10$$

AFN

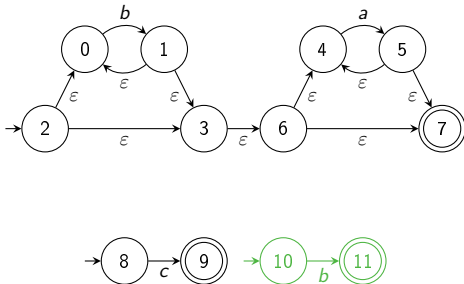


$$\text{expr} = b^*a^*(cb)^*$$

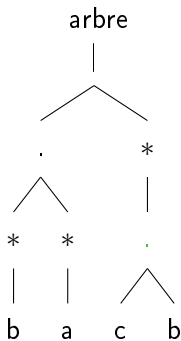


$i = 12$

AFN

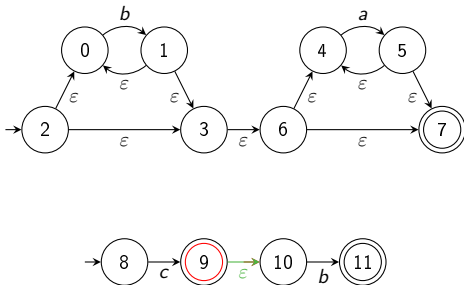


expr = b*a*|(cb)*

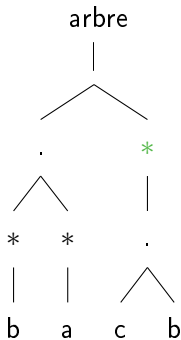


$i = 12$

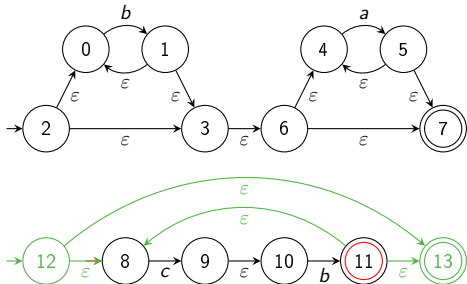
AFN



$\text{expr} = b^*a^*(cb)^*$

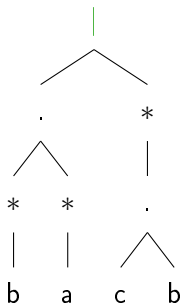


AFN



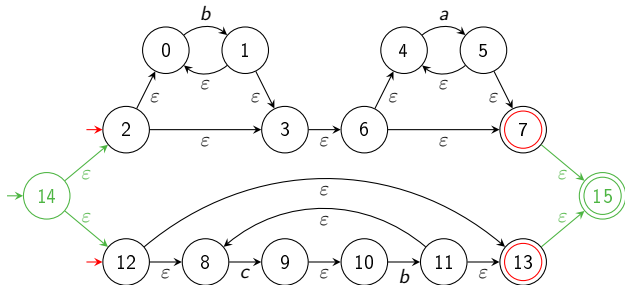
$$\text{expr} = b^*a^*(cb)^*$$

arbre



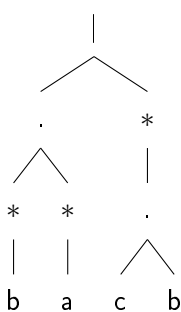
$i = 16$

AFN



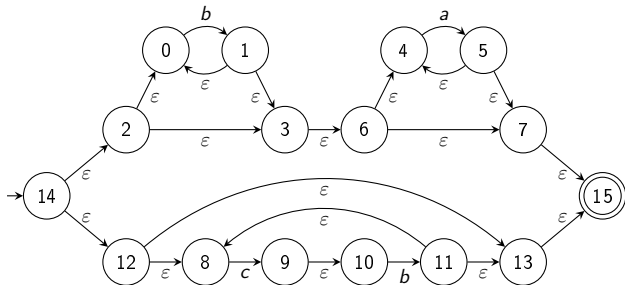
$$\text{expr} = b^*a^*(cb)^*$$

arbre



$i = 16$

AFN



AFN final à 16 états

$$\text{expr} = (a|b)|c$$

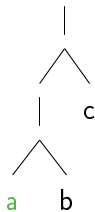
$\text{expr} = (a|b)|c$



$i = 0$

$\text{expr} = (a|b)|c$

arbre

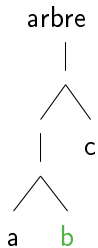


$i = 2$

AFN

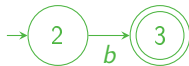
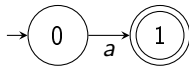


$\text{expr} = (a|b)|c$

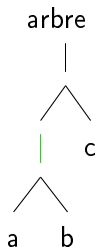


$i = 4$

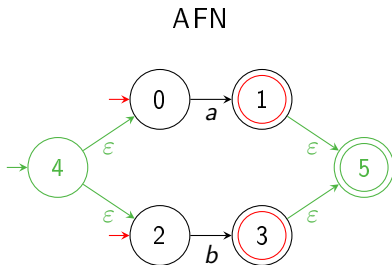
AFN



$\text{expr} = (a|b)|c$

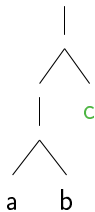


$i = 6$



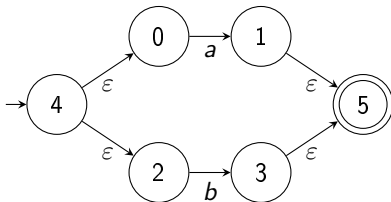
$$\text{expr} = (a|b)|c$$

arbre



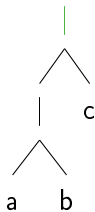
$$i = 8$$

AFN



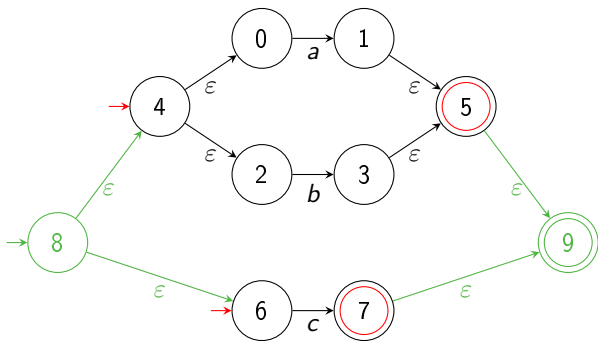
$$\text{expr} = (a|b)|c$$

arbre



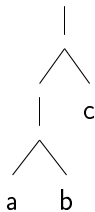
$i = 10$

AFN



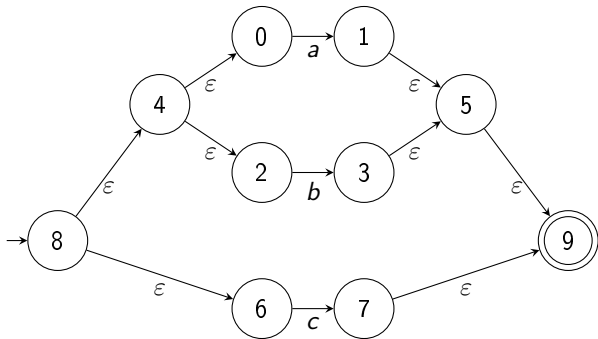
$\text{expr} = (a|b)|c$

arbre



$i = 10$

AFN



AFN final à 10 états

$$\text{expr} = a|(b|c)$$

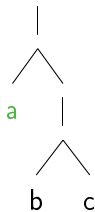
$\text{expr} = a|(b|c)$



$i = 0$

$\text{expr} = a|(b|c)$

arbre



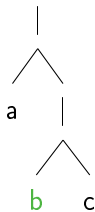
$i = 2$

AFN



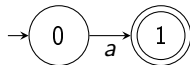
$\text{expr} = a|(b|c)$

arbre



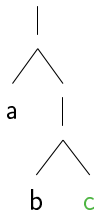
$i = 4$

AFN



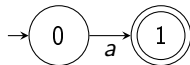
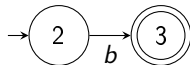
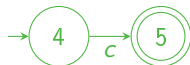
$\text{expr} = a|(b|c)$

arbre



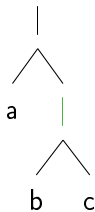
$i = 6$

AFN



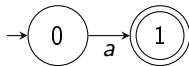
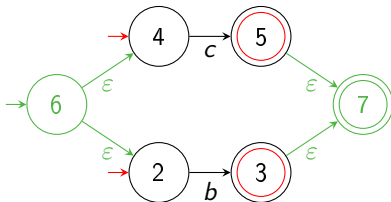
$\text{expr} = a|(b|c)$

arbre



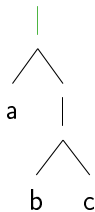
$i = 8$

AFN



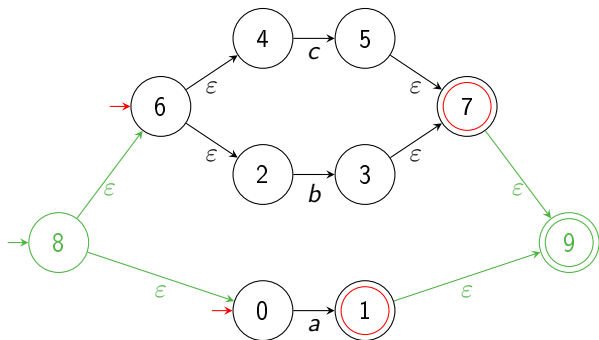
$\text{expr} = a|(b|c)$

arbre



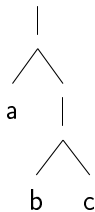
$i = 10$

AFN



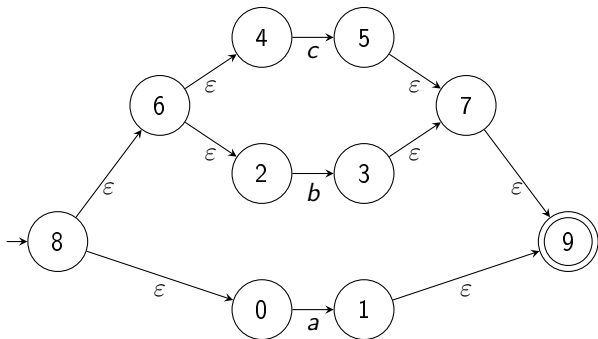
$\text{expr} = a|(b|c)$

arbre



$i = 10$

AFN



AFN final à 10 états

$\text{expr} = (ab)c$

$\text{expr} = (ab)c$

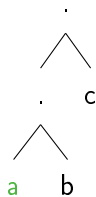
arbre



$i = 0$

$\text{expr} = (ab)c$

arbre



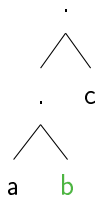
$i = 2$

AFN



$\text{expr} = (ab)c$

arbre



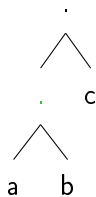
$i = 4$

AFN



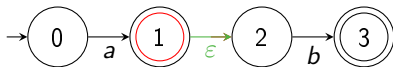
$\text{expr} = (ab)c$

arbre



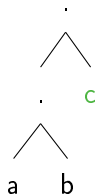
$i = 4$

AFN



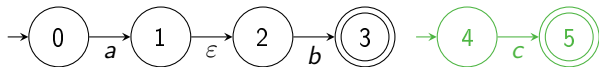
$$\text{expr} = (ab)c$$

arbre



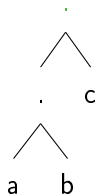
$$i = 6$$

AFN



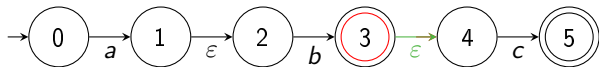
$$\text{expr} = (ab)c$$

arbre



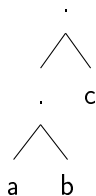
$$i = 6$$

AFN



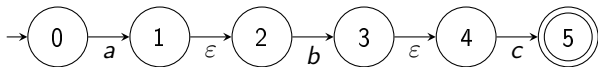
$$\text{expr} = (ab)c$$

arbre



$$i = 6$$

AFN



AFN final à 6 états

$\text{expr} = a(bc)$

$\text{expr} = a(bc)$

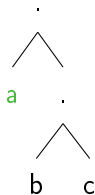
arbre



$i = 0$

$\text{expr} = a(bc)$

arbre



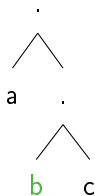
AFN



$i = 2$

$\text{expr} = a(bc)$

arbre



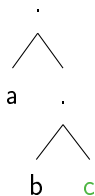
$i = 4$

AFN



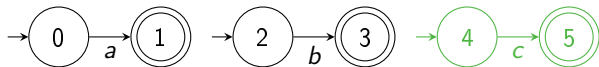
$\text{expr} = a(bc)$

arbre



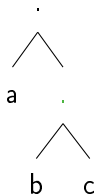
$i = 6$

AFN



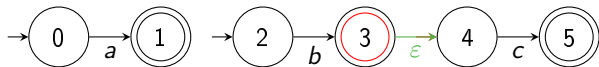
$\text{expr} = a(bc)$

arbre



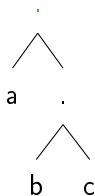
$i = 6$

AFN



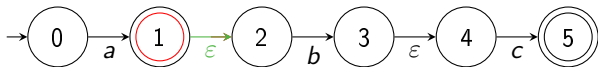
$\text{expr} = a(bc)$

arbre



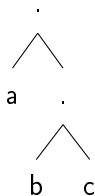
$i = 6$

AFN



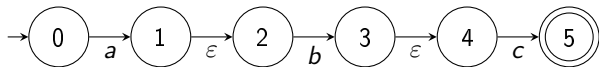
$\text{expr} = a(bc)$

arbre



$i = 6$

AFN



AFN final à 6 états

Les AFN des expressions régulières $(ab)c$ et $a(bc)$ sont strictement identiques. Cela montre que quels que soient a , b et c , la concaténation est associative. Il est donc possible de supprimer les parenthèses sans ambiguïté.

$$(ab)c = a(bc) = abc$$