Stability Analysis on Replicator Games with Perturbations

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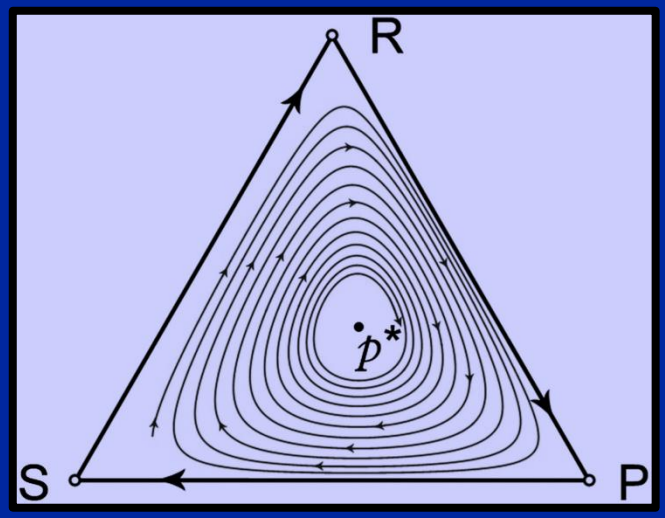
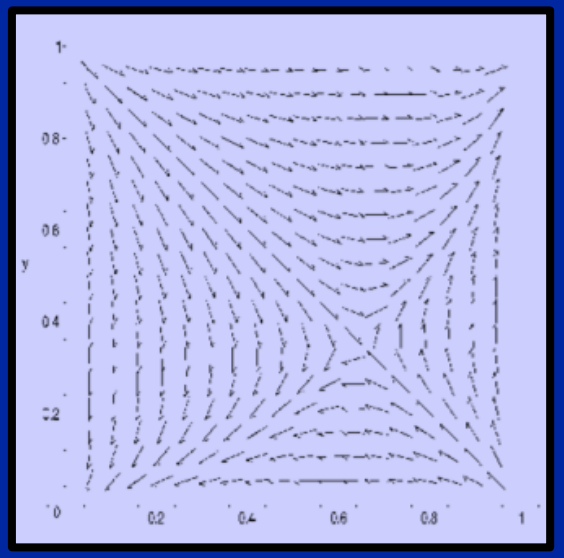
Motivation

- The oscillation of Shapley's Game (rock-paper-scissor game)
- The phase portrait of replicator dynamics
- Stability and smoothness with inclusion of perturbations (Gibbs entropy)





(Battle of Sexes)

Approach

- **Control theory approach – Stability Analysis**
  - What are the stability/asymptotic stability criteria?
  - Linearization around stationary points
  - State space analysis via eigenvalue decomposition
- **System modification under perturbation**
  - What are the effects/explanation of perturbation
  - Analysis on change of eigenvalues
  - MATLAB simulations
  - Using Gibbs entropy to modify payoffs

Tasks

- **Stability Analysis:**

  Meaning of stable/unstable stationary points.

  - 2x2 battle of sexes (saddle mixed strategy)
  - 2x2 matching pennies game (oscillatory)
  - 3x3 Shapley's game (oscillatory)
- **Stability with Gibbs entropy as Perturbation**
  - Discrete or continuous time game
  - Stability Criteria
- **An explanation for the perturbation**

Expected Results

- Description and meaning of:
  - Instability
  - Oscillations
- Demonstration of asymptotic stability with inclusion of perturbation
- Constraints on perturbation
- An physical explanation for the perturbation

Main References

- **Replicator Dynamics**
  - https://www.ma.imperial.ac.uk/~svanstri/GamesAndDynamics/The%20Replicator%20Dynamic%20(Draft).pdf
  - https://www.cs.ubc.ca/~kevinlb/teaching/cs532a%20-%202004-5/Class%20projects/Tim.pdf
  - https://www.pnas.org/doi/10.1073/pnas.1400823111
- **Smoothed Fictitious Play**
  - https://justinkang221.github.io/files/paper5.pdf
- **Prof. CCW's course on Linear Systems, NTU**