Stability Analysis on Replicator Games with Perturbations

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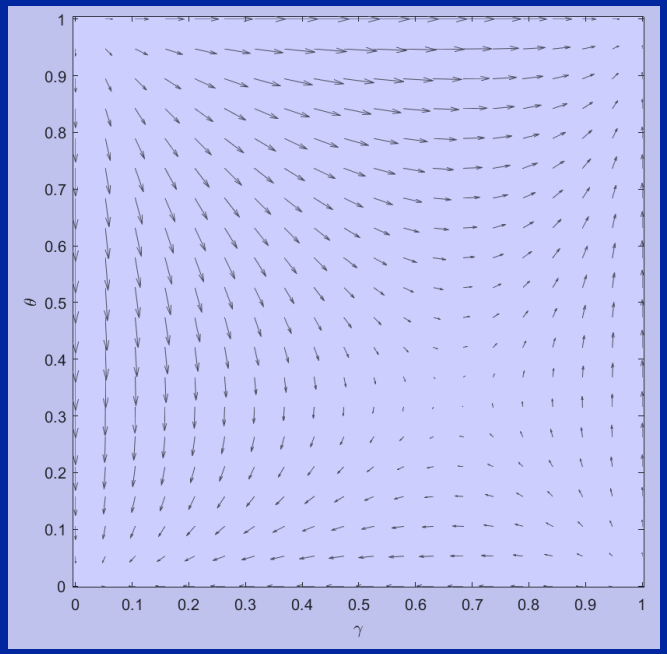
Motivation



- The oscillation of Shapley's Game (rock-paper-scissor game)
- The phase portrait of replicator dynamics $\frac{d}{dt}p_i^t = \alpha\beta \cdot p_i^t(u_{i\sigma}^t - \bar{u}_{\cdot\sigma}^t)$
- Convergence with inclusion of decreasing perturbations (Gibbs entropy)

Approach

- **Control theory approach – Stability Analysis**

  What are the stability/asymptotic stability criteria?

  – Linearization around stationary points

  – State space analysis via eigenvalue decomposition

    – Meaning of the game Nash equilibria.

- **System modification under perturbation**

  What are the effects/explanation of perturbation

  – Payoff modification via perturbations

  – MATLAB simulations

Tasks

- **Replicator Dynamics and Stability Analysis:**

  Meaning of stable/unstable stationary points.

  - 2x2 game with single Nash.
  - 2x2 the game of chicken (saddle mixed strategy, cold war).
  - 3x3 Shapley's game (oscillatory).

- **Stability with Gibbs entropy as Perturbation**
  - Reasoning and meaning.
  - Stability criteria.

Expected Results

- Description and meaning of:
  - Nash equilibrium in replicator dynamics.
  - Instability/Stability in replicator dynamics.
  - Oscillations
- Demonstration of asymptotic stability with inclusion of perturbation
- Constraints on perturbation
- An physical explanation for the perturbation

Main References

- **Replicator Dynamics**
  - https://www.ma.imperial.ac.uk/~svanstri/GamesAndDynamics/The%20Replicator%20Dynamic%20(Draft).pdf
  - https://www.cs.ubc.ca/~kevinlb/teaching/cs532a%20-%202004-5/Class%20projects/Tim.pdf
  - https://www.pnas.org/doi/10.1073/pnas.1400823111
- **Smoothed Fictitious Play**
  - https://justinkang221.github.io/files/paper5.pdf
- **Prof. CCW's course on Linear Systems, NTU**