G54FUZ/COMP4033 Coursework (2021/2022)

Designing and Tuning a Fuzzy Inference System

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Introduction

Five fuzzy inference systems will be described in this report about the effect of body temperature and headache level on the need to go to the hospital.
At the very beginning of the study, it was necessary to use terms to indicate the degree of headache and the different body temperatures. Based on the NHS and some academic literature, the system classifies the degree of headache, body temperature range, and the urgency of seeking medical attention into the following categories according to three terms or five terms.

Variables name	Terms numbers	Terms
Temperature	5	Hypothermia, Normal, Mild Fever, Severe Fever, Hyperpyrexia
	3	Hypothermia, Normal, Fever
Headache	5	No pain, Mild, Moderate, Severe, Excruciating
	3	Mild, Moderate, Severe
Urgency	5	No need, Mild, Medium, High, Emergency
	3	Mild, Medium, High

The next thing is to determine the range of each variable and terms. Based on the coursework instruction, I have ranged the three variables as [35,41], [0,10], and [0,100].
For functions like gbell and gauss, fis should specify the center of the function for for each term according to different numbers and variables. As shown in the picture below.

Variables name	Terms (5)	Center of function	Terms (3)	Center of function
Temperature	Hypothermia	35	Hypothermia	35
	Normal	37		
	Mild Fever	37.8	Normal	37
	Severe Fever	38.3	Fever	41
	Hyperpyrexia	41		
Headache	No Pain	0	Mild	0
	Mild	2.5		
	Moderate	5	Moderate	5
	Severe	7.5	Severe	10
	Excruciating	10		
Urgency	No Need	0	Mild	0
	Mild	20		
	Medium	50	Medium	50
	High	80	High	100
	Emergency	100		

Section 1 Description of Alternative Fuzzy Models

First FIS mamdani lightweight version

The first FIS to be created is a small mamdani system. The first time, I did not use five terms and used three terms for each variable instead. The aim of this mamdani system is to become familiar with the ideal output dataset and the nature of FIS system.
I chose Trapezoidal membership functions and generalized bell functions for this first FIS. The trapezoidal function requires four coordinates to be specified, the start of the function with a μ value of 0, the left and right of the shoulder, and the end of the function with a μ value of 0. The Mathematical formulae for gbell membership function is below.

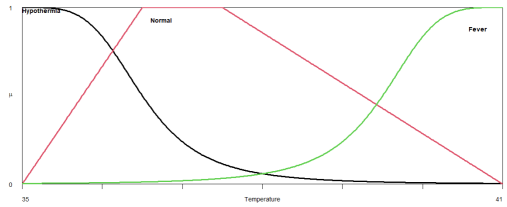
$$f(x;a,b,c) = \frac{1}{1+\left|\frac{x-c}{a}\right|^{2b}}$$

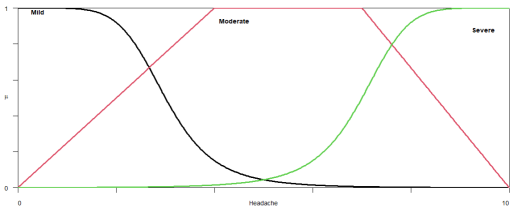
The generalized bell function depends on three parameters a, b and c. Each of these parameters has a physical meaning: c determines the centre of the corresponding membership function; a is the half-width; and b (together with a) controls the slopes at the crossover points. I need to specify the center and the width of the gbell function.

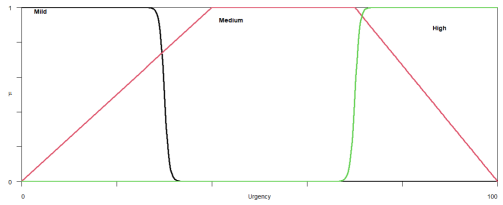
```
fis1 <- addmf(fis1, 'input', 1, 'Hypothermia', 'gbellmf', c(1.5,2, 35))
fis1 <- addmf(fis1, 'input', 1, 'Normal',      'trapmf', c(35,36.5,37.5,41))
fis1 <- addmf(fis1, 'input', 1, 'Fever',       'gbellmf', c(1.5,2,41))



fis1 <- addmf(fis1, 'input', 2, 'Mild',        'gbellmf', c(3,3, 0))
fis1 <- addmf(fis1, 'input', 2, 'Moderate',    'trapmf', c(0,4,7,10))
fis1 <- addmf(fis1, 'input', 2, 'Severe',      'gbellmf', c(3,3,10))



fis1 <- addmf(fis1, 'output', 1, 'Mild',      'gbellmf', c(30,30, 0))
fis1 <- addmf(fis1, 'output', 1, 'Medium',    'trapmf', c(0,40,70,100))
fis1 <- addmf(fis1, 'output', 1, 'High',      'gbellmf', c(30,30, 100))
```

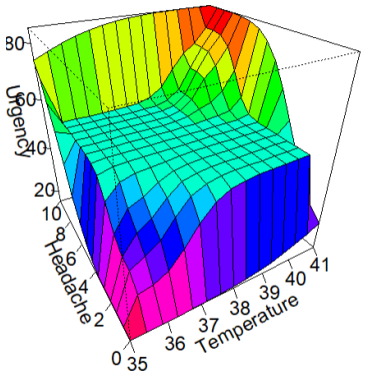
Next thing is to determine the rule set.
1. If (Temperature is Normal) and (Headache is Moderate) then (Urgency is Medium) (1)

2. If (Temperature is Fever) and (Headache is Severe) then (Urgency is High) (1)
3. If (Temperature is Hypothermia) and (Headache is Mild) then (Urgency is Mild) (1)
The membership function， GUI and surf figures for this FIS are showed below.







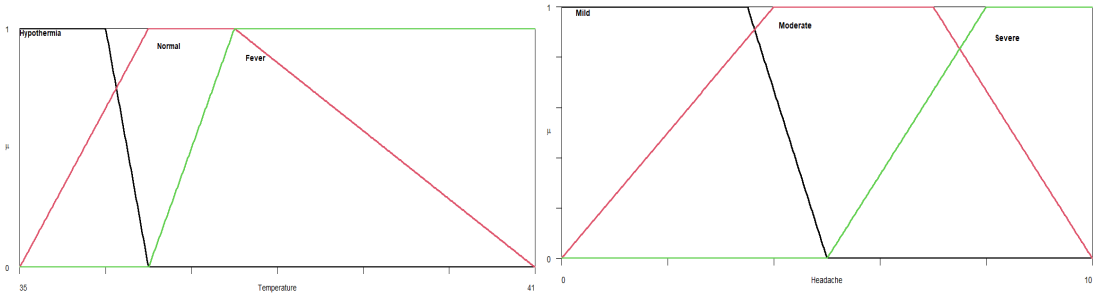


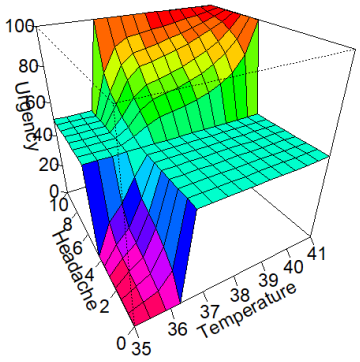
2nd FIS TSK Zeroth order

The second fis is a TSK Zeroth order one.
To created a TSK FIS, system need to specify the type of the system in function newfis().
Next thing is to specify the fuzzification method of the two input variables. System set the fuzzification method as singleton.fuzzification.
The Final thing is to specify the output membership function. Different from mamdani system, TSK system's output functions should be linear membership functions as it does not have the complex defuzzification process. Since this fis is a zeroth one, there is no need to specify the non-zero value for the p and q of the linear membership function, and only the r one needs to specify.
The rule sets stay the same as the former one.

```
   {r FIS2 TSK Zeroth order}
## FIS2

fis2 <- newfis('fis2',fisType = "tsk",andMethod = "prod")
fis2 <- addvar(fis2, 'input', 'Temperature', c(35, 41),fuzzification.method
<-"singleton.fuzzification")
fis2 <- addvar(fis2, 'input', 'Headache', c(0,10),fuzzification.method <-
"singleton.fuzzification")
fis2 <- addvar(fis2, 'output', 'Urgency', c(0, 100))

fis2 <- addmf(fis2, 'input', 1, 'Hypothermia', 'trapmf', c(35,35,36,36.5))
fis2 <- addmf(fis2, 'input', 1, 'Normal',      'trapmf', c(35,36.5,37.5,41))
fis2 <- addmf(fis2, 'input', 1, 'Fever',       'trapmf', c(36.5,37.5,41,41))

fis2 <- addmf(fis2, 'input', 2, 'Mild',        'trapmf', c(0,0,3.5,5))
fis2 <- addmf(fis2, 'input', 2, 'Moderate',    'trapmf', c(0,4,7,10))
fis2 <- addmf(fis2, 'input', 2, 'Severe',      'trapmf', c(5,8,10,10))


fis2 <- addmf(fis2, 'output', 1, 'Mild',      "linearmf", c(0, 0, 0))
fis2 <- addmf(fis2, 'output', 1, 'Medium',    "linearmf", c(50, 0, 0))
fis2 <- addmf(fis2, 'output', 1, 'High',      "linearmf", c(100, 0, 0))
```

The membership function，GUI and surf figures for this FIS are showed below.





The GUI can not show the figure for the linear membership function.

3rd FIS TSK First order

This fis introduces the sigmoid function for two terms: Fever and Severe.
The characteristic for sigmoid function is its S-shaped curve. The mathematic formulae for it is f(x) = 1/(1 + exp(−s ∗ (x − c))), while the curve coordinate is c for value of curve is 1.
The difference between zeroth TSK and First-order TSK is their output linear membership function. The fact that the p and q values of a linear function are no longer zero means that the linear function is no longer a vertical straight line but a line with a slope.
The rule sets stay the same as the former one.

```r
```{r FIS3 TSK First order}
fis3 <- newfis('fis3',fisType = "tsk")
fis3 <- addvar(fis3, 'input', 'Temperature', c(35, 41),fuzzification.method
<-"singleton.fuzzification")
fis3 <- addvar(fis3, 'input', 'Headache', c(0,10),fuzzification.method <-
"singleton.fuzzification")
fis3 <- addvar(fis3, 'output', 'Urgency', c(0, 100))

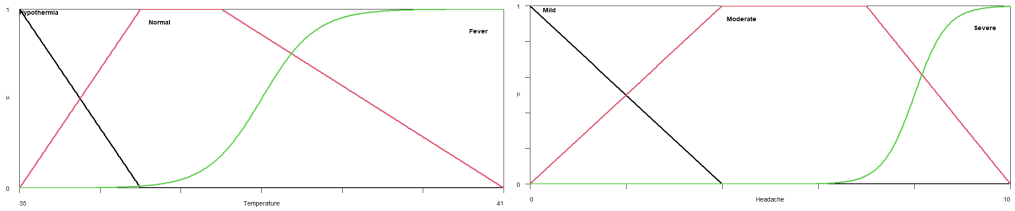
fis3 <- addmf(fis3, 'input', 1, 'Hypothermia', 'trimf', c(0,35,36.5))
fis3 <- addmf(fis3, 'input', 1, 'Normal',      'trapmf', c(35,36.5,37.5,41))
fis3 <- addmf(fis3, 'input', 1, 'Fever',       'sigmoidmf', c(3,38))

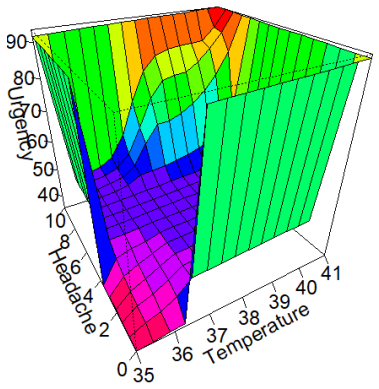
fis3 <- addmf(fis3, 'input', 2, 'Mild',        'trimf', c(0,0,4))
fis3 <- addmf(fis3, 'input', 2, 'Moderate',    'trapmf', c(0,4,7,10))
fis3 <- addmf(fis3, 'input', 2, 'Severe',      'sigmoidmf', c(3,8))



fis3 <- addmf(fis3, 'output', 1, 'Mild',      "linearmf", c(0, 1, 0.1))
fis3 <- addmf(fis3, 'output', 1, 'Medium',    "linearmf", c(30, 0.4, 0.4))
fis3 <- addmf(fis3, 'output', 1, 'High',      "linearmf", c(80, 0.3, 0.1))

rl  <- rbind(c(2,2,2,1,1), c(3,3,3,1,1),c(1,1,1,1,1))
```





4st FIS TSK First order

This FIS is the best fis and will be detailed in the next section.
Gauss membership functions are used in this FIS. Two parameters (standard deviation, centre) contained in gauss funtion while the first one can be considered as the width of the bell part of the gauss curve.
Five terms for each variable are used in this FIS while 17 rules are added.

5th FIS Mamdani complex version

This fis is created to test how accurate mamdani system can be.
The new function used here for variable headache is triangular membership function. The triangular membership function is easy to understand and construct, as I have specified where the highest point and the two legs of the triangle function are located.
This system used 15 terms totally like the former first-order TSK system and included
```

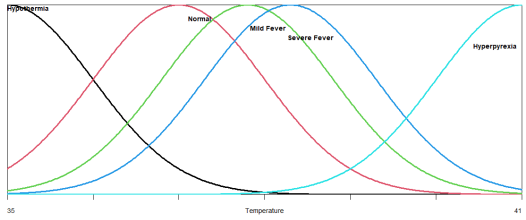
the same 17 rules.

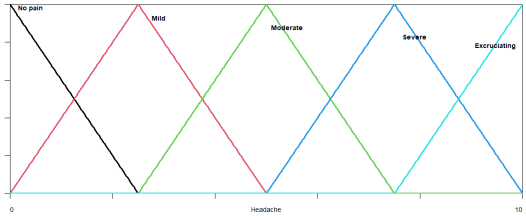
```
fis5 <- newfis('fis5')
fis5 <- addvar(fis5, 'input', 'Temperature', c(35, 41))
fis5 <- addvar(fis5, 'input', 'Headache', c(0,10))
fis5 <- addvar(fis5, 'output', 'Urgency', c(0, 100))

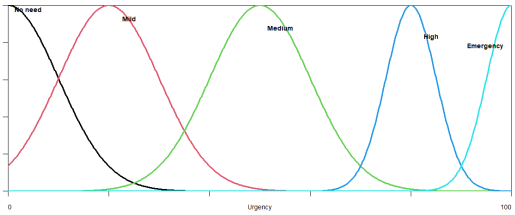
fis5 <- addmf(fis5, 'input', 1, 'Hypothermia', 'gaussmf', c(1, 35))
fis5 <- addmf(fis5, 'input', 1, 'Normal',      'gaussmf', c(1, 37))
fis5 <- addmf(fis5, 'input', 1, 'Mild Fever',  'gaussmf', c(1, 37.8))
fis5 <- addmf(fis5, 'input', 1, 'Severe Fever','gaussmf', c(1, 38.3))
fis5 <- addmf(fis5, 'input', 1, 'Hyperpyrexia','gaussmf', c(1, 41))

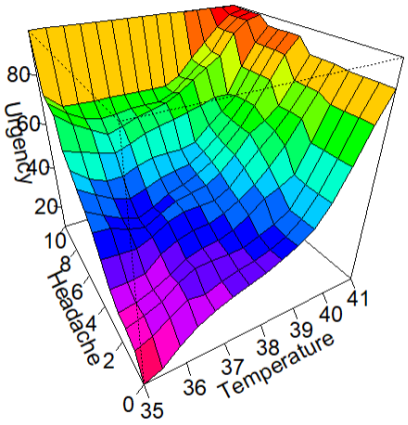
fis5 <- addmf(fis5, 'input', 2, 'No pain',      'trimf', c(0, 0, 2.5))
fis5 <- addmf(fis5, 'input', 2, 'Mild',         'trimf', c(0, 2.5, 5))
fis5 <- addmf(fis5, 'input', 2, 'Moderate',     'trimf', c(2.5, 5,7.5))
fis5 <- addmf(fis5, 'input', 2, 'Severe',       'trimf', c(5, 7.5, 10))
fis5 <- addmf(fis5, 'input', 2, 'Excruciating', 'trimf', c(7.5, 10, 10))

fis5 <- addmf(fis5, 'output', 1, 'No need',   'gaussmf', c(10, 0))
fis5 <- addmf(fis5, 'output', 1, 'Mild',      'gaussmf', c(10, 20))
fis5 <- addmf(fis5, 'output', 1, 'Medium',    'gaussmf', c(10, 50))
fis5 <- addmf(fis5, 'output', 1, 'High',      'gaussmf', c(5, 80))
fis5 <- addmf(fis5, 'output', 1, 'Emergency', 'gaussmf', c(5, 100))
```









Section 2 Detail of Final Fuzzy Model

This fis uses 15 terms in total. For each variable, there are five terms that describe the scope of that variable. The types and parameters of different membership functions are displayed in the chart below.

Variables name	Terms	Membership Function	1st parameter	2nd parameter	3rd parameter	Range
Temperature	Hypothermia	gaussmf	1	35		[35,41]
	Normal	gaussmf	1	37		
	Mild Fever	gaussmf	1	37.8		
	Severe Fever	gaussmf	1	38.3		
	Hyperpyrexia	gaussmf	1	41		
Headache	No Pain	gaussmf	1	0		[0,10]
	Mild	gaussmf	1	2.5		
	Moderate	gaussmf	1	5		
	Severe	gaussmf	1	7.5		
	Excruciating	gaussmf	1	10		
Urgency	No Need	linearmf	0	1	0.1	[0,100]
	Mild	linearmf	10	0.05	5	
	Medium	linearmf	30	0.4	0.3	
	High	linearmf	80	0.3	0.1	
	Emergency	linearmf	90	0.3	0.1	
FIS type	TSK		Andmethod	production		

The five different terms for variable temperature are hypothermia, normal, mild fever, severe fever, and hyperpyrexia. The five terms for headache are no pain, mild, moderate, severe, and excruciating. And the last five terms for urgency are no need, mild, medium, high, emergency.
This fis system use gauss membership for two input variables, Temperature and Headache, and linear membership for the ouput variable, urgency.
The gauss function is easy to set parameters because it only has two parameters, one is the center of the function, and the other one is the width of the bell part of curve. Mention the linear membership parameters, since it is linearly increasing, determined by the two parameters temperature and headache, care needs to be taken not to set the value of r to the maximum value of 100. The coefficients of the linear line are determined considering the effect of the slope on the line and the weights of the two input variables.

```
rl = rbind(c(1,1,1,1,1),c(1,2,2,1,1),c(1,3,2,1,1),c(1,4,3,1,1),
           c(2,1,1,1,1),c(2,2,2,1,1),c(2,3,2,1,1),c(2,4,3,2,1),
           c(3,1,2,1,1),c(3,2,2,1,1),c(3,3,3,1,1),c(3,4,3,1,1),
           c(4,1,3,1,1),c(4,2,3,1,1),c(4,3,4,1,1),c(4,4,5,1,1),
           c(5,5,5,1,2))

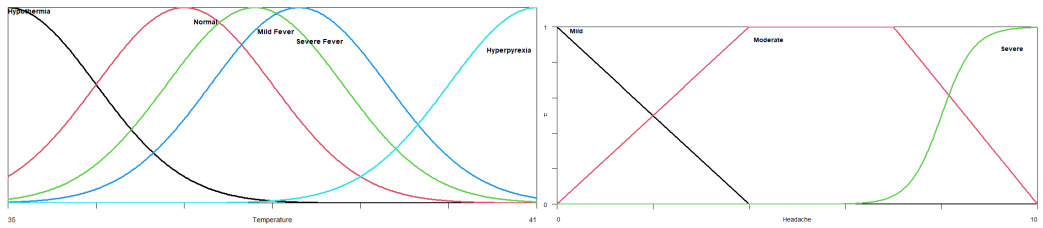
fis4 <- addrule(fis4, rl)
```

```
fis4 <- addmf(fis4, 'input', 1, 'Hypothermia', 'gaussmf', c(1, 35))
fis4 <- addmf(fis4, 'input', 1, 'Normal',      'gaussmf', c(1, 37))
fis4 <- addmf(fis4, 'input', 1, 'Mild Fever',  'gaussmf', c(1, 37.8))
fis4 <- addmf(fis4, 'input', 1, 'Severe Fever','gaussmf', c(1, 38.3))
fis4 <- addmf(fis4, 'input', 1, 'Hyperpyrexia','gaussmf', c(1, 41))

fis4 <- addmf(fis4, 'input', 2, 'No pain',      'gaussmf', c(1, 0))
fis4 <- addmf(fis4, 'input', 2, 'Mild',         'gaussmf', c(1, 2.5))
fis4 <- addmf(fis4, 'input', 2, 'Moderate',     'gaussmf', c(1, 5))
fis4 <- addmf(fis4, 'input', 2, 'Severe',       'gaussmf', c(1, 7.5))
fis4 <- addmf(fis4, 'input', 2, 'excruciating', 'gaussmf', c(1, 10))

fis4 <- addmf(fis4, 'output', 1, 'No need',   "linearmf", c(0, 1, 0.1))
fis4 <- addmf(fis4, 'output', 1, 'Mild',      "linearmf", c(10, 0.05, 5))
fis4 <- addmf(fis4, 'output', 1, 'Medium',    "linearmf", c(30, 0.4, 0.3))
fis4 <- addmf(fis4, 'output', 1, 'High',      "linearmf", c(80, 0.3, 0.1))
fis4 <- addmf(fis4, 'output', 1, 'Emergency', "linearmf", c(90, 0.3, 0.1))
```

The membership chart of two input parameters are shown below.

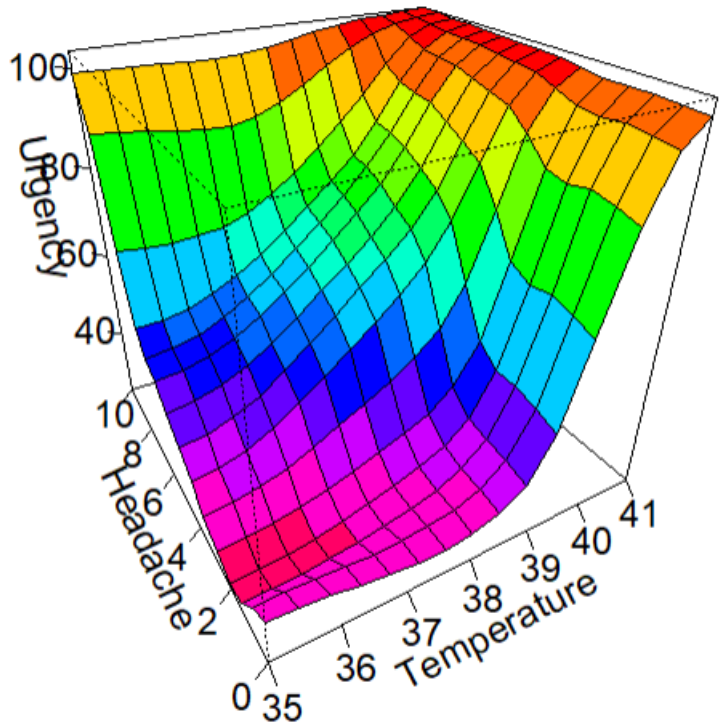


There are 17 rules set for this fis.

1. If (Temperature is Hypothermia) and (Headache is No pain) then (Urgency is No need) (1)
2. If (Temperature is Hypothermia) and (Headache is Mild) then (Urgency is Mild) (1)
3. If (Temperature is Hypothermia) and (Headache is Moderate) then (Urgency is Mild) (1)
4. If (Temperature is Hypothermia) and (Headache is Severe) then (Urgency is Medium) (1)
5. If (Temperature is Normal) and (Headache is No pain) then (Urgency is No need) (1)
6. If (Temperature is Normal) and (Headache is Mild) then (Urgency is Mild) (1)
7. If (Temperature is Normal) and (Headache is Moderate) then (Urgency is Mild) (1)
8. If (Temperature is Normal) and (Headache is Severe) then (Urgency is Medium) (2)
9. If (Temperature is Mild Fever) and (Headache is No pain) then (Urgency is Mild) (1)
10. If (Temperature is Mild Fever) and (Headache is Mild) then (Urgency is Mild) (1)
11. If (Temperature is Mild Fever) and (Headache is Moderate) then (Urgency is Medium) (1)

12. If (Temperature is Mild Fever) and (Headache is Severe) then (Urgency is Medium) (1)
13. If (Temperature is Severe Fever) and (Headache is No pain) then (Urgency is Medium) (1)
14. If (Temperature is Severe Fever) and (Headache is Mild) then (Urgency is Medium) (1)
15. If (Temperature is Severe Fever) and (Headache is Moderate) then (Urgency is High) (1)
16. If (Temperature is Severe Fever) and (Headache is Severe) then (Urgency is Emergency) (1)
17. If (Temperature is Hyperpyrexia) or (Headache is excruciating) then (Urgency is Emergency) (1)

Regarding the rule setting: at the beginning, I set 25 rules since there are two input variables and each variable has 5 terms, therefore the number of rules is 5x5 = 25. However, after setting this up successfully, there happened a problem that the surface was not flat, and there were depressions in the part of the surface where the headache was very low above 38 degrees. After several attempts to remove the depressions, it was found that 4x4+1 rules were needed, with the last rule being (5,1,5,1,5) to increase the weighting of severe headaches and high body temperatures on the range of output results.



FIS 4 output surface

In order to test the system, I created a dataset of input and output.

Dataset for testing		
Temperature	Headache	Urgency
35.5	1	30
35.8	3	45
36.0	3	30
36.5	8	60
36.7	1	10
36.7	3	20
36.9	6	30
36.8	8	70
37.3	1	10
37.8	3	30
38.3	1	50
38.2	6	70
40.1	1	80
40.2	5	90
40.6	10	100

The example of testing the accuracy is to use the RMSE method.
RMSE is a method used to measure the accuracy of models. The process of RMSE is calculating square root of the differences between the original and predicted values extracted by averaging the absolute difference over the data set.
The differences between the original dataset and predicted values are shown in the chart below.

	Output	Ideal output	Differences
1	28.59907	30	-1.400929
2	27.88588	45	-17.114117
3	28.38485	30	-1.615150
4	56.65460	60	-3.345397
5	28.41086	10	18.410861
6	30.98691	20	10.986905
7	53.20949	30	23.209489
8	58.65655	70	-11.343452
9	29.19850	10	19.198499
10	36.46406	30	6.464064
11	32.77173	50	-17.228273
12	66.65470	70	-3.345303
13	84.35280	80	4.352795
14	96.54637	90	6.546372
15	103.12954	100	3.129540

Section 3 Discussion of Final Fuzzy Model

In order to determine which FIS has the best performance, there are several different criteria for dicision-making process.

RMSE

RMSE method takes the differences and the weight of each input and output data into consideration. It is good to answer a question," how good should we expect the model to be?" obviously, the best performance of RMSE method measurement from FIS 4.

Running time

The less time the system runs, the better it is. FIS 2 and FIS 3 hav the least running time for their small number of terms and rules. Another thing can be concluded from the chart is mamnadi system's running time is usually less than TSK one. The method to record the running time is to call the function: proc.time() before the evalfis function predicts the value of input data, and recalls the proc.time() function again right after the completion of evalfis function.

Rules

The rules for the fis should not be too many or too few. Some light-weighted systems simply contain 3 or 4 rules, while some others contain too many rules, even up to 37. The rule number for FIS4 is 17, which is a reasonable amount.

Terms

If a variable's terms number is considerable, it means the range of this variable is divided into tiny realm: which makes system produce more accurate prediction.

Surface

The surface of FIS should be as smooth as possible. When there are breaks and jumps in the surface diagram, the connections between the logic within the system do not make sense.

FIS Number	RMSE	Running time	Rules	Terms	Surface
FIS1	22.14	0.029	3	9	Smooth
FIS2	25.33	0.01	3	9	Leap
FIS3	23.01	0.01	3	9	Leap
FIS4	12.18	0.029	17	15	Very Smooth
FIS5	14.57	0.029	17	15	Smooth

Conclusion

The best fuzzy inference system is FIS 4 so far due to its best performance on criteria RMSE, Rules, Terms, and Surface. However, there are still some problems with this system. For example, the system runs for period, and the number of terms could be increased to make the system more accurate in its predictions.