Report on "Smooth saddle-point representation of a non-smooth function"

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1 SP-MD

The Section 5.2 describes the saddle point mirror descent algorithm as applying mirror descent to a product space $Z = X \times Y$ using a specific mirror map $\Phi(z) = a\Phi_X(x) + b\Phi_Y(y)$. This approach utilizes a mirror map defined on the combined domain $D = D_X \times D_Y$, with coefficients a and b being positive real number that will be determined based on the problem's specific requirements. The vector filed $g : Z \to \mathbb{R}^n \times \mathbb{R}^m$ is defined based on the gradients with respect on the function ϕ at (x, y) positions.

The update formula for the algorithm is:

$$z_{t+1} \in \arg\min_{z \in Z \cap D} \left(\eta \langle g_t, z\rangle + D_\Phi(z, z_t)\right),$$

where g_t includes subgradients $g_{X,t}$ and $g_{Y,t}$ corresponding to the respective parts of the function ϕ. The implementation of the SP-MD algorithm is available at: https://github.com/WenbinWang024/Intern/blob/main/SP-MD.py. This algorithm applies the SP-MD method by transforming the problem into an iterative optimization over the product space $Z = X \times Y$ using a mirror map Φ. Here, Φ combines individual mirror maps on X and Y along with adjusted coefficients a and b, to accommodate the Lipschitz continuity properties of the two subspaces. The update step uses the Bregman divergence to measure distances between points, guiding the search direction for the next step.

2 SP-MP

The saddle point mirror prox is an advanced algorithm designed for scenarios where the function ϕ is smooth. The algorithm is detailed under the assumption that ϕ possesses certain smoothness properties defined by constants $\beta_{11}, \beta_{12}, \beta_{22}$ and β_{21}. These properties specify the smmothness of the partial derivatives of ϕ with respect to x and y over the domain X and Y. This method relies on smoothness to guarantee that the vector field g used in the algorithm maintains Lipschitz continuity under a suitably defined norm. The coefficients a and b in the mirror map $\Phi(z) = a\Phi_X(x) + b\Phi_Y(y)$ are chosen based on the relationship to the Lipschitz constants and some intrinsic measures of the sets X and Y, like their radius. The implementation of the SP-MP algorithm is available at: https://github.com/WenbinWang024/Intern/blob/main/SP-MP.py. This algorithm enhances the mirror descent method through a two-stage optimization process suitable for scenarios where the function φ is smooth. It begins by computing an intermediate point w_{t+1} which introduces an additional predictive step. Then, the main iteration point z_{t+1} is updated in the principal step. Each step involves solving an optimization problem to minimize a modified objective function that includes the gradients of φ and the Bregman divergence D_Φ. This method is designed to leverage the smoothness of φ to enhance the stability and convergence speed of the algorithm.

3 Application

The implementations of theapplications are available at: https://github.com/WenbinWang024/Intern/tree/main.