

Project for Course: Case Studies in Machine Learning and Finance

Comparing Stability of CAPM Beta and ERoD Beta Using Rolling Window Test

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Abstract

This project explores the stability of Capital Asset Pricing Model (CAPM) beta and Expected Regret of Drawdown (ERoD) beta by applying the rolling window test across two different time periods: from January 1, 2000, to January 1, 2024, and during the financial crisis from January 1, 2007, to January 1, 2009. Both CAPM beta and ERoD beta are crucial in measuring risk and the return of the Capital Asset Pricing Model, with ERoD beta incorporating a drawdown based adjustment. The analysis is conducted using the data from SP&500 with the selected notable stocks from different industries. This project contributes to the understanding of how CAPM beta and ERoD beta performs under different market trends and provides the insights of investment strategies and portfolio management.

1 Introduction

1.1 Problem Statement

This project will examine the stability of CAPM beta and ERoD beta through the rolling window test. Since ERoD beta serves the same function as the CAPM beta in the CAPM with drawdown measure, the beta with more stability will provide us with a more stable measure for CAPM or CAPM with drawdown measure. We will be comparing the beta for 2 time periods. Period 1: from 2000/01/01 to 2024/01/01. Period 2: 2007/01/01 to 2009/01/01. The goal of this project is to identify the stability of both betas in a long time period and in a time period where the market is going down such as financial crises.

1.2 Drawdown

Drawdown is a key factor in portfolio management, it measures the difference between the most recent peak of the portfolio's cumulative returns and the current portfolio's cumulative return. The definition of drawdown (Ding and Uryasev (2022)) can be written as:

Suppose that $\{r_t\}_{1 \leq t \leq T}$ is a sample path of scalar returns of some instrument. Let us denote:

$\{w_t\}_{1 \leq t \leq T}$ = vector of uncompounded cumulative returns.

$$w_t = \sum_{v=1}^t r_v, \quad 1 \leq t \leq T.$$

$\{d_t\}_{1 \leq t \leq T}$ = vector of drawdowns,

$$d_t = \max_{1 \leq v \leq t} \{w_v\} - w_t, \quad 1 \leq t \leq T$$

For every time moment t , d_t is the difference between the previous cumulative return peak and the current cumulative return.

1.3 Expected Regret of Drawdown (ERoD)

In this project, we consider using the Expected Regret of Drawdown (ERoD) as the drawdown. The definition of ERoD (Ding and Uryasev (2022)) can be written as:

Let's denote by $w(x)$ the vector of cumulative returns of a portfolio with weights vector x and

$D(w(x))$ be the random drawdown value for the portfolio x .

$$ERoD_{\varepsilon}(w(x)) = E[(D(w(x)) - \varepsilon)^+]$$

If we want to calculate the average of all positive drawdowns, we can set ε to a sufficiently small threshold. In this project, we will be looking at all positive drawdowns and drawdowns that are equal to or larger than 0%, $\varepsilon = 0$.

$ERoD_{\varepsilon}(w(x))$ with threshold ε can be calculated as follows:

Let's denote

- $x = (x^1, \dots, x^I)$ = vector of weights for n assets in the portfolio;
- $(w_{st}^1, \dots, w_{st}^I)$ = vector of uncompounded cumulative returns of portfolio assets at time moment t

- on scenario s ;
- p_s probability of the scenario (sample path of returns of securities);
- $w_{st}(x) = \sum_{i=1}^I w_{st}^i x^i$ = cumulative portfolio return at time moment t on scenario s ;
- $w(x)$ = vector of cumulative portfolio returns with components $w_{st}(x)$, $s=1, \dots, S$; $t=1, \dots, T$;
- $d_{st}(x) = \max_{1 \leq v \leq t} \{w_{sv}(x)\} - w_{st}(x)$ = drawdown of portfolio at time t on sample path s .

$$ERoD_{\epsilon}(w(x)) = \frac{1}{T} \sum_{s=1}^S \sum_{t=1}^T p_s (d_{st}(x) - \epsilon)^+.$$

This project considers a single path, in this case, path $s=1$ and $p_s = 1$.

1.4 CAPM and CAPM With Drawdown Measure

The Capital Pricing Model (CAPM) is a financial model that estimates the expected return on an investment based on its risk relative to the overall market, based on the Module_3_Slides (Uryasev), the definition of CAPM and CAPM beta can be written as:

$$E_{rj} = r_0 + \beta_j (E_{rM} - r_0), \quad \beta_j = \frac{cov(r_j, r_M)}{\sigma_M^2}$$

- E_{rj} = expected return of asset j ;
- r_0 = risk-free rate;
- β_j = beta of asset j ;
- E_{rM} = expected return of market;
- $E_{rM} - r_0$ = market risk premium.

With the measurement of ERoD, the CAPM can be expressed in the drawdown measure, the definition of CAPM with drawn down measure ((Ding and Uryasev (2022))) can be written as:

- $$\sum_{s=1}^S p_s w_{sT}^i = \beta_{ERoD}^i \sum_{s=1}^S p_s w_{sT}^M, \quad \hat{\beta}_{ERoD}^i = \frac{\frac{1}{T} \sum_{s=1}^S \sum_{t=1}^T p_s q_{st}^* (w_{s,\tau(s,t)}^i - w_{st}^i)}{\overline{E}_{\epsilon}(w^M)}$$
- $\overline{E}_{\epsilon}(w^M) = \frac{1}{T} \sum_{s=1}^S \sum_{t=1}^T p_s q_{st}^* (w_{s,\tau(s,t)}^M - w_{st}^M)$;
 - β_{ERoD}^i = ERoD Beta relating the total expected cumulative return, $\sum_{s=1}^S p_s w_{sT}^M$, of the optimal portfolio (market) and total expected cumulative return, $\sum_{s=1}^S p_s w_{sT}^i$;
 - $\tau(s, t) = \max\{K | 1 \leq k \leq t, w_{sk}^M = \max_{1 \leq \zeta \leq t} w_{s\zeta}^M\}$;

- $d_{st}^M = w_{s,\tau(s,t)}^M - w_{st}^M$ = drawdowns of the optimal portfolio;
- $q_{st}^* = 1(d_{st}^M \geq \epsilon)$ = indicator function which is equal to 1 for $d_{st}^M \geq \epsilon$, and 0 otherwise.

In this project, we consider a single path:

$$w_T^i = \beta_{ERoD}^i w_T^M, \quad \hat{\beta}_{ERoD}^i = \frac{\frac{1}{T} \sum_{t=1}^T p_s(w_{\tau(t)}^i - w_t^i)}{\overline{E_\epsilon(w^M)}}$$

- $\overline{E_\epsilon(w^M)} = \frac{1}{T} \sum_{t=1}^T q_{st}^* (w_{\tau(t)}^M - w_t^M) = \text{ERoD with threshold } \epsilon \text{ for return } w^M$;
- $q_{st}^* = 1(d_t^M \geq \epsilon) = \text{indicator for drawdowns } d_t^M = w_{\tau(t)}^M - w_t^M$;
- $\tau(t) = \max\{K | 1 \leq k \leq t, w_k^M = \max_{1 \leq \zeta \leq t} w_\zeta^M\}$.

1.5 CAPM Beta and ERoD Beta

CAPM beta for a security relates its expected return to the market's expected excess return over the risk-free rate, the ERoD Beta relates the average instrument losses during market drawdowns exceeding threshold over average market drawdowns exceeding threshold. In the CAPM and CAPM with drawdown measure, CAPM beta and ERoD Beta they both serve as the estimator. In this project, we will examine their stability and test which beta is more stable during different time periods.

2 Methodology

The approach to the solution is completely done in python with libraries: yfinance, pandas, numpy, and matplotlib. The coding will be in general, we will be able to change the starting time, ending time, tickers, threshold, and rolling window size easily by changing the content of the variables since we are going to repeat the process for different time periods and different thresholds to compare the stability of CAPM beta and ERoD beta through the rolling window test. The rest of the coding can be kept the same for two different periods. The tickers that we are using are: 'AAPL', 'JNJ', 'PG', 'XOM', 'JPM', 'KO', 'MSFT', 'UNH', 'MMM', '^GSPC', the '^GSPC' will serve as the market in this project.

Steps for rolling window test for CAPM beta:

- Step1: Import all the necessary libraries: yfinance, pandas, and matplotlib;
- Step 2: Define tickers and choose the tickers that we want to use for comparing the stability of CAPM beta and ERoD beta;
- Step 3: Download the data with the time periods that we want (starting time, ending time) to consider for comparing the stability of CAPM beta and ERoD beta;
- Step 4: Extract the data for adjusted closing price of the tickers;
- Step 5: Define the function for calculating CAPM Beta (1.4 CAPM Beta and ERoD Beta).
- Step 6: Create an empty dataframe.
- Step 7: Perform the rolling window test for CAPM Beta with a specific rolling window size that we want to use for the test.
- Step 8: Save the results from step 7 to the empty data frame created in step 6.
- Step 9: Plot the graphs of rolling window tests for CAPM beta.
- Step 10: Calculate the variances for CAPM betas

Steps for rolling window test for ERoD beta:

- Step1: Import all the necessary libraries: yfinance, numpy, pandas, and matplotlib;
- Step 2: Define tickers and choose the same tickers that we decided for CAPM beta.
- Step 3: Download the data with the time periods that we want (starting time, ending time) to consider for comparing the stability of CAPM beta and ERoD beta;
- Step 4: Extract the data for adjusted closing price of the tickers;
- Step 5: Define the function for calculating ERoD Beta (1.4 CAPM Beta and ERoD Beta) with the threshold that we want to calculate.
- Step 6: Create an empty dataframe.
- Step 7: Perform the rolling window test for ERoD beta with the same rolling window size for CAPM beta.
- Step 8: Save the results from step 7 to the empty data frame created in step 6.
- Step 9: Plot the graphs of rolling window tests for ERoD beta.
- Step 10: Calculate the variance for ERoD betas.

The next step for this project is to focus on comparing two betas with visualization.

- Plot stock's CAPM beta and ERoD beta separately.
- Combine all the rolling CAPM beta in one graph.
- Combine all the rolling ERoD beta in one graph.
- Combine each ticker's CAPM beta and ERoD in one graph.

We are going to repeat all the steps for different time periods and thresholds.

In the first run we are going to compare the stability of CAPM beta and ERoD beta from 2000-01-01 to 2024-01-01, with threshold $\varepsilon=0$ and rolling window size of 252.

In the second run, we are going to change the time period from 2007-01-01 to 2009-01-01, with the same threshold and rolling window size. It is important to see how the betas perform over a long time period and during time periods where markets are going down.

3 Experiment / Results

Provides data, graphical and statistical results for rolling window tests of CAPM beta and ERoD beta.

3.1 Data

The initial data is sp&500 index with tickers: 'AAPL', 'JNJ', 'JPM', 'KO', 'MMM', 'MSFT', 'PG', 'UNH', 'XOM', '^GSPC', from 2000/01/01 to 2024/01/01.

The data includes the dates, Open, High, Low, Close, Adjusted Close, and Volume for the tickers. For the rolling window test of CAPM beta and ERoD beta, we are only considering using Adjusted Close Price with dates for the tickers, therefore, we are going to extract the Adjusted Close Price with corresponding dates and tickers and drop other data. We are going to extract it two times, the first time is from 2000/01/01 to 2024/01/01, and the second time is from 2007/01/01 to 2009/01/01.

Figure 1 is the extracted data of adjusted closing price for the tickers from 2000/01/01 to 2024/01/01.

Figure 2 is the extracted data of adjusted closing price for the tickers from 2007/01/01 to 2009/01/01.

(Figure 1: Adjusted Closing Price for the Tickers From 2000/01/01 to 2024/01/01)

Ticker	AAPL	JNJ	JPM	KO	MMM \
Date					
2000-01-03	0.846127	24.698620	23.776054	14.434139	17.557297
2000-01-04	0.774790	23.794405	23.254374	14.450142	16.859659
2000-01-05	0.786128	24.045578	23.110825	14.578156	17.348019
2000-01-06	0.718097	24.799088	23.438942	14.594159	18.743296
2000-01-07	0.752113	25.854008	23.869564	15.554296	19.115364
...
2023-12-22	193.353287	154.288544	165.409225	57.857216	87.167412
2023-12-26	192.803986	154.963409	166.387451	58.095314	88.626625
2023-12-27	192.903839	155.171844	167.385437	58.244122	89.143089
2023-12-28	193.333298	155.400101	168.274734	58.283806	89.921883
2023-12-29	192.284637	155.558899	168.077118	58.462376	89.618561

Ticker	MSFT	PG	UNH	XOM	^GSPC
Date					
2000-01-03	36.065571	28.069103	5.400593	18.328701	1455.219971
2000-01-04	34.847260	27.529005	5.331516	17.977631	1399.420044
2000-01-05	35.214703	27.005247	5.318956	18.957701	1402.109985
2000-01-06	34.035065	28.249125	5.513628	19.937754	1403.449951
2000-01-07	34.479851	30.507757	6.160444	19.879248	1441.469971
...
2023-12-22	373.888580	143.442017	518.266907	100.971611	4754.629883
2023-12-26	373.968445	144.093689	517.988037	101.199486	4774.750000
2023-12-27	373.379547	144.212158	520.737183	100.723907	4781.580078
2023-12-28	374.587280	143.886322	522.838928	99.267441	4783.350098
2023-12-29	375.345886	144.686081	524.402710	99.059380	4769.830078

[6037 rows x 10 columns]

(Figure 2: Adjusted Closing Price for the Tickers From 2007/01/01 to 2009/01/01)

Ticker	AAPL	JNJ	JPM	KO	MMM	MSFT \
Date						
2007-01-03	2.533751	40.151031	30.657494	14.311197	34.797928	21.397421
2007-01-04	2.589991	40.652912	30.734028	14.317080	34.660091	21.361603
2007-01-05	2.571546	40.284061	30.478922	14.216932	34.424438	21.239777
2007-01-08	2.584245	40.217552	30.580956	14.308254	34.500000	21.447590
2007-01-09	2.798918	40.066372	30.453409	14.320037	34.540031	21.469086
...
2008-12-24	2.571244	37.360832	20.186495	13.710385	26.233116	14.180634
2008-12-26	2.594525	37.360832	20.152681	13.859618	26.346539	14.151044
2008-12-29	2.618714	37.099251	20.139156	13.806764	26.171669	14.025287
2008-12-30	2.609038	37.749996	20.970968	13.968424	27.022490	14.306395
2008-12-31	2.580616	38.171059	21.322617	14.074129	27.197371	14.380359

Ticker	PG	UNH	XOM	^GSPC
Date				
2007-01-03	39.077045	42.389317	40.657738	1416.599976
2007-01-04	38.780380	42.663479	39.895157	1418.339966
2007-01-05	38.447380	42.373207	40.180443	1409.709961
2007-01-08	38.532139	42.994076	39.856770	1412.839966
2007-01-09	38.435265	42.478020	39.549538	1412.109985
...
2008-12-24	38.216614	20.962343	43.098732	868.150024
2008-12-26	38.305222	21.236891	43.900646	872.799988
2008-12-29	38.096375	20.647425	44.372692	869.419983
2008-12-30	38.678574	21.834433	44.696873	890.640015
2008-12-31	39.121567	21.479141	45.402100	903.250000

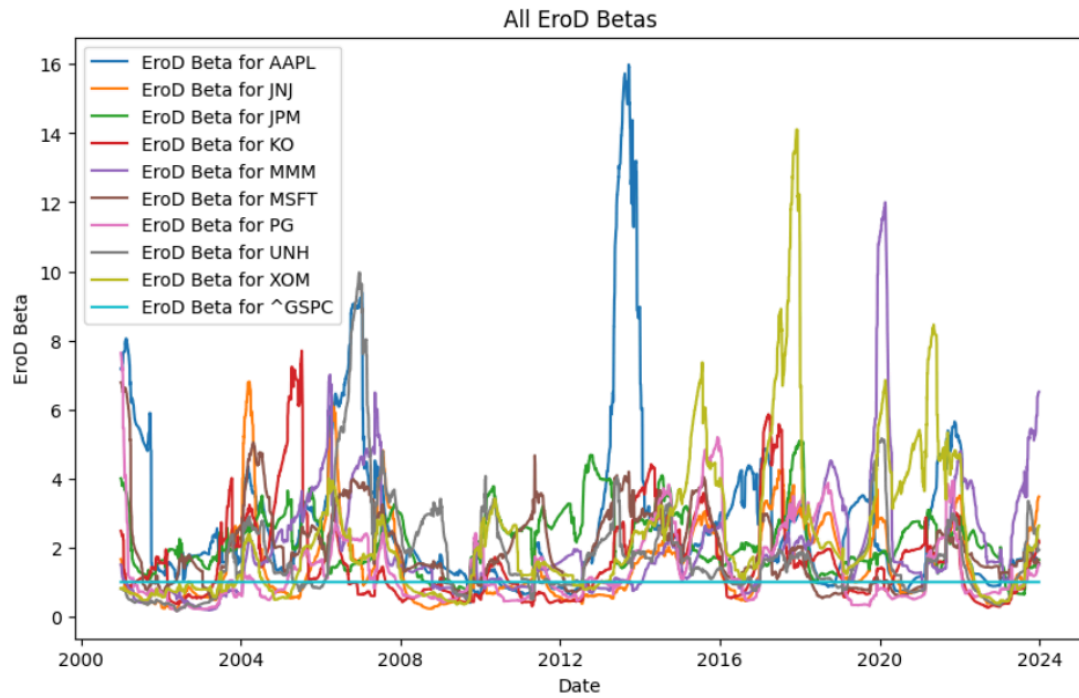
[504 rows x 10 columns]

3.2 Graphical Results

Figure 3 is all the tickers' rolling ERoD betas from 2000/01/01 to 2024/01/01.

Figure 4 is all the tickers' rolling CAPM betas from 2000/01/01 to 2024/01/01.

(Figure 3)



(Figure 4)

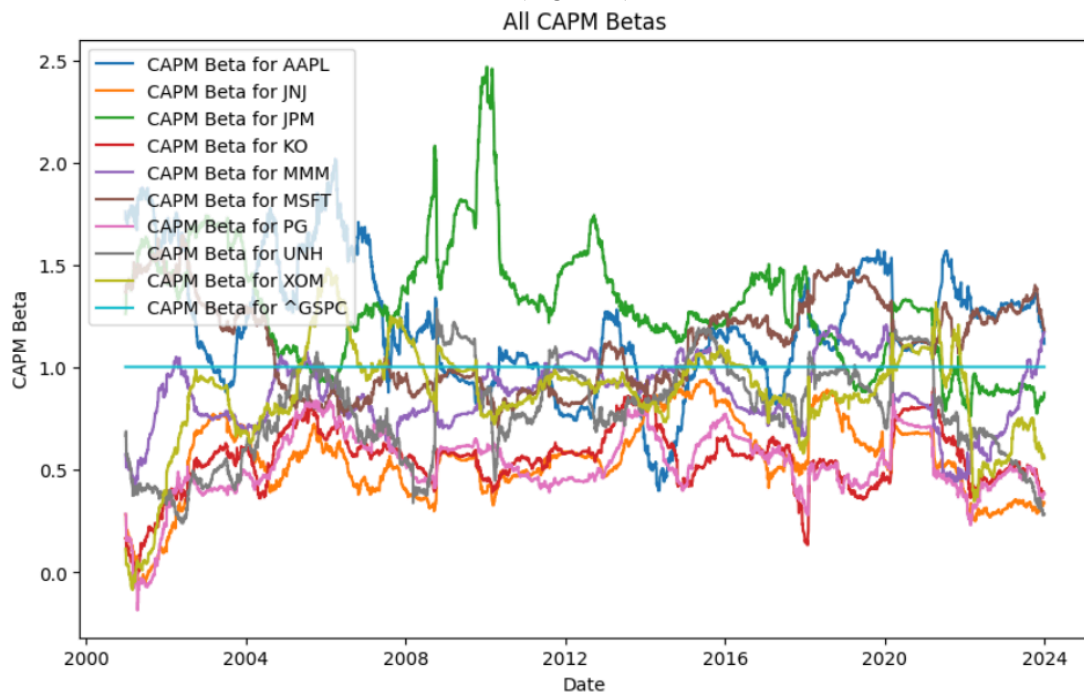
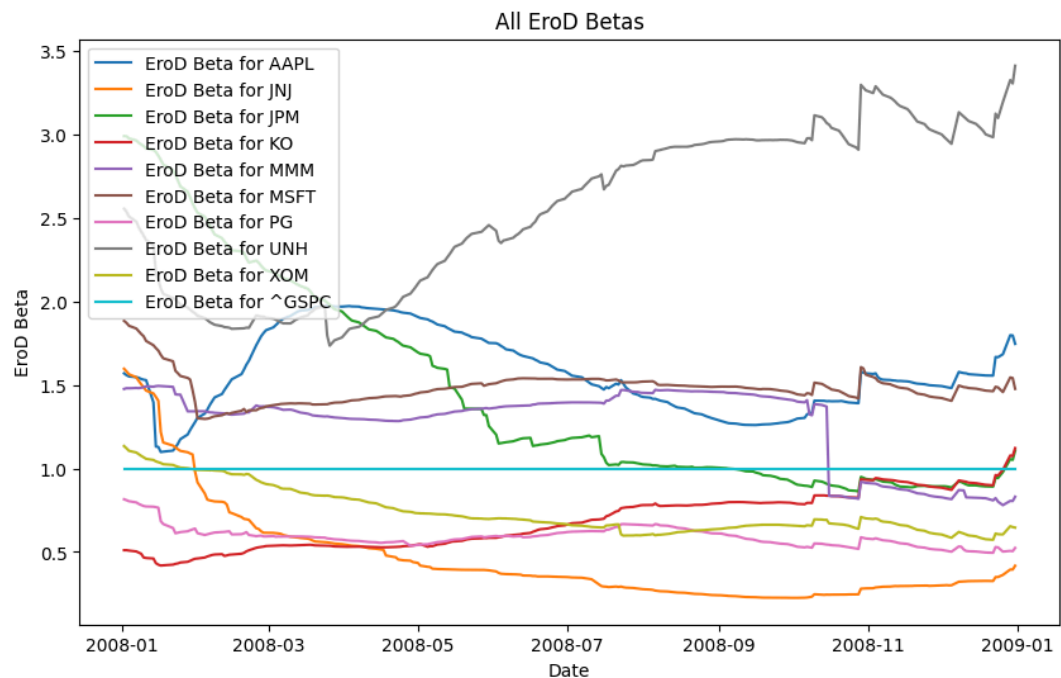
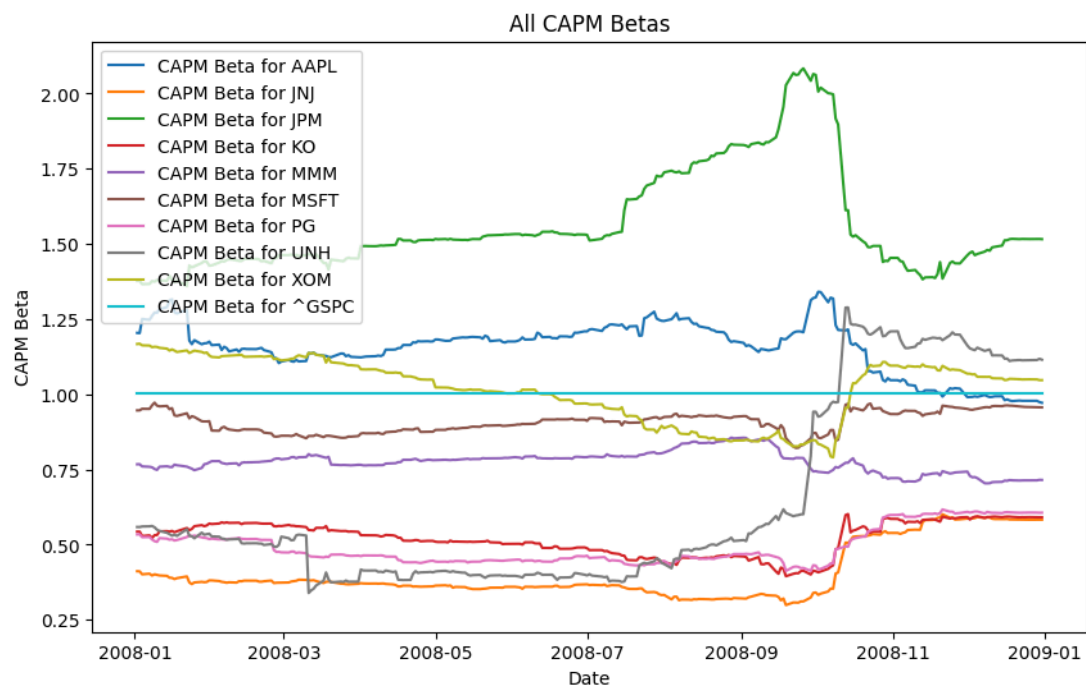


Figure 5 is all the tickers' rolling EROD betas from 2007/01/01 to 209/01/01.
 Figure 6 is all the tickers' rolling CAPM betas from 2007/01/01 to 2009/01/01.

(Figure 5)

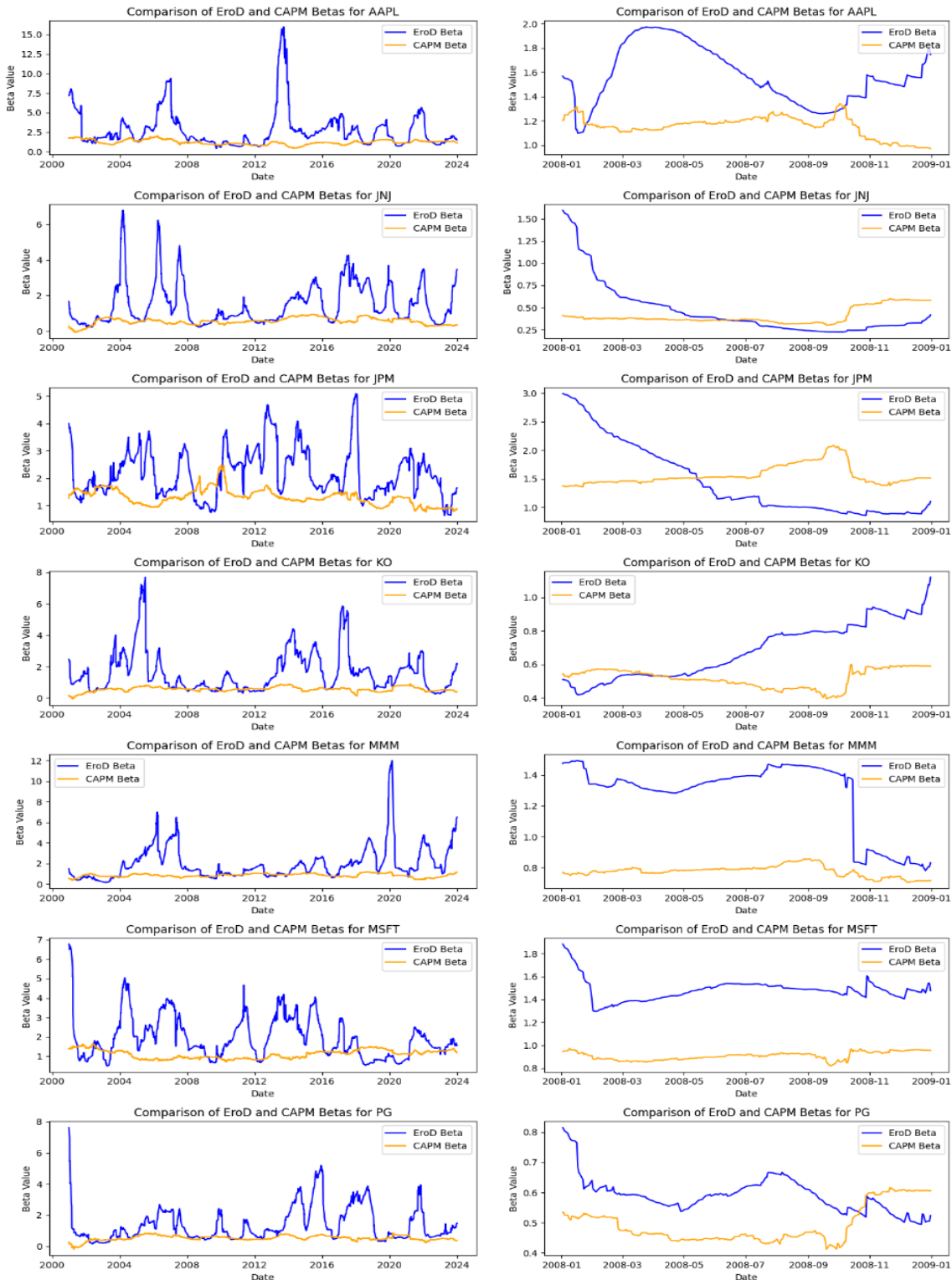


(Figure 6)



The figures on the left side is the comparison of rolling CAPM beta with rolling ERoD beta from 2000/01/01 to 2024/01/01 for some notable tickers that we computed.

The figures on the right side is the comparison of rolling CAPM beta with rolling ERoD beta from 2007/01/01 to 2009/01/01 for some notable tickers that we computed.



3.3 Statistical Results

Table 1 provides the variances for rolling CAPM beta and ERoD beta from 2000/01/01 to 2024/01/01
Table 2 provides the variances for rolling CAPM beta and ERoD beta from 2007/01/01 to 2009/01/01

(Table 1)

	CAPM Beta Variance	EroD Beta Variance
Ticker		
AAPL	0.094103	5.880929
JNJ	0.038017	1.331109
JPM	0.084197	0.750909
KO	0.022278	1.64258
MMM	0.027663	2.597611
MSFT	0.048648	1.181238
PG	0.026244	1.161085
UNH	0.053755	2.100743
XOM	0.060029	4.219792
^GSPC	0.0	0.0

(Table 2)

	CAPM Beta Variance	EroD Beta Variance
Ticker		
AAPL	0.007291	0.05643
JNJ	0.007976	0.091628
JPM	0.030241	0.393462
KO	0.003021	0.027011
MMM	0.001281	0.051027
MSFT	0.001224	0.008858
PG	0.003598	0.00352
UNH	0.095129	0.236647
XOM	0.010582	0.019145
^GSPC	0.0	0.0

4 Conclusion

In this project, we conducted the rolling window test for CAPM beta and ERoD beta for a long time period and the time period where the market is going down. It is important to see how stable the betas are during different market trends since the ERoD beta only considers the time periods where market has drawdown but the CAPM beta considers all time intervals.

In the rolling window test, we used 9 notable stocks from different industries and the SP&500 index as the market in two betas' calculations. In addition, the rolling window is 252 which is approximately the number of trading days in one year for two betas' rolling window tests.

Based on the rolling window tests for CAPM beta and ERoD beta during the time period from 2000/01/01 to 2024/01/01, CAPM beta performs more stable than ERoD beta. In the time periods from 2007/01/01 to 2009/01/01 which is the financial crisis, the variances of both betas decreased, especially for ERoD beta. The variance of ERoD beta decreased on a much larger scale compared to the CAPM beta. It suggests that the ERoD beta is more stable during the time periods when the market is going down compared to a longer time period. However, during this period, the variance of CAPM beta is still smaller than the variance of ERoD beta even though the variance of CAPM beta did not decrease that much. Overall, this project suggests that the CAPM beta provides a more stable relation between the stock's return and the market's return and the CAPM beta is a more stable estimator compared to ERoD beta in the CAPM.

References

Ding, R., & Uryasev, S. (2022). Drawdown beta and portfolio optimization. *Quantitative Finance*, 22(7), 1265–1276. <https://doi.org/10.1080/14697688.2022.2037698>

Uryasev, S. *Module 3 Slides: Calibrating Risk Preferences* [PowerPoint slides]. Stony Brook University.

Appendices

Coding in Python for this project

```
import yfinance as yf
import pandas as pd
import matplotlib.pyplot as plt
import numpy as np

#data
tickers = ['AAPL', 'JNJ', 'JPM', 'KO', 'MMM', 'MSFT', 'PG', 'UNH', 'XOM', '^GSPC']
data = yf.download(tickers, start="2000-01-01", end="2024-01-01")
print(data)

# Adjusted Closing Price
adj_close = data['Adj Close']
print(adj_close)

# Function to calculate ERoD Beta
def Calculate_EroD_Beta(Market_Return_Series, Stock_Return_Series):
    #daily returns
    stock_dreturns = Stock_Return_Series.pct_change()
    market_dreturns = Market_Return_Series.pct_change()

    #Uncompounded cumulative returns
    stock_uqr = stock_dreturns.cumsum()
    market_uqr = market_dreturns.cumsum()

    #Calculate 2 drawdown series
    market_drawdowns = market_uqr.cummax() - market_uqr
    stock_drawdowns = stock_uqr.cummax() - stock_uqr

    #compute and return beta (selected when drawdown exists for market first)
    threshold = market_drawdowns > 0
    market_drawdowns_selected = market_drawdowns[threshold]
    stock_drawdowns_selected = stock_drawdowns[threshold]
    beta = stock_drawdowns_selected.mean() / market_drawdowns_selected.mean()
    #return beta
    return beta
```

```
# Function to calculate CAPM Beta
def Calculate_CAPM_Beta(Market_Return_Series, Stock_Return_Series):
    # Calculate daily returns
    stock_returns = Stock_Return_Series.pct_change().dropna()
    market_returns = Market_Return_Series.pct_change().dropna()

    # Align the lengths of the return series
    common_dates = stock_returns.index.intersection(market_returns.index)
    stock_returns = stock_returns.loc[common_dates]
    market_returns = market_returns.loc[common_dates]

    # Calculate covariance between stock returns and market returns
    covariance = np.cov(stock_returns, market_returns)

    # The covariance between the stock and the market is the [0,1] element
    cov_stock_market = covariance[0, 1]

    # Calculate the variance of the market returns
    var_market = np.var(market_returns)

    # Beta is the ratio of covariance to variance
    beta = cov_stock_market / var_market
    return beta
```

```
# Rolling window size and creating a empty dataframe
window_size = 252
drawdown_betas = pd.DataFrame(index=adj_close.index[window_size-1:], columns=adj_close.columns)
capm_betas = pd.DataFrame(index=adj_close.index[window_size-1:], columns=adj_close.columns)
```

```
#rolling window test
for ticker in adj_close.columns:
    for end_index in range(window_size - 1, len(adj_close)):
        start_index = end_index - window_size + 1
        current_date = adj_close.index[end_index]
        market_return_series = adj_close['^GSPC'][start_index:(end_index + 1)]
        stock_return_series = adj_close[ticker][start_index:(end_index + 1)]

        # Calculate and store the EroD Beta
        drawdown_betas.at[current_date, ticker] = Calculate_EroD_Beta(market_return_series, stock_return_series)

        # Calculate and store the CAPM Beta
        capm_betas.at[current_date, ticker] = Calculate_CAPM_Beta(market_return_series, stock_return_series)
```

```
#plots for each stock's CAPM beta and EroD beta
```

```
num_stocks = 9 # The number of stocks you want to plot.
fig, axes = plt.subplots(num_stocks * 2, 1, figsize=(10, 40)) # Adjust the size as needed

for i, ticker in enumerate(drawdown_betas.columns[:num_stocks]):
    # EroD Beta subplot
    axes[i * 2].plot(drawdown_betas.index, drawdown_betas[ticker], label=f'EroD Beta for {ticker}')
    axes[i * 2].set_title(f'EroD Beta for {ticker}')
    axes[i * 2].set_xlabel('Date')
    axes[i * 2].set_ylabel('EroD Beta')
    axes[i * 2].legend()

    # CAPM Beta subplot
    axes[i * 2 + 1].plot(capm_betas.index, capm_betas[ticker], label=f'CAPM Beta for {ticker}', color='orange')
    axes[i * 2 + 1].set_title(f'CAPM Beta for {ticker}')
    axes[i * 2 + 1].set_xlabel('Date')
    axes[i * 2 + 1].set_ylabel('CAPM Beta')
    axes[i * 2 + 1].legend()

plt.tight_layout() # Adjust the layout to prevent overlap
plt.show()
```

```

#Plots for all CAPM beta in one graph and all ERoD beta in one graph
num_stocks = len(drawdown_betas.columns)

# Plotting all EroD Betas in one graph
fig, ax1 = plt.subplots(figsize=(10, 6))
for ticker in drawdown_betas.columns[:num_stocks]:
    ax1.plot(drawdown_betas.index, drawdown_betas[ticker], label=f'EroD Beta for {ticker}')

ax1.set_title('All EroD Betas')
ax1.set_xlabel('Date')
ax1.set_ylabel('EroD Beta')
ax1.legend(loc='upper left') # You may need to adjust the location of the Legend
plt.show()

# Plotting all CAPM Betas in one graph
fig, ax2 = plt.subplots(figsize=(10, 6))
for ticker in capm_betas.columns[:num_stocks]:
    ax2.plot(capm_betas.index, capm_betas[ticker], label=f'CAPM Beta for {ticker}')

ax2.set_title('All CAPM Betas')
ax2.set_xlabel('Date')
ax2.set_ylabel('CAPM Beta')
ax2.legend(loc='upper left') # You may need to adjust the location of the Legend
plt.show()

#Comparing each stock's CAPM beta and ERoD beta
num_stocks = len(drawdown_betas.columns) # Assuming you want to plot for all stocks

fig, axes = plt.subplots(num_stocks, 1, figsize=(7, 3 * num_stocks))

if num_stocks == 1:
    axes = [axes]

# Loop through each ticker and plot both EroD and CAPM Betas on the same subplot
for i, ticker in enumerate(drawdown_betas.columns):
    axes[i].plot(drawdown_betas.index, drawdown_betas[ticker], label='EroD Beta', color='blue')
    axes[i].plot(capm_betas.index, capm_betas[ticker], label='CAPM Beta', color='orange')
    axes[i].set_title(f'Comparison of EroD and CAPM Betas for {ticker}')
    axes[i].set_xlabel('Date')
    axes[i].set_ylabel('Beta Value')
    axes[i].legend()

# Adjust the Layout
plt.tight_layout()
plt.show()

# Calculate variance for each column in both DataFrames
capm_variances = capm_betas.var()
erod_variances = drawdown_betas.var()

variances_df = pd.DataFrame({
    'CAPM Beta Variance': capm_variances,
    'EroD Beta Variance': erod_variances
})

# Print the resulting DataFrame
print(variances_df)

```

The data will be submitted in a spreadsheet to Brightspace.