

Adaptive Fuzzy Consensus of Multiple Uncertain Euler-Lagrange Systems With Sampled-Data Output Interactions

Tianye Wang, Wencheng Zou, Jian Guo

Abstract—In this paper, the consensus problem for a class of multi-agent systems is investigated, where each agent is described by an Euler-Lagrange system and only sampled-data output interaction among agents is allowed. In addition, the Euler-Lagrange system considered possesses a higher degree of uncertainty; specifically, the regression matrix is also unknown. The intricate heterogeneity, inherent uncertainties, and stringent constraints of information interactions compel us to develop a new protocol. The protocol is synthesized by organically integrating theories and techniques such as graph theory, sampled-data cooperative control, fixed-time control, and fuzzy approximation. Each agent is equipped with a first-order difference trajectory generator that updates exclusively at sampling instants using the output information received from its neighbors. The virtual trajectory serves as the tracking target for the corresponding agent systems output. A fixed-time fuzzy adaptive tracker is proposed for each agent to track the generated trajectory. The problem of analyzing the protocol's effectiveness is deconstructed into two coupled subproblems: a synchronization problem for a perturbed first-order differentiator and a practical fixed-time tracking problem for a single Euler-Lagrange system. This problem is adequately addressed primarily based on the Lyapunov function method. Finally, the effectiveness of the proposed protocol is validated through numerical simulation and ROS experiment.

Keywords: Multi-agent systems, Euler-Lagrange systems, sampled-data protocol, uncertain systems, consensus

I. INTRODUCTION

THE consensus problem in multi-agent systems (MASs) has attracted considerable scholarly interest in recent years, due to its fundamental role in achieving distributed coordination. Many typical MASs have been extensively studied, such as single-integrator MASs [1], [2], double-integrator MASs [3], [4], high-order integrator MASs [5], [6], general linear MASs [7], [8], first-order nonlinear MASs [9], [10], second-order nonlinear MASs [11], [12], and high-order nonlinear MASs [13], [14]. Multiple Euler-Lagrange (EL) systems represent a crucial category of MASs, capable of modeling networks composed of multiple mechanical agents governed by Lagrangian dynamics (such as robots and unmanned aerial vehicles). Their dynamics typically exhibit strong nonlinearity, model uncertainties, and high coupling. These inherent

complexities pose significant challenges for consensus control, making them a widespread research focus. In [15], by designing the distributed position feedback control law, the leader-following consensus problem for the EL systems is solved. In [16], a modified optimization algorithm is proposed for the optimal formation problem with collision avoidance. A PD controller is designed to drive the nonidentical EL systems in [17]. Li et al. [18] consider the uncertain EL systems under the influence of deception attacks by using the auxiliary system. In [19], a bipartite consensus problem is investigated for a class of EL systems with external disturbances.

EL system equation is often subject to uncertainties. As a powerful tool to handle such uncertainties, fuzzy approximation technique has been widely adopted in the design of cooperative control protocols for multiple EL systems. In [20], an adaptive fuzzy consensus control method is proposed to enable a team of autonomous aircraft systems to cooperatively transport a cable-suspended rigid-body payload with unknown characteristics. In [21], an adaptive fuzzy sliding-mode distributed control law for multiple EL systems communicated via a directed topology is proposed. Xiao et al. [22] investigate the fixed-time formation control problem for a group of EL systems with unknown disturbances by applying fuzzy logic systems. In [23], the practical fixed-time consensus tracking problem for multiple EL systems with stochastic packet losses and input/output constraints is investigated, and an adaptive fixed-time control protocol based on fuzzy logic is designed. In [24], an adaptive fuzzy finite-time approach combining command filtering is proposed for multiple uncertain EL systems with unknown control directions. In [25], an adaptive fuzzy nonsingular terminal sliding mode protocol is designed for multiple EL systems with time-varying delays.

In the aforementioned works on multiple EL systems, the protocol implementations are mainly based on continuous information exchange, while the problem becomes more complex when only sampled-data communication is allowed. Currently, many excellent works have been reported for sampled-data interaction scenarios. For instance, Zhang et al. [26] investigate the consensus problem of multiple EL systems, and propose a sampled-data control strategy with dynamically adjustable sampling intervals. In [27], the sampled-data consensus of multiple EL systems with time-varying delays and packet dropouts is investigated. Ma et al. [28] consider the multiple EL systems with stochastic delays under directed topology, and propose an energy-efficient distributed sampled-data control strategy. In [29], the leaderless consen-

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sus problem of multiple EL systems under denial-of-service attacks is investigated, and a joint control protocol combining sampled-data control and adaptive control is proposed. In [30], the multiple EL systems with non-uniform sampling and probabilistic time-varying delays are considered. In [31], the synchronization problem of networked EL systems with time-varying delays under directed topology is investigated, and a sampled-data communication strategy based on a unified sampling sequence is proposed. It should be noted that the protocol designs in [26]–[31] rely on the knowledge of system dynamics or the regression matrix. How to design protocol for multiple EL systems with sampled-data output interaction and higher degree of uncertainties still remains unexplored.

In this paper, the consensus problem for a class of MASs subject to external disturbances is investigated, where each agent is described by an EL system and only sampled-data output interaction among agents is allowed. In addition, the Euler-Lagrange system considered possesses a higher degree of uncertainty; specifically, the regression matrix is also unknown. The main contributions are as follows:

- 1) A novel protocol is synthesized by organically integrating theories and techniques such as graph theory, sampled-data cooperative control, fixed-time control, and fuzzy approximation. Via the protocol, the practical consensus of multiple EL systems subject to sampled-data output interaction and unknown regression matrix can be achieved.
- 2) Compared with [23], which also concerns about the adaptive fuzzy consensus of multiple EL systems, the protocol proposed in this paper does not rely on continuous communication and can effectively reduce the communication load. Compared with [32], this paper addresses uncertain Euler-Lagrange systems rather than theoretical integrator systems, thereby enhancing the applicability of the protocol.
- 3) The dual-layer protocol design framework constructed in this paper achieves a substantial separation between the design of virtual trajectory dynamics and the agent input. Although some coupling effects continue to pose challenges for the design and analysis of the protocol, the original control problem is indeed simplified. The proposed framework demonstrates strong flexibility and portability, with the potential to be extended to solve other cooperative control problems of multiple EL systems.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. Graph Theory

For a team of N agents, the interaction among all agents can be represented by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{W})$, where $\mathcal{V} = \{1, 2, \dots, N\}$ and $\mathcal{W} \subseteq \mathcal{V}^2$ represent the agent set and the edge set, respectively. An edge in an undirected graph denoted as (i, j) means that agents i and j can obtain information from each other, i.e., agents i and j are neighbors of each other. We suppose that any agent is not a neighbor of itself. An undirected path is a sequence of undirected edges of the form $(i_1, i_2), (i_2, i_3), \dots$, where $i_j \in \mathcal{V}$. An undirected

graph is connected if there is an undirected path between every pair of distinct agents.

The adjacency matrix, along with Laplacian matrix, are often used to describe an undirected graph. The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ is defined such that $a_{ij} > 0$ if $(j, i) \in \mathcal{W}$ and $a_{ij} = 0$ otherwise. The Laplacian matrix $\mathcal{L} = [\ell_{ij}] \in \mathbb{R}^{N \times N}$ is defined such that $\ell_{ii} = \sum_{j=1, j \neq i}^N a_{ij}$ and $\ell_{ij} = -a_{ij}$, $i \neq j$.

B. Problem Formulation

The dynamic equation of each EL system considered in this paper is as follows:

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i + d_i, \quad (1)$$

where $i = 1, 2, \dots, N$, N is the number of the agents and $q_i \in \mathbb{R}^p$ is the vector of generalized coordinates. $M_i(q_i) \in \mathbb{R}^{p \times p}$, $C_i(q_i, \dot{q}_i) \in \mathbb{R}^{p \times p}$, $g_i(q_i) \in \mathbb{R}^p$ are inertia matrix, coriolis and centrifugal torques, gravitational torque respectively. We assume that the matrices $M_i(q_i)$, $C_i(q_i, \dot{q}_i)$ and $g_i(q_i)$ are bounded by positive constants ι_{inf} , ι_{sup} , ι_c , and ι_g in some sense, i.e., $M_i(q_i)$ is symmetric positive definite with $\iota_{inf}I_p \leq M_i(q_i) \leq \iota_{sup}I_p$, $\|C_i(q_i, \dot{q}_i)\| \leq \iota_c\|\dot{q}_i\|$, and $\|g_i(q_i)\| \leq \iota_g$, for any $q_i, \dot{q}_i \in \mathbb{R}^p$. $d_i \in \mathbb{R}^p$ refers to unknown external disturbance satisfying $\|d_i\| \leq \delta_d$ and δ_d is the unknown upper bound. $\tau_i \in \mathbb{R}^p$ is the control torque.

Assumption 1. [33] *The graph G is connected.*

Assumption 2. [34] *For any $x, y \in \mathbb{R}^p$ and $i = 1, 2, \dots, N$,*

$$M_i x_i + C_i y_i + g_i = Y_i \theta_i, \quad (2)$$

where θ_i represents the unknown constant vector, and Y_i represents the regressor matrix which is also unknown in this work.

Assumption 3. [35] *$\dot{M}_i - 2C_i$ is a skew-symmetric matrix. It means for all vectors $\Phi \in \mathbb{R}^p$, the equation $\Phi^T (\dot{M}_i - 2C_i) \Phi = 0$ is always met.*

Below are some useful lemmas for further development.

Lemma 1. [36] *If Assumption 1 is satisfied, the Laplacian matrix L is positive semidefinite. L possesses a simple eigenvalue 0 with $\mathbf{1}_N$ as the corresponding eigenvector. Let $0 = \lambda_1 < \lambda_2 \leq \dots \leq \lambda_N$ denote the eigenvalues of L . Moreover, for any vector x orthogonal to $\mathbf{1}_N$ (i.e., $\mathbf{1}_N^T x = 0$), the inequality $x^T L x \geq \lambda_2 x^T x$ holds.*

Lemma 2. [37] *We consider the system*

$$\dot{z} = g(t, z(t)), \quad z(0) = z_0, \quad (3)$$

with its equilibrium point being the origin. If there exists a continuous radially unbounded function $V(z(t))$ such that:

$$\begin{aligned} V(z(t)) &= 0 \Leftrightarrow z(t) = 0, \\ \dot{V}(z(t)) &\leq -c_1 V(z(t))^{r_1} - c_2 V(z(t))^{r_2} + \chi, \end{aligned}$$

for some positive constants c_1, c_2, χ, r_1 , and r_2 satisfying $0 < r_1 < 1 < r_2$. Then the following statement holds: The trajectory is practical fixed-time stable, and the residual set is given by $\{\lim_{t \rightarrow T} z|V(z) \leq \min\{c_1^{-1/r_1}(\chi/(1 -$

$\vartheta))^{1/r_1}, c_2^{-1/r_2}(\chi/(1-\vartheta))^{1/r_2}\}$ with $\vartheta \in (0, 1)$. The time needed to reach the residual set is bounded as $T(z_0) \leq \frac{1}{c_1(1-r_1)} + \frac{1}{c_2\vartheta(r_2-1)}$.

The fuzzy logic system is as follows:

$$Z(\xi) = \frac{\sum_{k=1}^m W_k \prod_{i=1}^n \mu_{F_i^k}(\xi_i)}{\sum_{k=1}^m \left[\prod_{i=1}^n \mu_{F_i^k}(\xi_i) \right]}, \quad (4)$$

where F_i^k is the fuzzy set, $\mu_{F_i^k}$ is the fuzzy membership function, $W_k = \max_{\xi \in \mathbb{R}} \mu_{F_i^k}(\xi)$, $k = 1, 2, \dots, m$ and m is the number of IF-THEN rules.

Let $S_i(\xi) = \frac{\prod_{i=1}^n \mu_{F_i^N}(\xi_i)}{\sum_{k=1}^m \left[\prod_{i=1}^n \mu_{F_i^k}(\xi_i) \right]}$, $W = [W_1, \dots, W_m]^T$ and $S = [S_1(\xi), \dots, S_m(\xi)]^T$, then the fuzzy logic system can be rewritten as $Z(\xi) = W^T S(\xi)$.

Lemma 3. [38] For a continuous function $\mathcal{F}(\xi)$ defined on a compact set Ω_ξ and a given constant $\epsilon > 0$, there exists a fuzzy logic system $Z(\xi) = W^{*T} S(\xi)$ such that

$$\sup_{\xi \in \Omega_\xi} |\mathcal{F}(\xi) - Z(\xi)| \leq \epsilon, \quad (5)$$

where $W^* = [W_1^*, W_2^*, \dots, W_N^*]^T$ and $S(\xi) = [S_1, S_2, \dots, S_N]^T$ are the ideal constant weight vector and basis function vector, respectively.

The control objective of the work is to design a distributed protocol such that the practical consensus of the multiple EL systems can be achieved, that is, $\lim_{t \rightarrow \infty} \|q_i - q_j\| \leq \varsigma$, where $\varsigma > 0$ is a constant.

III. PROTOCOL DESIGN

In this section, a protocol framework featuring a dual-layer structure is proposed. This framework consists of a virtual layer generating a virtual trajectory and a tracking layer performing precise tracking. This framework achieves a substantial separation between the design of virtual trajectory dynamics and the agent input. Although some coupling effects continue to pose challenges for the design and analysis of the protocol, the original control problem is indeed simplified. Specifically, at each sampling instant, a reference trajectory command is assigned to each agent. The agent's output is then driven toward this target via a tailored control policy. Fig. 1 illustrates the detailed framework diagram.

A. Design of Virtual Layer

The virtual layer generates consensus-driven reference trajectories z_i , $i = 1, \dots, N$, through discrete sampling communication. Let t_k , $k = 1, 2, \dots$, denote sampling instants, and $T_s = t_{k+1} - t_k > 0$ be the sampling period. At t_k , each agent receives neighbors' positional data $q_j(t_k)$. The virtual

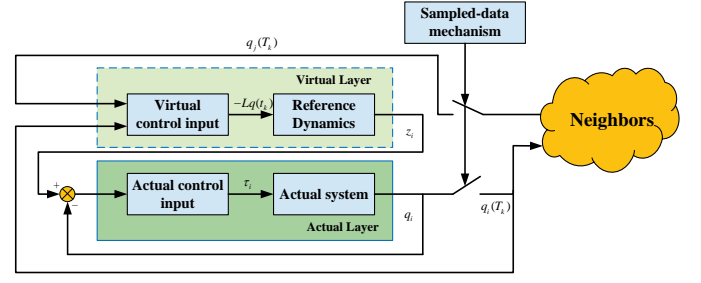


Fig. 1. Block diagram of the consensus protocol.

trajectory generator of agent i is governed by the following dynamics:

$$z_i(t_{k+1}) = z_i(t_k) - p \sum_{j \in \mathcal{N}_i} a_{ij}(q_i(t_k) - q_j(t_k)), \quad (6)$$

where $0 < p < \frac{\lambda_2}{\lambda_N}$ is a positive constant and $z_i(t_0) = q_i(t_0)$. The values of the solution of the following continuous system at the sampling instants are the same as those of system (6):

$$\dot{z}_i = -\frac{p}{T_s} \sum_{j \in \mathcal{N}_i} a_{ij}(q_i(t_k) - q_j(t_k)). \quad (7)$$

Operating on time-triggered updates, the virtual trajectory generator i is driven by sampled-data outputs from agent i and its neighbors. This design obviates the need for real-time communication and access to neighbors' internal states z_j , or complete state information-only positions are required. Consequently, guaranteeing effective convergence of virtual trajectories necessitates that the tracking controller suppresses the error $\|q_i - z_i\|$ throughout the interval $[t_k, t_{k+1})$. That is to say, at each sampling interval, the task of the tracking controller is to reach a certain fixed point in a timely manner.

Theorem 1. Consider N EL systems described by (1) communicating with each other over an undirected graph. The dynamics of virtual trajectory generator is given as in (6), and the agent input is designed in the following form:

$$\tau_i = \mathbf{g}_{i,1}(t, q_i, \dot{q}_i, q_i - z_i(t_{k+1}), \hat{\theta}_i), \quad (8)$$

$$\dot{\hat{\theta}}_i = \mathbf{g}_{i,2}(t, q_i, \dot{q}_i, q_i - z_i(t_{k+1}), \hat{\theta}_i), \quad t \in [t_k, t_{k+1}), \quad (9)$$

where $\mathbf{g}_{i,1}(\cdot)$ and $\mathbf{g}_{i,2}(\cdot)$ are two operators, $\hat{\theta}_i$ is an adaptive vector with appropriate dimensions. If Assumptions 1-3 hold, and there exist operators $\mathbf{g}_{i,1}(\cdot)$ and $\mathbf{g}_{i,2}(\cdot)$, and $T^* \in (0, T_s)$ such that

$$\|q_i - z_i(t_{k+1})\| \leq \nu_1, \quad \forall t \in [t_k + T^*, t_{k+1}), \quad (10)$$

with constant $\nu_1 > 0$, then there is a constant $\nu \propto \nu_1$ such that when $k \rightarrow \infty$,

$$\|z_i(t_k) - z_j(t_k)\| \leq \nu, \quad \forall i, j \in \{1, 2, \dots, N\}. \quad (11)$$

Proof: Rewriting (7), we can get the following equivalent

continuous-time system:

$$\begin{aligned}\dot{z}_i &= \frac{p}{T_s} \sum_{j=1}^N a_{ij}(z_j(t_k) - z_i(t_k)) \\ &+ \frac{p}{T_s} \sum_{j=1}^N a_{ij}(\mathbf{e}_j(t_k) - \mathbf{e}_i(t_k)),\end{aligned}\quad (12)$$

where $\mathbf{e}_i(t_k) = q_i(t_k) - z_i(t_k)$.

Let $\Xi = I_N - \frac{1_N \mathbf{1}_N^T}{N}$, $z = [z_1^T, z_2^T, \dots, z_N^T]^T$, $\phi = (\Xi \otimes I_p)z = [\phi_1^T, \phi_2^T, \dots, \phi_N^T]^T$, and $\mathbf{e} = [\mathbf{e}_1^T, \mathbf{e}_2^T, \dots, \mathbf{e}_N^T]^T$. The derivation of z is

$$\dot{z} = -\frac{p}{T_s}(L \otimes I_p)z(t_k) - \frac{p}{T_s}(L \otimes I_p)\mathbf{e}(t_k). \quad (13)$$

Consider the following Lyapunov function:

$$V_z = \frac{1}{2}\phi^T \phi. \quad (14)$$

It is noted that $\Xi^T = \Xi$ and $\Xi L = L = L\Xi$, then the derivation of (14) is

$$\begin{aligned}\dot{V}_z &= \phi^T \dot{\phi} = \phi^T (\Xi \otimes I_p) \dot{z} \\ &= -\frac{p}{T_s} \phi^T (\Xi L \otimes I_p) z(t_k) - \frac{p}{T_s} \phi^T (\Xi L \otimes I_p) \mathbf{e}(t_k) \\ &= -\frac{p}{T_s} \phi^T (L \otimes I_p) \phi - \frac{p}{T_s} \phi^T (L \otimes I_p) \mathbf{e}(t_k) \\ &+ \frac{p}{T_s} \phi^T (L \otimes I_p) (\phi - \phi(t_k)).\end{aligned}\quad (15)$$

Note that

$$\begin{aligned}\mathbf{1}_{N \times p}^T \phi &= \mathbf{1}_{N \times p}^T (\Xi \otimes I_p) z = (\mathbf{1}_N^T \otimes \mathbf{1}_p^T) (\Xi \otimes I_p) z \\ &= (\mathbf{1}_N^T \Xi \otimes \mathbf{1}_p^T I_p) z = 0.\end{aligned}\quad (16)$$

Utilizing Lemma 1 and (16) gives

$$-\frac{p}{T_s} \phi^T (L \otimes I_p) \phi \leq -\frac{p\lambda_2}{T_s} \phi^T \phi. \quad (17)$$

According to (10) and (13), one has

$$\|\dot{\phi}\| \leq \frac{p\lambda_N}{T_s} \|\phi(t_k)\| + \frac{p\nu_1 N \lambda_N}{T_s}. \quad (18)$$

Therefore, we have

$$\|\phi - \phi(t_k)\| \leq p\lambda_N \|\phi(t_k)\| + p\nu_1 N \lambda_N. \quad (19)$$

Combining (15), (17) and (19) yields

$$\begin{aligned}\dot{V}_z &\leq -\frac{p\lambda_2}{T_s} \phi^T \phi + \frac{(1 + p\lambda_N)pN\nu_1\lambda_N}{T_s} \|\phi\| \\ &+ \frac{p^2\lambda_N^2}{T_s} \|\phi\| \|\phi(t_k)\| \\ &\leq -\sigma_1 V_z + \sigma_2 \sqrt{V_z} + \sigma_3 \sqrt{V_z} \sqrt{V_z(t_k)},\end{aligned}\quad (20)$$

where $\sigma_1 = \frac{2p\lambda_2}{T_s}$, $\sigma_2 = \frac{\sqrt{2}(1+p\lambda_N)pN\nu_1\lambda_N}{T_s}$, and $\sigma_3 = \frac{2p^2\lambda_N^2}{T_s}$.

Letting $\Lambda = \sqrt{V_z}$, one has $\dot{\Lambda} = -\frac{\sigma_1}{2}\Lambda + \frac{\sigma_3}{2}\Lambda(t_k) + \frac{\sigma_2}{2}$. Hence, it can be easily obtain that

$$\Lambda \leq (\aleph(\bar{t}) + \frac{\sigma_3}{\sigma_1}(1 - \aleph(\bar{t})))\Lambda(t_k) + \frac{\sigma_2(1 - \aleph(\bar{t}))}{\sigma_1}, \quad (21)$$

where $\bar{t} = t - t_k$ and $\aleph(\bar{t}) = e^{-\frac{\sigma_1}{2}\bar{t}}$.

It is noted that $t_{k+1} - t_k = T_s$, as such, when $t = t_{k+1}$, (21) can be rewritten as

$$\Lambda(t_{k+1}) \leq (\aleph(T_s) + \frac{\sigma_3}{\sigma_1}(1 - \aleph(T_s)))\Lambda(t_k) + \frac{\sigma_2(1 - \aleph(T_s))}{\sigma_1}. \quad (22)$$

From (22), we can obtain that

$$\Lambda(t_k) \leq (\aleph(T_s) + \frac{\sigma_3}{\sigma_1}(1 - \aleph(T_s)))^k \Lambda(t_0) + \frac{\sigma_2 \mathcal{Y}(T_s)}{\sigma_1}, \quad (23)$$

where $\mathcal{Y}(T_s) = (1 - \aleph(T_s)) \frac{1 - (\aleph(T_s) + \frac{\sigma_3}{\sigma_1}(1 - \aleph(T_s)))^{k-1}}{1 - (\aleph(T_s) + \frac{\sigma_3}{\sigma_1}(1 - \aleph(T_s)))}$. Since $p < \frac{\lambda_2}{\lambda_N^2}$, we can get that

$$\aleph(T_s) + \frac{\sigma_3}{\sigma_1}(1 - \aleph(T_s)) = \aleph(T_s) + \frac{p\lambda_N^2}{\lambda_2}(1 - \aleph(T_s)) < 1. \quad (24)$$

Then, we can get that, when $k \rightarrow \infty$,

$$\Lambda(t_k) \leq \frac{\sigma_2(1 - \aleph(T_s))}{\sigma_1 - \sigma_1\aleph(T_s) - \sigma_3(1 - \aleph(T_s))} = \frac{\sigma_2}{\sigma_1 - \sigma_3}, \quad (25)$$

which implies that $\lim_{k \rightarrow \infty} V_z(t_k) \leq \frac{\sigma_2^2}{(\sigma_1 - \sigma_3)^2}$. Hence, $\lim_{k \rightarrow \infty} \|\phi(t_k)\| \leq \frac{\sqrt{2}\sigma_2}{\sigma_1 - \sigma_3} = \frac{\nu}{2}$, one has $\lim_{k \rightarrow \infty} \|\phi_i(t_k)\| \leq \frac{\nu}{2}$, $\forall i \in \mathcal{V}$. Noting that $\|\phi_i(t_k)\| = \|z_i(t_k) - \sum_{j=1}^N z_j(t_k)\|$, one has $\lim_{k \rightarrow \infty} \|z_i(t_k) - z_j(t_k)\| \leq \nu$. ■

Remark 1. In contrast to [40], the agents in this study share only their own outputs, excluding virtual states. This approach enhances privacy by design, as it avoids disclosing internal information and relies solely on the exchange of location data. Furthermore, robots can actively acquire neighbor status via onboard sensors (e.g., radar, cameras), eliminating dependence on communication modules and simplifying implementation. Compared with the open-loop strategy in [41], where the reference trajectory is determined solely by the initial states of the EL system and remains fixed, our method introduces a feedback loop between the tracking and virtual layers. This allows the virtual trajectory to update dynamically based on actual states, resulting in more timely adjustments and greater suitability for real-world applications.

Remark 2. The selection of the sampled-data period is independent of inter-system variations or external disturbances, allowing it to be freely determined. Longer sampling intervals reduce communication frequency but decelerate system convergence, whereas excessively short intervals elevate communication burdens and may generate disproportionately large control signals. In practical applications, engineers can select an appropriate sampling period to balance the desired convergence rate, permissible control input magnitude, and initial output discrepancies among agents.

B. Design of Tracking Layer

The task of tracking layer is to design a tracker such that

$$\lim_{t \rightarrow (t_k + T_s)} \|q_i - z_i(t_{k+1})\| \leq \nu_1 \quad \forall i \in \mathcal{V}. \quad (26)$$

Firstly, letting $e_{1,i} = q_i - z_i(t_{k+1})$, we can define the following piecewise function

$$\omega_{1,i,l} = \begin{cases} e_{1,i,l}^{2\beta-1}, & |e_{1,i,l}| \geq r, \\ \varpi_1(e_{1,i,l}), & |e_{1,i,l}| < r, \end{cases}$$

where $\varpi_1(e_{1,i,l}) = r^{2\beta-2}(2-\beta)e_{1,i,l} + r^{2\beta-4}(\beta-1)e_{1,i,l}^3$, $r > 0$, $\beta \in (0, 1)$, and $e_{1,i,l}$ denotes the l -th component of $e_{1,i}$. For convenience, define $\omega_{1,i} = [\omega_{1,i,1}^T, \omega_{1,i,2}^T, \dots, \omega_{1,i,p}^T]^T$. Furthermore, we define $\dot{q}_{ri} = -\alpha_1\omega_{1,i} - \alpha_2e_{1,i}^{2\varrho-1}$ where $\alpha_1 > 0$, $\alpha_2 > 0$ are adjustable parameters and $\varrho > 1$ is an integer. Then, define $e_{2,i} = \dot{q}_i - \dot{q}_{ri}$, and the following piecewise function can be given

$$\omega_{2,i,l} = \begin{cases} e_{2,i,l}^{2\beta-1}, & |e_{2,i,l}| \geq r, \\ \varpi_2(e_{2,i,l}), & |e_{2,i,l}| < r, \end{cases}$$

where $\varpi_2(e_{2,i,l}) = r^{2\beta-2}(2-\beta)e_{2,i,l} + r^{2\beta-4}(\beta-1)e_{2,i,l}^3$, and $e_{2,i,l}$ denotes the l -th component of $e_{2,i}$. For convenience, define $\omega_{2,i} = [\omega_{2,i,1}^T, \omega_{2,i,2}^T, \dots, \omega_{2,i,p}^T]^T$.

Inspired by the fuzzy approximation technique [38] and the fixed-time control technique [42], the adaptive fuzzy fixed-time tracker is designed as follows:

$$\begin{aligned} \tau_i = & -\sigma_1\omega_{2,i} - \sigma_2e_{2,i}^{2\varrho-1} - \frac{1}{2}e_{2,i} - e_{1,i} \\ & - \frac{1}{2a^2}\hat{\theta}_i S(q_i, \dot{q}_i, \dot{q}_{ri}, \ddot{q}_{ri})^T S(q_i, \dot{q}_i, \dot{q}_{ri}, \ddot{q}_{ri})e_{2,i}, \end{aligned} \quad (27)$$

and the adaptive update law is

$$\begin{aligned} \dot{\hat{\theta}}_i = & \frac{1}{2}S(q_i, \dot{q}_i, \dot{q}_{ri}, \ddot{q}_{ri})^T S(q_i, \dot{q}_i, \dot{q}_{ri}, \ddot{q}_{ri})e_{2,i}e_{2,i}^T \\ & - \kappa_1\hat{\theta}_i - \kappa_2\hat{\theta}_i^{2\varrho-1}, \end{aligned} \quad (28)$$

where σ_1 , σ_2 , κ_1 , κ_2 , and a are positive parameters.

For brevity, we refer to $M_i(q_i)$, $C_i(q_i, \dot{q}_i)$, and $g_i(q_i)$ as M_i , C_i , and g_i , respectively. It is noted that the analysis for each sampling period are same. For simplicity, we select the agent i at the initial interval $[0, T_s]$ for analysis.

Theorem 2. Consider the EL system (1) with the controller (27) and the adaptive update law (28), if Assumptions 2 and 3 hold, $T_s \geq \frac{1}{\pi_1\vartheta_i(1-\beta)} + \frac{1}{\pi_2\vartheta_i(1-\varrho)}$ with π_1 and π_2 being two computable parameters, then there is a $T^* \in (0, T_s]$ such that $\lim_{t \rightarrow T^*} \|q_i - z_i(t_1)\| \leq \nu_1$, where $\vartheta_i \in (0, 1)$.

Proof: Construct the following Lyapunov function

$$V_{tr,i} = V_{1,i} + V_{2,i} + V_{3,i}, \quad (29)$$

$$V_{1,i} = \frac{1}{2}e_{1,i}^T e_{1,i}, \quad (30)$$

$$V_{2,i} = \frac{1}{2}e_{2,i}^T M_i e_{2,i}, \quad (31)$$

$$V_{3,i} = \frac{1}{2a^2}\tilde{\theta}_i^2, \quad (32)$$

where $\tilde{\theta}_i = \hat{\theta}_i - \theta_i$ and θ_i is an unknown constant which will be designed later. The derivation of $V_{1,i}$ is

$$\begin{aligned} \dot{V}_{1,i} = & e_{1,i}^T \dot{q}_i \\ = & e_{1,i}^T (\dot{q}_i - \dot{q}_{ri} + \dot{q}_{ri}). \end{aligned} \quad (33)$$

Noting that $e_{2,i} = \dot{q}_i - \dot{q}_{ri}$, one has

$$\dot{V}_{1,i} = e_{1,i}^T e_{2,i} + e_{1,i}^T \dot{q}_{ri}. \quad (34)$$

Next, we will discuss the two cases. When $|e_{1,i,l}| < r$, the l -th component of $e_{1,i,l}\dot{q}_{ri,l}$ can be written as

$$\begin{aligned} e_{1,i,l}\dot{q}_{ri,l} = & -\alpha_1 r^{2\beta-2}(2-\beta)e_{1,i,l}^2 \\ & - \alpha_1 r^{2\beta-4}(\beta-1)e_{1,i,l}^4 - \alpha_2 e_{1,i,l}^{2\varrho} \\ = & -\alpha_1 r^{2\beta-2}(1-\beta)e_{1,i,l}^2 - \alpha_1 r^{2\beta-2}e_{1,i,l}^2 \\ & + \alpha_1 r^{2\beta-2}(1-\beta)e_{1,i,l}^2 \left(\frac{e_{1,i,l}}{r}\right)^2 - \alpha_2 e_{1,i,l}^{2\varrho} \\ \leq & -\alpha_1 r^{2\beta-2}(1-\beta)e_{1,i,l}^2 - \alpha_1 r^{2\beta-2}e_{1,i,l}^2 \\ & + \alpha_1 r^{2\beta-2}(1-\beta)e_{1,i,l}^2 - \alpha_2 e_{1,i,l}^{2\varrho} \\ = & -\alpha_1 r^{2\beta-2}e_{1,i,l}^2 - \alpha_2 e_{1,i,l}^{2\varrho} \\ = & -\alpha_1 e_{1,i,l}^{2\beta} \frac{r^{2\beta}}{e_{1,i,l}^{2\beta}} \frac{e_{1,i,l}^2}{r^2} - \alpha_2 e_{1,i,l}^{2\varrho} \\ = & -\alpha_1 e_{1,i,l}^{2\beta} \left(\frac{r}{e_{1,i,l}}\right)^{2\beta} \left(\left(\frac{e_{1,i,l}}{r}\right)^2 + 1\right) \\ & + \alpha_1 r^{2\beta} - \alpha_2 e_{1,i,l}^{2\varrho} \\ \leq & -\alpha_1 e_{1,i,l}^{2\beta} - \alpha_2 e_{1,i,l}^{2\varrho} + \alpha_1 r^{2\beta}. \end{aligned} \quad (35)$$

When $|e_{1,i,l}| \geq r$, one has

$$e_{1,i,l}\dot{q}_{ri,l} = -\alpha_1 e_{1,i,l}^{2\beta} - \alpha_2 e_{1,i,l}^{2\varrho}. \quad (36)$$

Combining with (35) and (36), one can get that

$$\begin{aligned} e_{1,i}^T \dot{q}_{ri} \leq & -\alpha_1 \sum_{l=1}^p e_{1,i,l}^{2\beta} - \alpha_2 \sum_{l=1}^p e_{1,i,l}^{2\varrho} + p\alpha_1 r^{2\beta} \\ \leq & -\alpha_1 \left(\sum_{l=1}^p e_{1,i,l}^2\right)^\beta - \alpha_2 p^{1-\varrho} \left(\sum_{l=1}^p e_{1,i,l}^2\right)^\varrho + p\alpha_1 r^{2\beta} \\ \leq & -\alpha_1 (e_{1,i}^T e_{1,i})^\beta - \alpha_2 p^{1-\varrho} (e_{1,i}^T e_{1,i})^\varrho + p\alpha_1 r^{2\beta}. \end{aligned}$$

Defining $\bar{\alpha}_1 = 2^\beta \alpha_1$ and $\bar{\alpha}_2 = 2^\varrho \alpha_2 p^{1-\varrho}$, one has

$$\dot{V}_{1,i} \leq e_{1,i}^T e_{2,i} - \bar{\alpha}_1 V_{1,i}^\beta - \bar{\alpha}_2 V_{1,i}^\varrho + p\alpha_1 r^{2\beta}. \quad (37)$$

According to Assumption 2, one has

$$M_i \ddot{q}_{ri} + C_i \dot{q}_{ri} + g_i = Y_i \Theta_i. \quad (38)$$

It is noted that both Y_i and Θ_i are unknown in this paper, so the fuzzy approximation technique is used to approximate $Y_i \Theta_i$, which is presented as

$$Y_i \Theta_i = W_i^T S(q_i, \dot{q}_i, \dot{q}_{ri}, \ddot{q}_{ri}) + \varepsilon_i, \|\varepsilon_i\| \leq \bar{\varepsilon}. \quad (39)$$

Subsequently, we refer to $S(q_i, \dot{q}_i, \dot{q}_{ri}, \ddot{q}_{ri})$ as S for brevity. According to (1), (38), and (39), it can be obtained that $M_i \dot{e}_{2,i} = -C_i e_{2,i} + \tau_i - W_i^T S_i - \varepsilon_i + d_i$. The derivative of $V_{2,i}$ is thus as follows:

$$\begin{aligned} \dot{V}_{2,i} = & e_{2,i}^T M_i \dot{e}_{2,i} + \frac{1}{2}e_{2,i}^T \dot{M}_i e_{2,i} \\ = & e_{2,i}^T (\tau_i + d_i - W_i^T S_i - \varepsilon_i) + \frac{1}{2}e_{2,i}^T (\dot{M}_i - 2C_i) e_{2,i} \\ \leq & e_{2,i}^T \tau_i + \frac{1}{2}e_{2,i}^T e_{2,i} + \frac{1}{2a^2}\theta_i e_{2,i}^T S_i^T S_i \\ & + \frac{a^2}{2} + \bar{\varepsilon}^2 + \delta_d^2, \end{aligned} \quad (40)$$

where $\theta_i = \max\{\|W_i\|^2\}$ and Assumption 3 is used. Next, substituting (27) into (40), one has

$$\begin{aligned} \dot{V}_{2,i} \leq & -\sigma_1 e_{2,i}^T \omega_{2,i} - \sigma_2 p^{1-\varrho} (e_{2,i}^T e_{2,i})^\varrho - e_{i,1}^T e_{i,2} \\ & - \frac{1}{2a^2} \tilde{\theta}_i S_i^T S_i e_{2,i}^T e_{2,i} + \frac{a^2}{2} + \bar{\varepsilon}^2 + \delta_d^2. \end{aligned} \quad (41)$$

Next, we will discuss the two cases of $-\sigma_1 e_{2,i,l} \omega_{2,i,l}$.

When $|e_{2,i,l}| < r$, it can be obtained that

$$\begin{aligned} -\sigma_1 e_{2,i,l} \omega_{2,i,l} &= -\sigma_1 r^{2\beta-2} (2-\beta) e_{2,i,l}^2 \\ &\quad - \sigma_1 r^{2\beta-4} (\beta-1) e_{2,i,l}^4 \\ &= -\sigma_1 r^{2\beta-2} (1-\beta) e_{2,i,l}^2 - \sigma_1 r^{2\beta-2} e_{2,i,l}^2 \\ &\quad + \sigma_1 r^{2\beta-2} (1-\beta) e_{2,i,l}^2 \left(\frac{e_{2,i,l}}{r}\right)^2 \\ &\leq -\sigma_1 r^{2\beta-2} (1-\beta) e_{2,i,l}^2 - \sigma_1 r^{2\beta-2} e_{2,i,l}^2 \\ &\quad + \sigma_1 r^{2\beta-2} (1-\beta) e_{2,i,l}^2 \\ &= -\sigma_1 e_{2,i,l}^{2\beta} \left(\frac{r}{e_{2,i,l}}\right)^{2\beta} \left(\left(\frac{e_{2,i,l}}{r}\right)^2 + 1\right) \\ &\quad + \sigma_1 r^{2\beta} \\ &\leq -\sigma_1 e_{2,i,l}^{2\beta} + \sigma_1 r^{2\beta}. \end{aligned} \quad (42)$$

When $|e_{2,i,l}| > r$, one has

$$-\sigma_1 e_{2,i,l} \omega_{2,i,l} = -\sigma_1 e_{2,i,l}^{2\beta}. \quad (43)$$

Define $\bar{\sigma}_1 = 2^\beta \sigma_1$ and $\bar{\sigma}_2 = 2^\varrho \sigma_2 p^{1-\varrho}$. Then, combining (42) with (43), (41) can be rewritten as

$$\begin{aligned} \dot{V}_{2,i} \leq & -\bar{\sigma}_1 V_{i,2}^\beta - \bar{\sigma}_2 V_{i,2}^\varrho - e_{i,1}^T e_{i,2} - \frac{1}{2a^2} \tilde{\theta}_i S_i^T S_i e_{2,i}^T e_{2,i} \\ & + p\sigma_1 r^{2\beta} + \frac{a^2}{2} + \bar{\varepsilon}^2 + \delta_d^2. \end{aligned} \quad (44)$$

The derivative of $V_{3,i}$ is

$$\dot{V}_{3,i} = \frac{1}{a^2} \tilde{\theta}_i^T \dot{\tilde{\theta}}_i. \quad (45)$$

Substituting (28) into (45), one has

$$\dot{V}_{3,i} = \frac{1}{2a^2} \tilde{\theta}_i^T S_i^T S_i e_{2,i}^T e_{2,i} - \frac{\kappa_1}{a^2} \tilde{\theta}_i^T \hat{\theta}_i - \frac{\kappa_2}{a^2} \tilde{\theta}_i^T \hat{\theta}_i^{2\varrho-1}, \quad (46)$$

where

$$\begin{aligned} -\frac{\kappa_1}{a^2} \tilde{\theta}_i \hat{\theta}_i &= -\frac{\kappa_1}{a^2} \tilde{\theta}_i (\tilde{\theta}_i + \theta_i) \\ &\leq -\frac{\kappa_1}{2a^2} \tilde{\theta}_i^2 + \frac{\kappa_1}{2a^2} \theta_i^2, \end{aligned} \quad (47)$$

and

$$-\frac{\kappa_2}{a^2} \tilde{\theta}_i \hat{\theta}_i^{2\varrho-1} = -\frac{\kappa_2}{a^2} \tilde{\theta}_i (\tilde{\theta}_i + \theta_i)^{2\varrho-1}. \quad (48)$$

By using Binomial Theorem, one has

$$(\tilde{\theta}_i + \theta_i)^{2\varrho-1} = \sum_{k=0}^{2\varrho-1} C_{2\varrho-1}^k \tilde{\theta}_i^{2\varrho-1-k} \theta_i^k. \quad (49)$$

By substituting (49) into (48) gives

$$-\frac{\kappa_2}{a^2} \tilde{\theta}_i \hat{\theta}_i^{2\varrho-1} = -\frac{\kappa_2}{a^2} \tilde{\theta}_i^{2\varrho} - \frac{\kappa_2}{a^2} \sum_{k=1}^{2\varrho-1} C_{2\varrho-1}^k \tilde{\theta}_i^{2\varrho-k} \theta_i^k. \quad (50)$$

It can be seen that

$$-C_{2\varrho-1}^k \tilde{\theta}_i^{2\varrho-k} \theta_i^k \leq \mathcal{T}_{\tilde{\theta}_i,k} + \mathcal{T}_{\theta_i,k}, \quad (51)$$

where $\mathcal{T}_{\tilde{\theta}_i,k} = C_{2\varrho-1}^k \frac{(2\varrho-k)\gamma_k}{2\varrho} \tilde{\theta}_i^{2\varrho}$, $\mathcal{T}_{\theta_i,k} = C_{2\varrho-1}^k \frac{k\gamma_k}{2\varrho} \theta_i^{2\varrho}$, $\gamma_k = \frac{1}{C_{2\varrho-1}^{2\varrho-k} (2\varrho-k)}$. Then, one has

$$-\kappa_2 C_{2\varrho-1}^k \tilde{\theta}_i^{2\varrho-k} \theta_i^k \leq \frac{\kappa_2}{2\varrho} \tilde{\theta}_i^{2\varrho} + \frac{\Delta_k}{2\varrho} \theta_i^{2\varrho}, \quad (52)$$

where $\Delta_k = \kappa_2 k C_{2\varrho-1}^k \exp(-\frac{2\varrho-k}{k} \ln \frac{1}{C_{2\varrho-1}^{2\varrho-k}})$.

Combining (46)-(52), we can get that:

$$\begin{aligned} & -\frac{\kappa_2}{a^2} \tilde{\theta}_i \hat{\theta}_i^{2\varrho-1} \\ & \leq -\frac{\kappa_2}{a^2} \tilde{\theta}_i^{2\varrho} + \frac{\kappa_2 (2\varrho-1)}{2\varrho a^2} \tilde{\theta}_i^{2\varrho} + \sum_{k=1}^{2\varrho-1} \frac{\Delta_k}{a^2} \theta_i^{2\varrho} \\ & = -\frac{\kappa_2}{2\varrho a^2} \tilde{\theta}_i^{2\varrho} + \sum_{k=1}^{2\varrho-1} \frac{\Delta_k}{a^2} \theta_i^{2\varrho}. \end{aligned} \quad (53)$$

Hence, one has

$$\begin{aligned} \dot{V}_{3,i} \leq & \frac{1}{2a^2} \tilde{\theta}_i S_i^T S_i e_{2,i}^T e_{2,i} - \frac{\kappa_1}{2a^2} \tilde{\theta}_i^2 \\ & - \frac{\kappa_2}{2\varrho a^2} \tilde{\theta}_i^{2\varrho} + \frac{\kappa_1}{2a^2} \theta_i^2 + \sum_{k=1}^{2\varrho-1} \frac{\Delta_k}{a^2} \theta_i^{2\varrho}. \end{aligned} \quad (54)$$

By Young's inequality, one has

$$\frac{\kappa_2}{2a^2} \tilde{\theta}_i^{2\varrho} \leq \frac{\kappa_1}{2a^2} \beta l_{\theta_i} \tilde{\theta}_i^2 + \frac{\kappa_1}{2a^2} (1-\beta) l_{\theta_i}^{-\frac{\beta}{1-\beta}}. \quad (55)$$

Letting $l_{\theta_i} = \frac{1}{\beta}$, we can further include that

$$\frac{\kappa_2}{2a^2} \tilde{\theta}_i^{2\varrho} \leq \frac{\kappa_1}{2a^2} \tilde{\theta}_i^2 + \frac{\kappa_1}{2a^2} (1-\beta) e^{\frac{\beta \ln \beta}{1-\beta}}. \quad (56)$$

So $\dot{V}_{3,i}$ can be further expanded into the following form

$$\begin{aligned} \dot{V}_{3,i} \leq & \frac{1}{2a^2} \tilde{\theta}_i S_i^T S_i e_{2,i}^T e_{2,i} - \bar{\kappa}_1 V_{i,3}^\beta - \bar{\kappa}_2 V_{i,3}^\varrho + \frac{\kappa_1}{2a^2} \theta_i^2 \\ & + \sum_{k=1}^{2\varrho-1} \frac{\Delta_k}{a^2} \theta_i^{2\varrho} + \frac{\kappa_2}{2a^2} (1-\beta) e^{\frac{\beta \ln \beta}{1-\beta}}, \end{aligned} \quad (57)$$

where $\bar{\kappa}_1 = \frac{\kappa_1 2^\beta a^{2\beta}}{2a^2}$ and $\bar{\kappa}_2 = \frac{\kappa_2 2^\varrho a^{2\varrho}}{2\varrho a^2}$. Combining (37), (44) and (57), $\dot{V}_{tr,i}$ can be rewritten as

$$\begin{aligned} \dot{V}_{tr,i} &= \dot{V}_{1,i} + \dot{V}_{2,i} + \dot{V}_{3,i} \\ &\leq -\bar{\alpha}_1 V_{i,1}^\beta - \bar{\alpha}_2 V_{i,1}^\varrho - \bar{\sigma}_1 V_{i,2}^\beta - \bar{\sigma}_2 V_{i,2}^\varrho \\ &\quad - \bar{\kappa}_1 V_{i,3}^\beta - \bar{\kappa}_2 V_{i,3}^\varrho + b \\ &\leq -\pi_1 V^\beta - \pi_2 V^\varrho + b, \end{aligned} \quad (58)$$

where $b = p\alpha_1 r^{2\beta} + p\sigma_1 r^{2\beta} + \frac{a^2}{2} + \bar{\varepsilon}^2 + \delta_d^2 + \frac{\kappa_1}{2a^2} \theta_i^2 + \sum_{k=1}^{2\varrho-1} \frac{\Delta_k}{a^2} \theta_i^{2\varrho} + \frac{\kappa_2}{2a^2} (1-\beta) e^{\frac{\beta \ln \beta}{1-\beta}}$, $\pi_1 = \min\{\bar{\alpha}_1, \bar{\sigma}_1, \bar{\kappa}_1\}$, $\pi_2 = 3^{1-\varrho} \min\{\bar{\alpha}_2, \bar{\sigma}_2, \bar{\kappa}_2\}$. Thus, one has

$$\lim_{t \rightarrow T^*} V_{tr,i} \leq \min \left\{ \pi_1^{-\frac{1}{\beta}} \left(\frac{b}{1-\vartheta_i} \right)^{\frac{1}{\beta}}, \pi_2^{-\frac{1}{\varrho}} \left(\frac{b}{1-\vartheta_i} \right)^{\frac{1}{\varrho}} \right\}. \quad (59)$$

According to Lemma 2, the setting time T^* satisfies

$$T^* \leq \frac{1}{\pi_1 \vartheta_i (1-\beta)} + \frac{1}{\pi_2 \vartheta_i (1-\varrho)}. \quad (60)$$

Letting $\nu_1 = \sqrt{2 \min \left\{ \pi_1^{-\frac{1}{\beta}} \left(\frac{b}{1-\vartheta_i} \right)^{\frac{1}{\beta}}, \pi_2^{-\frac{1}{\varrho}} \left(\frac{b}{1-\vartheta_i} \right)^{\frac{1}{\varrho}} \right\}}$, it can be obtained that $\lim_{t \rightarrow T^*} \|e_{i,1}\| = \lim_{t \rightarrow T^*} \|q_i - z_i(t_1)\| \leq \nu_1$. ■

C. Consensus Analysis

Theorem 3. Consider N EL systems described by (1) with the virtual trajectory generator (6), the controller (27) and the adaptive update law (28). If Assumptions 1-3 hold and choose $T_s \geq \frac{1}{\pi_1 \vartheta_i (1-\beta)} + \frac{1}{\pi_2 \vartheta_i (1-\varrho)}$, there exists a constant ς such that when $t \rightarrow \infty$, $\|q_i - q_j\| \leq \varsigma$.

Proof: From Theorem 2, at the initial interval $[0, T_s]$, if choose $T_s > T^*$, one has $\|q_i(T_s) - z_i(T_s)\| \leq \nu_1$. Similarly, in any sampling period $[t_k, t_{k+1}]$, it can be obtained that $\|q_i(t_{k+1}) - z_i(t_{k+1})\| \leq \nu_1$. It implies that the controller (27) can make system output meets the condition (10). Then according to Theorem 1, one has $\lim_{k \rightarrow \infty} \|z_i(t_k) - z_j(t_k)\| \leq \nu$, $\forall i, j \in \mathcal{V}$. It is noted that $z_i(t)$ will not change at time (t_k, t_{k+1}) , as such $\lim_{t \rightarrow \infty} \|z_i(t) - z_j(t)\| \leq \nu$, $\forall i, j \in \mathcal{V}$. In light of the above, one has

$$\|q_i - q_j\| \leq \underbrace{\|q_i - z_i\|}_{\leq \nu_1} + \underbrace{\|z_i - z_j\|}_{\leq \nu} + \underbrace{\|z_j - q_j\|}_{\leq \nu_1} \leq 2\nu_1 + \nu.$$

Hence, it can be obtained that $\|q_i - q_j\| \leq 2\nu_1 + \nu = \varsigma$. The proof is completed. ■

Remark 3. The lower bound for the sampling interval T_s derives from the minimum time required for the tracking controller's error convergence, leading to the constraint $T_s \geq \frac{1}{\pi_1 \vartheta_i (1-\beta)} + \frac{1}{\pi_2 \vartheta_i (1-\varrho)}$. Meanwhile, T_s is computable because all the parameters (π_1 , π_2 , β , q , ϑ_i) are user-specified constants.

IV. SIMULATION RESULTS

A. Numerical Simulation

A numerical simulation validates the proposed distributed control framework using a heterogeneous multiple EL system comprising 6 agents. The vector of generalized coordinates are considered in \mathbb{R}^2 , with initial positions

$$\begin{aligned} \mathbf{q}_1(0) &= [1, 2]^T, & \mathbf{q}_2(0) &= [3, 4]^T, \\ \mathbf{q}_3(0) &= [5, 6]^T, & \mathbf{q}_4(0) &= [7, 8]^T, \\ \mathbf{q}_5(0) &= [9, 10]^T, & \mathbf{q}_6(0) &= [11, 12]^T, \end{aligned}$$

zero initial velocities ($\dot{\mathbf{q}}_i(0) = \mathbf{0}_{2 \times 1}$). And virtual trajectories are initialized as $\mathbf{z}_i(0) = \mathbf{q}_i(0)$. Communication topology follows an undirected graph defined by adjacency matrix

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix},$$

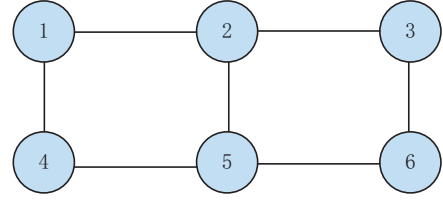


Fig. 2. Topology.

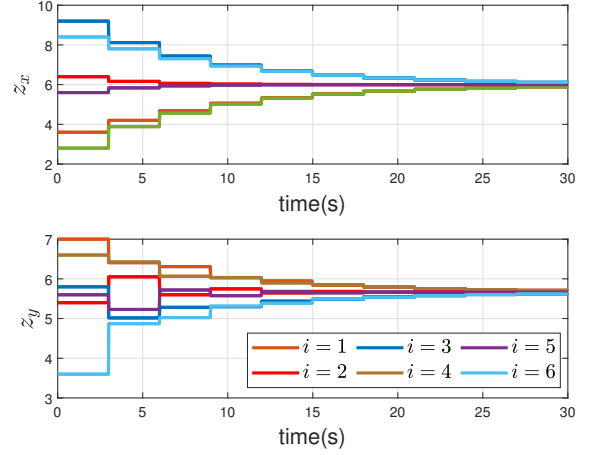


Fig. 3. The virtual reference trajectories z in x and y axes.

where the second smallest eigenvalue of \mathcal{L} can be computed as $\lambda_2(\mathcal{L}) = 1.382$, and the topology is shown in Fig. 2.

Agent dynamics incorporate heterogeneous parameters:

$$\begin{aligned} \mathbf{M}_i &= \begin{bmatrix} \eta_1 + \eta_2 + 4\eta_3 \cos(q_{i2}) & \eta_2 + 2\eta_3 \cos(q_{i2}) \\ \eta_2 + 2\eta_3 \cos(q_{i2}) & \eta_2 + \eta_4 \end{bmatrix}, \\ \mathbf{C}_i &= \begin{bmatrix} -2\eta_3 \dot{q}_{i2} \sin(q_{i2}) & -\eta_3 \dot{q}_{i2} \sin(q_{i2}) \\ -\eta_3 \dot{q}_{i2} \sin(q_{i2}) & 0 \end{bmatrix}, \\ \mathbf{g}_i &= \begin{bmatrix} \eta_5 \sin(q_{i,2}) \\ g\eta_6 \cos(q_{i2}) \sin(q_{i2}) \end{bmatrix}, \\ \mathbf{d}_i &= \begin{bmatrix} 0.2 \cos t \sin(0.3t) - 0.1 \sin(0.25t) \\ 0.2 \sin t \cos(0.3t) - 0.1 \cos(0.25t) \end{bmatrix}, \end{aligned}$$

where for $i = 1, 2, 3$, $\eta_i = [2.25, 0.1, 0.04, 1.5, 0.2, 0.1]^T$ and for $i = 3, 4, 5$, $\eta_i = [2.45, 0.2, 0.02, 1.5, 0.3, 0.1]^T$. Control parameters are configured as Table I:

TABLE I
CONTROL PARAMETERS CONFIGURATION

Parameter	Value	Description
T_s	3 s	Sampling period
β	0.7	Piecewise function exponent
r	0.1	Error switching threshold
α_1, α_2	1.0	Virtual velocity gains
σ_1, σ_2	1.0	Control input gains
$k_{\theta 1}, k_{\theta 2}$	1.0	Adaptation law coefficients
T_{\max}	20 s	Simulation duration
Step size	0.0001 s	Fixed-step solver resolution

Fig. 3 illustrates the virtual reference trajectories of all 6 agents, demonstrating that the designed virtual layer suc-

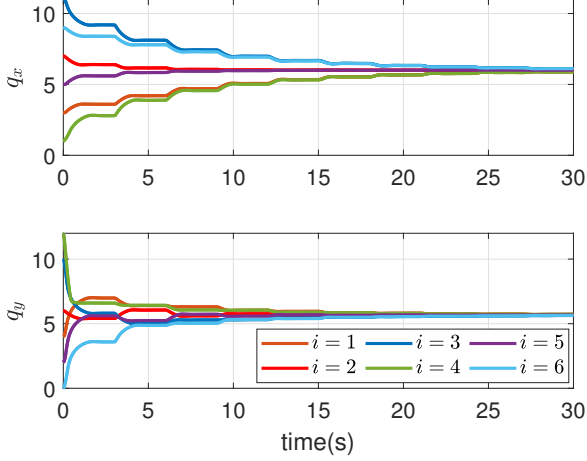


Fig. 4. The actual trajectories q in x and y axes.

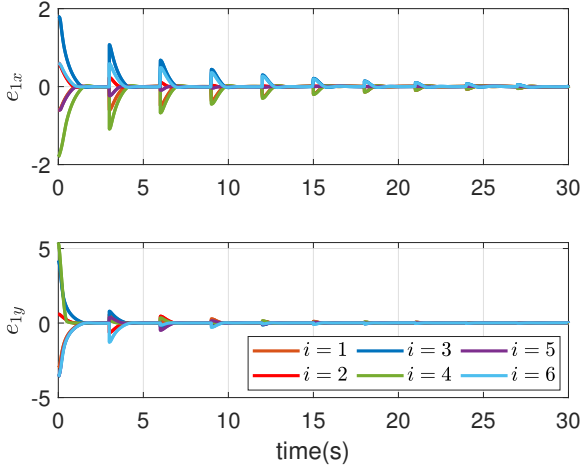


Fig. 5. The position errors $e_1 = q - z$ in x and y axes.

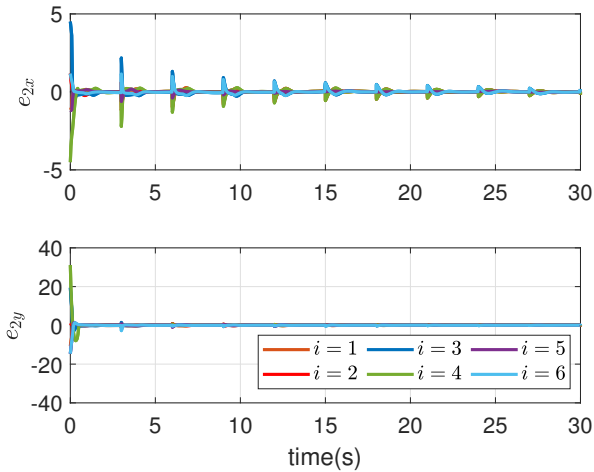


Fig. 6. The velocity errors $e_2 = \dot{q} - \dot{q}_r$ in x and y axes.

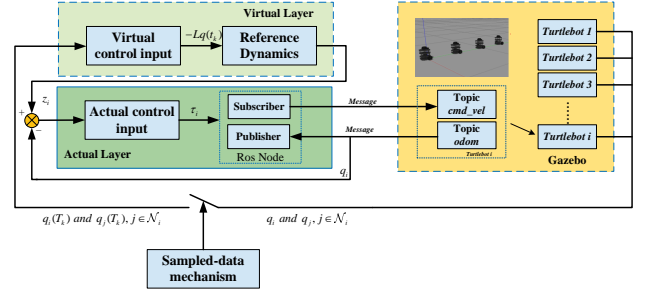


Fig. 7. The bolck diagram of the proposed protocol for ROS.

successfully achieves consensus among the virtual trajectories. These trajectories converge systematically through a discrete-time update mechanism at each sampling instant, validating the effectiveness of the distributed virtual signal generation protocol in coordinating the multi-agent reference targets. Fig. 4 shows the actual position trajectories. Actual position trajectories converge rapidly from initial scattered states to steady values. Figs. 5 and 6 plot the tracking errors with position and velocity signals, respectively. It can be seen that tracking errors are successfully suppressed. Some chatterings may appear in the figures, which is a normal phenomenon due to the samplings.

B. ROS Experiment

The Robot Operating System (ROS) is indispensable for validating distributed formation control algorithms under realistic conditions, leveraging its standardized communication framework (Topics/Services/Actions) and visualization tools to verify multi-agent convergence and robustness while preemptively exposing real-world challenges such as communication delays or node failures, thereby reducing development costs. The bolck diagram of the proposed protocol for ROS is shown in Fig. 7. The virtual layer on the left generates high-level commands and reference trajectories, while the cctual layer facilitates hardware interaction through ROS nodes. Real-time messaging mechanisms enable bidirectional instruction and feedback exchange between these layers. The Gazebo simulation environment on the right receives actual layer commands via ROS topics and simulates the physical responses of Turtlebot robots. The key parameters are configured through a fully connected bidirectional network defined by the adjacency matrix

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

enabling Agents $1 \leftrightarrow 2$, $2 \leftrightarrow 3$, $3 \leftrightarrow 4$ and $4 \leftrightarrow 1$ communication with cross-agent pose subscriptions, where initial positions Agent 1: $(4, 0)$, Agent 2: $(1, 3)$, Agent 3: $(1, -3)$, Agent 4: $(1, 0)$. In order to be more practical, the proposed protocol is transformed into a formation one, with the formation error calculation $e_{ij} = (q_i - q_j) - (\delta_i - \delta_j)$ with formation offsets $\delta = [0, -1; -1, -2; 0, 1; -1, 0]^T$.

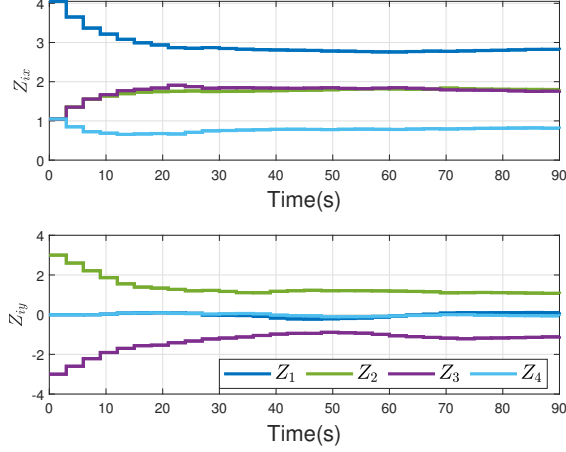


Fig. 8. The virtual reference trajectories z in x and y axes.

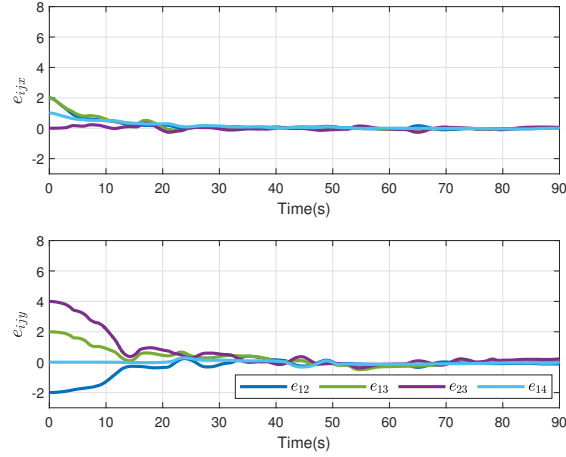


Fig. 9. The formation error in x and y axes.

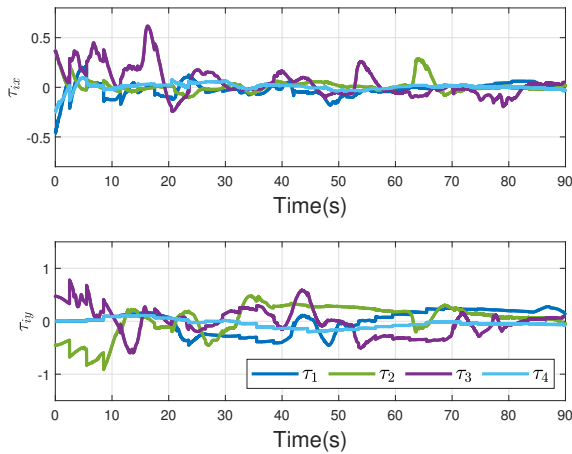


Fig. 10. The control input τ in x and y axes.

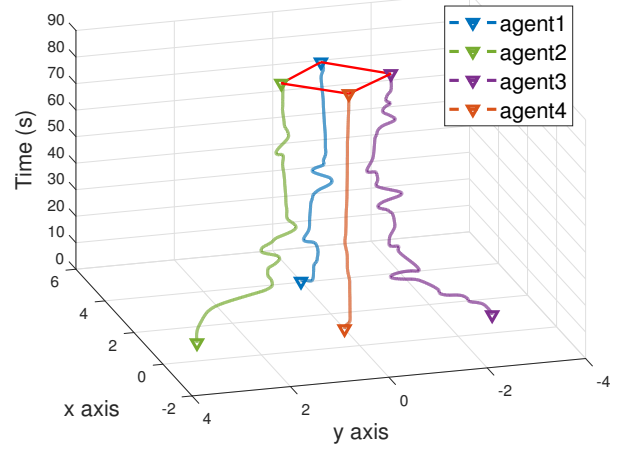


Fig. 11. The 3D trajectories of the four agents.

The comprehensive experimental results validate the formation control algorithm's efficacy across all performance dimensions. Fig. 8 illustrates the virtual trajectory evolution in x and y axes, where distinctive colored paths demonstrate systematic step-wise descent to stable plateaus at approximately 2.8, 1.8, 1.8 and 0.8 respectively by 30 seconds in x axes, and 80 seconds in y axes. Fig. 9 shows the formation errors of x and y , which stabilizes to constant values after approximately 30 seconds. Fig. 10 displays the control inputs of the x and y axes, while Fig. 11 illustrates the trajectories of four agents. These results verify that the system achieves parallelogram formation and demonstrate the rationality of the proposed framework.

V. CONCLUSION

This paper proposes a novel distributed control framework for achieving practical consensus in heterogeneous MASs governed by EL dynamics under practical constraints. The framework addresses three critical challenges: agents with unknown parameters and unknown regression matrices, heterogeneity in dynamic properties, and communication limited to sampled-data output interactions. This study establishes a theoretical framework for deploying heterogeneous EL systems in practical applications that demand high-precision and time-sensitive coordination under communication constraints. In future work, we will incorporate dynamic leaders to enhance the protocols adaptability to a broader range of engineering scenarios.

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