Notes for Vector Calculus

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October 12, 2021

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Chapter 1.

Particle Motion

§1.1 Particle Motion

Assume that we have a particle moving in space $\mathbb{R}^n$, as the motion of the particle is described by $\mathbf{r}(t)$ with respect to time t.

The *displacement* $\Delta\mathbf{r}$ between t_1 and t_0 ($t_0 < t_1$) is defined to be the position change over the period (t_0, t_1):

$$\Delta\mathbf{r} = \mathbf{r}(t_1) - \mathbf{r}(t_0) \quad \text{m}. \tag{1.1}$$

The *average velocity* $\bar{\mathbf{v}}$ is defined to be the average rate of position change during the period:

$$\bar{\mathbf{v}} = \frac{\Delta\mathbf{r}}{\Delta t} \quad \text{m/sec}, \tag{1.2}$$

where $\Delta t = t_1 - t_0$.

The *velocity*, or *instantaneous velocity*, $\mathbf{v}(t)$ at a certain time t is defined to be the rate of change during a very small period between t and $t + \Delta t$ ($|\Delta t| > 0$). That is, the limit of $\bar{\mathbf{v}}$ over neighbourhood of t as $\Delta t \to 0$:

$$\mathbf{v}(t) = \lim_{\Delta t \to 0} \frac{\mathbf{r}(t + \Delta t) - \mathbf{r}(t)}{\Delta t} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}(t) \quad \text{m/sec}. \tag{1.3}$$

The *distance traveled* s of the particle from time t_0 to t_1 is defined to be the total variation of $\mathbf{r}(t)$ over $[t_0, t_1]$; as (1.3) is given, we have

$$s = \int_{t_0}^{t_1} \|\mathrm{d}\mathbf{r}(t)\| = \int_{t_0}^{t_1} \|\mathbf{v}(t)\| \mathrm{d}(t) \quad \mathrm{m}. \tag{1.4}$$

The acceleration $\mathbf{a}(t)$ at a certain time t is defined to be the rate of velocity change over a very small interval between t and $t + \Delta t$:

$$\mathbf{a}(t) = \lim_{\Delta t \to 0} \frac{\mathbf{v}(t + \Delta t) - \mathbf{v}(t)}{\Delta t} = \frac{\mathrm{d}\mathbf{v}(t)}{\mathrm{d}t} = \frac{\mathrm{d}^2 \mathbf{r}(t)}{\mathrm{d}t^2} \quad \mathrm{m/sec}^2 \tag{1.5}$$

§1.2 Components of Velocity

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§1.3 Components of Acceleration

Following the condition in Section 1.1.

The *tangential acceleration* $\mathbf{a}_{\mathrm{T}}(t)$ at a certain time t is defined to be the projection of $\mathbf{a}(t)$ on any tangent vector of the motion curve at t. Thus,

$$\mathbf{a}_{\mathrm{T}}(t) = \mathbf{a}(t) \cdot \hat{\mathbf{v}}(t) \cdot \hat{\mathbf{v}}(t) \quad \mathrm{m/sec}^2. \tag{1.6}$$

Then *centripetal acceleration* $\mathbf{a}_{\mathrm{C}}(t)$ at t is defined to be the projection of $\mathbf{a}(t)$ at any right vector of the motion curve at t. As the right vector is orthogonal to the tangent vector, hence it is orthogonal to $\mathbf{a}(t)$. By the sum of vectors, we have

$$\mathbf{a}_{\mathrm{C}}(t) = \mathbf{a}(t) - \mathbf{a}_{\mathrm{T}}(t) \quad \mathrm{m/sec}^2. \tag{1.7}$$

As any right vector at t can be defined by the.........