Notes for General Topology

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Chapter 1.

Topological Spaces

§1.1 Basic Definitions

Definition 1.1.1. Let X be any set, and let $\mathcal{T} \subseteq 2^X$.

Then $\mathcal{T}$ is called a **topology on** X iff it satisfies the **open set axioms**. That is,

O1. $\emptyset, X \in \mathcal{T}$

O2. For any $\mathcal{U} \subseteq \mathcal{T}$, $\bigcup \mathcal{U} \in \mathcal{T}$; i.e., $\mathcal{T}$ is closed under arbitrary union.

O3. For any finite $\mathcal{V} \subseteq \mathcal{T}$, $\bigcap \mathcal{V} \in \mathcal{T}$; i.e., $\mathcal{T}$ is closed under finite intersection.

The ordered pair $\mathbb{X} = (X, \mathcal{T})$ is called a **topological space**.

A subset $U \subseteq X$ is said to be **open** iff it is an element of $\mathcal{T}$.

Rigorously, $\emptyset \in \mathcal{T}$ is not necessary for O1 in Definition 1.1.1, because it can be proved in a simple way.

As empty set is an element of any set, it is also an element of $\mathcal{T}$. Therefore,

$$\emptyset = \bigcup \emptyset \in \mathcal{T}.$$