Notes for University Physics

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Contents

Chapter 1.

Vector Spaces

§1.1 Linear Combinations

Definition 1.1.1. Let $\langle \mathbf{v}_i \rangle_{i=1}^{n}$ be a sequence such that for any $i \in \{1, \ldots, n\}$, $\mathbf{v}_i \in \mathbb{R}^n$.

The *linear span of* $\langle \mathbf{v}_i \rangle$, denoted $\operatorname{span}\langle \mathbf{v}_i \rangle$ is a subset of $\mathbb{R}^n$ defined as

$$\operatorname{span}\langle \mathbf{v}_i \rangle := \{\mathbf{a} \cdot \langle \mathbf{v}_i \rangle : \mathbf{a} \in \mathbb{R}^n\}.$$

An element $\mathbf{u} \in \mathbb{R}^n$ is a *linear combination of* $\langle \mathbf{v}_i \rangle$ iff

$$\mathbf{u} \in \operatorname{span}\langle \mathbf{v}_i \rangle.$$

Definition 1.1.2. With the conditions above, for any $i, j \in \{1, \ldots, n\}$, $\mathbf{v}_i$ and $\mathbf{v}_j$ are said to be *linearly dependent* iff there exists $t \in \mathbb{R}$, such that

$$\mathbf{v}_i = t\mathbf{v}_j.$$

$\mathbf{v}_i$ and $\mathbf{v}_j$ are *linearly independent* iff they are not linearly dependent.

Note 1.1.1. By definition of inner product, that is

$$\mathbf{u} = \sum_{i=1}^{n} a_i \mathbf{v}_i.$$

Note 1.1.2. Let $\langle \hat{\mathbf{e}}_i \rangle_{i=1}^{n}$ be a sequence, and for any $i \in \{1, \dots, n\}$,

$$\hat{\mathbf{e}}_i := \langle \underset{1}{0}, \dots, \underset{i}{1}, \dots, \underset{n}{0} \rangle.$$

Then, for any $\mathbf{u} \in \mathbb{R}^n$, the linear combination form of $\mathbf{u}$ is

$$\mathbf{u} = \sum_{i=1}^{n} u_i \hat{\mathbf{e}}_i.$$

Lemma 1.1.1. With the conditions in Definition 1.1.1,

$$\operatorname{span}\langle \mathbf{v}_i \rangle = \mathbb{R}^n$$

iff for all $i, j \in \{1, \dots, n\}$ with

§1.2 Line and Plane

Definition 1.2.1. Let $\mathbf{a}, \mathbf{b} \in \mathbb{R}^n$, where $\mathbf{a} \neq \mathbf{b}$. Let $L_{\mathbf{ab}} : \mathbb{R} \to \mathbb{R}^n$ be a mapping defined as

$$L_{\mathbf{ab}}(t) := \mathbf{a} + t(\mathbf{b} - \mathbf{a}).$$

The *line* $\overline{\mathbf{ab}}$ *through* $\mathbf{a}$ *and* $\mathbf{b}$ is defined as the image of $\mathbb{R}$ under L; i.e.,

$$\overline{\mathbf{ab}} := L_{\mathbf{ab}}[\mathbb{R}].$$

With $\mathbf{a}$ and $\mathbf{b}$ as *end points*, we define

(i) $L_{\mathbf{ab}}[[0, 1]]$ as *closed segment*,

(ii) $L_{\mathbf{ab}}[(0, 1)]$ as *open segment*,

(iii) $L_{\mathbf{ab}}[(0, 1]]$ as *half-open segment*,

(iv) $L_{\mathbf{ab}}[[0, 1]]$ as *half-closed segment*.

Definition 1.2.2. Let $\mathbf{a} \in \mathbb{R}^n$ and $\mathbf{u} \in \mathbb{R}^n \setminus \{\mathbf{0}\}$.

The *plane through* $\mathbf{a}$ *and orthogonal to* $\mathbf{u}$ is defined as

$$P := \{\mathbf{x} \in \mathbb{R}^n : \mathbf{u} \cdot (\mathbf{x} - \mathbf{a}) = \mathbf{0}\}.$$