Notes for University Physics

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Contents

Chapter 1.

Particle Motion

§1.1 Particle Motion

Assume that we have a particle moving in space $\mathbb{R}^n$, as the motion of the particle is described by $\mathbf{r}(t)$ with respect to time t.

The *displacement* $\Delta\mathbf{r}$ between t_1 and t_0 ($t_0 < t_1$) is defined to be the position change over the period (t_0, t_1):

$$\Delta\mathbf{r} = \mathbf{r}(t_1) - \mathbf{r}(t_0) \quad \text{m}. \tag{1.1}$$

The *average velocity* $\bar{\mathbf{v}}$ is defined to be the average rate of position change during the period:

$$\bar{\mathbf{v}} = \frac{\Delta\mathbf{r}}{\Delta t} \quad \text{m/sec}, \tag{1.2}$$

where $\Delta t = t_1 - t_0$.

The *velocity*, or *instantaneous velocity*, $\mathbf{v}(t)$ at a certain time t is defined to be the rate of change during a very small period between t and $t + \Delta t$ ($|\Delta t| > 0$). That is, the limit of $\bar{\mathbf{v}}$ over neighbourhood of t as $\Delta t \to 0$:

$$\mathbf{v}(t) = \lim_{\Delta t \to 0} \frac{\mathbf{r}(t + \Delta t) - \mathbf{r}(t)}{\Delta t} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}(t) \quad \text{m/sec}. \tag{1.3}$$

The *distance traveled* s of the particle from time t_0 to t_1 is defined to be the total variation of $\mathbf{r}(t)$ over $[t_0, t_1]$; as (1.3) is given, we have

$$s = \int_{t_0}^{t_1} |\mathrm{d}\mathbf{r}(t)| = \int_{t_0}^{t_1} |\mathbf{v}(t)| \mathrm{d}(t) \quad \text{m}. \tag{1.4}$$

The acceleration $\mathbf{a}(t)$ at a certain time t is defined to be the rate of velocity change over a very small interval between t and $t + \Delta t$:

$$\mathbf{a}(t) = \lim_{\Delta t \to 0} \frac{\mathbf{v}(t + \Delta t) - \mathbf{v}(t)}{\Delta t} = \frac{\mathrm{d}\mathbf{v}(t)}{\mathrm{d}t} = \frac{\mathrm{d}^2\mathbf{r}(t)}{\mathrm{d}t^2} \quad \text{m/sec}^2 \tag{1.5}$$

§1.2 Angular Velocity

Following the condition in Section 1.1.

The *direction of velocity* of the particle at time t is given by the unit vector of velocity $\mathbf{v}(t)$:

$$\hat{\mathbf{v}}(t) = \frac{\mathbf{v}(t)}{|\mathbf{v}(t)|}. \tag{1.6}$$

As the unit m/sec is canceled by the quotient between $\mathbf{v}(t)$ and $|\mathbf{v}(t)|$, $\hat{\mathbf{v}}(t)$ only remains the direction of $\mathbf{v}(t)$, with length 1.

The *angular velocity* $\boldsymbol{\omega}(t)$ at time t of the particle is defined to be the angular change of the particle during a very small time interval. That is,

$$\boldsymbol{\omega}(t) = \lim_{\Delta t \to 0} \frac{\hat{\mathbf{v}}(t + \Delta t) - \hat{\mathbf{v}}(t)}{\Delta t} = \frac{\mathrm{d}\hat{\mathbf{v}}(t)}{\mathrm{d}t} \quad \text{sec}^{-1}. \tag{1.7}$$

Problem 1.2.1. Explain circular motion and uniform circular motion.

§1.3 Curvature

Following the condition in Section 1.1.

The *curvature* $\kappa(t)$ of the particle at time t is defined to be

$$\kappa(t) = \left|\frac{\mathrm{d}\hat{\mathbf{v}}(t)}{\mathrm{d}s(t)}\right| \quad \sec^{-1}, \tag{1.8}$$

where $\mathrm{d}s(t)$ is the differential of distance traveled at t.

By (1.4), we have

$$\begin{aligned} \mathrm{d}s(t) &= \frac{\mathrm{d}s(t)}{\mathrm{d}t}\mathrm{d}t \\ &= \frac{\mathrm{d}}{\mathrm{d}t}\int_{t_0}^{t} |\mathbf{v}(\tilde{t})|\mathrm{d}\tilde{t} \cdot \mathrm{d}t \\ &= |\mathbf{v}(t)|\mathrm{d}t. \end{aligned}$$

As $\mathrm{d}\hat{\mathbf{v}}(t) = \boldsymbol{\omega}(t)\mathrm{d}t$, we have

$$\kappa(t) = \frac{|\boldsymbol{\omega}(t)|}{|\mathbf{v}(t)|} \quad \sec^{-1}. \tag{1.8'}$$

Problem 1.3.1. Prove that

$$\kappa(t) = \frac{|\mathbf{v}(t) \times \mathbf{a}(t)|}{|\mathbf{v}(t)|^3} \quad \sec^{-1}. \tag{1.8''}$$

§1.4 Components of Acceleration

Following the condition in Section 1.1.

The *tangential acceleration* $\mathbf{a}_{\mathrm{T}}(t)$ at a certain time t is defined to be the projection of $\mathbf{a}(t)$ on any tangent vector of the motion curve at t. Thus,

$$\mathbf{a}_{\mathrm{T}}(t) = \mathbf{a}(t) \cdot \hat{\mathbf{v}}(t) \cdot \hat{\mathbf{v}}(t) \quad \mathrm{m/sec}^2. \tag{1.9}$$

Then *centripetal acceleration* $\mathbf{a}_{\mathrm{C}}(t)$ at t is defined to be the projection of $\mathbf{a}(t)$ at any normal vector of the motion curve at t. That is,

$$\mathbf{a}_{\mathrm{C}}(t) = \mathbf{a}(t) \cdot \hat{\mathbf{n}}(t) \cdot \hat{\mathbf{n}}(t) \quad \mathrm{m/sec}^2, \tag{1.10}$$

where $\hat{\mathbf{n}}(t)$ is the unit normal vector given by

$$\hat{\mathbf{n}}(t) = \frac{\mathrm{d}\hat{\mathbf{v}}(t)}{\mathrm{d}t} \cdot \left| \frac{\mathrm{d}\hat{\mathbf{v}}(t)}{\mathrm{d}t} \right|^{-1}.$$

As the normal vector is orthogonal to the tangent vector, hence it is orthogonal to $\mathbf{a}(t)$. By the sum of vectors, we have

$$\mathbf{a}_{\mathrm{C}}(t) = \mathbf{a}(t) - \mathbf{a}_{\mathrm{T}}(t) \quad \mathrm{m/sec}^2. \tag{1.11}$$

Chapter 2.
Force

§2.1 Newton's Laws of Motion

First Law.

An object at rest remains at rest, and an object in motion remains in motion at constant speed and in a straight line unless acted on by an unbalanced force.

Second Law

The acceleration of an object depends on the mass of the object and the amount of force applied.

Third Law

Whenever one object exerts a force on another object, the second object exerts an equal and opposite on the first.

§2.2 Explanation for the Laws

The Second Law can be expressed as the equation

$$\mathbf{F}(t) = m\mathbf{a}(t) \quad \mathrm{N}, \tag{2.1}$$

where m is the mass of the object and the unit Newton is defined as

$$\mathrm{N} = \mathrm{kg} \cdot \mathrm{m/sec}.$$

In the First Law, the force is $\mathbf{0}$, thus it is the case as $\mathbf{a}(t) = \mathbf{0}$. In this case,

$$\mathbf{v}(t) = \int \mathbf{a}(t)\mathrm{d}t\Big|_{\mathbf{a}(t)=\mathbf{0}} = \mathbf{v}_0 \quad \mathrm{m/s},$$

where $\mathbf{v}_0$ is the initial velocity. Thus, the First Law can be considered as

$$\mathbf{a}(t) = \mathbf{0} \iff \mathbf{F}(t) = \mathbf{0}. \tag{2.2}$$

Denote $\mathbf{F}_{\mathbf{a}}(t)$ for the force exerted on the object, then the object also exerts a force

$$\mathbf{F}_{-\mathbf{a}}(t) = -\mathbf{F}_{\mathbf{a}}(t) \quad \mathrm{N}. \tag{2.3}$$

§2.3 Momentum

Definition 2.3.1. Given particle motion $\mathbf{r}(t)$, the momentum of the particle at a certain time t is defined to be the integral of fors

$$\mathbf{p}(t) := \int \mathbf{F}(t)\mathrm{d}t \quad \mathrm{kg} \cdot \mathrm{m/s},$$

where $\mathbf{F}(t)$ is the force at t, m is the mass of, and $\mathbf{v}_0$ is the initial velocity of the particle.

Proposition 2.3.1. Let $\mathbf{a}(t)$ describe the acceleration of a particle motion with respect to time t, and let $\mathbf{p}(t)$ describe the momentum of the particle motion. $\mathbf{p}(t) = \mathbf{0}$ iff

$$\mathbf{0} = \int_0^t \mathbf{a}(t)\mathrm{d}t + m\mathbf{v}_0 \quad \mathrm{m/s},$$

where $\mathbf{v}_0$ is the initial velocity of the particle.

Proof. By Definition 2.3.1,

$$\mathbf{0} = m \int \mathbf{a}(t_*) \mathrm{d}t_* \bigg|_{t_*=t} = m \int_0^t \mathbf{a}(t_*) \mathrm{d}t_* + m\mathbf{v}_0 \quad \mathrm{m/s}.$$

That is, the velocity $\mathbf{v}(t) = \mathbf{0}$. ∎

Proposition 2.3.2. ...

Proof.

$$\begin{aligned} m \left(\mathbf{v}(t) + \int \mathbf{a}_2 \mathrm{d}t_* \bigg|_{t_*=t} \right) = \mathbf{0} &\iff \mathbf{v}(t) + \int \mathbf{a}_2 \mathrm{d}t_* \bigg|_{t_*=t} = \mathbf{0} \\ &\iff \int \mathbf{a}(t) \mathrm{d}t + \int \mathbf{a}_2 \mathrm{d}t = \mathbf{0} \\ &\iff \end{aligned}$$

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