WENZHOU-KEAN UNIVERSITY

NUMERICAL ANALYSIS
MATH 3940

Dr. Puneet Rana, Assistant Professor

College of Science and Technology, Wenzhou-Kean University
Email: *prana@kean.edu, Mb: +91-9711333514*

WeChat ID: puneet-wku2020

Sep-21

UNIT 1: Error Analysis (Practice Session)

Rounding to Significant Figures?

Number	*Rounded-off to*		
	Three digits	*Four digits*	*Five digits*
00.543241 39.5255 69.4155 00.667676			

Absolute, Relative and Percentage Error

Question: Round of the numbers 865250 and 37.46235 to four significant figures and compute absolute error, relative error and percentage error in each case.

Error in Arithmetic Operations (Error Propagation)

General Formula $\left(E_R\right)_{\max} \le \left|\frac{\partial f}{\partial x}\frac{\Delta x}{u}\right| + \left|\frac{\partial f}{\partial y}\frac{\Delta y}{u}\right|$

- Addition: Let $u = x + y$

$$\left(E_R\right)_{\max} \le \left|1\times\frac{\Delta x}{u}\right| + \left|1\times\frac{\Delta y}{u}\right| \quad \text{i.e.} \quad \left(E_R\right)_{\max} \le \left|\frac{\Delta x}{u}\right| + \left|\frac{\Delta y}{u}\right|$$

- Subtraction: Let $u = x - y$

$$\left(E_R\right)_{\max} \le \left|1\times\frac{\Delta x}{u}\right| + \left|-1\times\frac{\Delta y}{u}\right| \text{ i.e.} \quad \left(E_R\right)_{\max} \le \left|\frac{\Delta x}{u}\right| + \left|\frac{\Delta y}{u}\right|$$

- Multiplication: Let $u = xy$

$$\left(E_R\right)_{\max} \le \left|y\times\frac{\Delta x}{u}\right| + \left|x\times\frac{\Delta y}{u}\right| \quad \text{i.e.} \quad \left(E_R\right)_{\max} \le \left|\frac{\Delta x}{x}\right| + \left|\frac{\Delta y}{y}\right|$$

Propagation of Error

Question : The area of cross-section of a rod is desired upto 0.2%. How accurately should the diameter be measured?

Propagation of Error

Question : *Compute the percentage error in the time period* $T = 2\pi\sqrt{\frac{l}{g}}$ *for* $l = 1$ *m if the error in the measurement of* l *is 0.01.*

Error

Question : *Evaluate the sum* $S = \sqrt{3} + \sqrt{5} + \sqrt{7}$ *to 4 significant digits and find its absolute and relative errors.*

Taylor Series and Truncation Error

Question: Let $f(x) = \cos x$ and $x_0 = 0$. Determine

(a) the second Taylor polynomial for f about x_0;

Taylor Series and Truncation Error

Question: *Use the series*

$$\log_e\left(\frac{1+x}{1-x}\right)=2\left(x+\frac{x^3}{3}+\frac{x^5}{5}+\ldots\ldots\right)$$

to compute the value of log (1.2) correct to seven decimal places and find the number of terms retained.

Machine Numbers to Decimals

Question: What is the decimal number of the machine number

01000001010000000000000000000000?

Machine Numbers to Decimals

Question: What is the decimal number of the machine number

$$0\underline{10000001}01000000000000000000000?$$

1. The leftmost bit is zero, which indicates that the number is positive.
2. The next 8 bits, 10000001, are equivalent to

$$c = 1 \cdot 2^7 + 0 \cdot 2^6 + \cdots + 0 \cdot 2^1 + 1 \cdot 2^0 = 129.$$

The exponential part of the number is $2^{129-127} = 2^2$.

3. The final 23 bits specify that the mantissa is

$$f = 0 \cdot (2)^{-1} + 1 \cdot (2)^{-2} + 0 \cdot (2)^{-3} + \cdots + 0 \cdot (2)^{-23} = 0.25.$$

4. Consequently, this machine number precisely represents the decimal number

$$(-1)^s 2^{c-127}(1+f) = 2^2 \cdot (1 + 0.25) = 5.$$

THANK YOU