Assignment: MATH 3490 Numerical Analysis

温州肯恩大学
WENZHOU-KEAN UNIVERSITY

Instructor: Dr. Puneet Rana **Deadline: Nov 28, 2021**

UNIT 8-9: AS1 Direct and Iterative Techniques (30 Points)
Note: All questions are of equal marks

Question 1: Which of the following statements are **TRUE**?
(i) Suppose matrices A and B commute, that is, $AB = BA$, then A^t and B^t also commute.
(ii) If A is strictly diagonally dominant in $Ax=b$, then $\|T_j\|_\infty < 1$.
(iii) If λ be an eigenvalue of the $n \times n$ matrix A, then λ is also an eigenvalue of A^{-1}.
(iv) If A is symmetric, then $\|A\|_2 = \rho(A)$.
(v) $A=\begin{bmatrix} 1/2 & 0 \\ 16 & 1/6 \end{bmatrix}$ is convergent matrix.

Question 2: If $ax^2+bx+c=0$ is divided by $x+3$, $x-5$ and $x-1$, the remainders are 21, 61 and 9 respectively. Use Gaussian elimination method to evaluate the value of a, b and c.

Question 3: Let $A=\begin{bmatrix} \alpha & 1 & 0 \\ \beta & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$, find the all the value of α and β for which

(a) A is singular
(b) A is strictly diagonal dominant
(c) A is symmetric
(d) A is positive definite*

*A matrix A is positive definite if it is symmetric and if $\boldsymbol{x^t A x} > 0$ for every n-dimensional vector $\boldsymbol{x} \neq \boldsymbol{0}$.

Question 4: Find the permutation matrix P so that PA can be factored into the product LU, where L is lower triangular with ones on its diagonal and U is upper triangular for these matrices. Consider the following matrix, $A=\begin{bmatrix} 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & -1 \\ 1 & 2 & -1 & 3 \\ 1 & 1 & 2 & 0 \end{bmatrix}$.

Question 5: Find $\|\boldsymbol{x}\|_\infty$ and $\|\boldsymbol{x}\|_2$ for the following vectors:
(a) $\boldsymbol{x} = (3, -4, 0, 3/2)^T$
(b) $\boldsymbol{x} = (\sin k, \cos k, 2^k)^T$ for a fixed positive integer k.

Question 6: (a) Verify that the function $\|.\|_1$, defined on $\mathbb{R}^n$ by $\|\boldsymbol{x}\|_1 = \sum_{i=1}^{n} |x_i|$ is a norm on $\mathbb{R}^n$.

(b) Show by example that $\|.\|_*$, defined by $\|A\|_* = \max_{1\le i,j\le n} |a_{ij}|$, does not define a matrix norm.

Question 7: Compute the eigenvalues, associated eigenvectors and spectral radius of the following matrix

$$\begin{bmatrix} -1 & 2 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 7 \end{bmatrix}$$

Question 8: The linear system

$$\begin{aligned} x_1 + 2x_2 - 2x_3 &= 7, \\ x_1 + x_2 + x_3 &= 2, \\ 2x_1 + 2x_2 + x_3 &= 5 \end{aligned}$$

has the solution $(1, 2, -1)^t$,
(a) Find the value of $\rho(T_j)$ and $\rho(T_g)$.
(b) Use the Jacobi method with $x(0) = 0$ to approximate the solution to the linear system to within 10^{-5} in the l_∞ norm.

Question 9: The linear system of equation is defined as

$$\begin{aligned} 10x_1 - x_2 &= 9, \\ -x_1 + 10x_2 - 2x_3 &= 7, \\ -2x_2 + 10x_3 &= 6. \end{aligned}$$

(a) Find the first two iterations of the SOR method with ω = 1.1, using $\boldsymbol{x}^{(0)} = \mathbf{0}$.

(b) If the above matrix is tridiagonal and positive definite, then Repeat (a) using the optimal choice of ω.

Question 10: Compute the condition number of the following matrix relative to $\|x\|_\infty$.

$$\begin{bmatrix} 0.04 & 0.01 & -0.01 \\ 0.2 & 0.5 & -0.2 \\ 1 & 2 & 4 \end{bmatrix}$$

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