Assignment: MATH 3490 Numerical Analysis

温州肯恩大学 WENZHOU-KEAN UNIVERSITY

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Deadline: Sept 30, 2021

UNIT 2: AS 1 Solution of Nonlinear Equation (30 Points)

Note: All questions are of equal marks

Question 1: (a) From Descartes's rule of signs, conclude that the equation $x^{2n}-1=0$ has $2n-2$ imaginary roots.

(b) Without actually obtaining the roots, show that $x^6-5x^5-7x^2+8x+20=0$ possess imaginary roots.

Question 2: The equation $f(x)=x^2-2e^x=0$ has a solution in the interval $[-1,1]$.

(a) With $p_0=-1$ and $p_1=1$, calculate p_2 using the Secant method.

(b) With p_2 from part (a), calculate p_3 using Newton's method.

Question 3: The equation $f(x)=2-x^2\sin x=0$ has a solution in the interval $[-1,2]$.

(a) Verify that the Bisection method can be applied to the function $f(x)$ on $[-1,2]$.

(b) Using the error formula for the Bisection method, find the number of iterations needed for accuracy 10^{-7}.

(c) Compute p_3 for the Bisection method.

Question 4: The following refer to the fixed-point problem:

(a) State the theorem which gives conditions for a fixed-point sequence to converge to a unique fixed point.

(b) Given $g(x)=\frac{2-x^3+2x}{3}$, use the theorem to show that the fixed-point sequence will converge to the unique fixed-point of $g(x)$ for any p_0 in $[-1,1.1]$.

(c) With $p_0=0.5$ generate p_3.

Question 5: What is the main difference between secant and regula falsi method? Compute the root of the equation $x^2e^{-x/2}=1$ in the interval [0, 2] correct to three decimal places using both secant method and regula falsi method.

Question 6: (a) Suppose the function $f(x)$ has a unique zero p in the interval $[a, b]$. Further, suppose $f''(x)$ exists and is continuous on the interval $[a, b]$. Under what conditions, Newton's method give a quadratically convergent sequence to p?

(b) Using Newton's method, construct an iterative formula to find $\sqrt[k]{N}$ *i.e.* k^{th} root of any number N.

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