Assignment: MATH 3490 Numerical Analysis

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UNIT 10-11: AS1 Approximating Eigenvalues (30 Points)

Question 1: Which of the following statements are **TRUE**?
(i) The vectors $\boldsymbol{v_1} = (2,-1)^t$, $\boldsymbol{v_2} = (1,1)^t$ and $\boldsymbol{v_3} = (1,3)^t$are linearly independent.
(ii) The pairs of matrices $A = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -1 \\ -2 & 2 \end{bmatrix}$ are not similar.
(iii) If A is similar to B, then det(A) = det(B).
(iv) Suppose that A is a symmetric $n \times n$ matrix, then the eigenvalues of A are real numbers.
(v) If A has the singular value decomposition $A = USV^t$, then Rank (A) ≠Rank (S).

Question 2: Use the Geršgorin Circle Theorem to determine bounds for the eigenvalues, and the spectral radius of the following matrix

$$\begin{bmatrix} 4.75 & 2.25 & -0.25 \\ 2.25 & 4.75 & 1.25 \\ -0.25 & 1.25 & 4.75 \end{bmatrix}$$

Question 3: Consider the follow sets of vectors. (***i***) Show that the set is linearly independent; (***ii***) use the Gram-Schmidt process to find a set of orthogonal vectors; (***iii***) determine a set of orthonormal vectors from the vectors in (***ii***).

(a) $\boldsymbol{v_1} = (1,1)^t$, $\boldsymbol{v_2} = (-2,1)^t$

(b) $\boldsymbol{v_1} = (1,1,1,1)^t$, $\boldsymbol{v_2} = (0,2,2,2)^t$, $\boldsymbol{v_3} = (1,0,0,1)^t$

Question 4: (i) For the following matrix determine if it diagonalizable and, if so, find P and D with $A = PDP^{-1}$.

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

(ii) Determine if the above matrix is positive definite, and if so, (ii) construct an orthogonal matrix Q for which $Q^tAQ = D$, where D is a diagonal matrix.

Question 5: (a) Find the first three iterations obtained by the Power method applied to the following matrices

$$\begin{bmatrix} 4 & 2 & 1 \\ 0 & 3 & 2 \\ 1 & 1 & 4 \end{bmatrix};$$

Use $\mathbf{x}^{(0)} = (1,2,1)^t$.

(b) Use the Power method to approximate the most dominant eigenvalue of the matrix. Iterate until a tolerance of 10^{-4} is achieved or until the number of iterations exceeds 25.

(c) Repeat above using Aitken's Δ^2 technique and the Power method for the most dominant eigenvalue.

Question 6: Use Householder's method to place the following matrices in tridiagonal form

$$\begin{bmatrix} 4 & -1 & -1 & 0 \\ -1 & 4 & 0 & -1 \\ -1 & 0 & 4 & -1 \\ 0 & -1 & -1 & 4 \end{bmatrix}$$

Question 7: Apply two iterations of the QR method without shifting to the following matrix.

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

Question 8: (a) Determine the singular values of the following matrix

$$A = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$$

(b) Determine a singular value decomposition for the above matrix.

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