Assignment: MATH 3490 Numerical Analysis

温州肯恩大学 WENZHOU-KEAN UNIVERSITY

Instructor: Dr. Puneet Rana

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UNIT 1: AS 1 Error Analysis (30 Points)

Note: All questions are of equal marks

Question 1: Round off the numbers 925150 and 26.36125 to four significant figures and compute absolute, relative and percentage errors.

Question 2: If $x = 5.675$, $y = 4.373$ and $z = 3.373$, calculate $x(y - z)$ and $xy - xz$, to four significant figures, which one is more accurate?

Question 3: Use three-digit rounding arithmetic to perform the following calculations. Compute the absolute error and relative error with the exact value determined to at least five digits.

(a) $-10\pi + 6e - \frac{3}{32}$ (b) $\frac{\sqrt{13}+\sqrt{11}}{\sqrt{13}-\sqrt{11}}$

Question 4: Evaluate $f(x) = x^3 - 6.1x^2 + 3.2x + 1.5$ at $x = 4.71$ using three-digit arithmetic

Question 5: Given $r = 3h(h^6 - 2)$, find the percentage error in r at h=1 if there is 5% error in h.

Question 6: The derivative of a function $f(x)$ at a particular value of x can be approximately calculated by $f'(x) \approx \frac{f(x+h)-f(x)}{h}$. For $f(x)$ =7e$^{0.5x}$, find the percentage error in calculating $f'(2)$ using values from $h = 0.3$ and $h = 0.15$.

Question 7: Use the 64-bit-long real format to find the decimal equivalent of the following floating-point machine numbers.

(a) 0 10000001010 1001001100000000000000000000000000000000000000000000000000

(b) 1 10000001010 1001001100000000000000000000000000000000000000000000000000

Question 8: Obtain a second-degree polynomial approximation to $f(x)=(1+x)^{1/2}$, $x \in [0, 0.1]$ using Taylor's series expansion about x=0. Use the expansion to approximate f (0.05) and bound the truncation error.

Question 9: If $f(x) = x^3 - e^{-x}, x_0 = 0.5$

(a) Find the Taylor Polynomial, $T_2(x)$, of degree at most 2 for $f(x)$ expanded about x_0.

(b) Evaluate $T_2(0.8)$ and compute the actual error $|f(0.8) - T_2(0.8)|$.

Question 10: Find the limit of each of following sequence and determine its rate of convergence.

(a) $\left\{\frac{2^n+3}{2^n+7}\right\}$ (b) $\left\{\frac{1-2n^2}{3n^2+n-1}\right\}$ (c) $\left\{ln\left(\frac{2n-1}{2n+1}\right)\right\}$ (d) $\left\{sin\left(\frac{1}{n}\right)\right\}$

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