Assignment: MATH 3490 Numerical Analysis

温州肯恩大学 WENZHOU-KEAN UNIVERSITY

Instructor: Dr. Puneet Rana **Deadline: Oct 28, 2021**

UNIT 4-5: AS1 Numerical Differentiation and Integration (30 Points)

Note: All questions are of equal marks

Question 1: If $f(x) = \frac{\cos x}{1+x^3}$, then find the approximate value of $f'(0.9)$ using the three-point midpoint formula with h=0.2.

Question 2: Use the most accurate three-point formula to determine each missing entry in the following table

x:	-0.3	-0.2	-0.1	0
$f(x)$	-0.27652	-0.25074	-0.16134	0
$f'(x)$	---	---	---	---

The above data is taken from the function $f(x) = e^{2x} - \cos 2x$. So, compute the actual errors.

Question 3: Let $f(x) = \cos \pi x$. Use the midpoint formula and the values of $f(x)$ at x=0.25, 0.5 and 0.75 to approximate $f''(0.5)$. Compare this result to the exact value.

Question 4: Apply the extrapolation process described to determine $N_3(h)$, an approximation to $f'(x_0)$, for the function $f(x) = x + e^x$, $x_0 = 0.0, h = 0.4$.

Question 5: Approximate the following integrals using Trapezoidal, Simpson's and midpoint rules.

(a) $\int_0^{\frac{\pi}{4}} e^{3x} \sin 2x \, dx$ (b) $\int_e^{e+1} \frac{dx}{x \ln(x)}$

Question 6: The Trapezoidal rule applied to $\int_0^2 f(x)dx$ gives the value 4, and Simpson's rule gives the value 2. What is the value of $f(1)$?

Question 7: Suppose that $f(0) = 1, f(0.5) = 2.5, f(1) = 2$ and $f(0.25) = f(0.75) = a$. Find the a, if the composite Trapezoidal rule with n=4 gives the value 1.75 for $\int_0^1 f(x)dx$.

Question 8: Determine the values of n and h required to approximate $\int_0^2 \frac{dx}{x+4}$ within 10^{-5} and compute the approximation using (a) composite Trapezoidal rule (b) composite Simpson's rule.

Question 9: Use Romberg integration to compute $R_{3,3}$ for the integral $\int_{-0.75}^{0.75} x \ln(x+1) \, dx$. Also, compare the results with exact value of the integral.

Question 10: Approximate the following integral $\int_0^{\frac{\pi}{4}} (\cos x)^2 \, dx$ using Gaussian quadrature with n = 2, 3 and 4. Also, compare your results to the exact values of the integral.

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