Assignment: MATH 3490 Numerical Analysis

温州肯恩大学 WENZHOU-KEAN UNIVERSITY

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UNIT 12-15: AS1 Nonlinear Equations and Boundary Value Problems (50 Points)

Question 1: The nonlinear system

$$5x_1^2 - x_2^2 = 0, \quad x_2 - 0.25(\sin x_1 + \cos x_2) = 0$$

has a solution near $\left(\frac{1}{4}, \frac{1}{4}\right)^t$.

a. Find a function **G** and a set D in $\mathbb{R}^2$ such that $\mathbf{G} : D \to \mathbb{R}^2$ and **G** has a unique fixed point in D.

b. Apply functional iteration to approximate the solution to within 10^{-5} in the l_∞ norm.

c. Does the Gauss-Seidel method accelerate convergence?

Question 2: Use Broyden's method with $\boldsymbol{x}^{(0)} = (1,1)^t$ to compute $\boldsymbol{x}^{(2)}$ for each of the following nonlinear systems.

$$3x_1^2 - x_2^2 = 0,$$
$$3x_1x_2^2 - x_1^3 - 1 = 0.$$

Question 3: Use the method of Steepest Descent to approximate minima to within 0.005 for the following function,

$$g(x_1, x_2) = \cos(x_1 + x_2) + \sin x_1 + \cos x_2$$

Question 4: The boundary-value problem (Use step size, **h=** π **/4**)

$$y'' = y' + 2y + \cos x, \quad 0 \le x \le \tfrac{\pi}{2}, \quad y(0) = -0.3, \quad y\left(\tfrac{\pi}{2}\right) = -0.1$$

has the solution $y(x) = -\frac{1}{10}(\sin x + 3\cos x)$. Use the Linear Shooting method to approximate the solution, and compare the results to the actual solution.

Question 5: Use the Linear Finite-Difference Algorithm to approximate the solution to the following boundary value problems (Use step size, ***h***=1/4)

$$y'' = -3y' + 2y + 2x + 3, \quad 0 \le x \le 1, y(0) = 2, y(1) = 1;$$

OR

Project:

(1) Solving any ordinary nonlinear differential equation by shooting or finite difference method.
(2) Write an algorithm (pseudocode) and source code in any programing language.
(3) Show graphical results.

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