Assignment: MATH 3490 Numerical Analysis

温州肯恩大学
WENZHOU-KEAN UNIVERSITY

Instructor: Dr. Puneet Rana

Issue Date: Oct 03, 2021
Deadline: Oct 09, 2021

UNIT 3: AS 1 Interpolation and Polynomial Approximation (30 Points)
Note: All questions are of equal marks

Question 1: Let $f(x) = x^3 - e^{-x}$, $x_0 = 0, x_1 = 0.7, x_2 = 1.0$.
(a) Find the Lagrange polynomial, $P_2(x)$, of degree at most 2 for $f(x)$ using x_0, x_1 and x_2.
(b) Evaluate $P_2(0.8)$ and compute the actual error, $|f(0.8) - P_2(0.8)|$.

Question 2: Let $f(x) = x^4 - 2x^3 + x^2 - 3$, $x_0 = 0, x_1 = 0.5, x_2 = 1.0, x_3 = 1.5$.
(a) Compute the interpolating polynomial, $P_3(x)$, of degree at most 3 for $f(x)$ using given nodes.
(b) Find the maximum error in using $P_3(x)$ to approximate $f(x)$ on the interval $[0, 2]$.

Question 3: Find the missing term in the following table using Lagrange's interpolation:

x:	0	1	2	3	4
y:	1	3	9	---	81

Question 4: Let $f(x) = x \sin 2x - x^2$, $x_0 = 0, x_1 = 0.3, x_2 = 0.7$.
(a) Find Newton's Divided-Difference form of the interpolating polynomial $P_2(x)$ for $f(x)$ using the three given nodes.
(b) Add a fourth node $x_3 = 0.9$ and compute the next interpolating polynomial $P_3(x)$.

Question 5: Let $f(x) = xe^{-x/2}$, $x_0 = 1, x_1 = 2$.
 (a) Construct the Hermite interpolating polynomial $H_3(x)$ for $f(x)$ using the given nodes.
(b) Approximate $f(1.4)$ using $H_3(1.4)$.
(c) Find the absolute error $| f(1.4) - H_3(1.4)|$
(d) Find a bound for the error using the error bound formula.

Question 6: The following values of x and y are given:

x:	1	2	3	4
y:	1	2	5	11

Find the natural cubic splines and evaluate $y(1.5)$ and $y'(3)$.

*****