Assignment: MATH 3490 Numerical Analysis

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UNIT 6-7: AS1 Initial Value Problem for Ordinary Differential Equations (30 Points)

Note: All questions are of equal marks

Question 1: Which of the following statements are **TRUE**?

(i) The differential equation $2y' + x^2 y = 2x + 3,\ \ y(0) = 5$ is linear.

(ii) A differential equation is considered to be ordinary if it has one dependent variable.

(iii) The exact of ordinary equation, $2y' + 3y = e^{-x},\ \ y(0) = 1$ is $y(x) = e^{-x}$.

(iv) The initial value problem, $y' = y - t\cos ty,\ 0 \le t \le 2,\ y(0) = 2$ has a unique solution.

Question 2: Given $3y' + 5y^2 = \sin x,\ y(0.3) = 5$ and using a step size of $h = 0.3$, the value of $y(0.9)$ using Euler's method (correct to three decimal places)?

Question 3: Euler's method can be derived by using the first two terms of the Taylor series of writing the value of y_{i+1}, that is the value of y at x_{i+1}, in terms of y_i and all the derivatives of y at x_i. If $h = x_{i+1} - x_i$, then write down the explicit expression for y_{i+1} if the first three terms of the Taylor series are chosen for the ordinary differential equation, $3y' + 2y = e^{-4x}, y(0) = 7$.

Question 4: Given the initial-value problem $\dfrac{dy}{dt} = \dfrac{1}{t^2} - \dfrac{y}{t} - y^2, 1 \le t \le 2,\ y(1) = -1$, with exact solution $y(t) = -1/t$. Use Taylor's method of order two with $h = 0.05$ to approximate the solution and compare it with the actual values at $y(1.1)$.

Question 5: Given $3y' + 5\sqrt{y} = e^{0.1x}, y(0.3) = 5$ and using a step size of $h = 0.3$, the best estimate of $y'(0.6)$ using the Runge-Kutta 2nd order midpoint method (Correct to two decimal places)?

Question 6: Use the Modified Euler method to approximate the solutions to following initial-value problems and compare the results to the actual values:

$\dfrac{dy}{dt} = \dfrac{y^2}{1+t}, 1 \le t \le 2,\ y(1) = \dfrac{-1}{\ln 2}$ with h =0.1; actual solution $y(t) = \dfrac{-1}{\ln(t+1)}$.

Question 7: Show that the Midpoint method and the Modified Euler method give the same approximations to the initial-value problem, $y' = -y + t + 1,\ 0 \le t \le 1,\ y(0) = 1$ for any choice of h. Why is this true? What is the relationship between these methods?

Question 8: In numerical methods, as h is decreased the calculation takes longer but is more accurate. However, decreasing h too much could cause significant errors. Why does this occur?

Question 9: Use the Runge-Kutta Fehlberg Algorithm with tolerance $TOL = 10^{-4}$ to approximate the solution to the following initial-value problems, $y' = \sin t + e^{-t}$, $0 \le t \le 1,\ y(0) = 0$ with *hmax*=0.25 and *hmin* =0.02.

Question 10: Use all the Adams-Bashforth methods to approximate the solutions to the initial-value problems: $y' = 1 + y/t, 1 \le t \le 2,\ y(1) = 2$ with h=0.2; actual solution, $y(t) = t\ln t + 2t$. Use exact starting values and compare the results to the actual values.

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