Python 3 玩儿转机器学习

liuyubobobo

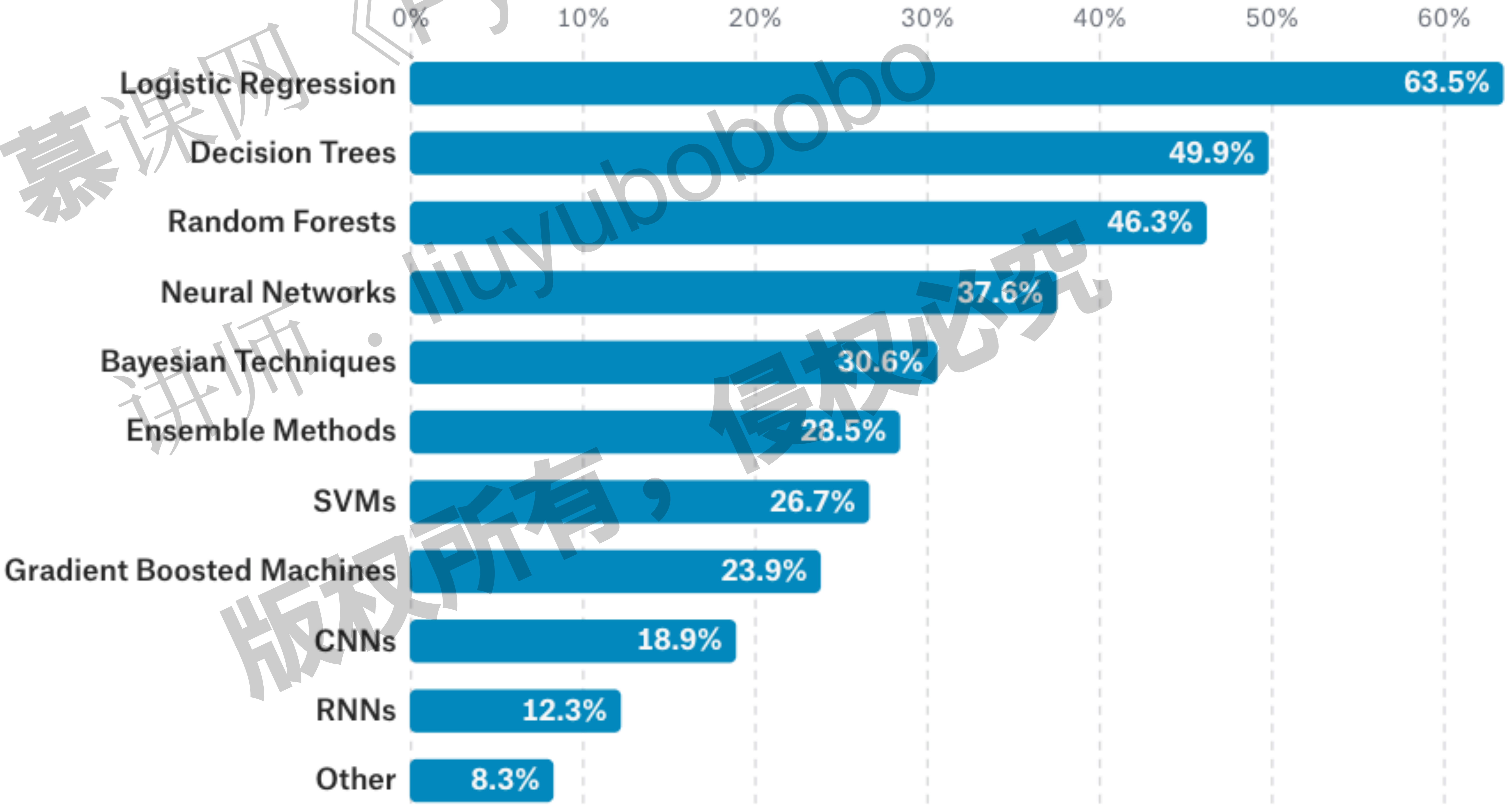
逻辑回归

Logistic Regression

What data science methods are used at work?

Logistic regression is the most commonly reported data science method used at work for all industries *except* Military and Security where Neural Networks are used slightly more frequently.

Company Size | Industry | Job Title



什么是逻辑回归

逻辑回归 Logistic Regression

逻辑回归：解决分类问题

回归问题怎么解决分类问题?

将样本的特征和样本发生的概率联系起来，概率是一个数

逻辑回归 Logistic Regression

$$\hat{y} = f(x)$$

$$\hat{p} = f(x) \qquad \hat{y} = \begin{cases} 1, & \hat{p} \geq 0.5 \\ 0, & \hat{p} \leq 0.5 \end{cases}$$

逻辑回归既可以看做是回归算法，也可以看做是分类算法

通常作为分类算法用，只可以解决二分类问题

逻辑回归 Logistic Regression

$$\hat{y} = f(x)$$

$$\hat{y} = \theta^T \cdot x_b$$

值域 (-infinity, +infinity)

概率的值域为 [0, 1]

$$\hat{p} = \sigma(\theta^T \cdot x_b)$$

Sigmoid 函数

$$\hat{p} = \sigma(\theta^T \cdot x_b)$$

$$\sigma(t) = \frac{1}{1+e^{-t}}$$

实践：sigmoid函数

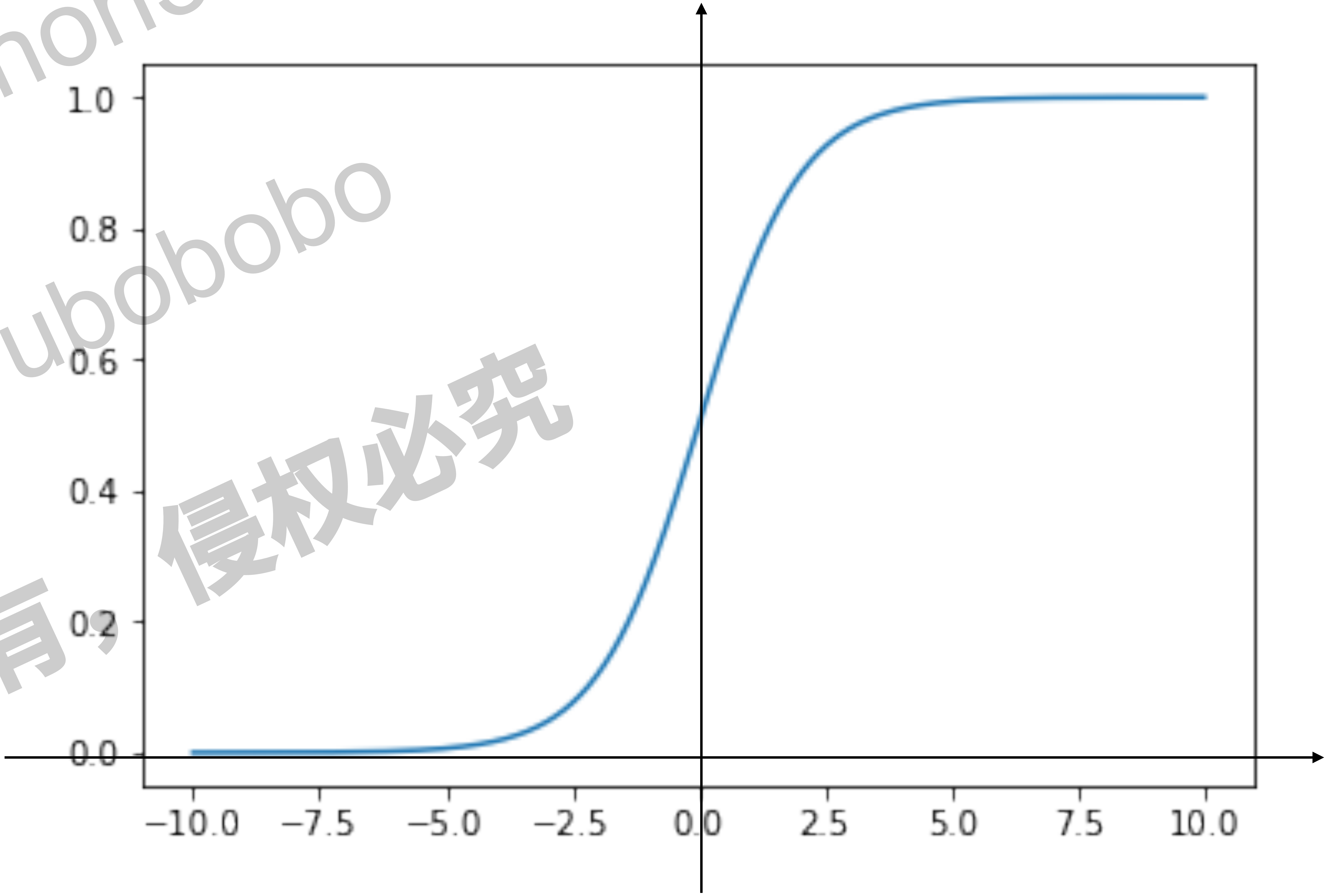
Sigmoid 函数

$$\sigma(t) = \frac{1}{1+e^{-t}}$$

值域 (0, 1)

t > 0 时，p > 0.5

t < 0 时，p < 0.5



逻辑回归 Logistic Regression

$$\hat{p} = \sigma(\theta^T \cdot x_b) = \frac{1}{1+e^{-\theta^T \cdot x_b}}$$

$$\hat{y} = \begin{cases} 1, & \hat{p} \geq 0.5 \\ 0, & \hat{p} \leq 0.5 \end{cases}$$

逻辑回归 Logistic Regression

$$\hat{p} = \sigma(\theta^T \cdot x_b) = \frac{1}{1+e^{-\theta^T \cdot x_b}}$$

$$\hat{y} = \begin{cases} 1, & \hat{p} \geq 0.5 \\ 0, & \hat{p} \leq 0.5 \end{cases}$$

问题：

对于给定的样本数据集X,y,

我们如何找到参数theta,

使得用这样的方式,

可以最大程度获得样本数据集X

对应的分类输出y?

逻辑回归中损失函数的定义

逻辑回归 Logistic Regression

$$\hat{p} = \sigma(\theta^T \cdot x_b) = \frac{1}{1 + e^{-\theta^T \cdot x_b}}$$

$$\hat{y} = \begin{cases} 1, & \hat{p} \geq 0.5 \\ 0, & \hat{p} \leq 0.5 \end{cases}$$

逻辑回归 Logistic Regression

$$\hat{p} = \sigma(\theta^T \cdot x_b) = \frac{1}{1 + e^{-\theta^T \cdot x_b}}$$

$$\hat{y} = \begin{cases} 1, & \hat{p} \geq 0.5 \\ 0, & \hat{p} \leq 0.5 \end{cases}$$

$$\text{cost} = \begin{cases} \text{如果}y=1, p\text{越小，cost越大} \\ \text{如果}y=0, p\text{越大，cost越大} \end{cases}$$

逻辑回归 Logistic Regression

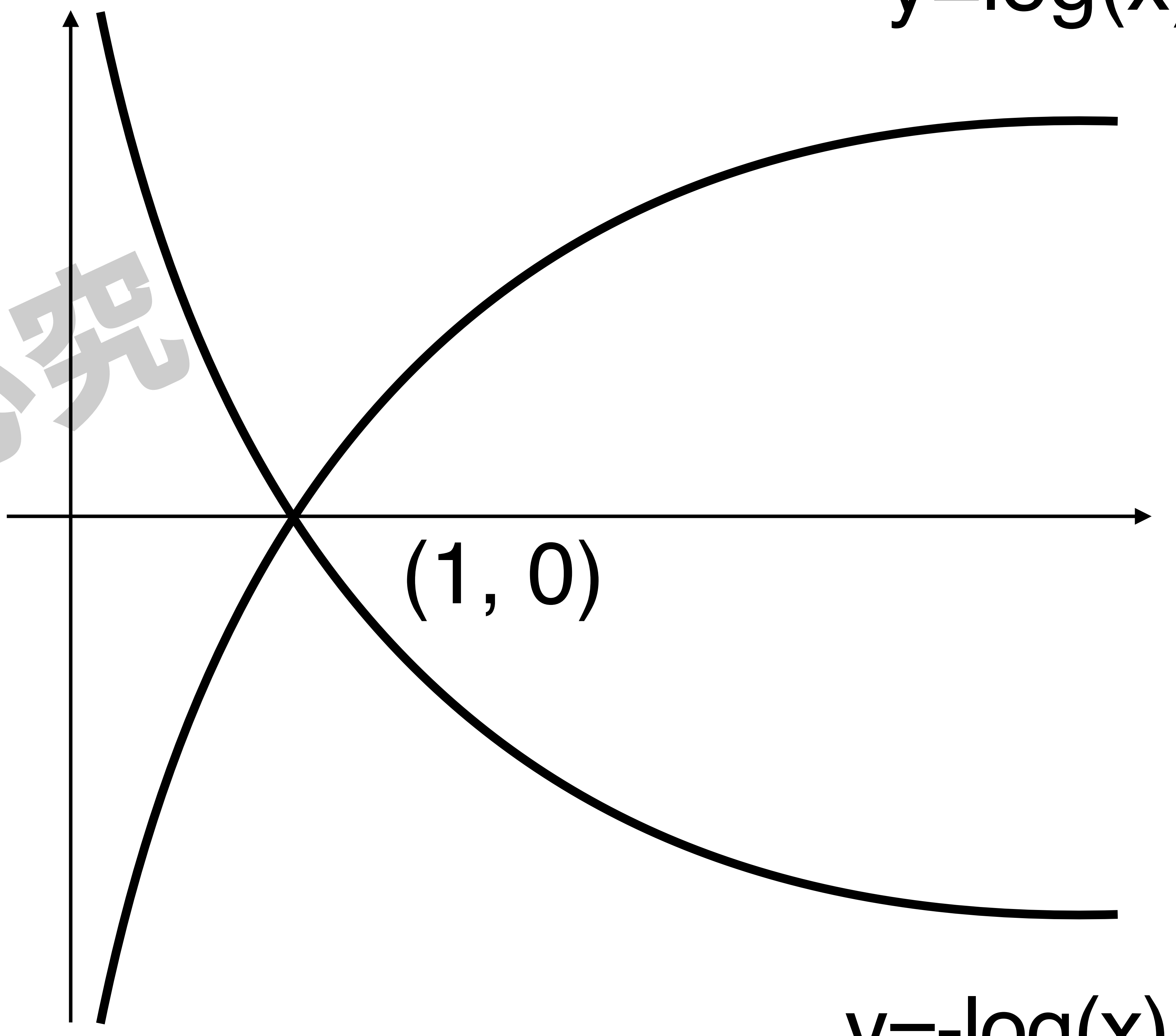
$$\text{cost} = \begin{cases} \text{如果y=1, p越小，cost越大} \\ \text{如果y=0, p越大，cost越大} \end{cases}$$

$$\Downarrow$$

$$\text{cost} = \begin{cases} -\log(\hat{p}) & if \quad y=1 \\ -\log(1-\hat{p}) & if \quad y=0 \end{cases}$$

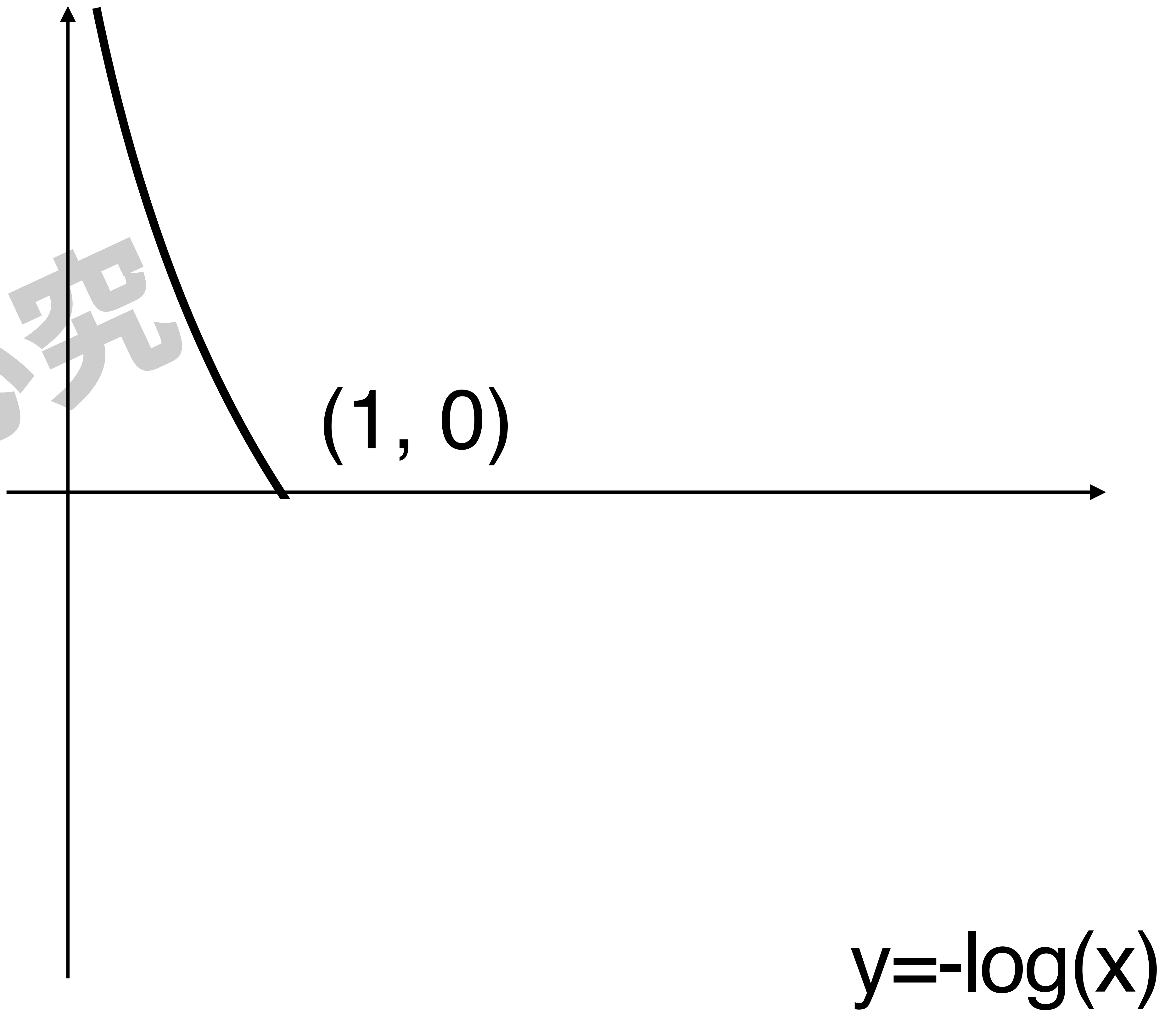
逻辑回归 Logistic Regression

$$\text{cost} = \begin{cases} -\log(\hat{p}) & if \quad y=1 \\ -\log(1-\hat{p}) & if \quad y=0 \end{cases}$$



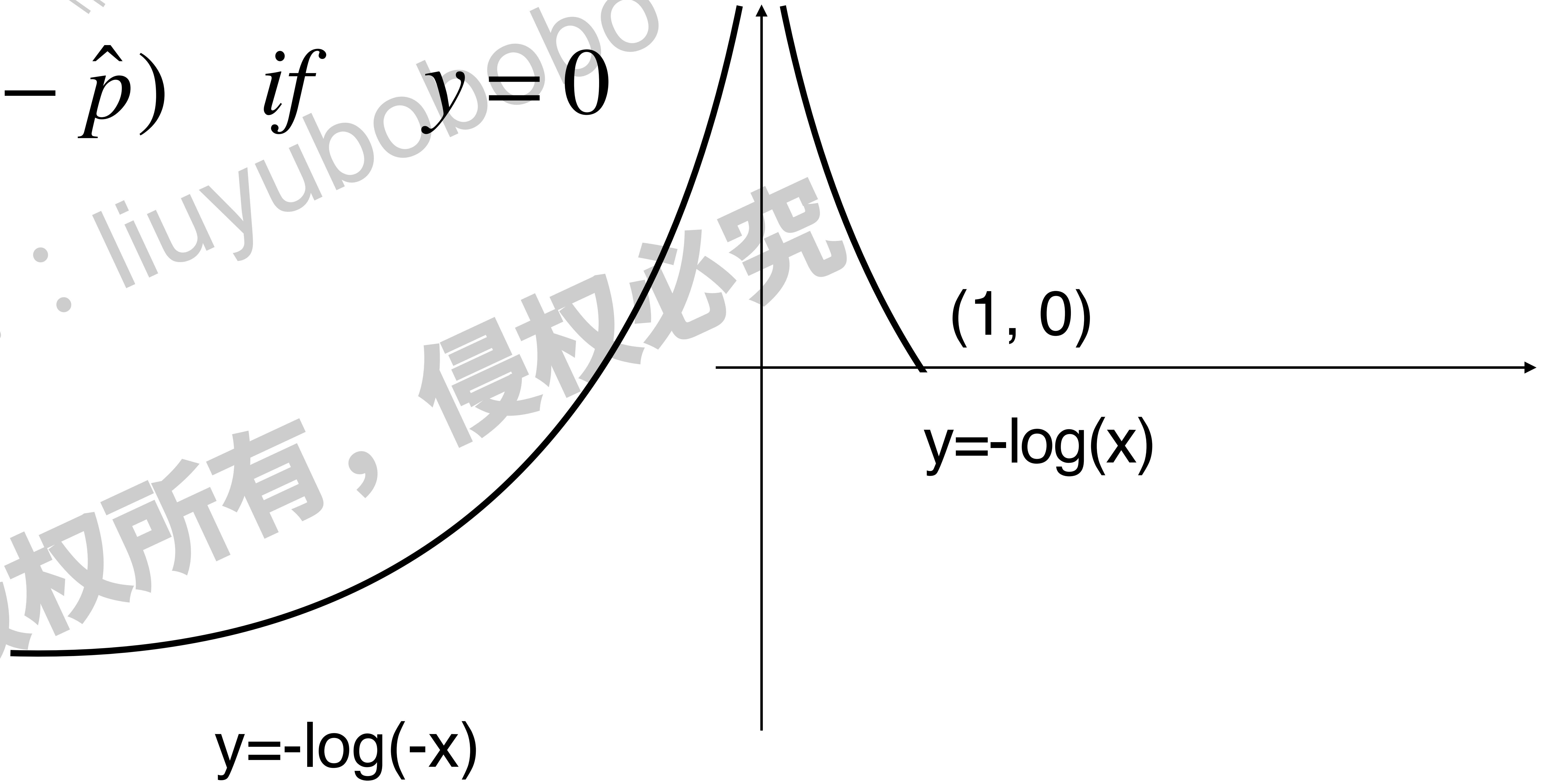
逻辑回归 Logistic Regression

$$\text{cost} = \begin{cases} -\log(\hat{p}) & if \quad y=1 \\ -\log(1-\hat{p}) & if \quad y=0 \end{cases}$$



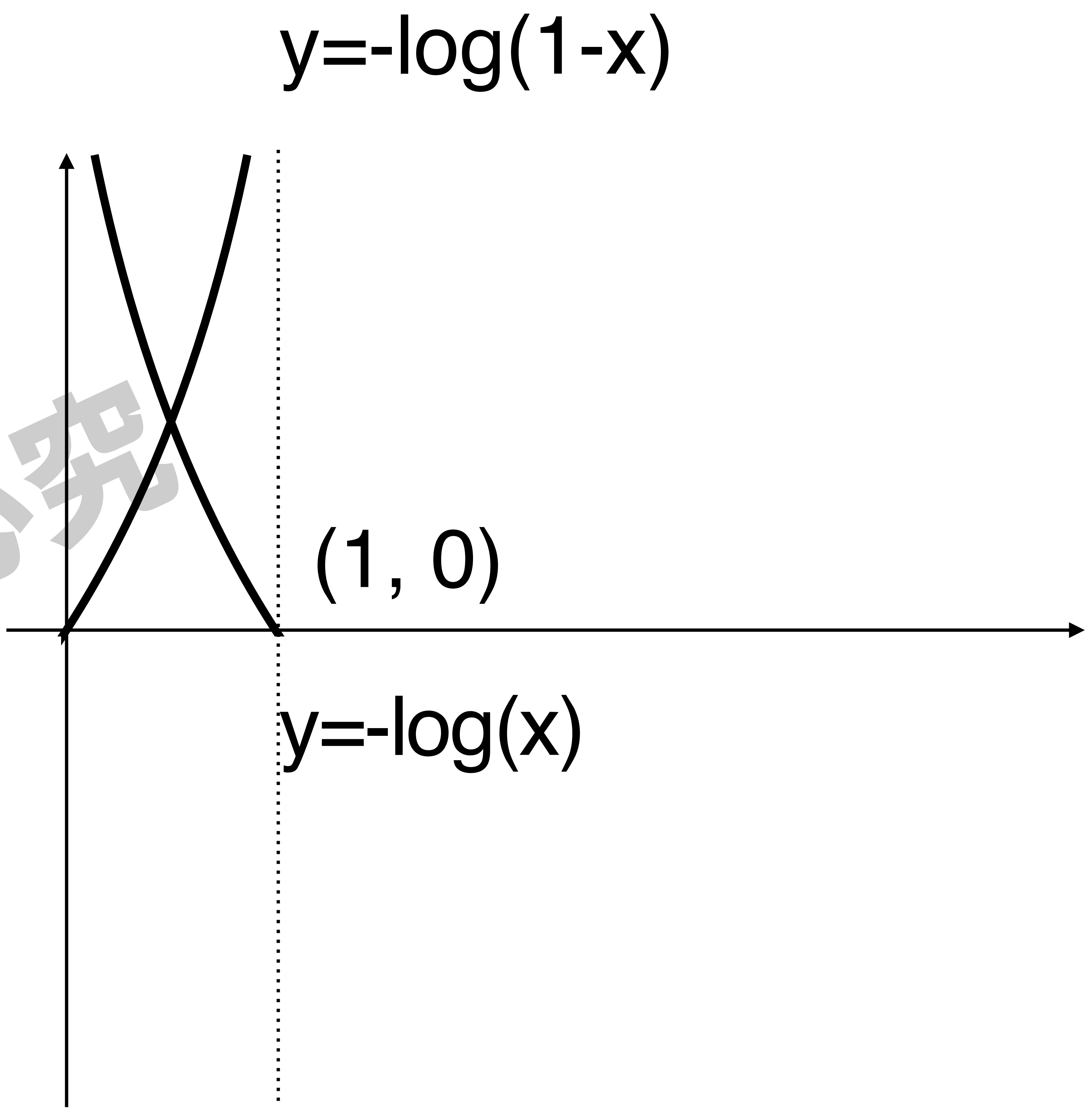
逻辑回归 Logistic Regression

$$\text{cost} = \begin{cases} -\log(\hat{p}) & if \quad y=1 \\ -\log(1-\hat{p}) & if \quad y=0 \end{cases}$$



逻辑回归 Logistic Regression

$$\text{cost} = \begin{cases} -\log(\hat{p}) & if \quad y = 1 \\ -\log(1-\hat{p}) & if \quad y = 0 \end{cases}$$



逻辑回归 Logistic Regression

$$\text{cost} = \begin{cases} -\log(\hat{p}) & if \quad y = 1 \\ -\log(1-\hat{p}) & if \quad y = 0 \end{cases}$$

$$\text{cost} = -y\log(\hat{p}) - (1-y)\log(1-\hat{p})$$

逻辑回归 Logistic Regression

$$\text{cost} = -y\log(\hat{p}) - (1-y)\log(1-\hat{p})$$

$$J(\theta) = -\frac{1}{m}\sum_{i=1}^{m} y^{(i)}\log(\hat{p}^{(i)}) + (1-y^{(i)})\log(1-\hat{p}^{(i)})$$

逻辑回归 Logistic Regression

$$J(\theta) = -\frac{1}{m}\sum_{i=1}^{m} y^{(i)}\log(\hat{p}^{(i)}) + (1-y^{(i)})\log(1-\hat{p}^{(i)})$$

$$\hat{p}^{(i)} = \sigma(X_b^{(i)}\theta) = \frac{1}{1+e^{-X_b^{(i)}\theta}}$$

逻辑回归 Logistic Regression

$$J(\theta) = -\frac{1}{m}\sum_{i=1}^{m} y^{(i)} \log(\hat{p}^{(i)}) + (1 - y^{(i)}) \log(1 - \hat{p}^{(i)})$$

$$J(\theta) = -\frac{1}{m}\sum_{i=1}^{m} y^{(i)} \log(\sigma(X_b^{(i)}\theta)) + (1 - y^{(i)}) \log(1 - \sigma(X_b^{(i)}\theta))$$

逻辑回归 Logistic Regression

$$J(\theta) = -\frac{1}{m}\sum_{i=1}^{m} y^{(i)}\log(\hat{p}^{(i)}) + (1-y^{(i)})\log(1-\hat{p}^{(i)})$$

$$J(\theta) = -\frac{1}{m}\sum_{i=1}^{m} y^{(i)}\log(\sigma(X_b^{(i)}\theta)) + (1-y^{(i)})\log(1-\sigma(X_b^{(i)}\theta))$$

没有公式解，只能使用梯度下降法求解

逻辑回归损失函数的梯度

逻辑回归 Logistic Regression

$$J(\theta) = -\frac{1}{m}\sum_{i=1}^{m} y^{(i)}\log(\sigma(X_b^{(i)}\theta)) + (1-y^{(i)})\log(1-\sigma(X_b^{(i)}\theta))$$

$$\nabla J(\theta) = \begin{pmatrix} \frac{\partial J(\theta)}{\partial \theta_0} \\ \frac{\partial J(\theta)}{\partial \theta_1} \\ \cdots \\ \frac{\partial J(\theta)}{\partial \theta_n} \end{pmatrix}$$

逻辑回归 Logistic Regression

$$\sigma(t) = \frac{1}{1+e^{-t}} = (1+e^{-t})^{-1}$$

$$\sigma(t)' = -(1+e^{-t})^{-2} \cdot e^{-t} \cdot (-1) \ = (1+e^{-t})^{-2} \cdot e^{-t}$$

逻辑回归 Logistic Regression

$$J(\theta) = -\frac{1}{m}\sum_{i=1}^{m} y^{(i)} \log(\sigma(X_b^{(i)}\theta)) + (1-y^{(i)})\log(1-\sigma(X_b^{(i)}\theta))$$

$$\sigma(t) = \frac{1}{1+e^{-t}} = (1+e^{-t})^{-1} \qquad \sigma(t)' = (1+e^{-t})^{-2} \cdot e^{-t}$$

$$(\log\sigma(t))' = \frac{1}{\sigma(t)} \cdot \sigma(t)' = \frac{1}{\sigma(t)} \cdot (1+e^{-t})^{-2} \cdot e^{-t}$$

$$= \frac{1}{(1+e^{-t})^{-1}} \cdot (1+e^{-t})^{-2} \cdot e^{-t} \quad = (1+e^{-t})^{-1} \cdot e^{-t}$$

逻辑回归 Logistic Regression

$$\sigma(t) = \frac{1}{1+e^{-t}} = (1+e^{-t})^{-1}$$

$$(\log\sigma(t))' = (1+e^{-t})^{-1} \cdot e^{-t}$$

$$= \frac{e^{-t}}{1+e^{-t}} = \frac{1+e^{-t}-1}{1+e^{-t}} = 1 - \frac{1}{1+e^{-t}}$$

$$= 1 - \sigma(t)$$

逻辑回归 Logistic Regression

$$J(\theta) = -\frac{1}{m}\sum_{i=1}^{m} y^{(i)} \log(\sigma(X_b^{(i)}\theta)) + (1-y^{(i)})\log(1-\sigma(X_b^{(i)}\theta))$$

$$(\log \sigma(t))' = 1 - \sigma(t)$$

$$\frac{d(y^{(i)} \log \sigma(X_b^{(i)}\theta))}{d\theta_j} = y^{(i)}(1-\sigma(X_b^{(i)}\theta)) \cdot X_j^{(i)}$$

逻辑回归 Logistic Regression

$$J(\theta) = -\frac{1}{m}\sum_{i=1}^{m} y^{(i)}\log(\sigma(X_b^{(i)}\theta)) + (1-y^{(i)})\log(1-\sigma(X_b^{(i)}\theta))$$

$$\sigma(t) = \frac{1}{1+e^{-t}} = (1+e^{-t})^{-1}$$

$$\sigma(t)' = (1+e^{-t})^{-2}\cdot e^{-t}$$

$$(\log(1-\sigma(t)))' = \frac{1}{1-\sigma(t)}\cdot(-1)\cdot\sigma(t)' = -\frac{1}{1-\sigma(t)}\cdot(1+e^{-t})^{-2}\cdot e^{-t}$$

逻辑回归 Logistic Regression

$$\sigma(t) = \frac{1}{1+e^{-t}} = (1+e^{-t})^{-1} \qquad \sigma(t)' = (1+e^{-t})^{-2} \cdot e^{-t}$$

$$(\log(1-\sigma(t)))' = \frac{1}{1-\sigma(t)} \cdot (-1) \cdot \sigma(t)' = -\frac{1}{1-\sigma(t)} \cdot (1+e^{-t})^{-2} \cdot e^{-t}$$

$$-\frac{1}{1-\sigma(t)} = -\frac{1}{\frac{1+e^{-t}}{1+e^{-t}} - \frac{1}{1+e^{-t}}} = -\frac{1+e^{-t}}{e^{-t}}$$

逻辑回归 Logistic Regression

$$\sigma(t) = \frac{1}{1+e^{-t}} = (1+e^{-t})^{-1}$$

$$\sigma(t)' = (1+e^{-t})^{-2} \cdot e^{-t}$$

$$(\log(1-\sigma(t)))' = \frac{1}{1-\sigma(t)} \cdot (-1) \cdot \sigma(t)' = -\frac{1}{1-\sigma(t)} \cdot (1+e^{-t})^{-2} \cdot e^{-t}$$

$$-\frac{1}{1-\sigma(t)} = -\frac{1+e^{-t}}{e^{-t}}$$

$$= -\frac{1+e^{-t}}{e^{-t}} \cdot (1+e^{-t})^{-2} \cdot e^{-t}$$

$$= -(1+e^{-t})^{-1} \quad = -\sigma(t)$$

逻辑回归 Logistic Regression

$$J(\theta) = -\frac{1}{m}\sum_{i=1}^{m} y^{(i)}\log(\sigma(X_b^{(i)}\theta)) + (1-y^{(i)})\log(1-\sigma(X_b^{(i)}\theta))$$

$$(\log(1-\sigma(t)))' = -\sigma(t)$$

$$\frac{d((1-y^{(i)})\log(1-\sigma(X_b^{(i)}\theta)))}{d\theta_j} = (1-y^{(i)})\cdot(-\sigma(X_b^{(i)}\theta))\cdot X_j^{(i)}$$

逻辑回归 Logistic Regression

$$J(\theta) = -\frac{1}{m}\sum_{i=1}^{m} y^{(i)}\log(\sigma(X_b^{(i)}\theta)) + (1-y^{(i)})\log(1-\sigma(X_b^{(i)}\theta))$$

$$\frac{d(y^{(i)}\log\sigma(X_b^{(i)}\theta))}{d\theta_j} = y^{(i)}(1-\sigma(X_b^{(i)}\theta))\cdot X_j^{(i)}$$

$$+$$

$$\frac{d((1-y^{(i)})\log(1-\sigma(X_b^{(i)}\theta)))}{d\theta_j} = (1-y^{(i)})\cdot(-\sigma(X_b^{(i)}\theta))\cdot X_j^{(i)}$$

$$y^{(i)}X_j^{(i)} - y^{(i)}\sigma(X_b^{(i)}\theta)\cdot X_j^{(i)} \qquad -\sigma(X_b^{(i)}\theta)\cdot X_j^{(i)} + y^{(i)}\sigma(X_b^{(i)}\theta)\cdot X_j^{(i)}$$

逻辑回归 Logistic Regression

$$J(\theta) = -\frac{1}{m}\sum_{i=1}^{m} y^{(i)}\log(\sigma(X_b^{(i)}\theta)) + (1 - y^{(i)})\log(1 - \sigma(X_b^{(i)}\theta))$$

$$y^{(i)}X_j^{(i)} - y^{(i)}\sigma(X_b^{(i)}\theta)\cdot X_j^{(i)} - \sigma(X_b^{(i)}\theta)\cdot X_j^{(i)} + y^{(i)}\sigma(X_b^{(i)}\theta)\cdot X_j^{(i)}$$

$$= y^{(i)}X_j^{(i)} - \sigma(X_b^{(i)}\theta)\cdot X_j^{(i)} \quad = (y^{(i)} - \sigma(X_b^{(i)}\theta))\cdot X_j^{(i)}$$

$$\frac{J(\theta)}{\theta_j} = \frac{1}{m}\sum_{i=1}^{m}(\sigma(X_b^{(i)}\theta) - y^{(i)})X_j^{(i)}$$

逻辑回归 Logistic Regression

$$J(\theta) = -\frac{1}{m}\sum_{i=1}^{m} y^{(i)} \log(\sigma(X_b^{(i)}\theta)) + (1 - y^{(i)})\log(1 - \sigma(X_b^{(i)}\theta))$$

$$\frac{J(\theta)}{\theta_j} = \frac{1}{m}\sum_{i=1}^{m}(\sigma(X_b^{(i)}\theta) - y^{(i)})X_j^{(i)}$$

$$= \frac{1}{m}\sum_{i=1}^{m}(\hat{y}^{(i)} - y^{(i)})X_j^{(i)}$$

逻辑回归 Logistic Regression

$$\frac{J(\theta)}{\theta_j} = \frac{1}{m}\sum_{i=1}^{m}(\sigma(X_b^{(i)}\theta) - y^{(i)})X_j^{(i)}$$

$$\nabla J(\theta) = \begin{pmatrix} \partial J / \partial \theta_0 \\ \partial J / \partial \theta_1 \\ \partial J / \partial \theta_2 \\ \cdots \\ \partial J / \partial \theta_n \end{pmatrix} = \frac{1}{m} \cdot \begin{pmatrix} \sum_{i=1}^{m}(\sigma(X_b^{(i)}\theta) - y^{(i)}) \\ \sum_{i=1}^{m}(\sigma(X_b^{(i)}\theta) - y^{(i)}) \cdot X_1^{(i)} \\ \sum_{i=1}^{m}(\sigma(X_b^{(i)}\theta) - y^{(i)}) \cdot X_2^{(i)} \\ \cdots \\ \sum_{i=1}^{m}(\sigma(X_b^{(i)}\theta) - y^{(i)}) \cdot X_n^{(i)} \end{pmatrix} = \frac{1}{m} \cdot \begin{pmatrix} \sum_{i=1}^{m}(\hat{y}^{(i)} - y^{(i)}) \\ \sum_{i=1}^{m}(\hat{y}^{(i)} - y^{(i)}) \cdot X_1^{(i)} \\ \sum_{i=1}^{m}(\hat{y}^{(i)} - y^{(i)}) \cdot X_2^{(i)} \\ \cdots \\ \sum_{i=1}^{m}(\hat{y}^{(i)} - y^{(i)}) \cdot X_n^{(i)} \end{pmatrix}$$

逻辑回归 Logistic Regression

$$\nabla J(\theta) = \frac{1}{m} \cdot \begin{pmatrix} \sum_{i=1}^{m} (\hat{y}^{(i)} - y^{(i)}) \\ \sum_{i=1}^{m} (\hat{y}^{(i)} - y^{(i)}) \cdot X_1^{(i)} \\ \sum_{i=1}^{m} (\hat{y}^{(i)} - y^{(i)}) \cdot X_2^{(i)} \\ \cdots \\ \sum_{i=1}^{m} (\hat{y}^{(i)} - y^{(i)}) \cdot X_n^{(i)} \end{pmatrix}$$

回忆线性回归

$$\nabla J(\theta) = \frac{2}{m} \cdot \begin{pmatrix} \sum_{i=1}^{m} (X_b^{(i)}\theta - y^{(i)}) \\ \sum_{i=1}^{m} (X_b^{(i)}\theta - y^{(i)}) \cdot X_1^{(i)} \\ \sum_{i=1}^{m} (X_b^{(i)}\theta - y^{(i)}) \cdot X_2^{(i)} \\ \cdots \\ \sum_{i=1}^{m} (X_b^{(i)}\theta - y^{(i)}) \cdot X_n^{(i)} \end{pmatrix}$$

逻辑回归 Logistic Regression

回忆线性回归

$$\nabla J(\theta) = \frac{2}{m} \cdot \begin{pmatrix} \sum_{i=1}^{m} (X_b^{(i)}\theta - y^{(i)}) \\ \sum_{i=1}^{m} (X_b^{(i)}\theta - y^{(i)}) \cdot X_1^{(i)} \\ \sum_{i=1}^{m} (X_b^{(i)}\theta - y^{(i)}) \cdot X_2^{(i)} \\ \cdots \\ \sum_{i=1}^{m} (X_b^{(i)}\theta - y^{(i)}) \cdot X_n^{(i)} \end{pmatrix} = \frac{2}{m} \cdot X_b^T \cdot (X_b\theta - y)$$

逻辑回归 Logistic Regression

$$\nabla J(\theta) = \frac{1}{m} \cdot \begin{pmatrix} \sum_{i=1}^{m} (\hat{y}^{(i)} - y^{(i)}) \\ \sum_{i=1}^{m} (\hat{y}^{(i)} - y^{(i)}) \cdot X_1^{(i)} \\ \sum_{i=1}^{m} (\hat{y}^{(i)} - y^{(i)}) \cdot X_2^{(i)} \\ \cdots \\ \sum_{i=1}^{m} (\hat{y}^{(i)} - y^{(i)}) \cdot X_n^{(i)} \end{pmatrix} = \frac{1}{m} \cdot X_b^T \cdot (\sigma(X_b\theta) - y)$$

实现我们自己的逻辑回归

实践：实现我们自己的逻辑回归

决策边界

逻辑回归 Logistic Regression

$$\hat{p} = \sigma(\theta^T \cdot x_b) = \frac{1}{1 + e^{-\theta^T \cdot x_b}}$$

$$\hat{y} = \begin{cases} 1, & \hat{p} \geq 0.5 \\ 0, & \hat{p} < 0.5 \end{cases}$$

逻辑回归 Logistic Regression

$$\hat{p} = \sigma(\theta^T \cdot x_b) = \frac{1}{1+e^{-\theta^T \cdot x_b}}$$

$$\sigma(t) = \frac{1}{1+e^{-t}}$$

$$\hat{y} = \begin{cases} 1, & \hat{p} \geq 0.5 \\ 0, & \hat{p} < 0.5 \end{cases}$$

$t > 0$ 时，$p > 0.5$

$t < 0$ 时，$p < 0.5$

逻辑回归 Logistic Regression

$$\hat{p} = \sigma(\theta^T \cdot x_b) = \frac{1}{1 + e^{-\theta^T \cdot x_b}}$$

$$\hat{y} = \begin{cases} 1, & \hat{p} \geq 0.5 \quad\quad \theta^T \cdot x_b \geq 0 \\ 0, & \hat{p} < 0.5 \quad\quad \theta^T \cdot x_b < 0 \end{cases}$$

决策边界

$$\theta^T \cdot x_b = 0$$

逻辑回归 Logistic Regression

决策边界

$$\theta^T \cdot x_b = 0$$

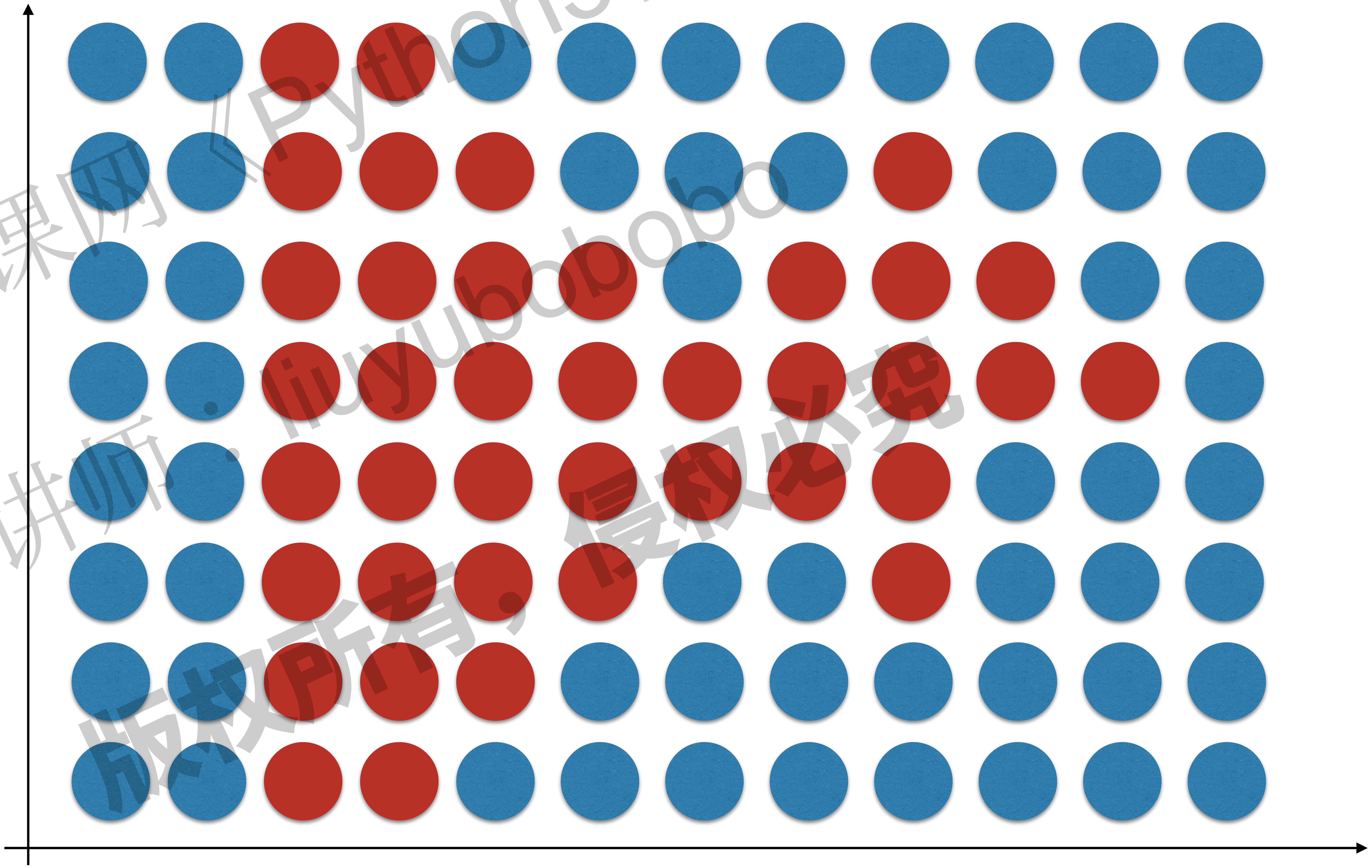
如果X有两个特征

$$\theta_0 + \theta_1 x_1 + \theta_2 x_2 = 0$$

$$x_2 = \frac{-\theta_0 - \theta_1 x_1}{\theta_2}$$

实践：绘制决策边界

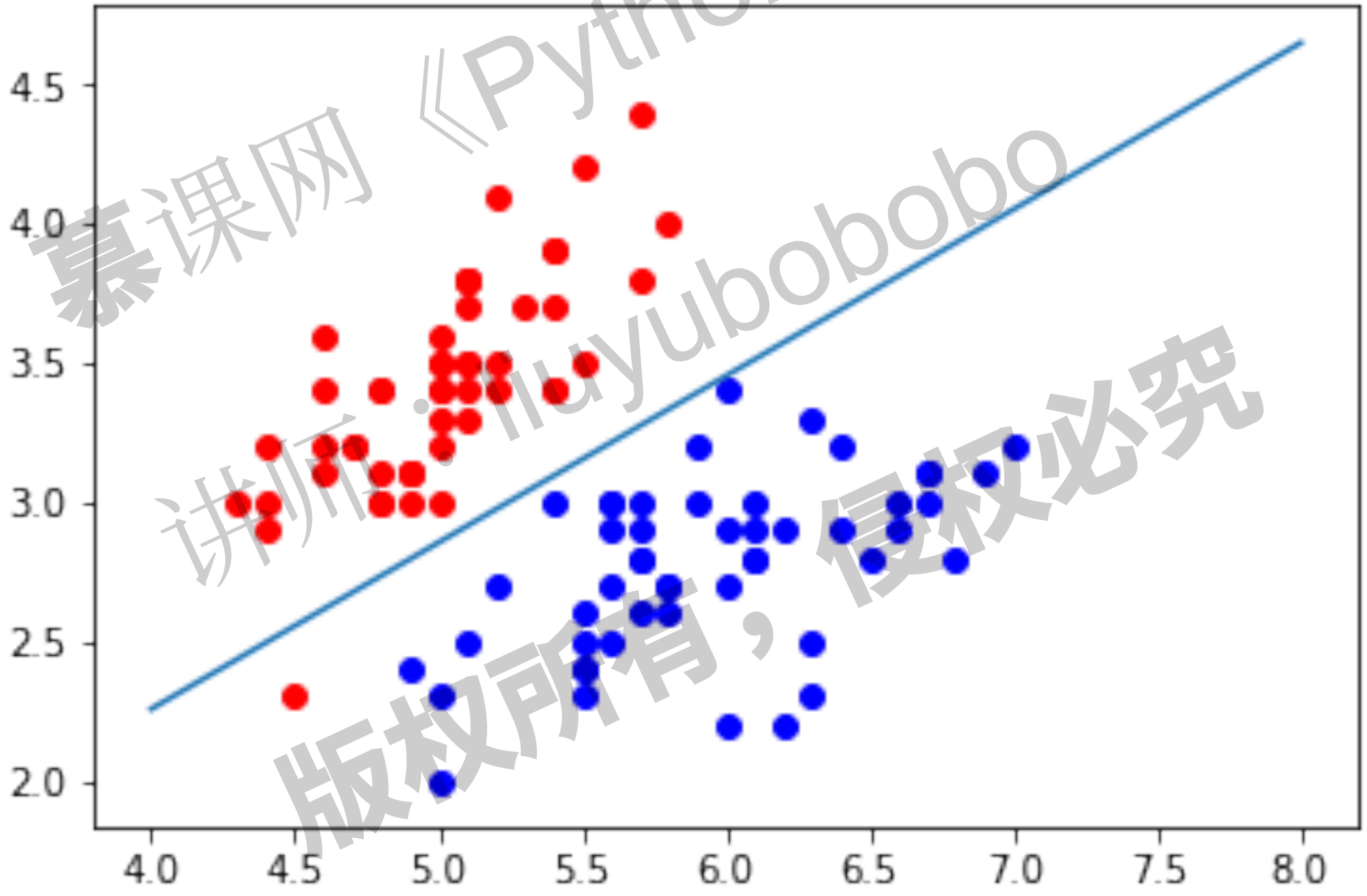
不规则的决策边界的绘制方法



实践：另一种绘制决策边界的方法

逻辑回归中使用多项式项

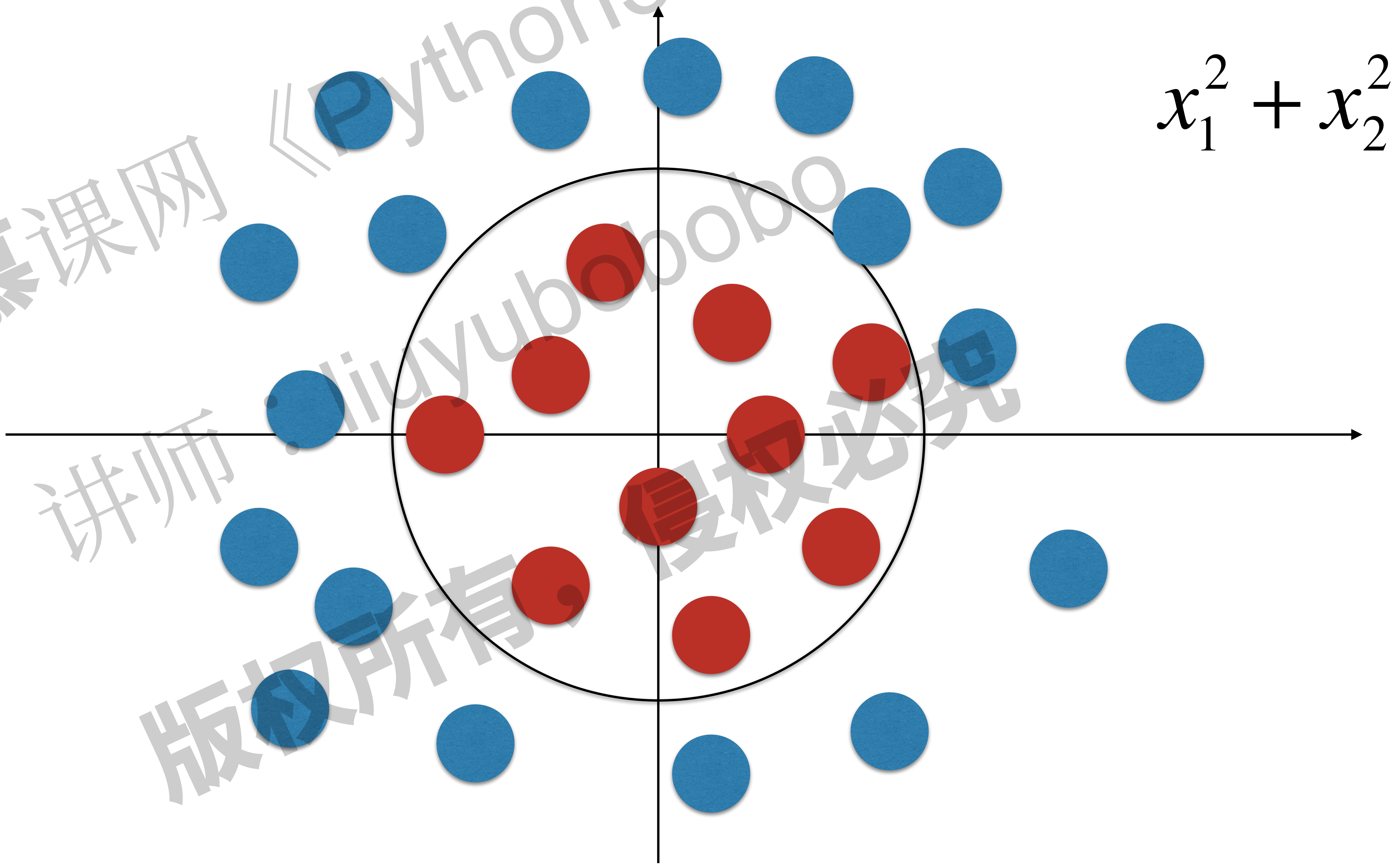
逻辑回归 Logistic Regression



决策边界

$$\theta^T \cdot x_b = 0$$

逻辑回归 Logistic Regression



$$x_1^2 + x_2^2 - r^2 = 0$$

实践：在逻辑回归中添加多项式项

scikit-learn中的逻辑回归

逻辑回归中使用正则化

$$J(\theta) + \alpha L_2$$

$$J(\theta) + \alpha L_1$$

逻辑回归中使用正则化

$$J(\theta) + \alpha L_2 \qquad C \cdot J(\theta) + L_1$$

$$J(\theta) + \alpha L_1 \qquad C \cdot J(\theta) + L_2$$

scikit-learn中使用的方式

实践：scikit-learn中的逻辑回归

OvR 和 OvO

逻辑回归 Logistic Regression

逻辑回归只可以解决二分类问题

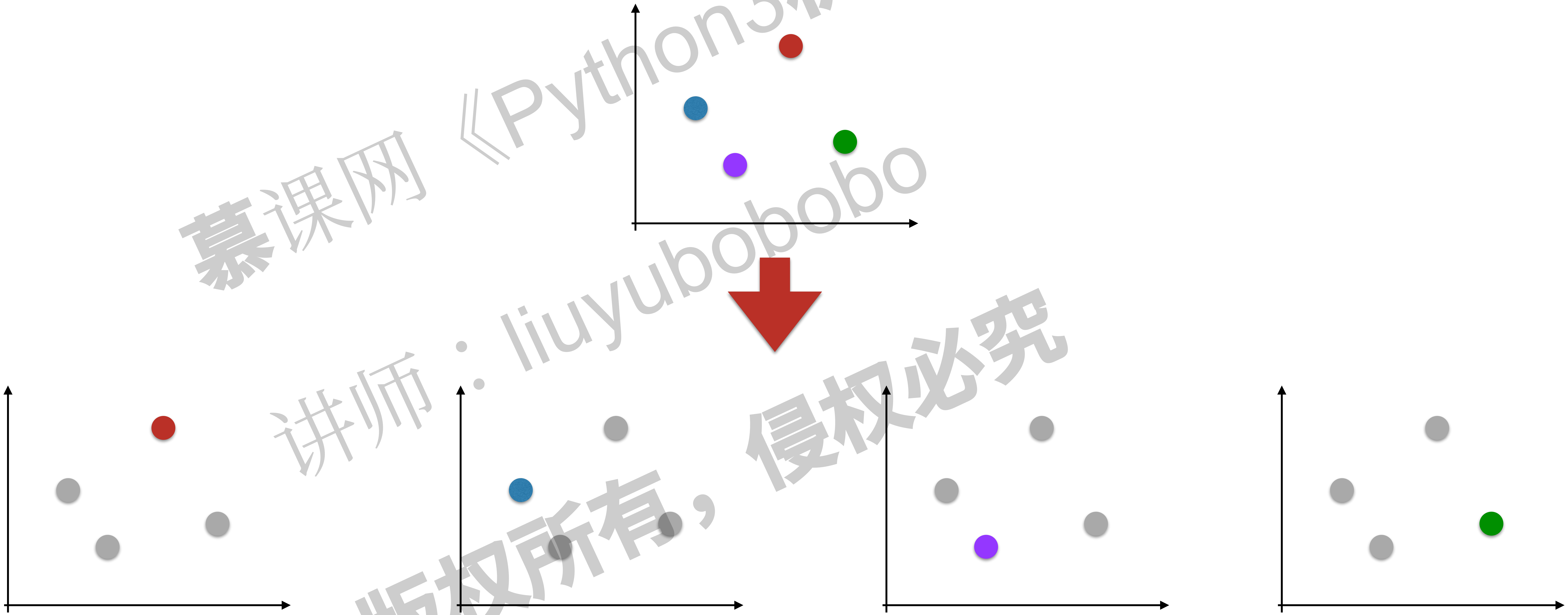
解决多分类问题：

- OvR
- OvO

OvR (One vs Rest)

OvR (One vs Rest)

OvR (One vs Rest)

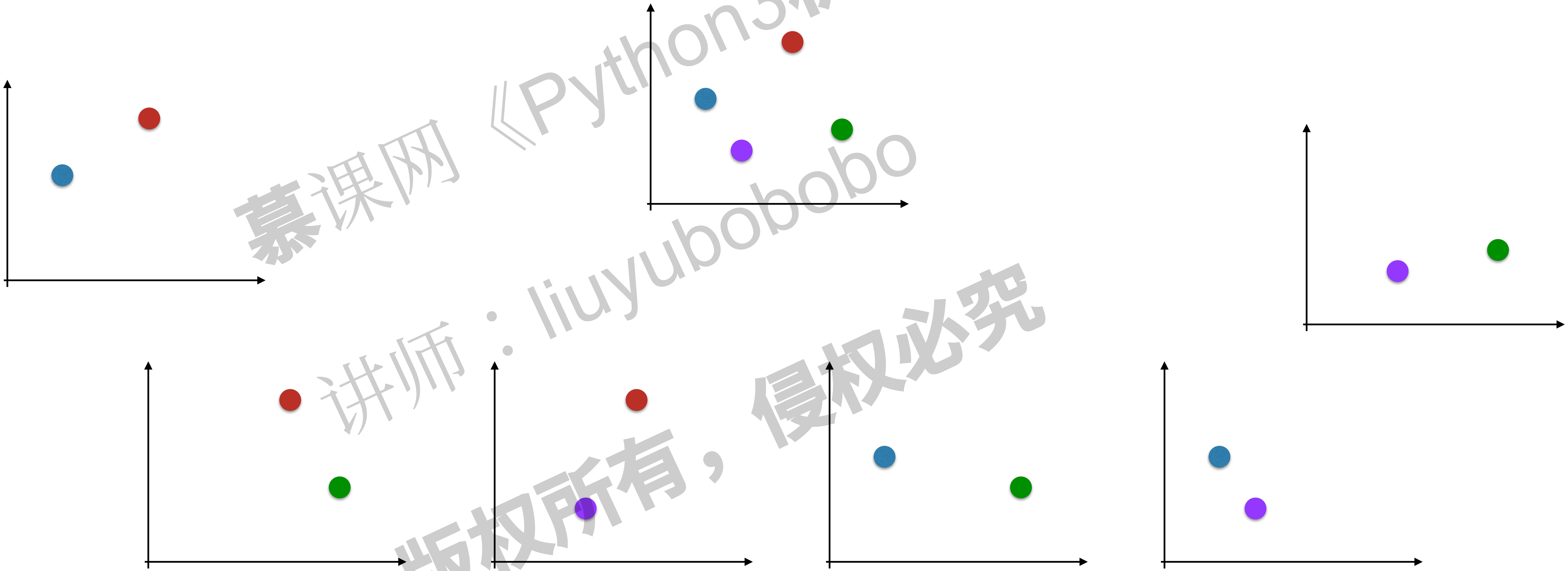


n个类别就进行n次分类，选择分类得分最高的

OvO (One vs One)

OvO (One vs One)

OvO (One vs One)



n个类别就进行C(n,2)次分类，选择赢数最高的分类

实践：OvR and OvO

其他

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