Theory of Electromagnetic Fields, Experiment 3

Author: Ying Yiwen
Number: 12210159

Contents

1 Introduction

According to Biot-Savart law, we can calculate the strength of the magnetic field generated by the current element, which is:

$$d\mathbf{H} = \frac{\mathbf{I}d\mathbf{L} \times \mathbf{a}_R}{4\pi R^2} = \frac{\mathbf{I}d\mathbf{L} \times \mathbf{R}}{4\pi R^3} \tag{1}$$

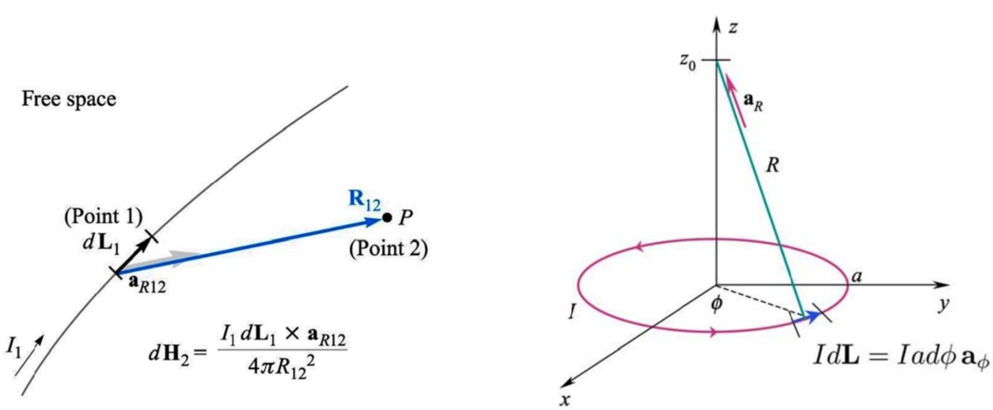
where $\mathbf{H}$ is the magnetic field intensity vector, $\mathbf{I}d\mathbf{L}$ is the current element vector, $\mathbf{R}$ is the vector from the current element $\mathbf{I}d\mathbf{L}$ to the field point P, and its magnitude is R.

For the magnetic field generated by the current ring in space, the distribution of the magnetic field intensity on the central axis of the current ring is derived using the Biot-Savart law, and the results are as follows:

$$\mathbf{H} = \frac{I(\pi a^2)\mathbf{a}_z}{2\pi(a^2 + z_0^2)^{3/2}} \tag{2}$$

where a is the radius of the current ring, I is the size of the current flowing through the current ring, and z_0 is the coordinate of the field point (located on the central axis, the z axis).

Similar to the electric field, the magnetic field also follows the superposition principle, so we can also divide the current-carrying conductor into many current elements, and the magnetic field generated by the entire current-carrying conductor is the superposition of the magnetic field generated by these current elements. Based on this idea, we can use Matlab programming to solve the distribution of the magnetic field established by the current loop at any field point.



(a) The Strength of the Magnetic Field Generated by the Current Element

(b) The Strength of the Magnetic Field on the Central Axis of the Current Loop

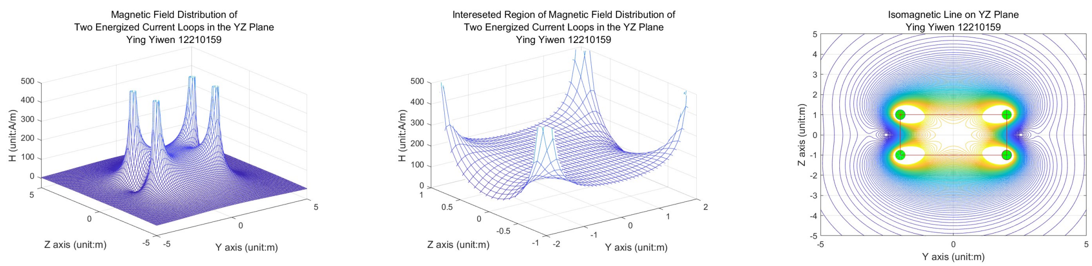
Fig. 1: Sketch Map for Biot-Savart Law

2 Condition 1 - The Helmholtz Coil

Two current rings with radius of 2m, the same current direction, the size of the current is 500A, parallel to XY plane, the center of the circle is located respectively at O_1 $(0, 0, -1)$, O_2 $(0, 0, 1)$.

2.1 Results

By Matlab simulation, we can find out the magentic field, as follows:



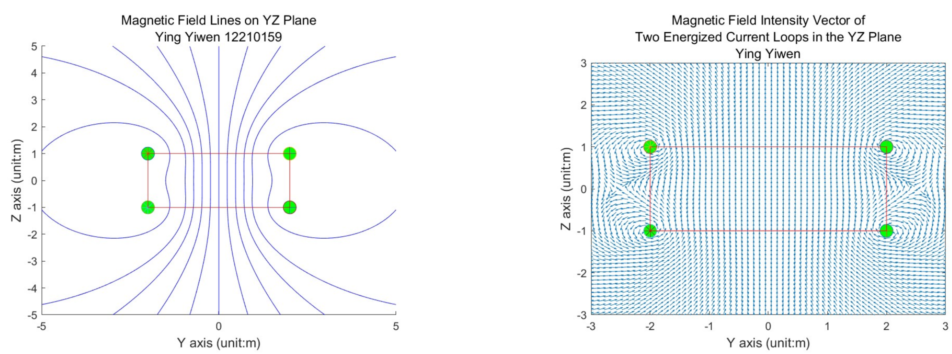
(a) Magnetic Field Distribution (Overall Perspective)

(b) Magnetic Field Distribution (Interested Region)

(c) Isomagnetic Line

Fig. 2: Distribution of Magnetic Field Strength Values, the Helmholtz Coil

Similarly, we can also find out the magnetic field intensity vector distribution, as follows:



(a) Magnetic Field Lines

(b) Magnetic Field Intensity Vector (By arrows)

Fig. 3: Magnetic Field Intensity Vector Distribution, the Helmholtz Coil

From the results above, we can find that, in the Helmholtz coil, the magnetic field inside the coils (y in [-2,2], z in [-1,1] in this case) is almost uniform magnetic field.

2.2 Code

2.2.1 Set Parameters

```
% r=2,I=500,O1(0,0,-1),O2(0,0,1),same direction
clc,clear,close all;
a=2; % radius of circuit
I=500; % current
C=I/(4*pi); % merge constant
N=200; % segments of circuit loop
ym=5; % y region
zm=5; % z region
n=100; % divide segments
```

2.2.2 Calculate the Magnetic Field

```
y=linspace(-ym,ym,n); % divide y
z=linspace(-zm,zm,n); % divide z
[Y,Z]=meshgrid(y,z); % form coordinates
theta0=linspace(0,2*pi,N+1); % divide the circle
theta1=theta0(1:N); % start angle
x1=a*cos(theta1); % start x
y1=a*sin(theta1); % start y
theta2=theta0(2:N+1); % end angle
x2=a*cos(theta2); % end x
y2=a*sin(theta2); % end y
xc=(x1+x2)/2; % midpoint x
yc=(y1+y2)/2; % midpoint y
zc1=-ones(N); % midpoint z1
zc2=ones(N); % midpoint z2
dlx=x2-x1; % dl x
dly=y2-y1; % dl y
dlz=0; %dl z

Hy=zeros(n); % build H for y component
Hz=zeros(n); % build H for z component
H=zeros(n); % build H (total)
for i = 1:N
    X=zeros(n); % x are all zeros because on yz plane
    rx=X-xc(i); % r x
    ry=Y-yc(i); % r y
    rz1=Z-zc1(i); % r z1
    rz2=Z-zc2(i); % r z2
    r31=sqrt(rx.^2+ry.^2+rz1.^2).^3; % r^3 1
    r32=sqrt(rx.^2+ry.^2+rz2.^2).^3; % r^3 2
```

```
    dlXr_y1=-dlx(i).*rz1; % y component of dl×r 1
    dlXr_y2=-dlx(i).*rz2; % y component of dl×r 2
    dlXr_z=dlx(i).*ry-dly(i).*rx; % z component of dl×r
    Hy=Hy+C.*dlXr_y1./r31; % add to Hy 1
    Hy=Hy+C.*dlXr_y2./r32; % add to Hy 2
    Hz=Hz+C.*dlXr_z./r31; % add to Hz 1
    Hz=Hz+C.*dlXr_z./r32; % add to Hz 2
    H=(Hy.^2+Hz.^2).^0.5; % compute H
end
```

2.2.3 Magnetic Field Distribution

```
figure; % magnetic distribution (whole)
mesh(Y,Z,H); % draw distribution figure
hold on; % keep the same graph
title(sprintf('Magnetic Field Distribution of\nTwo Energized Current Loops in the YZ Plane\nYing Yiwen 12210159'),"FontSize",12); % title
axis([-5,5,-5,5,-50,500]); % xyz limits
xlabel('Y axis (unit:m)','FontSize',12); % x label
ylabel('Z axis (unit:m)','FontSize',12); % y label
zlabel('H (unit:A/m)','FontSize',12); % z label
saveas(gcf,'fig1-1.jpg'); % save figure
figure; % magnetic distribution (-2,2,-1,1)
mesh(Y,Z,H); % draw distribution figure
hold on; % keep the same graph
title(sprintf('Intereseted Region of Magnetic Field Distribution of\nTwo Energized Current Loops in the YZ Plane\nYing Yiwen 12210159'),"FontSize",12); % title
axis([-2,2,-1,1,-0,500]); % xyz limits
xlabel('Y axis (unit:m)','FontSize',12); % x label
ylabel('Z axis (unit:m)','FontSize',12); % y label
zlabel('H (unit:A/m)','FontSize',12); % z label
saveas(gcf,'fig1-2.jpg'); % save figure

figure; % isomagnetic
Hmin=0; % minimum H
Hmax=200; % maximum H
H0=linspace(Hmin,Hmax,100); % divide isomagnetic line
contour(Y,Z,H,H0); % draw isomagnetic line
grid on; % form grid
hold on; % keep the same graph
title(sprintf('Isomagnetic Line on YZ Plane\nYing Yiwen 12210159'),"FontSize",12); % title
axis([-5,5,-5,5]); % xy limits
```

```
plot(-2,1,'o','MarkerFaceColor','g','MarkerSize',12); % (-2,1) bisect
plot(-2,1,'*','MarkerFaceColor','k','MarkerSize',12); % current flow direction
plot(2,1,'o','MarkerFaceColor','g','MarkerSize',12); % (2,1) bisect
plot(2,1,'+','MarkerFaceColor','k','MarkerSize',12); % current flow direction
plot(-2,-1,'o','MarkerFaceColor','g','MarkerSize',12); % (-2,1) bisect
plot(-2,-1,'*','MarkerFaceColor','k','MarkerSize',12); % current flow direction
plot(2,-1,'o','MarkerFaceColor','g','MarkerSize',12); % (2,1) bisect
plot(2,-1,'+','MarkerFaceColor','k','MarkerSize',12); % current flow direction
rectangle('Position',[-2,-1,4,2],'EdgeColor','r'); % the interested region
xlabel('Y axis (unit:m)','FontSize',12); % x label
ylabel('Z axis (unit:m)','FontSize',12); % y label
saveas(gcf,'fig1-3.jpg'); % save figure
```

2.2.4 Magnetic Field Lines

```
figure; % distribution of magnetic field lines
theta=[0 50 60 70 80 90 100 110 120 130 180].*pi/180; % radian value
ys=1.5*cos(theta); % field line origin coordinates x
zs=1.5*sin(theta); % field line origin coordinates y
streamline(Y,Z,Hy,Hz,ys,zs); % draw magnetic field lines outward
streamline(Y,Z,-Hy,-Hz,ys,zs); % draw magnetic field lines inward
title(sprintf('Magnetic Field Lines on YZ Plane\nYing Yiwen 12210159'),"FontSize",12); % title
hold on; % keep the same graph
axis([-5,5,-5,5]); % xy limits
plot(-2,1,'o','MarkerFaceColor','g','MarkerSize',12); % (-2,1) bisect
plot(-2,1,'*','MarkerFaceColor','k','MarkerSize',12); % current flow direction
plot(2,1,'o','MarkerFaceColor','g','MarkerSize',12); % (2,1) bisect
plot(2,1,'+','MarkerFaceColor','k','MarkerSize',12); % current flow direction
plot(-2,-1,'o','MarkerFaceColor','g','MarkerSize',12); % (-2,1) bisect
plot(-2,-1,'*','MarkerFaceColor','k','MarkerSize',12); % current flow direction
plot(2,-1,'o','MarkerFaceColor','g','MarkerSize',12); % (2,1) bisect
```

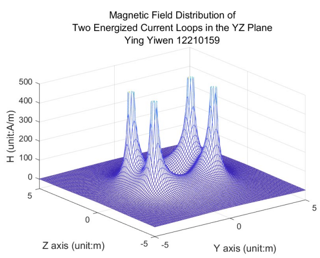
```matlab
plot(2,-1,'+','MarkerFaceColor','k','MarkerSize',12); % current flow direction
rectangle('Position',[-2,-1,4,2],'EdgeColor','r'); % the interested region
xlabel('Y axis (unit:m)','FontSize',12); % x label
ylabel('Z axis (unit:m)','FontSize',12); % y label
saveas(gcf,'fig1-4.jpg'); % save figure

figure; % magnetic field strength vector diagram
Hy=Hy./H; % normalization
Hz=Hz./H; % normalization
quiver(Y,Z,Hy,Hz); % draw vector diagram
hold on; % keep the same graph
title(sprintf('Magnetic Field Intensity Vector of\nTwo Energized Current Loops in the YZ Plane\nYing Yiwen'),"FontSize",12); % title
axis([-3,3,-3,3]); % xy limits
plot(-2,1,'o','MarkerFaceColor','g','MarkerSize',12); % (-2,1) bisect
plot(-2,1,'*','MarkerFaceColor','k','MarkerSize',12); % current flow direction
plot(2,1,'o','MarkerFaceColor','g','MarkerSize',12); % (2,1) bisect
plot(2,1,'+','MarkerFaceColor','k','MarkerSize',12); % current flow direction
plot(-2,-1,'o','MarkerFaceColor','g','MarkerSize',12); % (-2,1) bisect
plot(-2,-1,'*','MarkerFaceColor','k','MarkerSize',12); % current flow direction
plot(2,-1,'o','MarkerFaceColor','g','MarkerSize',12); % (2,1) bisect
plot(2,-1,'+','MarkerFaceColor','k','MarkerSize',12); % current flow direction
rectangle('Position',[-2,-1,4,2],'EdgeColor','r'); % the interested region
xlabel('Y axis (unit:m)','FontSize',12); % x label
ylabel('Z axis (unit:m)','FontSize',12); % y label
saveas(gcf,'fig1-5.jpg'); % save figure
```

3 Condition 2 - The Helmholtz Coil with Oppsite Direction Loops

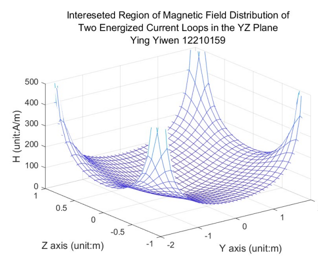
Two current rings with radius of 2m, the oppsite current direction, the size of the current is 500A, parallel to XY plane, the center of the circle is located respectively at O_1 $(0, 0, -1)$, O_2 $(0, 0, 1)$.

3.1 Results

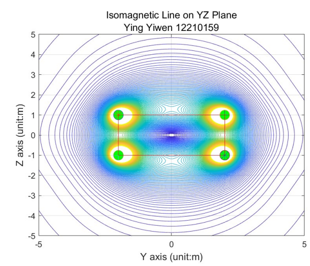
By Matlab simulation, we can find out the magentic field, as follows:



(a) Magnetic Field Distribution (Overall Perspective)



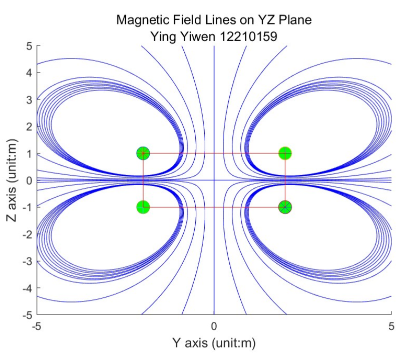
(b) Magnetic Field Distribution (Interested Region)



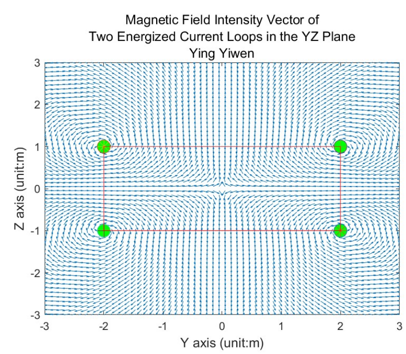
(c) Isomagnetic Line

Fig. 4: Distribution of Magnetic Field Strength Values, the Helmholtz Coil with Oppsite Direction

Similarly, we can also find out the magnetic field intensity vector distribution, as follows:



(a) Magnetic Field Lines



(b) Magnetic Field Intensity Vector (By arrows)

Fig. 5: Magnetic Field Intensity Vector Distribution, the Helmholtz Coil with Oppsite Direction

From the results above, we can find that, when the circuit directions are oppsite, the magnetic field inside the coils (y in [-2,2], z in [-1,1] in this case) is symmetric in y and z axis.

3.2 Code

3.2.1 Set Parameters

```
% r=2,I=500,O1(0,0,-1),O2(0,0,1),oppsite direction
clc,clear,close all;
a=2; % radius of circuit
I=500; % current
C=I/(4*pi); % merge constant
N=200; % segments of circuit loop
ym=5; % y region
zm=5; % z region
n=100; % divide segments
```

3.2.2 Calculate the Magnetic Field

```
y=linspace(-ym,ym,n); % divide y
z=linspace(-zm,zm,n); % divide z
[Y,Z]=meshgrid(y,z); % form coordinates
theta0=linspace(0,2*pi,N+1); % divide the circle
theta1=theta0(1:N); % start angle
x1=a*cos(theta1); % start x
y1=a*sin(theta1); % start y
theta2=theta0(2:N+1); % end angle
x2=a*cos(theta2); % end x
y2=a*sin(theta2); % end y
xc=(x1+x2)/2; % midpoint x
yc=(y1+y2)/2; % midpoint y
zc1=-ones(N); % midpoint z1
zc2=ones(N); % midpoint z2
dlx=x2-x1; % dl x
dly=y2-y1; % dl y
dlz=0; %dl z

Hy=zeros(n); % build H for y component
Hz=zeros(n); % build H for z component
H=zeros(n); % build H (total)
for i = 1:N
    X=zeros(n); % x are all zeros because on yz plane
    rx=X-xc(i); % r x
    ry=Y-yc(i); % r y
    rz1=Z-zc1(i); % r z1
    rz2=Z-zc2(i); % r z2
    r31=sqrt(rx.^2+ry.^2+rz1.^2).^3; % r^3 1
    r32=sqrt(rx.^2+ry.^2+rz2.^2).^3; % r^3 2
```

```
    dlXr_y1=dlx(i).*rz1; % y component of dl×r 1
    dlXr_y2=-dlx(i).*rz2; % y component of dl×r 2
    dlXr_z1=-dlx(i).*ry+dly(i).*rx; % z component of dl×r 1
    dlXr_z2=dlx(i).*ry-dly(i).*rx; % z component of dl×r 2
    Hy=Hy+C.*dlXr_y1./r31; % add to Hy 1
    Hy=Hy+C.*dlXr_y2./r32; % add to Hy 2
    Hz=Hz+C.*dlXr_z1./r31; % add to Hz 1
    Hz=Hz+C.*dlXr_z2./r32; % add to Hz 2
    H=(Hy.^2+Hz.^2).^0.5; % compute H
end
```

3.2.3 Magnetic Field Distribution

```
figure; % magnetic distribution (whole)
mesh(Y,Z,H); % draw distribution figure
hold on; % keep the same graph
title(sprintf('Magnetic Field Distribution of\nTwo Energized Current Loops in the YZ Plane\nYing Yiwen 12210159'),"FontSize",12); % title
axis([-5,5,-5,5,-50,500]); % xyz limits
xlabel('Y axis (unit:m)','FontSize',12); % x label
ylabel('Z axis (unit:m)','FontSize',12); % y label
zlabel('H (unit:A/m)','FontSize',12); % z label
saveas(gcf,'fig2-1.jpg'); % save figure
figure; % magnetic distribution (-2,2,-1,1)
mesh(Y,Z,H); % draw distribution figure
hold on; % keep the same graph
title(sprintf('Intereseted Region of Magnetic Field Distribution of\nTwo Energized Current Loops in the YZ Plane\nYing Yiwen 12210159'),"FontSize",12); % title
axis([-2,2,-1,1,-0,500]); % xyz limits
xlabel('Y axis (unit:m)','FontSize',12); % x label
ylabel('Z axis (unit:m)','FontSize',12); % y label
zlabel('H (unit:A/m)','FontSize',12); % z label
saveas(gcf,'fig2-2.jpg'); % save figure

figure; % isomagnetic
Hmin=0; % minimum H
Hmax=200; % maximum H
H0=linspace(Hmin,Hmax,100); % divide isomagnetic line
contour(Y,Z,H,H0); % draw isomagnetic line
grid on; % form grid
hold on; % keep the same graph
title(sprintf('Isomagnetic Line on YZ Plane\nYing Yiwen 12210159'),"FontSize",12); % title
```

```
axis([-5,5,-5,5]); % xy limits
plot(-2,1,'o','MarkerFaceColor','g','MarkerSize',12); % (-2,1) bisect
plot(-2,1,'*','MarkerFaceColor','k','MarkerSize',12); % current flow direction
plot(2,1,'o','MarkerFaceColor','g','MarkerSize',12); % (2,1) bisect
plot(2,1,'+','MarkerFaceColor','k','MarkerSize',12); % current flow direction
plot(-2,-1,'o','MarkerFaceColor','g','MarkerSize',12); % (-2,1) bisect
plot(-2,-1,'+','MarkerFaceColor','k','MarkerSize',12); % current flow direction
plot(2,-1,'o','MarkerFaceColor','g','MarkerSize',12); % (2,1) bisect
plot(2,-1,'*','MarkerFaceColor','k','MarkerSize',12); % current flow direction
rectangle('Position',[-2,-1,4,2],'EdgeColor','r'); % the interested region
xlabel('Y axis (unit:m)','FontSize',12); % x label
ylabel('Z axis (unit:m)','FontSize',12); % y label
saveas(gcf,'fig2-3.jpg'); % save figure
```

3.2.4 Magnetic Field Lines

```
figure; % distribution of magnetic field lines
theta=[0 50 60 70 80 90 100 110 120 130 180 230 240 250 260 270 280 290 300 310].*pi/180; % radian value
ys=1.5*cos(theta); % field line origin coordinates x
zs=1.5*sin(theta); % field line origin coordinates y
streamline(Y,Z,Hy,Hz,ys,zs); % draw magnetic field lines outward
streamline(Y,Z,-Hy,-Hz,ys,zs); % draw magnetic field lines inward
title(sprintf('Magnetic Field Lines on YZ Plane\nYing Yiwen 12210159'),"FontSize",12); % title
hold on; % keep the same graph
axis([-5,5,-5,5]); % xy limits
plot(-2,1,'o','MarkerFaceColor','g','MarkerSize',12); % (-2,1) bisect
plot(-2,1,'*','MarkerFaceColor','k','MarkerSize',12); % current flow direction
plot(2,1,'o','MarkerFaceColor','g','MarkerSize',12); % (2,1) bisect
plot(2,1,'+','MarkerFaceColor','k','MarkerSize',12); % current flow direction
plot(-2,-1,'o','MarkerFaceColor','g','MarkerSize',12); % (-2,1) bisect
plot(-2,-1,'+','MarkerFaceColor','k','MarkerSize',12); % current flow direction
```

```
plot(2,-1,'o','MarkerFaceColor','g','MarkerSize',12); % (2,1) bisect
plot(2,-1,'*','MarkerFaceColor','k','MarkerSize',12); % current flow direction
rectangle('Position',[-2,-1,4,2],'EdgeColor','r'); % the interested region
xlabel('Y axis (unit:m)','FontSize',12); % x label
ylabel('Z axis (unit:m)','FontSize',12); % y label
saveas(gcf,'fig2-4.jpg'); % save figure

figure; % magnetic field strength vector diagram
Hy=Hy./H; % normalization
Hz=Hz./H; % normalization
quiver(Y,Z,Hy,Hz); % draw vector diagram
hold on; % keep the same graph
title(sprintf('Magnetic Field Intensity Vector of\nTwo Energized Current Loops in the YZ Plane\nYing Yiwen'),"FontSize",12); % title
axis([-3,3,-3,3]); % xy limits
plot(-2,1,'o','MarkerFaceColor','g','MarkerSize',12); % (-2,1) bisect
plot(-2,1,'*','MarkerFaceColor','k','MarkerSize',12); % current flow direction
plot(2,1,'o','MarkerFaceColor','g','MarkerSize',12); % (2,1) bisect
plot(2,1,'+','MarkerFaceColor','k','MarkerSize',12); % current flow direction
plot(-2,-1,'o','MarkerFaceColor','g','MarkerSize',12); % (-2,1) bisect
plot(-2,-1,'+','MarkerFaceColor','k','MarkerSize',12); % current flow direction
plot(2,-1,'o','MarkerFaceColor','g','MarkerSize',12); % (2,1) bisect
plot(2,-1,'*','MarkerFaceColor','k','MarkerSize',12); % current flow direction
rectangle('Position',[-2,-1,4,2],'EdgeColor','r'); % the interested region
xlabel('Y axis (unit:m)','FontSize',12); % x label
ylabel('Z axis (unit:m)','FontSize',12); % y label
saveas(gcf,'fig2-5.jpg'); % save figure
```

4 Experience

Master the ability to simulate the magnetic field generated by paralleled loops. Observe the magnetic field distribution of Helmholtz coils, and also the condition when the current direction is oppsite. Be rather proficient in using latex for report writing.

A Helmholtz coil is a device that creates a uniform magnetic field over a small area. The open nature of the Helmholtz coil makes it easy to place other instruments in or out of it, as well as to make direct visual observations. In some applications, Helmholtz coils can be used to counteract geomagnetic fields, creating areas of near-zero magnetic field.

In this experiment, I verified the properties of the Helmholtz coil by simulating the abstract theoretical results and observed that the Helmholtz coil was indeed able to construct a uniform magnetic field. In turn, it is able to construct a symmetrical magnetic field when the coil currents are in opposite directions. Such interesting properties may be used frequently in experiments where a magnetic field needs to be established.