Theory of Electromagnetic Fields, Experiment 4

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Contents

1 Introduction **2**
1.1 Physics Principle 2
1.2 Magnetic Focusing 3

2 Magnetic Focusing Experiment **3**
2.1 Results 3
2.2 Code 6
2.2.1 Set Parameters 6
2.2.2 Three Dimension Figure 6
2.2.3 Top View 7
2.2.4 Front View 7

3 Experiment Experience **8**
3.1 Findings 8
3.2 Gains 8

1 Introduction

1.1 Physics Principle

Charges are subjected to the Lorentz force in an electromagnetic field, as follows:

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B} \tag{1}$$

where $\mathbf{F}$ is the combined force vector, $\mathbf{E}$ is the electric field strength vector, $\mathbf{B}$ is the magnetic flux density vector, $\mathbf{v}$ is the velocity vector of the charge motion, and q is the charge carried by the charge.

From Newton's Laws of Motion, it is known that charges accelerate under the action of a combined force, resulting in changes in velocity and displacement. In a three-dimensional Cartesian coordinate system, this process can be described by the following vector equation:

$$\mathbf{E}(t) = E_x(t)\mathbf{a}_x + E_y(t)\mathbf{a}_y + E_z(t)\mathbf{a}_z \tag{2}$$

$$\mathbf{B}(t) = B_x(t)\mathbf{a}_x + B_y(t)\mathbf{a}_y + B_z(t)\mathbf{a}_z \tag{3}$$

$$\mathbf{F}(t) = q\mathbf{E}(t) + q\mathbf{v} \times \mathbf{B}(t) \tag{4}$$

$$\mathbf{a}(t) = \frac{\mathbf{F}(t)}{m} \tag{5}$$

$$\mathbf{v}(t) = \mathbf{v}(1) + \int_0^t \mathbf{a}(t)dt \tag{6}$$

$$\mathbf{r}(t) = \mathbf{r}(1) + \int_0^t \mathbf{v}(t)dt \tag{7}$$

among which, m is the mass of the charge, $\mathbf{a}$ is the acceleration vector, $\mathbf{r}$ is the position vector.

It can be seen that the motion of a charge in an electromagnetic field is a process that develops over time. This process can be solved by solving the differential equations to obtain analytical solutions for the velocity vector and position vector at each moment in time.

To facilitate the solution, we discretise the time by introducing a very small time step $\triangle$t and assuming that the acceleration vector remains constant during that time fragment, such that Eqation (5) and (6) can be written in the following discrete form:

$$\mathbf{v}(t + \triangle t) = \mathbf{v}(t) + \mathbf{a}(t)\triangle t \tag{8}$$

$$\mathbf{r}(t + \triangle t) = \mathbf{r}(t) + \mathbf{v}(t)\triangle t \tag{9}$$

In this way, we can analyse the velocity and position vectors of each time segment through Matlab programming, thus depicting the trajectory of the charge over a period of time.

1.2 Magnetic Focusing

For a beam of charged particles with a small divergence angle, when they have the same velocity component in the direction of the magnetic field **B**, they have the same pitch of the trajectory, after a period of time they will be reunited at another point. The phenomenon of this dispersed particles east converge to a point with the lens will focus the beam phenomenon is very similar, so called magnetic focusing.

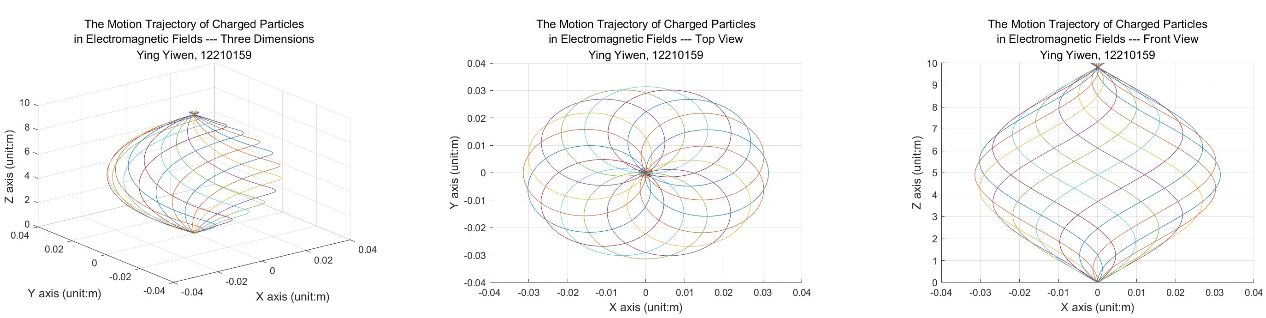
Conditions for magnetic focussing:
(1) The initial velocity **v** of each electron is approximately equal in magnitude.
(2) The angle between **v** and **B** is small enough such that each electron is in solenoidal motion.

2 Magnetic Focusing Experiment

Scenario:
There are 16 charges, with same mass m=0.02kg, same charge q=0.016C.
The initial position is the same **r**(1)=0 (all located at the origin).
The electric field strength in space is **E**=0, and the magnetic flux density is $B = 8\mathbf{a}_z Wb/m^2$.
The initial velocities of these 16 charges have the same component in the z-axis $v_z(1) = 10m/s$, and the components of their initial velocities in the x-axis and y-axis can be expressed as $v_x(1) = 0.1sin(\frac{k\pi}{8})m/s$, $v_y(1) = 0.1cos(\frac{k\pi}{8})m/s$, where k=0,1,2,......,15.

2.1 Results

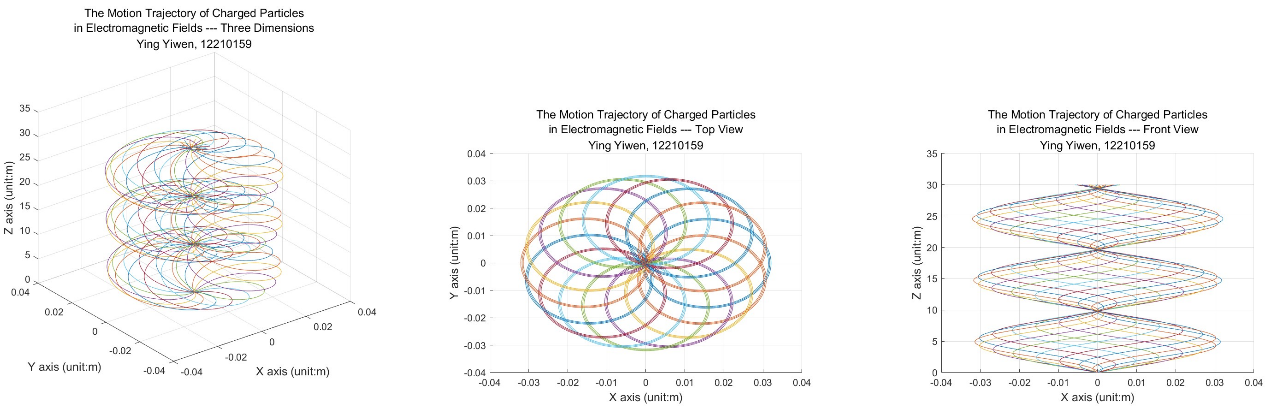
To clearly observe the results, we can set t to be various values, so as to observe the motion in different scales. For each case, we can observe the phenomenon through three dimension graph(xyz graph) to learn about the whole motion, top view(xy graph) to learn about the motion for each particle, and front view(xz graph) to learn about the magnetic focusing phenomenon.



(a) Three Dimension (b) Top View (c) Front View

Fig. 1: The Motion Trajectory of Charged Particlesin Electromagnetic Fields — t=1

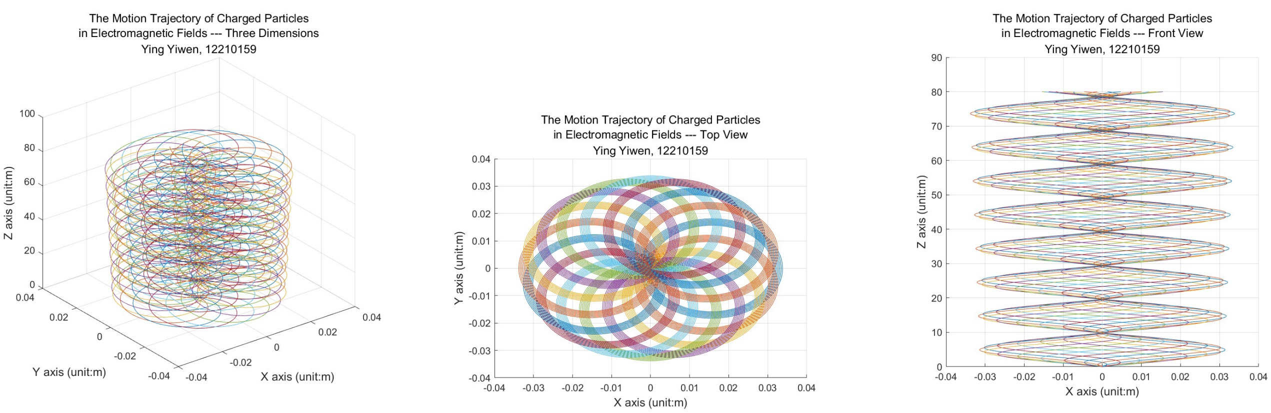
According to the figures above (t=1), we can know localised phenomena in electron motion. As the 16 particles are symmetry about the origin, the trajectories of the 16 electrons are symmetric about the origin.



(a) Three Dimension (b) Top View (c) Front View

Fig. 2: The Motion Trajectory of Charged Particlesin Electromagnetic Fields — t=3

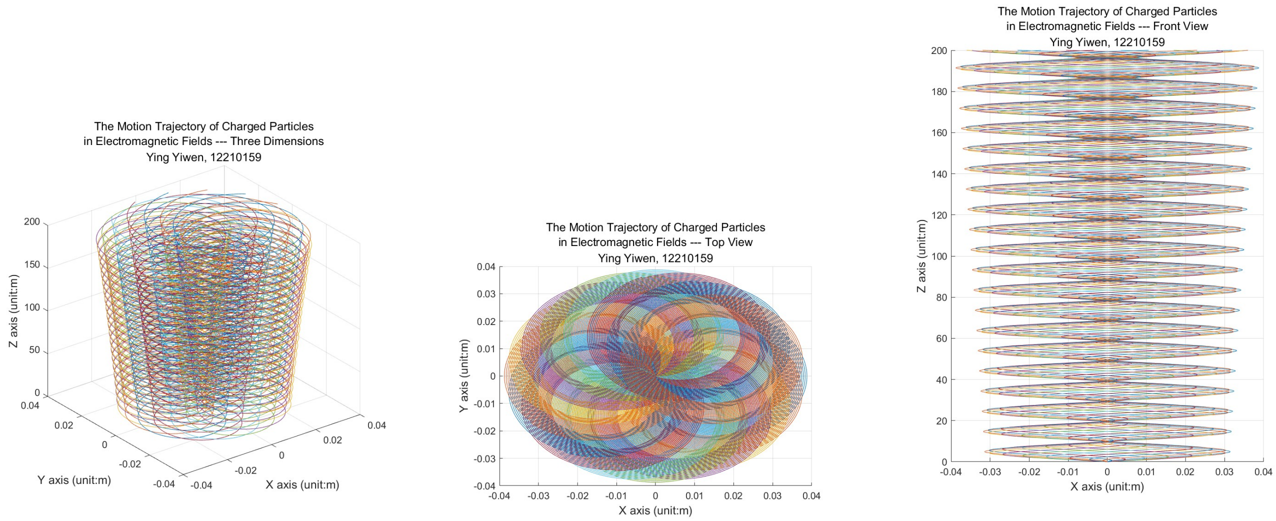
According to the figures above (t=3), we can learn from the front view that the particles really focus to a point after a period. Besides, we can know that, for a specific charge, it continuously moves around the same circle.



(a) Three Dimension (b) Top View (c) Front View

Fig. 3: The Motion Trajectory of Charged Particlesin Electromagnetic Fields — t=8

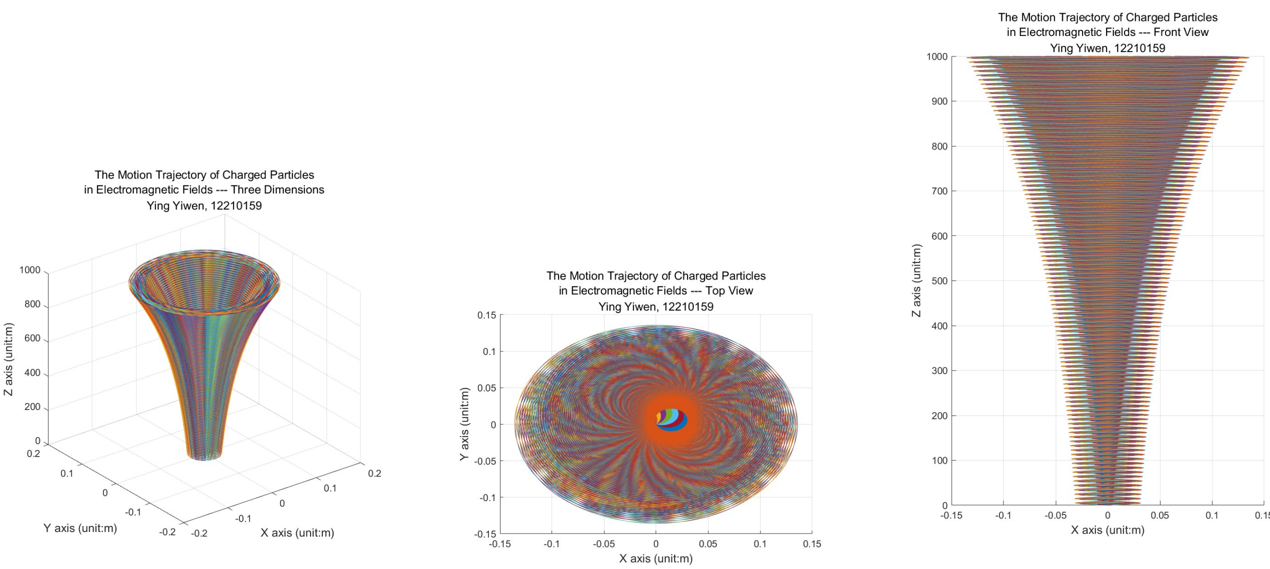
According to the figures above (t=8), we can learn from the top view that, in each period, the particles go inwards. No matter how many periods the charges experienced, they still focus on a point after each period.



(a) Three Dimension (b) Top View (c) Front View

Fig. 4: The Motion Trajectory of Charged Particlesin Electromagnetic Fields — t=20

According to the figures above (t=20), we can suggest the trend that, when charges focusing to the center after each period, in the next cycle, its trajectory is closer to the centre than in the previous cycle.



(a) Three Dimension (b) Top View (c) Front View

Fig. 5: The Motion Trajectory of Charged Particlesin Electromagnetic Fields — t=100

According to the figures above (t=100), on a large scale, the motion of the 16 electrons appears to be like a cone, with the electrons focusing to the center after each cycle and the motion gradually converging towards the center. It can be surmised that after a long enough time, all the electrons will eventually converge at one point.

2.2 Code

For each experiments, there are some small changes in the parameters, but the logic and implementation of the code is the same. Thus, I've only included a sample of the code.

2.2.1 Set Parameters

```
% parameters
m=0.02; % charge mass
q=1.6e-2; % charge quantity
dt=0.001; % time division
t=0:dt:1; % time array
vx=linspace(0,0,length(t)); % velocity x
vy=vx; % velocity y
vz=vx; % velocity z
rx=linspace(0,0,length(t)); % position x
ry=rx; % position y
rz=rx; % position z
Ex=0; % electric field intensity x
Ey=0; % electric field intensity y
Ez=0; % electric field intensity z
Bx=0; % magnetic field intensity x
By=0; % magnetic field intensity y
Bz=8; % magnetic field intensity z
Fx=linspace(0,0,length(t)); % force vector x
Fy=Fx; % force vetor y
Fz=Fx; % force vetor z
ax=linspace(0,0,length(t)); % accelerate x
ay=ax; % accelerate y
az=ax; % accelerate z
```

2.2.2 Three Dimension Figure

```
%draw
figure;
hold on; % keep the figure
grid on; % open the grid
title(sprintf('The Motion Trajectory of Charged Particles\nin Electromagnetic 
    Fields -- Three Dimensions'),'Ying Yiwen, 12210159','FontSize',12); % title
xlabel('X axis (unit:m)','FontSize',12); % x label
ylabel('Y axis (unit:m)','FontSize',12); % y label
zlabel('Z axis (unit:m)','FontSize',12); % z label
view(3); % set the graph to be three dimension
set(gcf,'Position',[360 178 560 420]); % adjust graph size

%compute
for k = 0:15
vx(1)=0.1*sin(k*pi/8); % velocity x init
vy(1)=0.1*cos(k*pi/8); % velocity y init
vz(1)=10; % velocity z init
for i = 1:(length(t)-1)
Fx(i)=q*Ex+q*(vy(i)*Bz-vz(i)*By); % force x i
Fy(i)=q*Ey+q*(vx(i)*Bx-vx(i)*Bz); % force y i
```

```
Fz(i)=q*Ez+q*(vx(i)*By-vy(i)*Bx); % force z i
ax(i)=Fx(i)/m; % accelerate x i
ay(i)=Fy(i)/m; % accelerate y i
az(i)=Fz(i)/m; % accelerate z i
vx(i+1)=vx(i)+ax(i)*dt; % velocity x i+1
vy(i+1)=vy(i)+ay(i)*dt; % velocity y i+1
vz(i+1)=vz(i)+az(i)*dt; % velocity z i+1
rx(i+1)=rx(i)+vx(i)*dt; % position x i+1
ry(i+1)=ry(i)+vy(i)*dt; % position y i+1
rz(i+1)=rz(i)+vz(i)*dt; % position z i+1
end
plot3(rx,ry,rz); % plot the figure
end
saveas(gcf,'fig-1-1.jpg'); % save figure
```

2.2.3 Top View

```
%draw
figure;
hold on; % keep the figure
grid on; % open the grid
title(sprintf('The Motion Trajectory of Charged Particles \nin Electromagnetic Fields -- Top View'),'Ying Yiwen, 12210159','FontSize',12); % title
xlabel('X axis (unit:m)','FontSize',12); % x label
ylabel('Y axis (unit:m)','FontSize',12); % y label

%compute
for k = 0:15
vx(1)=0.1*sin(k*pi/8); % velocity x init
vy(1)=0.1*cos(k*pi/8); % velocity y init
vz(1)=10; % velocity z init
for i = 1:(length(t)-1)
Fx(i)=q*Ex+q*(vy(i)*Bz-vz(i)*By); % force x i
Fy(i)=q*Ey+q*(vx(i)*Bx-vx(i)*Bz); % force y i
Fz(i)=q*Ez+q*(vx(i)*By-vy(i)*Bx); % force z i
ax(i)=Fx(i)/m; % accelerate x i
ay(i)=Fy(i)/m; % accelerate y i
az(i)=Fz(i)/m; % accelerate z i
vx(i+1)=vx(i)+ax(i)*dt; % velocity x i+1
vy(i+1)=vy(i)+ay(i)*dt; % velocity y i+1
vz(i+1)=vz(i)+az(i)*dt; % velocity z i+1
rx(i+1)=rx(i)+vx(i)*dt; % position x i+1
ry(i+1)=ry(i)+vy(i)*dt; % position y i+1
rz(i+1)=rz(i)+vz(i)*dt; % position z i+1
end
plot(rx,ry); % plot the figure
end
saveas(gcf,'fig-1-2.jpg'); % save figure
```

2.2.4 Front View

```
%draw
```

```
figure;
hold on; % keep the figure
grid on; % open the grid
title(sprintf('The Motion Trajectory of Charged Particles\nin Electromagnetic Fields —— Front View'),'Ying Yiwen, 12210159','FontSize',12); % title
xlabel('X axis (unit:m)','FontSize',12); % x label
ylabel('Z axis (unit:m)','FontSize',12); % z label
set(gcf,'Position',[360 178 560 420]); % adjust graph size

%compute
for k = 0:15
vx(1)=0.1*sin(k*pi/8); % velocity x init
vy(1)=0.1*cos(k*pi/8); % velocity y init
vz(1)=10; % velocity z init
for i = 1:(length(t)-1)
Fx(i)=q*Ex+q*(vy(i)*Bz-vz(i)*By); % force x i
Fy(i)=q*Ey+q*(vx(i)*Bx-vx(i)*Bz); % force y i
Fz(i)=q*Ez+q*(vx(i)*By-vy(i)*Bx); % force z i
ax(i)=Fx(i)/m; % accelerate x i
ay(i)=Fy(i)/m; % accelerate y i
az(i)=Fz(i)/m; % accelerate z i
vx(i+1)=vx(i)+ax(i)*dt; % velocity x i+1
vy(i+1)=vy(i)+ay(i)*dt; % velocity y i+1
vz(i+1)=vz(i)+az(i)*dt; % velocity z i+1
rx(i+1)=rx(i)+vx(i)*dt; % position x i+1
ry(i+1)=ry(i)+vy(i)*dt; % position y i+1
rz(i+1)=rz(i)+vz(i)*dt; % position z i+1
end
plot(rx,rz); % plot the figure
end
saveas(gcf,'fig-1-3.jpg'); % save figure
```

3 Experiment Experience

3.1 Findings

For the solenoidal motion of a beam of charged particles with a small divergence angle in a magnetic field, they do manage to reconverge at another point after one cycle.
After a large number of cycles, the solenoidal motion trajectory gradually moves closer to the centre, forming an overall conical motion trajectory.

Magnetic focusing is widely used in many electro-vacuum systems (e.g., electron microscopes), and in practice it is more commonly used for magnetic focusing of non-uniform magnetic fields in short coils.

3.2 Gains

Understood the manifestations of the magnetic focusing phenomenon and verified the conclusions of the magnetic focusing phenomenon.
Mastered the Matlab simulation of the motion trajectory of the magnetic focusing phenomenon.

In addition, when observing the phenomenon, different values of dt and t will give very different images, and appropriate values should be chosen to observe the results of the desired conclusion scale.