信号与系统互助课堂

常用公式及考点

$x(t) \to x(at+b)$ 先平移，再缩放

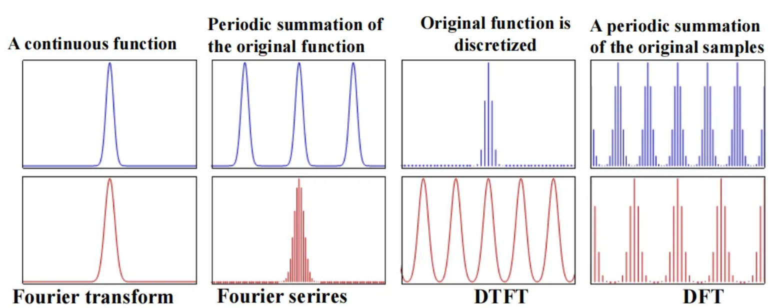
系统性质

- 时不变time-invariance $x(t-\tau)$ 输出为 $y(t-\tau)$
- 线性linear $\sum a_k x_k(t)$ 输出为 $\sum a_k y_k(t)$
- 因果causality $h(t) = 0 \text{ for } t < 0$
- 稳定stable $\sum\limits_{-\infty}^{+\infty} |h[k]| < \infty$
- 无记忆memoryless $h(t) = K\delta(t)$

傅里叶变换/级数

- 时域周期，频域离散，傅里叶级数
- 时域非周期，频域连续，傅里叶变换
- 时域连续，频域非周期
- 时域离散，频域周期

Time Domain	Non-Periodic	Periodic	
Continuous	Continuous Time Fourier Transform (CTFT)	Fourier Series (FS)	Non-Periodic
Discrete	Discrete Time Fourier Transform (DTFT)	Discrete Fourier Transform (DFT)	Periodic
	Continuous	Discrete	Frequency Domain



傅里叶级数

$x(t)$ real and even, a_k real and even

$x(t)$ real and odd, a_k imaginary and odd

傅里叶变换

时域连续 $H(e^{jw}) = \int_{-\infty}^{+\infty} h(t)e^{-jwt}dt$ 时域离散 $H(e^{jw}) = \sum_{-\infty}^{+\infty} h[n]e^{-jwn}$

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jw_0 t} \leftrightarrow X(jw) = \sum_{k=-\infty}^{+\infty} 2\pi a_k \delta(w - kw_0)$$

$$x(t) = \sum_{k=-\infty}^{+\infty} \delta(t - nT) \leftrightarrow X(jw) = \sum_{k=-\infty}^{+\infty} \frac{2\pi}{T} \delta(w - kw_0)$$

$$x(t) = \begin{cases} 1, |t| < T_1 \\ 0, |t| > T_1 \end{cases} \leftrightarrow X(jw) = \frac{2\sin(wT_1)}{w}$$

$$x(t) = \frac{\sin(Wt)}{\pi t} \leftrightarrow X(jw) = \begin{cases} 1, |w| < W \\ 0, |w| > W \end{cases}$$

$h(t), H(jw)$ 表示连续； $h[n], H(e^{jw})$ 表示离散

绝大部分公式通用

Lec 1–4 Signals and Systems, LTI

transformation:

- time shift $x(t - t_0)$
- time reversal $x(-t)$
- time scaling $x(at)$
  - $a > 1$ 收缩
  - $a < 1$ 膨胀

奇偶函数

- $x_{even} = \dfrac{x(t) + x(-t)}{2}$

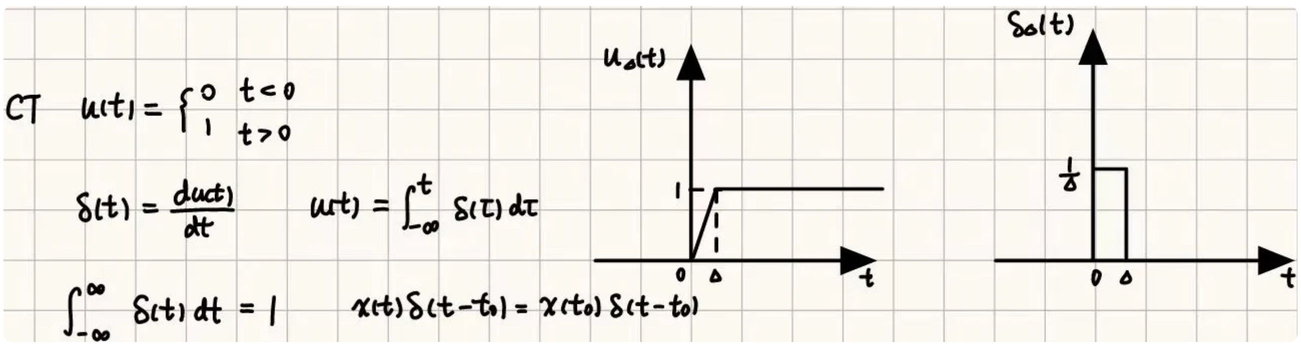
- $x_{odd} = \dfrac{x(t) - x(-t)}{2}$

欧拉公式

- $e^{jx} = \cos x + j \sin x$
- $\cos x = \dfrac{e^{jx} + e^{-jx}}{2}$
- $\sin x = \dfrac{e^{jx} - e^{-jx}}{2}$

离散信号periodic when $N = m \cdot \dfrac{2\pi}{w_0}$

$\delta(t)$



线性时不变系统，本课程讨论内容

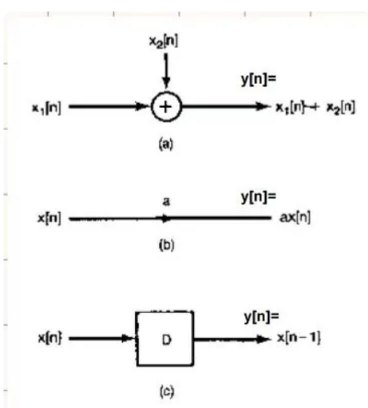
卷积 $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$

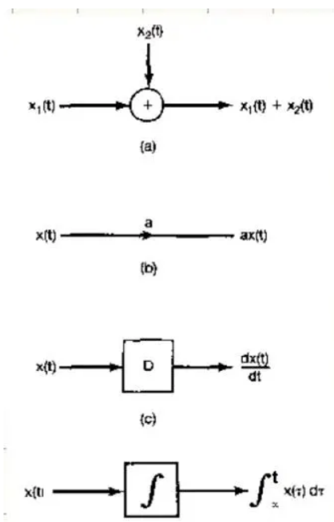
flip, shift, multiply, sum

$x[n] = \{1.2.3\}$　　$h[n] = \{4.5.6\}$

		1	2	3			
6	5	4					4
	6	5	4				13
		6	5	4			28
			6	5	4		27
				6	5	4	18

$\therefore x[n] * h[n] = \{4.13.28.27.18\}$





Lec 5-10 Fourier Series & Transform

连续时间傅里叶级数（CTFS）

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi k}{T}t}$$

$$c_k = \frac{1}{T}\int_T x(t)e^{-j\frac{2\pi k}{T}t}\,dt$$

连续时间傅里叶变换（CTFT）

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega$$

离散时间傅里叶变换（DTFT）

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

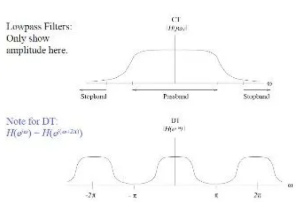
$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}\,d\omega$$

CTFS**常用**性质：（仅为个人做题经验常用公式，不是全部公式）

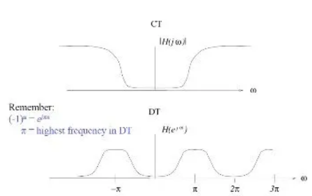
- $x(t)$ real $\leftrightarrow \quad a_k^* = a_{-k}$
- $x(t-t_0) \leftrightarrow a_k e^{-jk2\pi t_0/T}$
- $x(t)\cdot y(t) \leftrightarrow c_k = a_k * b_k$
- $\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$
- $e^{jMw_0t}x(t) \leftrightarrow a_{k-M}$
- $\frac{dx}{dt} \leftrightarrow jkw_0a_k$

magnitude and phase $H(e^{jkw_0}) = |H(e^{jkw_0})|e^{j\phi}$

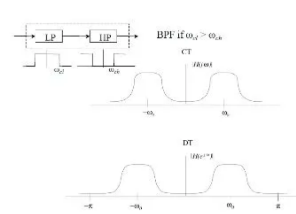
lowpass filter



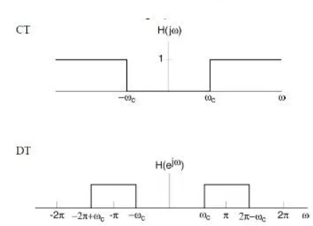
highpass filter



bandpass filter



idealized filter



finite energy can do Fourier Transform

1. absolutely integrable $\int_{-\infty}^{+\infty} |x(t)|^2 dt < \infty$
2. finite interval, finite number of maxima, minima
3. finite interval, finite number of discontinuities

定义 $\delta(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{jwt}dw$

- $x(t)=\delta(t-t_0) \quad \leftrightarrow \quad X(j\omega)=e^{-j\omega t_0}$
- $x(t)=e^{j\omega_0 t} \quad \leftrightarrow \quad X(j\omega)=2\pi\delta(\omega-\omega_0)$
- $x(t)=\sum_{k=-\infty}^{\infty}a_k e^{j\omega_0 t} \quad \leftrightarrow \quad X(j\omega)=\sum_{k=-\infty}^{\infty}2\pi a_k\delta(\omega-k\omega_0)$
- $x(t)=\sum_{k=-\infty}^{\infty}\delta(t-kT) \quad \leftrightarrow \quad X(j\omega)=\frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta(\omega-k\omega_0)$

CTFT**常用**性质：（仅为个人做题经验常用公式，不是全部公式）

- $x(t)$ real $\leftrightarrow \quad X(jw)=X^*(-jw)$
- $x(t-t_0)\leftrightarrow e^{-jkwt_0}X(jw)$
- $e^{jw_0t}x(t)\leftrightarrow X(j(w-w_0))$
- $x(t)\cdot h(t)\leftrightarrow \frac{1}{2\pi}X(jw)*H(jw)$
- $x(t)*y(t)\leftrightarrow$
- $\int_{-\infty}^{+\infty}|x(t)|^2dt=\frac{1}{2\pi}\int_{k=-\infty}^{+\infty}|X(jw)|^2dw$
- $\frac{dx}{dt}\leftrightarrow jwX(jw)$
- $x(at)\leftrightarrow \frac{1}{|a|}X(j\frac{w}{a})$
- $even\{x(t)\}\leftrightarrow Re\{X(jw)\}, odd\{x(t)\}\leftrightarrow Im\{X(jw)\}$
- $\cos w_0t\leftrightarrow \pi[\delta(w-w_0)+\delta(w+w_0)]$
- $\sin w_0t\leftrightarrow \frac{\pi}{j}[\delta(w-w_0)-\delta(w+w_0)]$
- $e^{-at}u(t)\leftrightarrow \frac{1}{a+jw}$

DTFT性质与CTFT极为相似

采样sampling

$$x[n]=\sum_{-\infty}^{+\infty}\delta[n-kN]$$

$$a_k = \sum_{n=<N>} \delta[n] e^{-jk\frac{2\pi}{N}n} = \frac{1}{N}$$

$$X(e^{jw}) = \frac{2\pi}{N} \sum_{k=-\infty}^{\infty} \delta(w - \frac{2\pi k}{N})$$

$$X_p(e^{jw}) = \frac{1}{N} \sum_{k=0}^{N-1} X(e^{j(w-kw_s)})$$

Problems

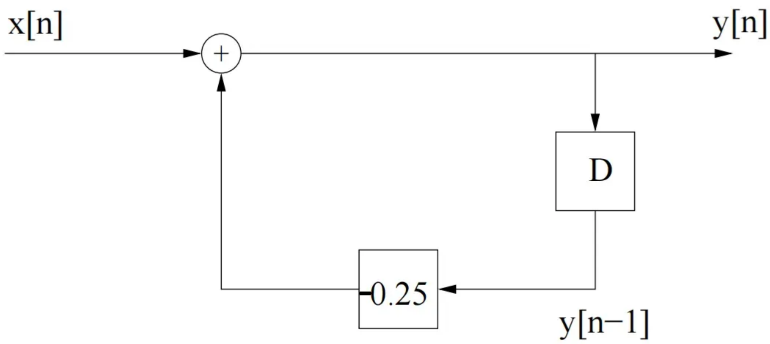
以下哪个函数可以作为线性时不变系统的冲激响应?

- A. $h(t) = e^{-at}$
- B. $h(t) = te^{-at}$
- C. $h(t) = \int_{t-1}^{t+1} x(\tau)d\tau$
- D. $h(t) = \sin(3t)$

以下哪个信号是非周期的?

- A. $x(t) = \sin(7\pi t) + \cos(\frac{3}{2}\pi t)$
- B. $x[n] = \sin(2n) + \cos(3n)$
- C. $x[n] = e^{j\frac{\pi}{3}n} + e^{j\frac{2\pi}{9}n}$
- D. $x(t) = 2\cos(\frac{2\pi t}{3}) + 3\cos(\frac{2\pi t}{6})$

$$X(jw) = \frac{2 + \frac{13}{3}e^{-jw} - \frac{29}{24}e^{-2jw}}{1 - \frac{1}{3}e^{-jw} + \frac{1}{48}e^{-2jw}} \text{ 求 } x[n]$$



$h[n]$ 为以上系统，求 $y[n] = x[n] * h[n]$

$x[n] = \{3, 0, 1, -2, -3, 4, 1, 0, -1\}$ 求：

$$X(e^{j0}) \quad X(e^{j\pi}) \quad \int_{-\pi}^{\pi} X(e^{j\omega})d\omega \quad \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega \quad \int_{-\pi}^{\pi} \left|\frac{dX(e^{j\omega})}{d\omega}\right|^2 d\omega$$

- 答案

  C，A非线性，B时变，D非线性

  B，离散的N，无法整除

  $$x[n] = 2\delta[n] + 3(\frac{1}{4})^{n-1}u[n-1] + 2(\frac{1}{12})^{n-1}u[n-1]$$

  $$y[n] = -\frac{17}{4}(\frac{1}{4})^n u[n] + \frac{25}{4}(\frac{1}{12})^n u[n]$$

  3 ， -1 ， 6π ， 82π ， 1368π