translational

$$ {}^{i+1}V_{i+1} = {}^{i+1}_{i}R \cdot {}^{i}V_i + {}^{i+1}_{i}R({}^{i}\omega_i \times {}^{i}P_{i+1}) + \dot{d}_{i+1} \cdot {}^{i+1}\hat{Z}_{i+1} $$

$$ {}^{i+1}\omega_{i+1} = {}^{i+1}_{i}R \cdot {}^{i}\omega_i $$

rotational

$$ {}^{i+1}V_{i+1} = {}^{i+1}_{i}R \cdot {}^{i}V_i + {}^{i+1}_{i}R({}^{i}\omega_i \times {}^{i}P_{i+1}) $$

$$ {}^{i+1}\omega_{i+1} = {}^{i+1}_{i}R \cdot {}^{i}\omega_i + \dot{\theta}_{i+1} \cdot {}^{i+1}\hat{Z}_{i+1} $$

$$ {}^{0}J_v(\theta) = \frac{\partial V}{\partial \dot{\theta}} \qquad {}^{0}J_\omega(\theta) = \begin{bmatrix} {}^{0}_{1}r_3 & {}^{0}_{2}r_3 & {}^{0}_{3}r_3 \end{bmatrix} $$

$$ {}^{A}J(\theta) = \begin{bmatrix} {}^{A}_{B}R & 0 \\ 0 & {}^{A}_{B}R \end{bmatrix} {}^{B}J(\theta) $$

$$ \tau = J^T F $$

$$ \begin{bmatrix} {}^{A}V_A \\ {}^{A}\omega_A \end{bmatrix} = \begin{bmatrix} {}^{A}_{B}R & {}^{A}P_{BORG} \times {}^{A}_{B}R \\ 0 & {}^{A}_{B}R \end{bmatrix} \begin{bmatrix} {}^{B}V_B \\ {}^{B}\omega_B \end{bmatrix} \qquad {}^{A}V_A = {}^{A}_{B}T_v \, {}^{B}V_B $$

$$ \begin{bmatrix} {}^{A}F_A \\ {}^{A}N_A \end{bmatrix} = \begin{bmatrix} {}^{A}_{B}R & 0 \\ {}^{A}P_{BORG} \times {}^{A}_{B}R & {}^{A}_{B}R \end{bmatrix} \begin{bmatrix} {}^{B}F_B \\ {}^{B}N_B \end{bmatrix} \qquad {}^{A}F_A = {}^{A}_{B}T_f \, {}^{B}F_B $$

$$ {}^{A}_{B}T_v{}^{T} = {}^{A}_{B}T_f $$

$$ {}^{A}I = \begin{bmatrix} \frac{m}{3}(l^2+h^2) & -\frac{m}{4}wl & -\frac{m}{4}hw \\ -\frac{m}{4}wl & \frac{m}{3}(w^2+h^2) & -\frac{m}{4}hl \\ -\frac{m}{4}hw & -\frac{m}{4}hl & \frac{m}{3}(l^2+w^2) \end{bmatrix} $$

$$ {}^{A}I = {}^{C}I + m(P_c^T P_c I_3 - P_c P_c^T) $$

$$ F_i = m\dot{V}_{c_i} \qquad N_i = {}^{C_i}I\dot{\omega}_i + \omega_i \times {}^{C_i}I\,\omega_i $$

$$ \tau = M(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta) $$

$$ k_i = \frac{1}{2}m_i V_{C_i}^T V_{C_i} + \frac{1}{2}{}^{i}\omega_i^T \, {}^{C}I_i \, \omega_i $$

$$ k(\theta,\dot{\theta}) = \frac{1}{2}\dot{\theta}^T M(\theta)\dot{\theta} $$

$$ u_i = -m_i \, {}^{0}g^T \, {}^{0}P_{C_i} $$

$$ \tau = \frac{d}{dt}\frac{\partial k}{\partial \dot{\theta}} - \frac{\partial k}{\partial \theta} + \frac{\partial u}{\partial \theta} $$

$$ F = M_x(\theta)\ddot{x} + V_x(\theta,\dot{\theta}) + G_x(\theta) $$

$$ M_x(\theta) = J^{-T}(\theta) M(\theta) J^{-1}(\theta) $$

$$ V_x(\theta,\dot{\theta}) = J^{-T}(\theta)\left(V(\theta,\dot{\theta}) - M(\theta)J^{-1}(\theta)\dot{J}(\theta)\dot{\theta}\right) $$

$$ G_x(\theta) = J^{-T}(\theta)G(\theta) $$

$$ m\ddot{x} + b\dot{x} + kx = f $$

$$ f = \alpha f' + \beta \qquad f' = \ddot{x} = -k_v\dot{x} - k_p x $$

$$ k_v = 2\sqrt{k_p} $$

恒等式 $\frac{1}{2}\dot{\theta}^T \dot{M}(\theta)\dot{\theta} = \dot{\theta}^T V(\theta,\dot{\theta})$.

$\ddot{x} = f' = \underbrace{\ddot{x}_d + k_v \dot{e} + k_p e}_{\text{servo}} + k_i \int e\,dt$

$^A_B R = [{}^A\hat{X}_B \quad {}^A\hat{Y}_B \quad {}^A\hat{Z}_B]$

pieper's theorem. 六轴IK封闭解、后三轴交于一点

workspace interior singularities:

充分条件: $|{}^0J| = 0$ / $rank(J) < m$

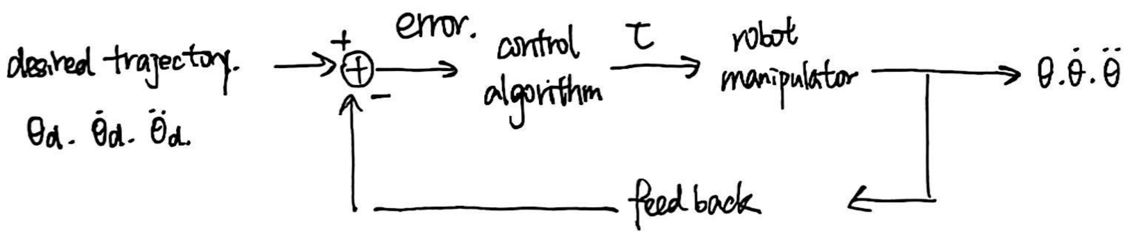
必要条件: $J\dot{q} = 0$ / $rank(J) < n$

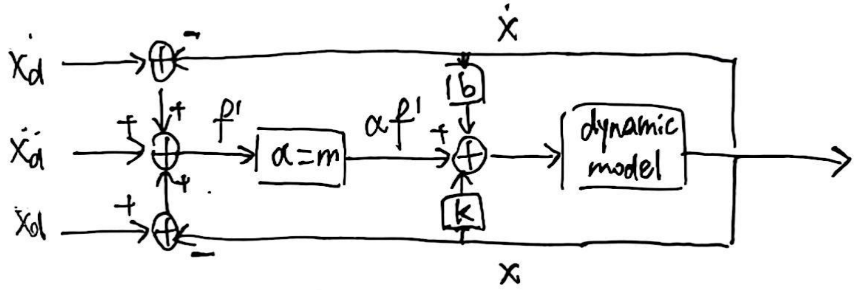
$\boxed{{}^0V} = {}^0J\,\dot{\theta}$

$\tau = {}^0J^T\,\boxed{{}^0F}$

joint cartesian

feedback control





$f = \alpha f' + \beta$ model-based

$= \underbrace{m[\ddot{x}_d + k_v(\dot{x}_d - \dot{x}) + k_p(x_d - x)] + b\dot{x}}_{\text{servo}} + kx$

PID. $\tau' = \ddot{\theta}_d + k_p(\theta_d - \theta) + k_i \int (\theta_d - \theta)\,dt + k_v(\dot{\theta}_d - \dot{\theta})$

lyapunov stability theory.

一阶连续导数. $V(x) > 0$ for $x > 0$. $V(0) = 0$.

$\dot{V}(x) \le 0$. lyapunov 稳定. $V = \frac{1}{2}\dot{\theta}^T M(\theta)\dot{\theta} + \frac{1}{2}\theta^T k_p \theta$.

$\dot{V}(x) < 0$. 渐近稳定.

α_{i-1} 从 X_{i-1} 转 Z.

a_{i-1} 沿 X_{i-1} 从 i-1 移到 i

d_i 沿 Z_i 从 i-1 移到 i

θ_i 从 Z_i 转 X.

X_i轴 从 Z_i 指向 Z_{i+1}.

$^{i-1}_{i}T = ROT(\hat{X}_{i-1}, \alpha_{i-1}) \cdot TRANS(\hat{X}_{i-1}, a_{i-1}) \cdot$

$ROT(\hat{Z}_i, \theta_i) \cdot TRAS(\hat{Z}_i, d_i)$

动力学精确 → 反馈控制

动力学不精确 → 自适应控制

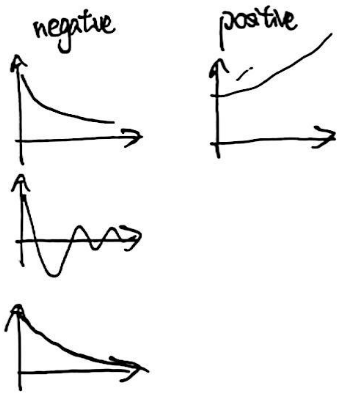
二阶线性.
$\alpha f' + \beta$ 轨迹跟踪
力矩跟踪
PID.

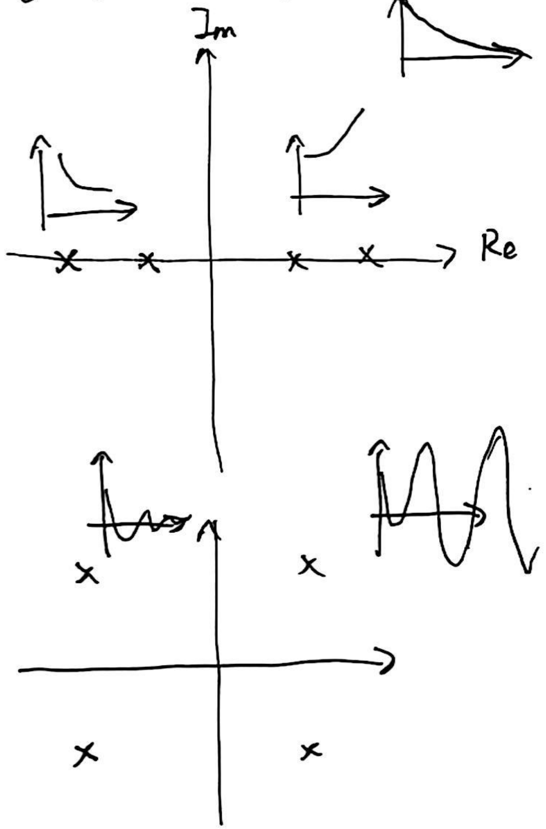
$m\ddot{x} + b\dot{x} + x = 0.$

$b^2 - 4mk > 0.$ 过阻尼.

$b^2 - 4mk < 0.$ 欠阻尼.

$b^2 - 4mk = 0.$ 临界阻尼.





控制律分解

$f = \alpha f' + \beta =$ servo + model 轨迹跟踪.

力矩控制. $\tau = M(\theta)\ddot{\theta}_d + C(\theta,\dot{\theta})\dot{\theta}_d + G(\theta) + K_p(\theta_d - \theta) + K_d(\dot{\theta}_d - \dot{\theta})$

PID 控制 $\tau' = \ddot{\theta}_d + k_p(\theta_d - \theta) + k_i\int(\theta_d - \theta)dt + k_v(\dot{\theta}_d - \dot{\theta})$ $\alpha = I.\ \beta = 0.$

重力补偿PD控制 $\tau = K_p e + K_d\dot{e} + G(\theta)$

adaptive control.

$$M(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta) = \tau = \Phi(\theta,\dot{\theta},\ddot{\theta})\cdot\theta$$

$$\tau = \underbrace{\hat{M}(\theta)\ddot{\theta}_d + \hat{C}(\theta,\dot{\theta})\dot{\theta}_d + \hat{G}(\theta)}_{\text{model}} + \underbrace{K_p(\theta_d - \theta) + K_d(\dot{\theta}_d - \dot{\theta})}_{\text{servo}}$$

$$\hat{\theta}_p(t+1) = \hat{\theta}_p(t) - \alpha\Phi^T(\theta(t),\dot{\theta}(t),\ddot{\theta}(t))e(t)$$

$$y(\theta,\dot{\theta},\ddot{\theta})\varphi = \hat{M}(\theta)(\ddot{E} + K_v\dot{E} + K_p E) + y(\theta,\dot{\theta},\ddot{\theta})\hat{\varphi}$$

real model = servo part + estimated model

$$\ddot{E} + K_v\dot{E} + K_p E = \hat{M}^{-1}(\theta)y(\theta,\dot{\theta},\ddot{\theta})\tilde{\varphi}$$

$$\dot{\tilde{E}} = A\tilde{E} + B\hat{M}^{-1}(\theta)y(\theta,\dot{\theta},\ddot{\theta})\tilde{\varphi} \quad \tilde{E} = \begin{bmatrix} E \\ \dot{E} \end{bmatrix} \quad A = \begin{bmatrix} 0 & I \\ -K_p & -K_v \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ I \end{bmatrix}$$

select lyapunov function. $V = \tilde{E}^T P \tilde{E} + \tilde{\varphi}^T\Gamma^{-1}\tilde{\varphi}$

$$\tau = M(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta).$$

$$\ddot{\theta} = M^{-1}(\theta)[\tau - V(\theta,\dot{\theta}) - G(\theta)] \quad \dot{\theta}(t+\Delta t) = \dot{\theta}(t) + \ddot{\theta}(t)\Delta t$$

$$\theta(t+\Delta t) = \theta(t) + \dot{\theta}(t)\Delta t + \frac{1}{2}\ddot{\theta}(t)\Delta t^2$$