Could a gravitational wave knock down the Rock of Gibraltar?

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1 Introduction

This essay aims to defend the position that gravitational waves (GWs) could not knock down the Rock of Gibraltar like EM waves. This position is based on the observation that in GR, the gravitational field is intrinsically different from any other matter fields (e.g. EM field). This first section gives a mathematical definition and derivation of GWs in contrast to EM waves, and concludes they do stand on equal grounds on the mathematical level at least until the derivation of their wave pattern. Then I move on to discuss whether gravitational waves/fields have the same physical effects as EM waves/fields do, particularly on how they carry energy.

I discuss both Duerr (2019) and Gomes and Rovelli (2023) positions on this matter. As I argue that Gomes and Rovelli (2023) does not respond to the key distinction between the tensorial nature of gravitational energy and EM energy, Duerr (2019)'s disanalogy remains successful. Therefore GWs should not be said to be able to knock down the rock of Gibraltar if some energy similar to the EM energy is required for a mountain to be 'knocked down'. This position might be objected that, as Gomes and Rovelli (2023) puts it, the account of energy transport via GWs is good enough for an *effective notion*. Nevertheless, such an effective notion of GWs as matter waves is a much weaker one compared to its EM counterpart and it requires a tweak of the conventional use of the phrase 'knock down'.

2 What are gravitational waves?

2.1 Deriving the wave pattern

It is important to see the mathematical object that represents gravitational waves before discussing the nature of its physical reality. In short, gravitational waves are spotted in the Einstein equation in the weak-field approximation or the linearized gravity approximation. We will adopt Creighton and Anderson (2011)'s approach for the derivation of gravitational waves, and later argue that, mathematically speaking it is on the same grounds as an EM wave.

The approximation is such that the metric g_{ab} is assumed to be the flat Minkowski metric η_{ab} plus a small perturbation h_{ab}. Therefore we have

$$g_{ab} = \eta_{ab} + h_{ab}. \tag{1}$$

The approximation is called the weak-field approximation as we assume $|h_{ab}| << 1$, $|\partial h_{ab}| << a$ and $|\partial^2 h_{ab}| << 1$ (Duerr, 2019). Note that objects labelled with lowercase Roman indices in this essay are tensor fields, and the component fields in some coordinate system are labelled with Greek indices. This approximation allows us to abandon all the terms on the order of h^2 in the affine connection, and therefore in the Ricci tensor, which is why the approximation is also called the linearised gravity approximation. Mathematically one more convention is assumed in the calculation: tensor indices are raised and lowered using $\eta_{\alpha\beta}$ and $\eta^{\alpha\beta}$ instead of the actual metric $g_{\alpha\beta}$ and $g^{\alpha\beta}$ (except when raising $g^{\alpha\beta}$ itself).

Now if we substitute the linearized gravity approximation into the Einstein field equation, surprisingly it will the simple form of a wave equation:

$$\Box \bar{h}_{\alpha\beta} = -\frac{16\pi G}{c^4} T_{\alpha\beta} \tag{2}$$

where $\Box$ is the d'Alembertian operator, $T_{\alpha\beta}$ is the stress-energy tensor, and $\bar{h}_{\alpha\beta} := h_{\alpha\beta} - \frac{1}{2}\eta_{\alpha\beta} h$ is the trace-reversed perturbation as a function of the real perturbation $h_{\alpha\beta}$.

Equivalently this leads to the wave equation of the real perturbation $h_{\alpha\beta}$:

$$\Box h_{\alpha\beta} = -\frac{16\pi G}{c^4} S_{\alpha\beta} \tag{3}$$

where $S_{\alpha\beta} := T_{\alpha\beta} - \frac{1}{2}\eta_{\alpha\beta} T^{\lambda}_{\lambda}$. This equation reveals the wave-like nature of $h_{\alpha\beta}$, which is also the mathematical definition of gravitational waves. For GW travelling in vacuum, the left hand side of Eq. 3 is zero. Hence we have the plane wave solutions:

$$h_{\alpha\beta} = \int d^3\vec{k} A_{\alpha\beta}(\vec{k}) e^{ik_\lambda x^\lambda} \tag{4}$$

where $A_{\alpha\beta}$ is a generic function dependent on the wave-vector, called the polarization tensor.

However before arriving at Eq. 3, there is an important step of taking the Lorenz gauge. This step is usually the place where most textbooks claim that the gravitational waves 'behave like' EM waves. Motivated by the Lorenz gauge condition in electromagnetism, the Lorenz gauge condition for metric perturbation is that the divergence of the trace-reversed metric vanishes:

$$\frac{\partial \bar{h}^{\mu\alpha}}{\partial x^\mu} = 0. \tag{5}$$

How is this freedom of gauge granted? It is granted with by the general covariant nature of Einstein field equation. We will use Weinberg (1972)'s formulation of general covariance:

> The equation is generally covariant when it preserves its form under a general coordinate transformation.

Knowing that the Einstein equation is a general covariant equation. A coordinate transformation (i.e. a harmonic coordinate system $g^{\mu\nu}\Gamma^{\alpha}_{\mu\nu}$) can always be found such that Eq.5 is solved. These harmonic coordinate systems ensure the freedom of taking the Lorenz gauge.

2.2 Analogies with electromagnetic wave

Now it naturally calls a comparison with the Lorenz gauge condition in electromagnetism. As it might be guessed from the sharing of names, the construction of Lorenz gauge condition in GR is in the same style of its EM predecessor.

The Lorenz gauge condition in electromagnetism is:

$$\partial_\mu A^\mu = 0. \tag{6}$$

This gauge freedom is granted as the differential equations satisfied by charged fields $\psi(x)$ and the EM potential $A_\alpha(x)$ retain the same form under the *local gauge transformations* (Weinberg, 1972)

$$\psi(x) \to \psi(x)e^{ie\phi(x)} \tag{7}$$

$$A_\alpha(x) \to A_\alpha(x) + \frac{\partial}{\partial x^\alpha}\phi(x) \tag{8}$$

where e is the charge of the particle represented by ψ , $\phi(x)$ is an arbitrary function of the space-time coordinates x^α and A_α is the vector potential. This property of EM field is called the *local gauge invariance*. Eq. 6 is solved with the transformation associated with the function ϕ that solves $\Box^2\phi = -\frac{\partial A^\alpha}{\partial x^\alpha}$. This Lorenz condition reduces Maxwell's equations into $\Box^2 A_\alpha = -J_\alpha$, which is an equation of plane waves.

Both Lorenz gauge conditions are chosen as (1) this choice of gauge simplifies the equation such that we are left with clean wave equations; and (2)both choices of gauge can be done with a transformation that does not take the solution out of the solution space of physical equations (i.e. they form 'dynamical' symmetry in Weinberg (1972)'s words). They both require a free choice of gauge for gravitational/EM wave to be derived. This gauge freedom originates from general covariance in GR, and local gauge invariance in electromagnetism respectively.

The mathematical analogy between gravitational field and EM field extends to the Peeling theorem (see Appendix in Gomes and Rovelli (2023)). The Peeling theorem describes the fall-off rate for each components in the Faraday tensor for EM field, and that of each components in the Weyl curvature (tensor) for gravitational field. I do not intend to go to the complicated mathematical details of the Peeling theorem. However the important observation is that, within the far zone of both fields, a component can be distinguished to represent radiation as it is the only term that decays with $1/r$. This analogy implies that the separation of the dynamical degrees of freedom of the radiation from the source, is as easy/difficult for gravitational field as it is for EM field.

Therefore, from the mathematical aspects, at the stage of deriving the wave pattern, it does not seem like the EM wave is anything more 'physical' than gravitational waves.

3 Do GWs carry energy like EM waves do?

We will now carefully look into the original claim that associates gravitational waves with the rock of Gibraltar.

> A strong burst of gravitational waves could come from the sky and knock down the rock of Gibraltar, precisely as strong burst of EM radiation could. (Rovelli, 1998)

Now it seems like this quote has already offered a positive answer to the question. After all, how is this a philosophical question? Shouldn't it be a physical fact that gravitational waves can or cannot knock out a rock? If fact, the GW detection from LIGO already proves that GW could have a physical effect of stretching its arms, so why can't we theoretically propose it to be so much stronger such that it could destroy the rock of Gibraltar? As it turns out, the questions pins on whether it is the best explanation that GW is the cause for the stretching of LIGO's arms, as one might question if GW even carries energy like EM waves do.

I will be mainly analysing the debate between Duerr (2019) and Gomes and Rovelli (2023) with the following positions:

1. The claim that GW carries energy is not the inference to the best explanation of current observation from binary systems. There is a more attractive alternative account that

solely appeals to the GR equation of motion and Einstein field equations. (Duerr, 2019)

2. All the skepticism faced by gravitational waves results from its property of being radiation, which is shared by EM waves. GW 'carries' energy in the same sense that EM waves 'carry' energy. Therefore GW can knock down the rock of Gibraltar just as EM waves can. (Gomes and Rovelli, 2023)

4 Duerr (2019)'s criticism on energy-carrying GWs

The account of GW as the gravitational energy-momentum carrier contributes as the standard explanation of for binary system's decreasing orbital period. I revised Duerr (2019)'s structure the explanation in the following logic:

P1 An isolated binary system's total energy-momentum is conserved unless the system radiates (i.e. GW carries energy away).

P2 If binary system's orbital period decreases then its total energy-momentum is not conserved.

observation Binary system's orbital period decreases.

explanation Binary system's orbital period decreases because GW carries away its energy-momentum.

Duerr (2019)'s concerns about this account of GW carrying energy comes from three angles:

1. Does energy conservation even hold for isolated binary system even without radiation such that there is any energy for GWs to carry away?

2. Is there even a satisfactory account on expressing gravitational energy?

3. Does an account of energy transport have more explanatory value than just 'ripples of energy' simpliciter?

Duerr (2019)'s answer is three straight NOs. Let's look into them one by one.

4.1 Energy conservation

The first angle involves reference to $\nabla_\mu T^\mu_\nu = 0$ which is the 'energy-momentum conservation' equation in GR. However it does not offer global energy-momentum conservation (which would require the covariant derivative to be a partial derivative) but quite the opposite. The global time-translation invariance of energy-momentum (defined by the energy-momentum tensor) is straightforwardly ruled out as this equation describes how 'energy and momentum evolve in a precisely specified way in response to the behavior of spacetime around them' (Carroll, 2010). Locally as the spacetime geometry is flat, the total energy would be be constant. Therefore P1 in the explanation of why GW carries energy would not hold.

4.2 Expressing gravitational energy

The other option is to adopt another object that locally conserves in a way similar to EM energy in GR, and define energy-momentum based on this object such that conservation of energy-momentum is still valid. However this option creates a further disanalogy between the gravitational field and the EM field: while the EM energy-momentum is coordinate-independent (i.e. it transforms tensorially), the gravitational energy-momentum is not so.

Einstein pseudotensor $t_{LL}^{\mu\nu}$ [1] is an object that is found to conserve locally (i.e. its total 4-divergence vanishes) when being added to the non-gravitational stress-energy tensor $T^{\mu\nu} + t_{LL}^{\mu\nu}$ (Landau et al., 2010 - 1975; Weinberg, 1972).

$$\frac{\partial}{\partial x^{\nu}} \tau^{\nu\lambda} = \frac{\partial}{\partial x^{\nu}} (\eta^{\nu\mu} \eta^{\lambda\kappa} [T^{\mu\nu} + t_{LL}^{\mu\nu}]) = 0 \tag{9}$$

As $t_{LL}^{\mu\nu}$ is purely constructed from the metric tensor, it thought to represent the 'pure energy' of the gravitational field. This has been attempted and it was found that the only way to define an integral energy-momentum obeying an exact conservation law is via the Einstein pseudotensors (Bertschinger, 2000; Duerr, 2019). The Einsteín pseudotensor come from the Noetherian approach of finding conservative quantities. From the Euler-Lagrange equations we can derive a continuity equation of Einstein pseudotensor that looks like a conservation principle, and take the Einstein pseudotensor as canonical energy-momentum of the metric. Under this characterisation of energy, pseudotensors can be used to derive the energy-momentum radiated by a localized source of gravitational radiation. The GW energy-momentum is extracted from the pseudotensor by selecting the part of the canonical energy-momentum that is associated with GW's degrees of freedom. This selected contribution represents the energy carried in the form of gravitational radiation.

However the derivation of energy-momentum from Einstein pseudotensor involves two questionable steps: (1) the assumption that the metric flattens out at infinity ($g^{\mu\nu} \to \eta^{\mu\nu}$) for the integral averaging the pseudotensor to be well-defined (Misner et al., 2017; Hoefer, 2000; Duerr, 2019) and (2) a clear separation of degrees of freedom associated with the GW and with the background (Duerr, 2019). Nevertheless later from Gomes and Rovelli (2023)'s arguments we can see the same problems occur in electromagnetism as definitive problems for radiation in general.

More seriously, Duerr (2019) questions whether the Einstein pseudotensor can even be interpreted as energy-momentum in the conventional sense because of its mathematical properties. The Einstein pseudotensor is not coordinate-independent as the pseudotensor's invariance group is smaller than the symmetry group of the spacetime on which they dwell. It is unnatrural to treat it as the energy-momentum held by the gravitational field (whose physics can be expressed coordinate-independently) when it is not an invariant under arbitrary coordinate transformation. The result of this is that, in a gravity-free world, the components of pseudotensor can be non-zero if a curvilinear coordinate system is chosen, which makes it pointless to speak of gravitational energy (Weyl, 1970). From another perspective, EM energy-momentum curves space as it appears on the RHS of the Einstein equations, while the "local gravitational energy-momentum" is not a source term in the same way. This reflects on how a reference frame can always be found such that all Christoffle symbols disappear (i.e. no local 'gravitational field') and 'gravitational energy' is supposed to be locally absent. However this is not possible for EM fields (Misner et al., 2017; Hoefer, 2000).

This creates a disanalogy between gravitational field and EM field. Duerr (2019) points out that unlike gravitational energy, the energy emitted by an EM radiating system has a tensorial representation: $T^{\mu\nu} = \frac{1}{\mu_0} \left[F^{\mu\alpha} F^{\nu}{}_{\alpha} - \frac{1}{4} \eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right]$. More straightfowardly An-

[1] LL stands for Landau and Lifshitz. It also signifies that this is not a tensor.

derson and Bergmann (1967) states that pseudotensorial gravitational energy doesn't form a geometric object like EM energy-momentum does.

The underdetermination problem constitutes another objection for using Einstein pseudotensor to represent gravitational energy. Misner et al. (2017) argues that while a localized electromagnetic energy and momentum in an element of 3-volume can be properly defined, the so-called gravitational energy-momentum fails. There is a unique formula for the localised EM energy-momentum, while the "local gravitational energy-momentum" has many distinct formulas.

4.3 A better alternative explanation

Lastly he argued that the explanation that binary system slows down because GW carries away the energy is not the best inference. He proposed a dynamical explanation in contrary. GR's equation of motion (EoMs, i.e. the field equations for the non-gravitational matter fields) and the Einstein equations tell us that binary pulsars orbital time is bound to decay. If GW is explained as the energy carrier, then some parts of the full EoMs have to be picked out to represent the conserved system (e.g. isolated binary systems) that are no longer conserved. However we could simply abandon this artificial, pre–general-relativistic way of splitting systems and energies and attribute the period changing of binary pulsars of the necessity of the mathematical formality of GR. This dynamical account is argued to be "superior on grounds of parsimony, universal scope, depth and unificatory power".

We can compare the dynamical and standard explanations in the following way:

standard QUA –EoMs & EEs –> GW emission – *D*–> GW depletes the system's energy – cEoMs –> binary pulsar's orbit decay

dynamical QUA –EoMs & EEs –> binary pulsar's orbit decay

where QUA denotes the time-varying quadrupole momentum distribution of matter, and *D* denotes the split of the EoMs into the conservative parts (cEoMs) associated with the dynamics of the system, and nonconservative parts associated with the dissipative gravitational radiation.

We can see the explanatory advantage of they dynamical account at the first glance: it requires fewer explanatory machinery. Solid concepts of gravitational energy, conservation of energy, a splitting of EoMs into the conservative and the non-conservative are required for the standard explanation to take off. It is in this sense that Duerr (2019) claimed the standard explanation to be beaten by the dynamical explanation at *parsimony*. Meanwhile, the standard interpretation *covers a smaller scope* in the sense that at higher orders of perturbation, the gravitational and radiative degrees of freedom become non-separable such that *D* is hard to be unambiguously attained. On the other hand, the dynamical interpretation shares the same scope of GR-EoMs, which is not limited by the approximation scheme. Duerr (2019)'s idea of *depth* is a measure of how fundamental the principles and concepts are as the working posit of each explanation. The dynamical explanation is deeper as it does not rely on the non-fundamental, 'artificial' and secondary principles or concepts in GR like cEoMs and energy conservation. This value of explanation seems to coincide with parsimony as they both disfavour the standard explanation for its reference to cEoMs and energy conservation principle. The distinction is that parsimony concerns mostly about the simplicity of the explanation (i.e. fewer steps, fewer concepts/principles), while depth concerns about the intrinsic role of the relying concepts and principles played in the explanation. It happens to be that the dynamical explanation trumps in both evaluation. Lastly, the dynamical explanation has more *unificatory power* as an explanation for pre-GR physics can be structured similarly. In pre-GR physics the full EoMs happen to be conservative.

4.4 Summary

Overall Duerr (2019) argued that the orbit decay of binary pulsars (or any other observation of GW) can be described without the notion of energy transport. He offers many reasons for why the standard interpretation of observation is questionable and should be replaced by the dynamical explanation, and they all source to 4 questionable technical moves as I summarise:

- T1 An unsatisfactory account of 'energy-momentum' conservation via the Einstein pseudotensor. Einstein pseudotensor is unsatisfactory in the sense that it is not coordinate-independent, which undermines the disanalogy with EM energy-momentum.
- T2 For energy flux to be properly defined for GW in the Notherian account, an integral is well-defined only with assumed asymptotic flatness at infinities.
- T3 A separation between the dynamical degrees of freedom associated with the source and the radiation is not always guaranteed.
- T4 A better alternative explanation that leaves out GW energy is found.

5 Gomes and Rovelli (2023)'s comeback

Gomes and Rovelli (2023) acknowledges the mathematical observation made in Duerr (2019) and even quotes the three questions asked by Duerr (2019). However Gomes and Rovelli (2023) argues that EM waves do as badly as GWs at encountering these concerns, which should be attributed to the nature of waves/radiation but not the nature of gravity. In this section, I present Gomes and Rovelli (2023)'s argument showing that a definition of EM waves indeed involves technical concerns T2 and T3. However as I have not seen a direct counterargument made towards the non-tensorial feature of Einstein pseudotensor, I conclude that EM waves still do better on this matter and the analogy fails. Therefore GWs cannot be said to be able to knock down the rock of Gibraltar for the reason that it can do as well as EM waves.

5.1 T1

T1 questions whether this 'gravitational energy-momentum' expressed by Einstein pseudotensor is even some energy-momentum in line with the EM energy-momentum. Gomes and Rovelli (2023) does not disagree with the claim that 'local gravitational energy-momentum' does not exist. I make the following attempt to resolve the concern T1. By observation, the Einstein pseudotensor is a very special mathematical object that has many features of the traditional energy-momentum. Weinberg (1972) lists 10 of these features in total in Section 7.6.

The Einstein pseudotensor is fortunately a Lorentz covariant and the energy derived from it after integration is a Lorentz invariant. In fact, they are covariant/invariant in general asymptotically Minkowskian spacetime. Therefore this notion of gravitational energy-momentum reduces to its traditional sense when we move back to the 'traditional' playground of Minkowskian spacetime. Duerr (2019) argues that in GR's general spacetime, linear transformations are no longer privileged. We could argue that, similarly, energy-momentum in special relativity gains its notion as it recovers its Newtonian predecessor at low velocity limit. Therefore if the notion of energy-momentum can be inherited by special relativity from Newtonian mechanics, then the same inheritance should be accepted for the 'energy-momentum' expressed by the Einstein pseudotensor as well.

However the disanalogy with EM field still exists as EM field has not encountered such problem. I conclude that the concern T1 is not satisfactorily resolved. Gomes and Rovelli (2023) then argues that for an effective notion in physics, well-definition is not necessary.

I will come back to this point after attempting to resolve the concern T3.

5.2 T2

Duerr (2019) questions whether gravitational energy maintains as a fundamental notion in GR if it can only be well-defined based on asymptotic flatness, which is an unrealistic idealisation. Gravitational energy, if defined on non-fundamental grounds, must be as non-fundamental if not more. Gomes and Rovelli (2023)'s gives a convincing counter-argument to this point:

> What undergirds the assumption of asymptotic flatness is just dynamical isolation of subsystems; conceptually, dynamical isolation is what grounds both an unambiguous separation of radiative and Coulombic modes and conservation of energy. To talk about energy transfer, we need to be able to clearly distinguish subsystems within the theory. And again, this condition (of dynamical isolation) is necessary to discuss energy conservation in general —even in the familiar case of Newtonian mechanics—not just in general relativity. (Gomes and Rovelli, 2023)

Asymptotic flatness may seem ad hoc but it is mathematically necessary for the isolation of subsystems. If we want to study 'pure' gravitational field energy from the GWs, then the playfield has to be cleared out first. The same kind of *general* assumption can be found in Newtonian mechanics. The Newtionian energy/momentum conservation is not valid for any realistic subsystem (e.g. Newton's cradle) if the assumption of no external disturbances is not in place. However this does not mean energy/momentum conservation is any less fundamental in Newtonian mechanics. Similarly energy defined on the ground of asymptotic flatness does not make the notion less general.

5.3 T3

The response to T3 is closely connected to the response to T2. As discussed at the end of Section 2.2, within the far field of the gravitational field, the component that corresponds to be radiation part of the Weyl curvature can be cleanly distinguished. One might argue that this separation is not possible in the close zone where the necessary approximation is no longer available. The analogy is such that the same thing could be said for EM radiation, and we certainly do not want to raise objection to EM waves. Therefore as Gomes and Rovelli (2023) would argue, the notion of radiation for gravitational field is as *effective* as it is for EM field.

However the above only applies to the mathematical derivation of the wave pattern, but not the energy carried by the wave. A separation of the source and radiation contribution to the gravitational energy-momentum must meet unsolved challenges, as whether an explicit expression of gravitational energy-momentum exists remains a question. No convincing separation can be done if the full expression itself is in question. The concern T3 is not properly resolved.

Indeed Gomes and Rovelli (2023) realises the difficulty of a solid ground for energy transport via GWs. As we have seen above, T1 and T3 are not resolved because such 'gravitational energy-momentum' remains a doubtful concept in GR. The following quote is their response to these concerns:

> Our theoretical commitment to robust wave patterns is not conditional on their being subject to energy conservation. Nor should we seek such sanction: 'energeticism'—the XIXth century hope of reducing all natural phenomena to 'manifestations of energy'—is long dead in theoretical physics, and for very good reasons. (Gomes and Rovelli, 2023)

This leads to a contradiction. Suppose we forfeit the efforts of explaining the stretching of LIGO arms or pulsars' orbit decay with energy-momentum, then GWs play no causal role in this physical process. On the other hand, suppose we seek for an energy-momentum explanation, gravitational energy (and therefore energy transport via GW) is not cleanly defined. The wave patterns themselves were never questioned. Therefore I understands this claim as saying 'the knock down of the rock of Gibraltar should not be considered a manifestation of gravitational energy', and hence we can confidently say that GWs can knock it down without a solid account of gravitational energy. The statement requires a big tweak of the conventional meaning of 'knock down' and seems to further contradict with their later arguments on how energy transport via GWs is an effective notion in physics historically.

Therefore we conclude that the overall response to T3 (and previously T1) is unsatisfactory.

5.4 T4

The response to T4 is that the account of energy transport of GWs is *historically, an effective* notion of physics. Duerr (2019)'s version of the standard explanation is still a good explanation for this reason. One is still justified to make reference to the energy carried by GWs despite the unresolved concern T1 and T3. Gomes and Rovelli (2023) acknowledges the difficulty of well-defining energy owned by GWs, and make the following claim:

> Physics, and indeed science more generally, trades in effective notions—like the energy of the wave, the horizon, the orbit, the black hole—that are perfectly well-defined for specific solutions, or for specific regimes, and that have exceptional explanatory value. A philosophy of science that denies any ontological status to these notions leads to an impoverished picture of science, that we must reject. (Gomes and Rovelli, 2023)

Despite a problematic definition of the energy owned by the *general* gravitational field, the energy carried by gravitational waves, under the weak-field approximation is well expressed by the quadruple formula. This effective notion of energy transport by GWs has been producing fruitful and useful predictions though being approximate.

The physical effects on the LIGO detector (i.e. how it becomes stretched and gains energy) can be explained as 'ripples of energy' in the style of the dynamical explanation. However Gomes and Rovelli (2023) points out that Duerr (2019) is 'mistaken to say that we cannot tell a causal story about transfer of energy from the wave to the detector' as there is an explicit 'radiation-reaction' account of how the GW interact with the detector just like the 'radiation-reaction' account for EM waves. If GWs have not encountered the LIGO detector, they would not leave the detector with less energy.

However it seems to me that the dynamical explanation is still better in terms of parsimony, scope, depth and unificatory power as Duerr (2019) claims. A historical choice is not good enough to outweigh all of these good features. The quadruple formula, as a formula of calculating the 'gravitational energy' radiated out from the source, should be considered as an effective **mathematical tool** rather than a **notion** under the weak-field approximation scheme. In comparison, the energy-momentum of EM field is a better *effective* notion.

6 Conclusion

I conclude this essay with the following statements:

A The non-tensorial feature of the Einstein pseudotensor greatly undermines the analogy between EM field/wave and gravitational field/wave, as 'gravitational energy-momentum' is not well-defined.

B Taking the energy-momentum transport via GWs as an effective but not well-defined notion in GR helps validating the choice of the standard explanation. However the dynamical explanation still wins on parsimony, scope, depth and unificatory power. Meanwhile the energy-momentum of the EM field is beyond an effective notion.

C Some concerns about the approximation scheme (e.g. asymptotic flatness, separation of the dynamical degrees of freedom, derivation of the wave pattern) are recognised as shared challenges for radiation in general, which strengthen the analogy between EM field/wave and gravitational field/wave.

So could a gravitational wave knock down the Rock of Gibraltar? No because it does not carry energy-momentum in the same way as EM waves do. However if we are willing to accept the energy-momentum expression given by Einstein pseudotensor, or tweak the conventional meaning of 'knock down' (i.e. knocking down something does not require the EM-like energy), then a postivie answer might be given.

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