Telescope time for Hubble constant measurement

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1 Deriving the number function

The schechter function is:

$$\phi(L)dL = \phi^* \left(\frac{L}{L^*}\right)^\alpha \exp\left(-\frac{L}{L^*}\right) dL \tag{1}$$

As the osbervational data are given in terms of absolute magnitude in the B band, we make the following substitution:

$$L = 10^{-0.4M_B}$$

$$dL = -0.4ln(10)10^{-0.4M_B} dM_B$$

Hence:

$$\phi(M_B)dM_B = 0.4\ln(10)\phi^* 10^{0.4(\alpha+1)(M_B^* - M_B)} \exp(-10^{0.4(\alpha+1)(M_B^* - M_B)}) dM_B \tag{2}$$

where I take $\alpha = -1.07$, $M_B^* = -20.6$ and $\phi^* = 1$ (normalisation is done all at once in the last step)

Now to derive the number function, we have:

$$n(L)dL = \phi(L)V(L)dL. \tag{3}$$

For a galaxy with intrinsic luminosity L, its apparent flux F at the luminosity distance D_L is given by:

$$F = \frac{L}{4\pi D_L^2}, D_L = \sqrt{\frac{L}{4\pi F}}$$

where D_L is the luminosity distance.

This equation shows that for a fixed flux limit F_{lim}, the maximum observable distance D_L grows proportionally to $\sqrt{L}$, and thus the observable volume V for a galaxy of luminosity L is proportional to:

$$V(L) \propto D_L^3 \propto \left(\sqrt{L}\right)^3 = L^{1.5}$$

This result, $V(L) \propto L^{1.5}$, means that as galaxy luminosity increases, the **volume of space within which we can detect that galaxy grows with $L^{1.5}$**. More luminous galaxies can be observed over larger volumes due to the greater distances at which they remain detectable.

Thus, the number function becomes:

$$n(L)dL = \phi(L)V(L)dL = \phi^* \left(\frac{L}{L^*}\right)^\alpha \exp\left(-\frac{L}{L^*}\right) L^{1.5} dL$$

Switching the variable to M_B, we have:

$$\phi(M_B)dM_B = 0.4\ln(10)\phi^* 10^{0.4(\alpha+1+0.6/0.4)(M_B^*-M_B)} \exp(-10^{0.4(\alpha+1)(M_B^*-M_B)})dM_B \tag{4}$$

because $L^{1.5} \propto 10^{-0.4*1.5M_B}$.