ELEC 321 / STAT 321

HOMEWORK 1-1

Problems marked with (*) have a computational component. For these problems, computations can be done using R or Matlab. Please, submit a copy of your computer script and display your results using tables, pictures, etc. when convenient.

Problem 1:
(a) Does $P(A) + P(B) + P(C) = 1$ imply that $P(A \cup B \cup C) = 1$? Why?
(b) Does $P(A \cup B \cup C) = 1$ imply that $P(A) + P(B) + P(C) = 1$? Why?
(b) Is $P(A|B)\,P(B) + 1 - P(B) \geq P(A)$ always true? Why?
(c) For what values of $P(A_0)$ can $P(A_{n+1}) = 2P(A_n)$ for all $n = 1, ..., 30$? example??
(d) Suppose that $P(A)$ and $P(B)$ are at least 0.60. Can $P(A \cap B)$ be equal to 0?

Problem 2: Let A, B and C be three events. Find expressions for the events so that of A, B and C :
(a) A and B occur;
(b) both A and B but not C occurs;
(c) exactly one of the three events occurs; how to show exactly one of them, which one?
(d) exactly two of the three events occur;
(e) all three events occur;
(f) none of the three events occurs;
(g) at most one of them occurs;
(h) at most two of them occurs;

solved

Problem 3:
(a) Prove the following inequality:

$$P\left(\cup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i) \tag{1}$$

Hint: use induction.
(b) Suppose that $P(A_i) \geq 0.95$ for all $i = 1, ..., 10$. Find a lower bound for $P\left(\cap_{i=1}^{10} A_i\right)$.

how to figure out the n number

Problem 4 (Total Probability): Suppose that $A_1, A_2, ..., A_n$ are a partition of the sample space. That is $A_i \cap A_j = \phi$ for $i \neq j$ and $A_1 \cup A_2 \cup \cdots \cup A_n = \Omega$. Show that, for all B,

$$P(B) = \sum_{i=1}^{n} P(B|A_i)\, P(A_i)$$

(*) Problem 5 : (a) A coded message can arrive through three possible noisy channels, called a, b and c. The probability that it arrives through each of them are 0.5, 0.3 and 0.2, respectively. Signals traveling through a, b and c have probabilities 0.8, 0.9 and 0.7 of being correctly decoded. What is the probability that a signal is correctly decoded.

(b)Simulate this communication system and numerically validate the probability you derived in Part (a)

Problem 6: Consider a system of n antennae arranged in a linear order. Communication flows through the system provided no two consecutive antennae are down.

does it mean linear ordering of pairs?

(a) Suppose that $m < n$ antennae are down and the remaining $n - m$ are functional. How many linear orderings are there in which no pair of consecutive antennae are down?

(b) Suppose that there are $n = 10$ antennae and the probability that m of them are down are as in the following table:

m	0	1	2	3	4	5	6
Probability	0.11	0.27	0.30	0.20	0.09	0.02	0.01

Calculate the probability that communication flows through this system.

[**Hint:** use the result of Problem 4 (a)]

Problem 7: Suppose that the sample space has a countably infinite number of points.

(a) Show that not all points can be equally likely.

does it include 0?

(b) Can all the points have positive probability of occurring? Why?

(*) Problem 8: Let f_n denote the number of ways of tossing a coin n times such that successive heads never appear.

(a) Argue that

$$f_n = f_{n-1} + f_{n-2}, \quad n \geq 2 \text{ where } f_0 = 1, \text{ and } f_1 = 2$$

Hint: how many outcomes are there that start with a head, and how many start with a tail?

(b) Suppose that all possible outcomes of n tosses are equally likely. If P_n denotes the probability that successive heads never appearing when a coin is tossed n times, find P_n (in terms of f_n).

(c) Complete the following table.

n	P_n
0	1
1	1
2	
$\vdots$	
25	

(*) Problem 9: Suppose that Paul has \$50 and Linda has \$50 dollars. Linda flips a fair coin. If it lands Head up, she pays Paul \$2 dollars If it lands Tail up, Paul pays her \$2. They play this game until one of them is ruined (has no money left). Use simulations to accurately estimate:

(a) The probability that Paul ends up ruined.

(b) The expected number of games they will play.

(c) What if Linda has \$500? \$5000? \$5000000?

(*) Problem 10: use a simulation model to represent a person making \$5000 per month who plays 50 even games worth \$100 each every weekend against a very wealthy casino? In fact, they are all very wealthy!