

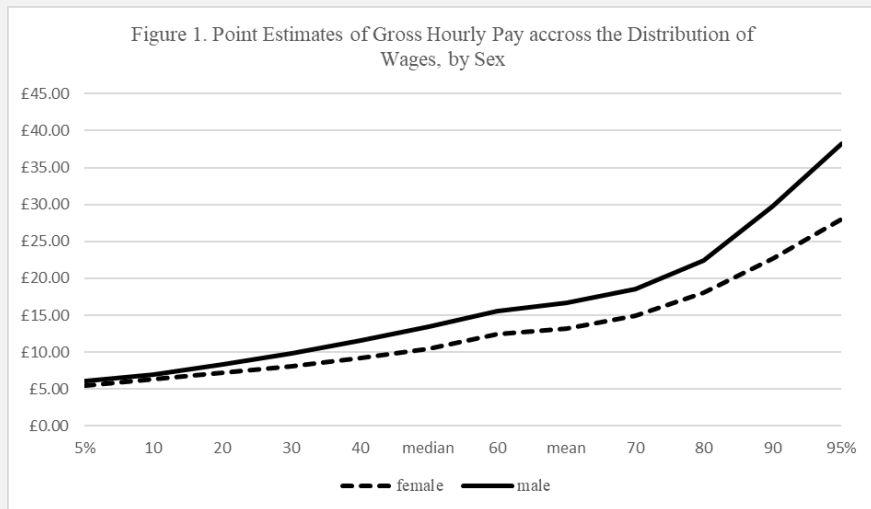


Social statistics away-  
day May 2024

# PAY GAP REPRESENTATION

*By Wendy Olsen with thanks to  
Myong Sook Kim*





## A PAY GAP COULD BE AT THE MEAN OR THE MEDIAN.

*It is inherently an aggregate statistic.*

characteristic or coefficient effects. In contrast to the underlying assumptions of simulation, Oaxaca models, simulation assumes integrated labour markets and specifies a pooled distribution across the sexes. Here the gender pay gap is calculated as the average difference between male and female characteristics as a function of an *undifferentiated slope* (Eq. B).

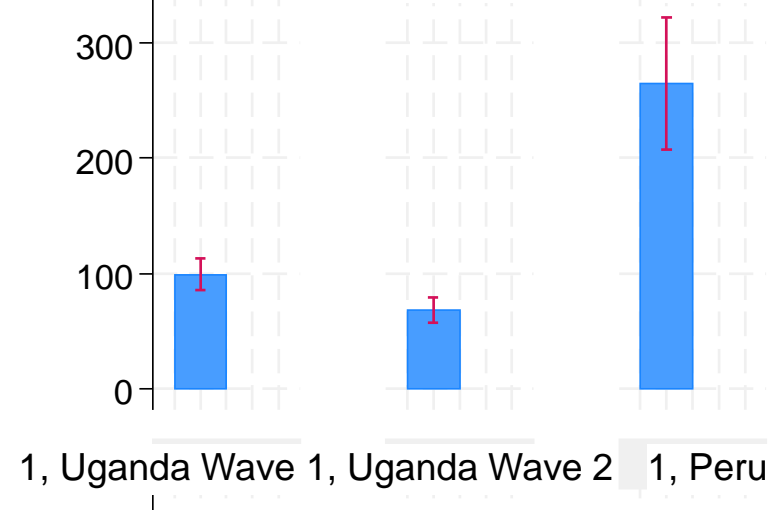
$$\ln y_f - \ln y_m = \sum (\bar{X}_f - \bar{X}_m) \beta_{\text{overall}} + (\text{Sex}) \beta_{\text{overall}} + (\varepsilon_f - \varepsilon_m) \quad (\text{Eq. B})$$

In its pooled estimates of all workers, simulation allows us to include sex as a *measured covariate* and we conceptualise sex as a ‘measured residual’. Unlike Oaxaca, however, simulation permits us to examine *unobserved* residual error ( $\varepsilon_f - \varepsilon_m$ ) independent of the *observed* measure of sex.

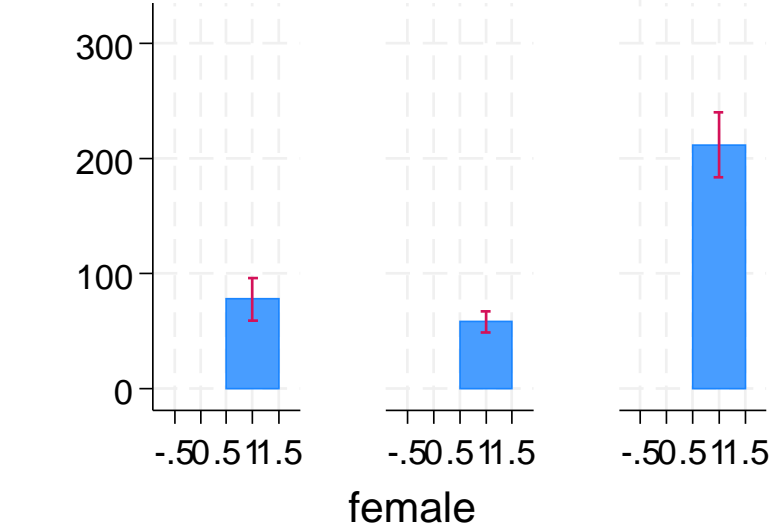
Example: male mean vs female mean, omitting the 0’s (non-earners). The distribution of earnings that we use is often “usual” earnings per hour, in the main job.

- This saves confusion and invalid comparisons
- We use the logarithm of pay if everyone has a job
- “Actual earnings” is considered more accurate.

0, Uganda Wave 0, Uganda Wave 2 0, Peru



1, Uganda Wave 1, Uganda Wave 2 1, Peru



female

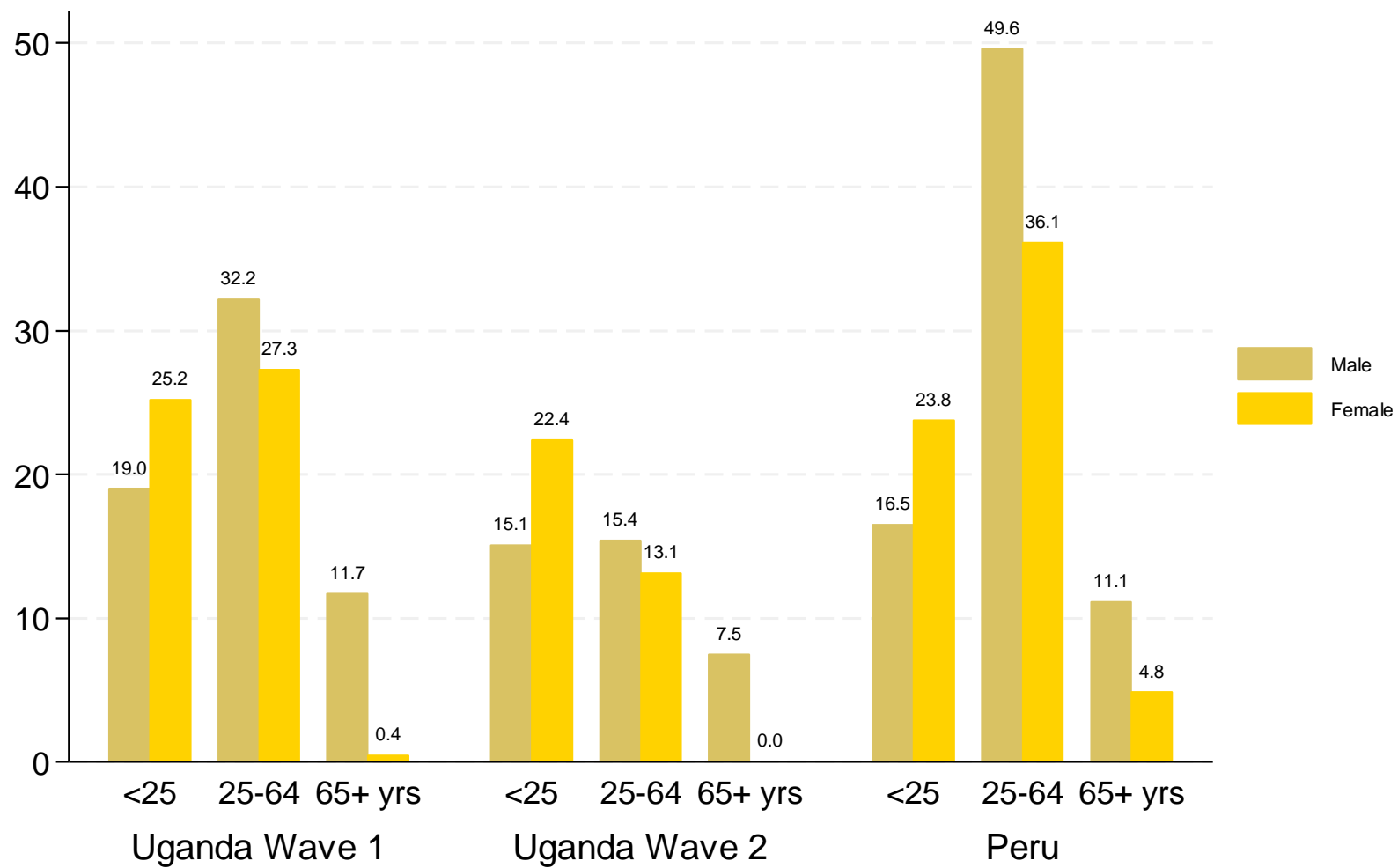
The second panel is the Females  
We have waves 1 and 2 in Uganda  
At right, Peru has just one wave.

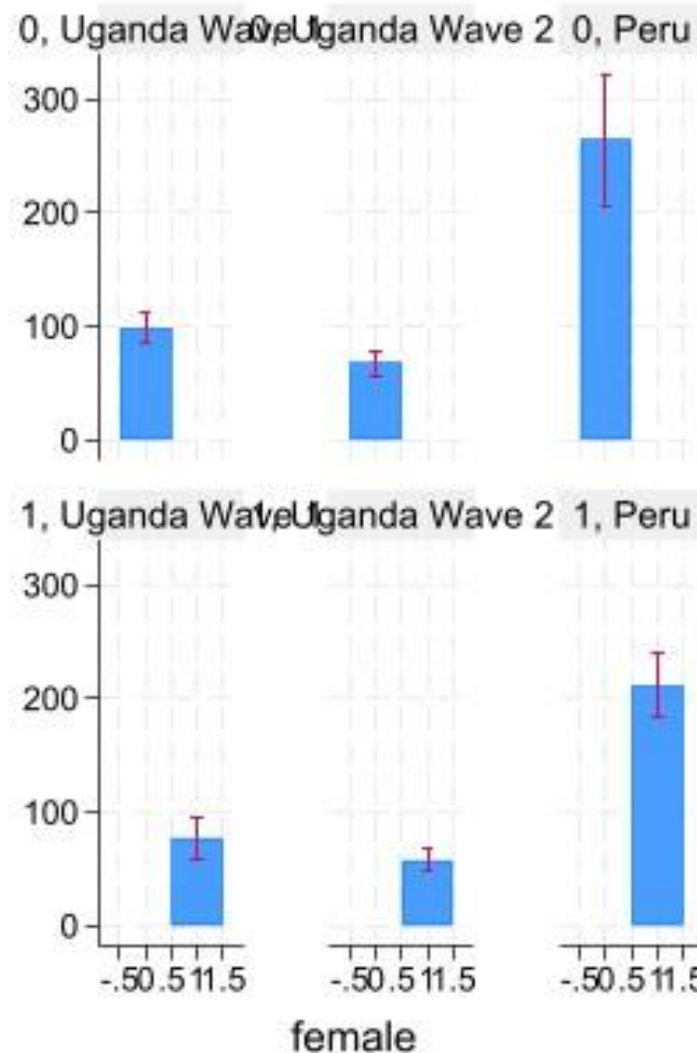
```
)) (rcap ul ll female, by(female wave ))
*Here we can also use the pay median*
graph export "results\comparemeanearnsBysexInwave.wmf",
replace
```

# HOW TO ADJUST COLOR AND LABELS ON BARS

```
graph bar
EARN_LSTMNTH_TOT_D_INCFW_USD
[pweight=WEIGHT_FINAL] if
inlist(ICSE18_MJJ,3,4,5) , over(sex)
over(agegroup3) over(wave) bar(1,
color(sand)) bar(2,color(gold))
legend(size(vsmall)) blabel(bar,
format(%9.1f) size(2))
yttitle("USD/Month, Females Yellow,
Males Darker")
```

So after you collapse, you will need to adjust the twoway graph to use a **categorical variable**, so that **over** can refer to **Bar 1, Bar 2**, etc. This is awkward in twoway graph but it can be manipulated.





Source: International Labour Organisation

Primary survey data, experimental research on questionnaire design, 2022.

**TRICKSY NOT AN ADEQUATE PAYGAP TILL PER-HOUR**

Here, pay is per month, estimated from the last paypacket, among only those who had pay. It refers only to adults in the main job.

Furthermore it includes the in-kind pay, converted by respondent to local currency.

The currency is US \$ per person per month.

female

Graphs by female and Which Part of the Survey

The pay is per month, estimated from the last paycheck, among only those who had pay. It refers only to adults in the main job.

Furthermore it includes the in-kind pay, converted by respondent to local currency.

TRICK: If the pay is per month, estimated from the last paycheck, something has been left out of the analysis.

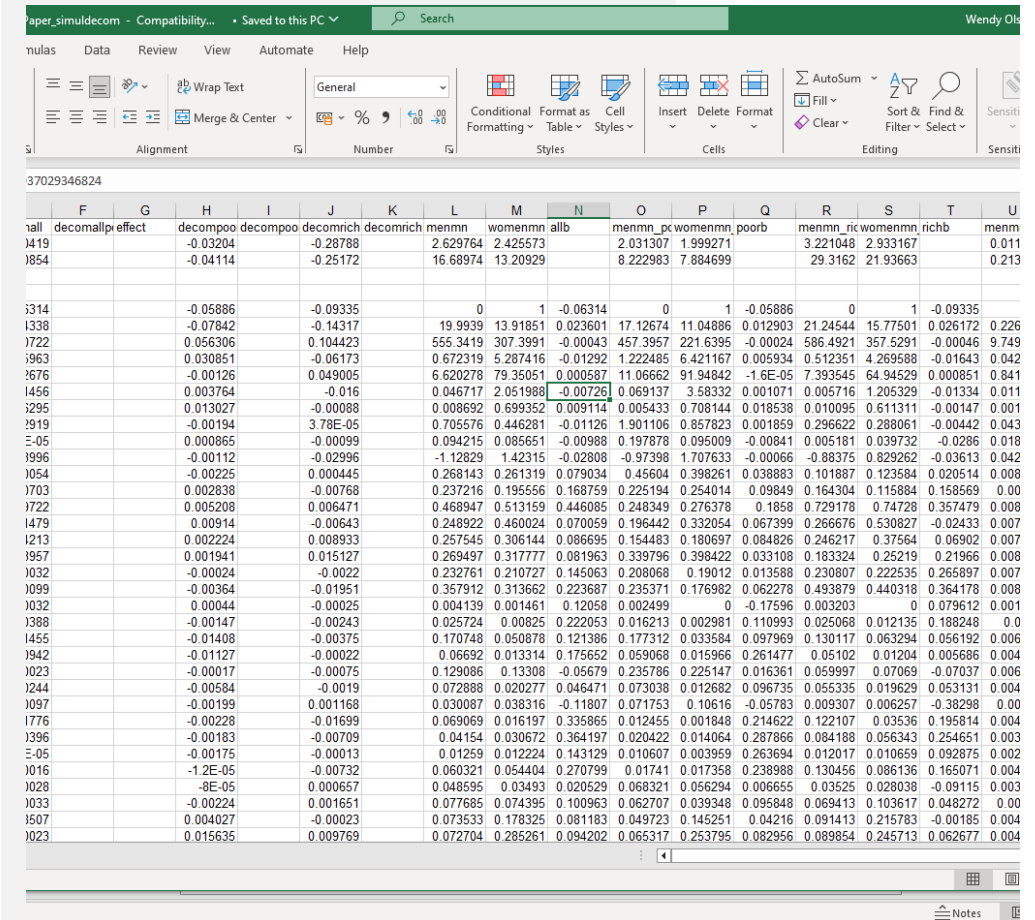
You do not see significant gender difference (red marks the 95% Wald confidence interval) – WHY NOT!?!? There IS a large and significant pay gap in Uganda and Peru - - why not shown?



# + ALSO, NEED TO DEAL WITH MISSING DATA

What about the inactive people with no job, and the unemployed?

- You can possibly impute to them a £1 or \$1 payment (per hour).
- This safely places them at the far left of the distribution.
- Regression calculations are sometimes done this way.
- A hurdle regression has two steps. First, what predicts them being in the zero group, and second, what predicts the value of the Y variable for non-zero cases.



	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
all	decomall	effect	decompoo	decompoo	decomrich	decomrich	menmn	womenmn	allb	menmn	pcwomenmn	poorb	menmn	ricwomenmn	richb	menmn
1419			-0.03204		-0.28788		2.629764	2.425573		2.031307	1.999271		3.221048	2.933167		0.011
1854			-0.04114		-0.25172		16.68974	13.20929		8.222983	7.884699		29.3162	21.93663		0.213
3314			-0.05886		-0.09335		0	1	-0.06314	0	1	-0.05886	0	1	-0.09335	
1338			-0.07842		-0.14317		19.9939	13.91851	0.023601	17.12674	11.04886	0.012903	21.24544	15.77501	0.026172	0.226
1722			0.056306		0.104423		555.3419	307.3991	-0.00043	457.3957	221.6395	-0.00024	586.4921	357.5291	-0.00046	9.749
1963			0.030851		-0.06173		0.672319	5.287416	-0.01292	1.222485	6.421167	0.005934	0.512351	4.269588	-0.01643	0.042
1676			-0.00126		0.049005		6.620278	79.35051	0.000587	11.06662	91.94842	-1.6E-05	7.393545	64.94529	0.000851	0.841
1456			0.003764		-0.016		0.046717	2.051988	-0.00726	0.069137	3.58332	0.001071	0.005716	1.205329	-0.01334	0.011
1295			0.013027		-0.00088		0.008692	0.699352	0.009114	0.005433	0.708144	0.018538	0.010095	0.611311	-0.00147	0.001
1919			-0.00194		3.78E-05		0.705576	0.446281	-0.01126	1.901106	0.857823	0.001859	0.296622	0.288061	-0.00442	0.043
105			0.000865		-0.00099		0.094215	0.085651	-0.00988	0.197878	0.095009	-0.00841	0.005181	0.039732	-0.0286	0.018
1996			-0.00112		-0.02996		-1.12829	1.42315	-0.02808	-0.97398	1.707633	-0.00066	-0.88375	0.829262	-0.03613	0.042
1054			-0.00225		0.000445		0.268143	0.261319	0.079034	0.45604	0.398261	0.038883	0.101887	0.123584	0.020514	0.008
1703			0.002838		-0.00768		0.237216	0.195556	0.168759	0.225194	0.254014	0.09849	0.164304	0.115884	0.158569	0.00
1722			0.005208		0.006471		0.468947	0.513159	0.446085	0.248349	0.276378	0.1858	0.729178	0.74728	0.357479	0.008
1479			0.00914		-0.00643		0.248922	0.460024	0.070059	0.196442	0.332054	0.067399	0.266676	0.530827	-0.02433	0.007
1213			0.002224		0.008933		0.257545	0.306144	0.086695	0.154483	0.180697	0.084826	0.246217	0.37564	0.06902	0.007
1957			0.001941		0.015127		0.269497	0.317777	0.081963	0.339796	0.398422	0.033108	0.183324	0.25219	0.21966	0.008
1032			-0.00024		-0.0022		0.232761	0.210727	0.145063	0.208068	0.19012	0.013588	0.230807	0.222535	0.265897	0.007
1099			-0.00364		-0.01951		0.357912	0.313662	0.223687	0.235371	0.176982	0.062278	0.493879	0.440318	0.364178	0.008
1032			-0.00044		-0.00025		0.004139	0.001461	0.12058	0.002499	0.0	-0.17596	0.003203	0	0.079612	0.001
1388			-0.00147		-0.00243		0.025724	0.00825	0.222053	0.016213	0.002981	0.110993	0.025068	0.012135	0.188248	0.0
1455			-0.01408		-0.00375		0.170748	0.050878	0.121386	0.177312	0.033584	0.097969	0.130117	0.063294	0.056192	0.006
1942			-0.01127		-0.00022		0.06692	0.013314	0.175652	0.059086	0.015966	0.261477	0.05102	0.01204	0.005686	0.004
1023			-0.00017		-0.00075		0.129086	0.13308	-0.05679	0.235786	0.225147	0.016361	0.059997	0.07069	-0.07037	0.006
1244			-0.00584		-0.0019		0.072888	0.020277	0.046471	0.073038	0.012682	0.096735	0.055335	0.019629	0.053131	0.004
1097			-0.00199		0.001168		0.030087	0.038316	-0.11807	0.071753	0.10616	-0.05783	0.009307	0.006257	-0.38298	0.00
1776			-0.00228		-0.01699		0.069069	0.016197	0.335865	0.012455	0.001848	0.214622	0.122107	0.03536	0.195814	0.004
1396			-0.00183		-0.00709		0.04154	0.030672	0.364197	0.020422	0.014064	0.287866	0.084188	0.056343	0.254651	0.003
105			-0.00175		-0.00013		0.01259	0.012224	0.143129	0.010607	0.003959	0.263694	0.012017	0.010659	0.092875	0.002
1016			-1.2E-05		-0.00732		0.060321	0.054404	0.270799	0.01741	0.017358	0.238988	0.130456	0.086136	0.165071	0.004
1028			-8E-05		0.000657		0.048595	0.03493	0.020529	0.068321	0.056294	0.006655	0.03525	0.028038	-0.09115	0.003
1033			-0.00224		0.001651		0.077685	0.074395	0.100963	0.062707	0.039348	0.095848	0.069413	0.103617	0.048272	0.00
1507			0.004027		-0.00023		0.073533	0.178325	0.081183	0.049723	0.145251	0.04216	0.091413	0.215783	-0.00185	0.004
1023			0.015635		0.009769		0.072704	0.285261	0.094202	0.065317	0.253795	0.082956	0.089854	0.245713	0.062677	0.004



# COMPARISON USING RCAP IN STATA

*A simple comparison of the mean wage-rate by sex gives the 'gender pay gap'*

## The pay gap at the mean

- In Stata, you will:
- Preserve
- Collapse
- Twoway graph ... (Hbar ) ..(rcap...)
- The same categorical variable but different continuous variables. I use llmen and ulmen, and llwomen and ulwomen as variable names in the collapse command
- You need to use your Pweights or Aweights in the collapse command
- Then graph ... (rcap ulmen llmen) and so on.

## Variant Pay-Gaps

- Most obviously, you can do the pay gap at the median wage-rate.
  - Monthly earnings is not a good idea because it ignores the part-time working hours. Therefore, it will exaggerate the gender pay gap.
- You also want to create a sexual-orientation pay gap? Make sure you give the confidence interval for the smaller group, especially if  $N < 30$  for the calculation of one of the means.
- Ethnicity Pay Gaps
- Disability Pay Gaps ... etc.
- All part of inequalities research.

# FIGURE 1 LOG PAY GAP REGRESSION & DECOMPOSITION

CJE article – version before the revisions – The Poor group is lowest 12%.

Table 3. Decomposition of the Gender Pay Gap by Household Type. ( OPTION B)

	Analysis at the Mean	Poor Households	Wealthy Households
	% Contributor of total	% Contributor of total	% Contributor of total
<i>Cumulative Work History in Years</i>			
Full-time work	18%	69%	13%
Part-time work	8%	-92%	4%
Unpaid Family Care	7%	-12%	6%
Parental Leave	-3%	-41%	0%
Unemployment	-1%	6%	0%
Illness	0%	-3%	0%
<i>Key Work Indicators</i>			
Occupational sex-segregation	20%	3%	10%
Public Sector	-7%	-29%	2%
Union member	-2%	-7%	-3%
Bonus receipt	6%	-13%	8%
<i>Residuals</i>			
Measured Female Residual	31%	184%	32%
Unobserved Residuals	16%	2%	12%
Total Coefficient Effects	-0.2	-0.03	-0.29

The working poor are defined as workers living in households with equivalised income <60% of median earned income (£1,335 or less). Here, it is the bottom 12% of households. We apply a similar cut-off on the right-hand side, >160% of median, N=10,275

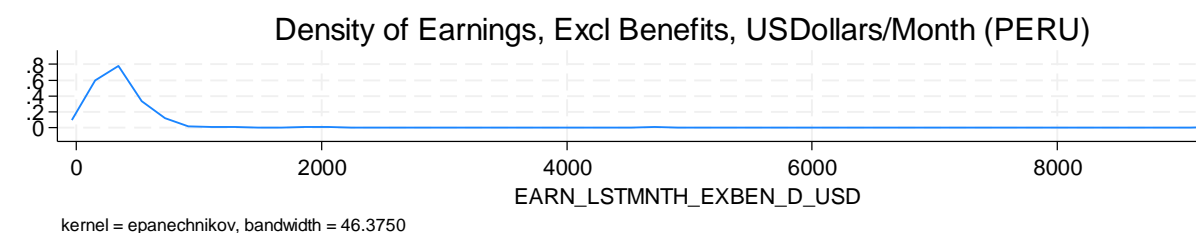
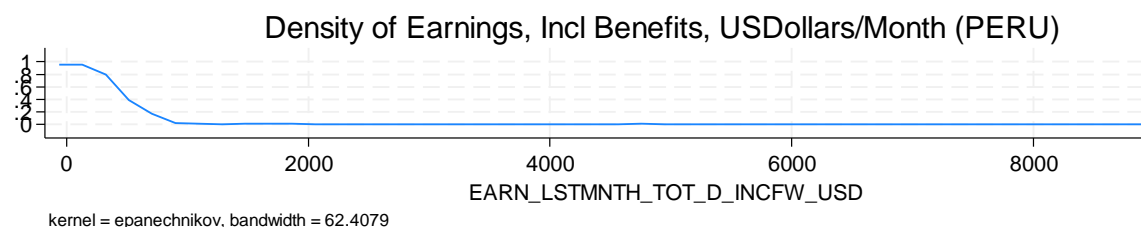
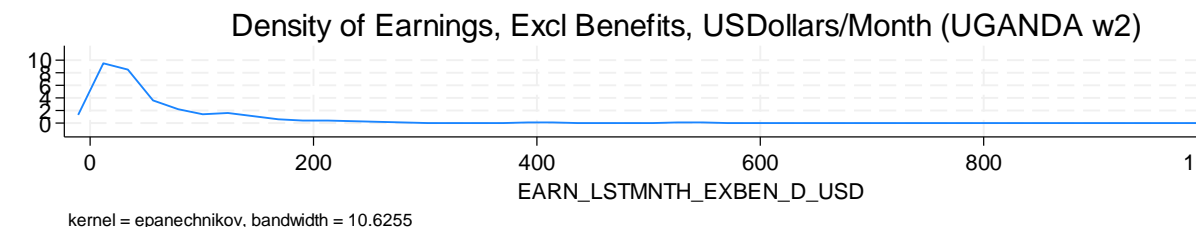
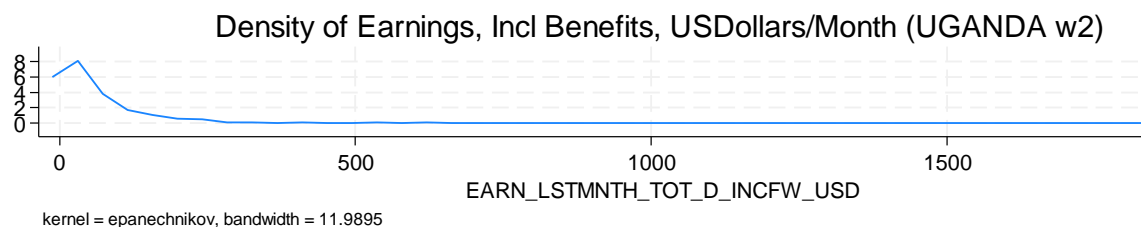
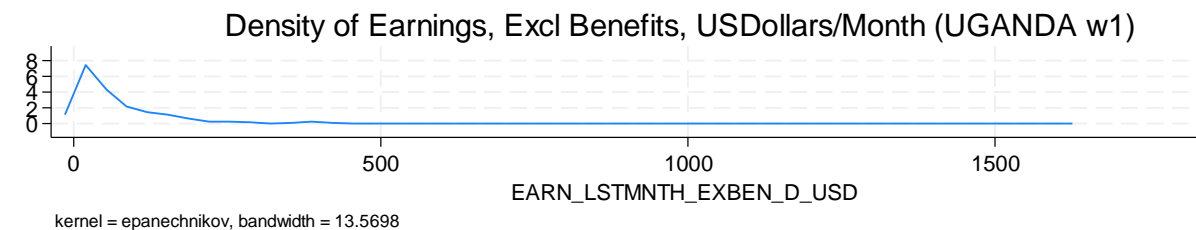
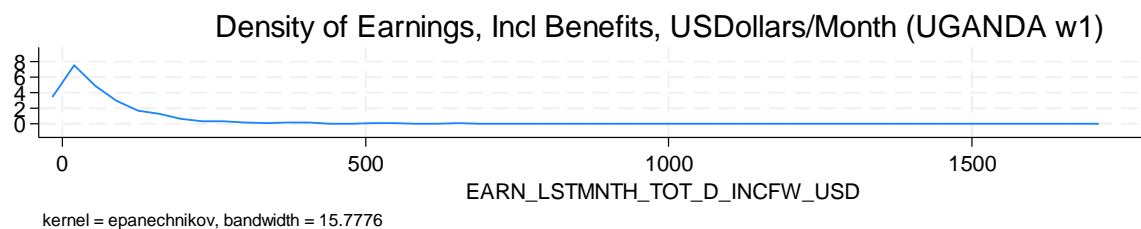
# SOOK KIM MADE GREAT CONTRIBUTIONS

*Work histories measured as one-month units on annual recall, overlapping, corrected.*

*All code reproducible and open-access.*

Table 1. Key Descriptive Statistics, weighted by cross-sectional weights

	All Households		Poor Households		Wealthy Households	
	Men	Women	Men	Women	Men	Women
Gross Hourly Pay	£16.67	£13.21	£8.22	£7.90	£29.20	£21.95
Gender Pay Gap	20%		4%		28%	
Cumulative Work History, in years and proportion of time in each status:						
Full-time work	19.98 (0.93)	13.93 (0.62)	17.13 (0.83)	11.06 (0.49)	21.17 (0.96)	15.84 (0.71)
Part-time work	0.67 (0.03)	5.28 (0.23)	1.22 (0.06)	6.39 (0.28)	0.5 (0.02)	4.26 (0.19)
Unemployment	0.71 (0.03)	0.45 (0.02)	1.90 (0.09)	0.86 (0.04)	0.30 (0.01)	0.29 (0.01)
Unpaid family care	0.05 (0.00)	2.05 (0.03)	0.07 (0.00)	3.57 (0.16)	0.01 (0.00)	1.2 (0.03)
Parental Leave	0.01 (0.00)	0.7 (0.03)	0.00(0.00)	0.70 (0.03)	0.01 (0.00)	0.61 (0.03)
Illness	0.09 (0.00)	0.09 (0.00)	0.20 (0.01)	0.10 (0.00)	0.01 (0.00)	0.04 (0.00)



This set of diagrams is not men/women. It is inclusive of in-kind (LEFT) versus excluding in-kind payments (RIGHT).

# DECOMPOSING THE BARRIERS TO EQUAL PAY: EXAMINING DIFFERENTIAL PREDICTORS OF THE GENDER PAY GAP BY SOCIO-ECONOMIC GROUP'

*By Vanessa Gash, Sook Kim, Nadine Zweiner, and Wendy Olsen*

*Submitted to Cambridge Journal of Economics 2022*

*Revised and resubmitted to CJE 2024*

**Keywords:** gender pay gap, sex-segregation, work-history, working-time.

**JEL:** B54, Feminist Economics, E24, Employment and Wages, J31, Wage Differentials.

*We submitted the paper in 2022. We received 9 pages single spaced editorial & reviewer comments in Sept. 2023. We resubmitted in Feb 2024. We await a response now (May 2024).*

*The equations in the paper cover the Decomposition of Pay Gaps by the Blinder-Oaxaca two-term method. This method has enduring interest. One reason is that any linearised model can be decomposed, but when we use nonlinear Generalised Linear modelling we often cannot decompose the factors' influence amounts upon the Y variable. Authors who don't know GLM use Blinder-Oaxaca. Possibly it is a blind alley because then, we are not investing time in better models.*

*A hurdle model or a Tobit model was avoided in this paper.*

# TABLE 1 K-S TEST OF THE DIFFERENCE OF TWO DISTRIBUTIONS (MEN’S VS WOMEN’S)

*The non-parametric Kolmogorov-Smirnov test is often used if one of the variables is ordinal. Here the distribution is so awkward it is treated as if it were ordinal; or you can do Spearman’s on the ranks.*

						Company revenue
Quiz						£0
Sketch the logged wage distribution of men						£1,013
Sketch the same for women						£5,063
Sketch them on the same graph						£20,250
Sketch the difference-distribution						£40,500



# GOOD EXPLANATORY POWER ON PAY-GAPS

*In the OECD, the bonus culture has created some explosive salary levels. Taking logarithms does not fully solve this problem. In regression, you can offer a bonus binary variable. This gets a high positive coefficient and has a role as a % of the explained variation of pay. This is useful for decomposing two sub-groups  $Y_a$  and  $Y_b$ .*

*When the variable is highly skewed, the error term in regression will not be normally distributed. Therefore, treat the variable using some of the three options a, b, and c:*

- a) Transform it using logs;*
- b) Add a binary variable to explain a key part of the right-hand skewness;*
  - a) Overtime explains higher wages*
  - b) Union membership explains higher wages*
  - c) Having a degree explains higher wages*
  - d) Usually in regression we get up to 30-35 variables.*

*In India, the exclusion of the informal labour relationships creates many, many zeroes for 'wage'.*

- c) Add a hurdle model to explain the zeroes part. A Tobit model, a negative binomial model, or a zero-inflated binomial model could also work. All these are harder/ impossible to decompose.*

*After these steps, your Wage Equation residuals may be normally distributed and your explanatory %'s in the decomposition make sense.*

*One can standardise all the variables. You then have also the same units in all variables. But standardising the binary variables is very much argued about. See Gelman (2008) about standardising binary variables. Good luck to you with research!*

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# THANK YOU

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