北京师范大学
香港浸会大学 联合国际学院

DS4003 Optimization Methods

Assignment Individual Project: Standard Form of Linear Programming — 2023 Spring

Student No.: **Student Name:**

Practical projects: in this project, you are expect to

1. familiar with the Python Optimization package.
2. familiar with the standard form of linear programming
3. be able to transform a linear programming problem to the standard from so that the python `linprog` can be applied
4. all problems should be solved with the **Jupyter notebook** and export itas PDF.

Reference

1. NumPy:SciPy:Optimize:linprog
2. D. G. Luenberger, Yinyu Ye, Linear and Nonlinear Programming, Spinger, Chapter 1.

A standard form of linear programming problem can be formulated as

$$\begin{aligned} &\min_x c^T x \\ &\text{such that} \\ &A_{ub} x \le b_{up} \\ &A_{eq} x = b_{eq} \\ &\ell \le x \le u \end{aligned} \tag{1}$$

where x is a vector of decision variables; c, b_{up}, b_{eq},ℓ and u are vectors; and A_{up} and A_{eq} are matrices.

1. (10 points) Read the `linprog` guide example and complete following questions.

$$\max_{x_1,x_2,x_3,x_4} 29x_1 + 45x_2$$

such that

$$\begin{aligned}
x_1 - x_2 - 3x_3 &\leq 5 \\
2x_1 - 3_x2 - 7x_3 + 3x_4 &\geq 10 \\
2x_1 + 8x_2 + x_3 &= 60; \\
4x_1 + 4x_2 + x_4 &= 60 \\
0 \leq x_1 & \\
0 \leq x_2 &\leq 5 \\
x_3 &\leq 0.5 \\
-3 &\leq x_4
\end{aligned}$$

   (a) (7 points) Transform the problem to the standard form (1) of linear programming. Specify that c, A_{up}, A_{eq}, b_{up}, b_{eq}, ℓ and u.

   (b) (3 points) Exclude the python code in the guide.

2. (20 points) Consider the following problem

$$\begin{aligned}
&\max\ x_1 + 4x_2 + x_3 \\
&\text{subject to :} 2x_1 - 2x_2 + x_3 = 4 \\
&x_1 - x_3 = 1 \\
&x_2 \geq 0, x_3 \geq 0
\end{aligned}$$

   (a) (14 points) Convert the following problem to standard form.

   (b) (6 points) Solve it with `linprog`

3. (30 points) Follow the steps of in question 1. Solve the following to linear programming program

$$\min\ x + 2y + 3z$$

subject to:

$$\begin{aligned}
2 \leq x + y &\leq 3 \\
4 \leq x + z &\leq 5 \\
x \geq 0, y &\geq 0, z \geq 0
\end{aligned}$$

   (Hints: you can use addition variables to make the equality constraints.)

   (a) (21 points) Convert the following problem to standard form.

   (b) (9 points) Solve it with `linprog`

4. (30 points) A large textile firm has two manufacturing plants, two sources of raw material, and three market centers. The transportation costs between the sources and the plants and between the plants and the markets are as follows

	plant A	plant B
source 1	¥100/ton	¥150/ton
source 2	¥200/ton	¥150/ton

	market 1	market 2	market 3
plant A	¥400/ton	¥200/ton	¥100 /ton
plant B	¥300/ton	¥400 /ton	¥200/ton

Ten tons are available from source 1 and 15 tons from source 2. The three market centers require 8 tons, 14 tons and 3 tons. The plants have unlimited processing capacity.

(a) (20 points) Formulate the problem of finding the shipping patterns from sources to plants to markets that minimizes the total transportation cost.

(b) (10 points) Solve the problem with `linprog`.

5. (20 points) A businessman is considering an investment project. The project has a lifetime of four years, with cash flows of -¥100,000, +¥50,000, +¥70,000, +¥30,000 in each of the four years, respectively. At any time he may borrow funds at the rates of 12%, 22%,and 34%(total) for 1, 2, 3 periods, respectively. He may also loan fund at 10% per period. He calculate the *present value* of a project as the maximum amount of money he would pay now, to another party, for the project, assuming that he has no cash on hand and must borrow and lend to pay the other party and operate the project while maintaining a non-negative cash balance after all debts are paid. Formulate the project valuation problem in a linear programming framework.