Homework 1

In this class, we refer our textbooks as

Book 1: Numerical Approximation of Partial Differential Equations by Soren Bartels
Book 2: The Finite Element Methods Theory, Implementation and Applications by Mats G. Larson and Fredrik Bengzon

Assignments with simple responses such as "I did it" or "I read it" or "I just started":

A1: Download the textbooks from VT's library.

A2: Read Sections 1.2.1, 1.2.3, 1.3.1, 1.3.3, 1.4.1, 1.4.2, 1.5.1, 1.5.2, 7.2.1, and 8.1.1 from Book 1.

Mathematical Problems:

B1: Let $u(X) = (u_1(X), u_2(X), u_3(X))$ be the displacement vector define on $\Omega \subset \mathbb{R}^3$ and let $X = (x, y, z) = (x_1, x_2, x_3)$ be an arbitrary point of Ω. Rewrite equation for linear elastostatics in (11.9a) (Book 2) explicitly as a system of PDEs about functions $u_1(X), u_2(X), u_3(X)$.

B2: Consider a BVP for the Euler-Bernoulli Beam: find $w(x)$ such that

$$(EIw(x)'')'' = q(x), x \in (0, L), \tag{1}$$
$$w(0) = w_0, w'(0) = w_1, \tag{2}$$
$$(EIw'')(L) = w_2, (EIw'')'(L) = w_3. \tag{3}$$

Here $EI(x)$ is the bending rigidity of the beam.

(a) Derive a weak form for this BVP.

(b) Classify the boundary conditions as natural and essential boundary conditions.

Programming/Computing Problems:

C1: Consider the following BVP: find $u(x)$ such that

$$\begin{aligned} -u''(x) + u'(x) + u(x) &= (x+1)^2 + x - 2 - \sin(x) + 2\cos(x), \quad x = (0,1), \\ u(0) &= 1, \quad u'(1) = 3 - \sin(1) \end{aligned}$$

(a) Derive the weak form for this BVP, and identify each boundary condition by marking a suitable slot in the following table:

Boundary Cond.	Natual	Essential
$u(0) = 1$		
$u'(1) = 3 - \sin(1)$		

(b) Use the following function spaces to compute the a Petrov-Galerkin solution $u_{pg}(x)$ for this BVP:

$$\mathcal{S}_3 = span\{\cos(x), e^x, x\}, \quad \mathcal{T}_2 = span\{\sin(x), x\}$$

Present your solution in the following table:

x	$u_{pg}(x)$
$\pi/5.7$	

Remark: Yes, as in the examples, we can use Matlab's `integral` function to carry out all the integrations needed in the computation for $u_{pg}(x)$ in this problem.