Homework 2

Remark: In default, equations mentioned by an equation number such as (11.9a) are from Book 2.

Assignments with simple responses such as "I did it" or "I read it":

A1: Read discussions in Lecture Slides about the programming for the following Matlab functions:

```
mesh_generator_1D
shape_fun_1D_Lagrange
fem_generator_Lagrange_1D
```

A2: Implement Matlab `shape_fun_1D_Lagrange_ref`.

A3: Let $\mathcal{T}_h$ be a mesh of a domain $\Omega \subset \mathbb{R}^1$ and let Π_p be the space of polynomials of degree p or less. Prove that for every element $K \in \mathcal{T}_h$,

$$V_h^p(K) = span\{L_{K,j}(x), \quad j = 1, 2, \cdots, p, p+1\} = \Pi_p.$$

A4: Let $\mathcal{T}_h$ be a mesh of $\Omega \subset \mathbb{R}^1$ and let $\mathcal{N}_{h,g}$ be the set of finite element nodes for the p-th degree finite element space. Consider the following set of functions:

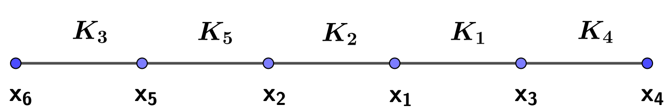
$$U_h^p(\Omega) = \left\{\phi \in C^0(\overline{\Omega}) : \phi|_K \in V_h^p(K) \ \ \forall K \in \mathcal{T}_h\right\}$$

(a) Show that $V_h^p(\Omega) \subseteq U_h^p(\Omega)$ where $V_h^p(\Omega)$ is the C^0 p-th degree finite element space spanned by the C^0 p-th degree finite element basis functions.

(b) Is $U_h^p(\Omega) \subseteq V_h^p(\Omega)$ true? Why or why not?

Programming/Computing Problems:

C1: Consider the following mesh as follows:



Assume that the edges are ordered from the left to right. Find the edge matrix/arry for this mesh.

C2: Implement a Matlab function that can generate a uniform mesh for a 1D domain $\Omega = (x_l, x_r)$ with n elements. This function should be in the following format:

```
function mesh = mesh_generator_1D(domain, n)
```

Then generate a mesh for the domain $\Omega = (2, 3)$ with $n = 512$ elements. Present numerical results in the following table:

p_{323}	
x_l	
x_r	

where x_l and x_r are the vertices of element K_{173}.

C3: Implement the Lagrange shape functions on an element with a Matlab function in the following format:

```
function f = shape_fun_1D_Lagrange(x, elem, degree, ...
                                    shape_index, d_index)
```

This Matlab function should work at least for `degree = 0, 1, 2, 3`. Then, generate a mesh $\mathcal{T}_h$ for the domain $\Omega = (2, 3)$ with $n = 512$ elements and use this Matlab function to evaluate the Lagrange shape functions on $K = K_{171} \in \mathcal{T}_h$. Present your numerical results by filling the following table:

	$p = 1$	$p = 2$	$p = 3$
$L^p_{K,1}(M)$			
$L^p_{K,2}(M)$			
$L^p_{K,3}(M)$	NA		
$(L^p_{K,1})'(M)$			
$(L^p_{K,2})'(M)$			
$(L^p_{K,3})'(M)$	NA		
$L^p_{K,4}(M)$	NA	NA	
$(L^p_{K,4})'(M)$	NA	NA	

where $M = (1/3)z_l + (2/3)z_r$ assuming $K = (z_l, z_r)$.

C4: Assume `fem` is for the 3rd degree C^0 finite element space generated on the mesh $\mathcal{T}_h$ for the domain $\Omega = (1, 2)$ with $n = 128$ elements.

(a) How many nodes does this mesh have? How many nodes does this finite element space have?

(b) Which finite element nodes are in element K_{73}?

(c) Find coordinates for all finite element nodes in $K_{73} \in \mathcal{T}_h$.