A1: Read discussions in Lecture Slides about the programming for the following Matlab functions:

30/32

```
mesh_generator_1D
shape_fun_1D_Lagrange
fem_generator_Lagrange_1D
```

A2: Implement Matlab `shape_fun_1D_Lagrange_ref`.

A1, A2 I did it

A3: Let $\mathcal{T}_h$ be a mesh of a domain $\Omega \subset \mathbb{R}^1$ and let Π_p be the space of polynomials of degree p or less. Prove that for every element $K \in \mathcal{T}_h$,

$$V_h^p(K) = span\{L_{K,j}(x), \quad j = 1, 2, \cdots, p, p+1\} = \Pi_p.$$

Assume $\sum_{j=1}^{p+1} u_j L_{k,j}(x) = 0 \quad \forall x \in K$

$$L_{k,j}^p(x) = \prod_{k=1, k \neq j}^{p} \frac{x - t_{k,k}}{t_{k,j} - t_{k,k}}$$

if $u_m \neq 0 \quad 1 \leq m \leq p+1$

then $u_m L_{k,m}(x) = -\sum_{j=1, j \neq m}^{p+1} u_j L_{k,j}(x)$

Let $x = t_{k,m}$, thus $L_{k,m}(t_{k,m}) = 1$

and $-\sum_{j=1, j \neq m}^{p+1} u_j L_{k,j}(t_{k,m}) = 0$ since $j \neq m$

as a result,

$$U_m \cdot L_{km}(t_{k,m}) = 0 \quad \text{with } L_{k,m}(t_{k,m}) = 1$$

$\Rightarrow U_m = 0$

However, in the very begining, we assume $U_m \neq 0$
which is in conflict with $U = 0$

thus, to make $\sum_{j=1}^{p+1} U_j L_{k,j}(x) = 0 \quad x \in k$

all the U_j should be 0

thus, $L_{k,j}(x)$ can be a group of basis for the polynominal space.

the order of $L_{k,j}(x)$ is x^p, thus $\text{span}\{L_{k,j}(x), j = 1, 2, \cdots p, p+1\}$ can construct the space of polynominal of degree p or less

A4: Let $\mathcal{T}_h$ be a mesh of $\Omega \subset \mathbb{R}^1$ and let $\mathcal{N}_{h,g}$ be the set of finite element nodes for the p-th degree finite element space. Consider the following set of functions:

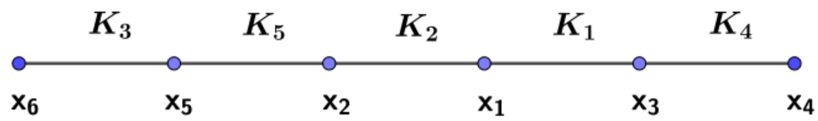
$$U_h^p(\Omega) = \left\{\phi \in C^0(\overline{\Omega}) : \phi|_K \in V_h^p(K) \quad \forall K \in \mathcal{T}_h\right\}$$

(a) Show that $V_h^p(\Omega) \subseteq U_h^p(\Omega)$ where $V_h^p(\Omega)$ is the C^0 p-th degree finite element space spanned by the C^0 p-th degree finite element basis functions.

(b) Is $U_h^p(\Omega) \subseteq V_h^p(\Omega)$ true? Why or why not?

-1 Your response?

C1: Consider the following mesh as follows:



Assume that the edges are ordered from the left to right. Find the edge matrix/arry for this mesh.

the edge matrix is

$$\begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 5 & 1 & 4 & 3 & 0 \\ 1 & 2 & 4 & 0 & 5 & 3 \end{matrix}$$

C2: Implement a Matlab function that can generate a uniform mesh for a 1D domain $\Omega = (x_l, x_r)$ with n elements. This function should be in the following format:

```
function mesh = mesh_generator_1D(domain, n)
```

Then generate a mesh for the domain $\Omega = (2, 3)$ with $n = 512$ elements. Present numerical results in the following table:

p_{323}	
x_l	
x_r	

```
function mesh = mesh_generator_1D(domain, n)
x_l = domain(1); x_r = domain(2);
p = x_l:(x_r - x_l)/n:x_r;
t = zeros(2,n); t(:,1) = [1;2];
for i=2:n
t(:,i) = t(:,i-1) + 1;
end
e = zeros(3, n+1);
e(1,:) = 1:n+1; e(2,:) = 0:n; e(3,:) = [1:n,0];
mesh = struct('p', p, 't', t, 'e',e);
```

Not enough input arguments.

Error in mesh_generator_1D (line 2)
x_l = domain(1); x_r = domain(2);

Published with MATLAB® R2015b

$P_{323} = 2.62890625000000$

$x_l = 173$

-1

the coordinates of xl and xr???

$x_r = 174$

C3: Implement the Lagrange shape functions on an element with a Matlab function in the following format:

```
function f = shape_fun_1D_Lagrange(x, elem, degree, ...
                                   shape_index, d_index)
```

This Matlab function should work at least for `degree = 0, 1, 2, 3`. Then, generate a mesh $\mathcal{T}_h$ for the domain $\Omega = (2,3)$ with $n = 512$ elements and use this Matlab function to evaluate the Lagrange shape functions on $K = K_{171} \in \mathcal{T}_h$. Present your numerical results by filling the following table:

	$p = 1$	$p = 2$	$p = 3$
$L^p_{K,1}(M)$			
$L^p_{K,2}(M)$			
$L^p_{K,3}(M)$	NA		
$(L^p_{K,1})'(M)$			
$(L^p_{K,2})'(M)$			
$(L^p_{K,3})'(M)$	NA		
$L^p_{K,4}(M)$	NA	NA	
$(L^p_{K,4})'(M)$	NA	NA	

where $M = (1/3)z_l + (2/3)z_r$ assuming $K = (z_l, z_r)$.

```
function f = shape_fun_1D_Lagrange_ref(t, shape_index, d_index,degree)
if degree == 0
    if shape_index == 1
        if d_index == 0
            f = ones(size(t));
        elseif d_index > 0
            f = zeros(size(t));
        end
    else
        disp(['shape_index = ', int2str(shape_index)])
        disp('This program is not for this value of shape_index')
        disp('Terminate this program by Ctrl+C')
        pause
    end
elseif degree == 1
    if shape_index == 1
        if d_index == 0
            f = (1-t)/2;
        elseif d_index == 1
            f = (-1/2)*ones(size(t));
        elseif d_index > 1
            f = zeros(size(t));
        end
    elseif shape_index == 2
        if d_index == 0
            f = (1+t)/2;
        elseif d_index == 1
            f = (1/2)*ones(size(t));
        elseif d_index > 1
            f = zeros(size(t));
        end
    else
        disp(['shape_index = ', int2str(shape_index)])
        disp('This program is not for this value of shape_index')
        disp('Terminate this program by Ctrl+C')
        pause
    end
elseif degree == 2
    if shape_index == 1
        if d_index == 0
            f = -(1/2)*(1-t).*t;
        elseif d_index == 1
            f = -1/2+t;
        elseif d_index == 2
            f = ones(size(t));
        elseif d_index > 2
            f = zeros(size(t));
        end
    elseif shape_index == 2
        if d_index == 0
            f = 1-t.^2;
        elseif d_index == 1
```

```
                f = -2*t;
            elseif d_index == 2
                f = -2*ones(size(t));
            elseif d_index > 2
                f = zeros(size(t));
            end
        elseif shape_index == 3
            if d_index == 0
                f = (1/2)*t.*(1+t);
            elseif d_index == 1
                f = 1/2 + t;
            elseif d_index == 2
                f = ones(size(t));
            elseif d_index > 2
                f = zeros(size(t));
            end
        else
            disp(['shape_index = ', int2str(shape_index)])
            disp('This program is not for this value of shape_index')
            disp('Terminate this program by Ctrl+C')
            pause
        end
    % degree = 3 added by WH
    elseif degree == 3
        if shape_index == 1
            if d_index == 0
                f = -(9*(t - 1).*(t - 1/3).*(t + 1/3))/16;
            elseif d_index == 1
                f = - (9*(t - 1).*(t - 1/3))/16 - (9*(t - 1).*(t +
 1/3))/16 - (9*(t - 1/3).*(t + 1/3))/16;
            elseif d_index == 2
                f = 9/8 - (27*t)/8;
            elseif d_index == 3
                f = -27/8*ones(size(t));
            elseif d_index > 3
                f = zeros(size(t));
            end
        elseif shape_index == 2
            if d_index == 0
                f = (27*(t - 1).*(t + 1).*(t - 1/3))/16;
            elseif d_index == 1
                f = (27*(t - 1).*(t + 1))/16 + (27*(t - 1).*(t - 1/3))/16
 + (27*(t + 1).*(t - 1/3))/16;
            elseif d_index == 2
                f = (81*t)/8 - 9/8;
            elseif d_index == 3
                f = 81/8*ones(size(t));
            elseif d_index > 3
                f = zeros(size(t));
            end
        elseif shape_index == 3
            if d_index == 0
                f = -(27*(t - 1).*(t + 1).*(t + 1/3))/16;
            elseif d_index == 1
```

```
            f = - (27*(t - 1).*(t + 1))/16 - (27*(t - 1).*(t +
1/3))/16 - (27*(t + 1).*(t + 1/3))/16;
        elseif d_index == 2
            f = - (81*t)/8 - 9/8;
        elseif d_index == 3
            f = - 81/8*ones(size(t));
        elseif d_index > 3
            f = zeros(size(t));
        end
    elseif shape_index == 4
        if d_index == 0
            f = (9*(t + 1).*(t - 1/3).*(t + 1/3))/16;
        elseif d_index == 1
            f = (9*(t + 1).*(t - 1/3))/16 + (9*(t + 1).*(t + 1/3))/16
+ (9*(t - 1/3).*(t + 1/3))/16;
        elseif d_index == 2
            f = (27*t)/8 + 9/8;
        elseif d_index == 3
            f = 27/8*ones(size(t));
        elseif d_index > 3
            f = zeros(size(t));
        end
    else
        disp(['shape_index = ', int2str(shape_index)])
        disp('This program is not for this value of shape_index')
        disp('Terminate this program by Ctrl+C')
        pause
    end
%
elseif degree > 3
    disp(['degree = ', int2str(degree)])
    disp('This program is not for this value of degree')
    disp('Terminate this program by Ctrl+C')
    pause
end
```

```
Not enough input arguments.

Error in shape_fun_1D_Lagrange_ref (line 2)
if degree == 0
```

Published with MATLAB® R2015b

	$p = 1$	$p = 2$	$p = 3$
$L_{K,1}^p(M)$	0.333333333333485	-0.111111111111061	-7.578659921847475e-14
$L_{K,2}^p(M)$	0.666666666666515	0.888888888889091	4.547195953109519e-13
$L_{K,3}^p(M)$	NA	0.222222222221970	0.999999999999773
$(L_{K,1}^p)'(M)$	-512	-1.706666666669771e+02	2.560000000000000e+02
$(L_{K,2}^p)'(M)$	512	-6.826666666660458e+02	-1.536000000000699e+03
$(L_{K,3}^p)'(M)$	NA	8.533333333330229e+02	7.680000000013972e+02
$L_{K,4}^p(M)$	NA	NA	-1.515731984368461e-13
$(L_{K,4}^p)'(M)$	NA	NA	5.119999999993015e+02

C4: Assume `fem` is for the 3rd degree C^0 finite element space generated on the mesh $\mathcal{T}_h$ for the domain $\Omega = (1, 2)$ with $n = 128$ elements.

(a) How many nodes does this mesh have? How many nodes does this finite element space have?

(b) Which finite element nodes are in element K_{73}?

(c) Find coordinates for all finite element nodes in $K_{73} \in \mathcal{T}_h$.

(a), the mesh has 129 nodes

the finite element space has 385 nodes

(b) 217, 218, 219, 220 nodes

are in element K_{73}

(c) 1.5625000100000000

1.5651041666667

1.567708333333334

1.5703125000000000

are the coordinates for the 4 nodes in element K_{73}