AMME 3500 Design Project 1

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I. Introduction

Automotive vehicles have been an important topic and investment as the increasing pace of living and advancement of technology. Two main features, cruise control, and lane change are crucial to maintaining a safe driving experience with efficiency. The former is responsible for the change of any desired speed while the latter is for lane changing. In this project, these two features are discussed starting from linearizing the given non-linear systems. And then, a controller is chosen for each scenario, followed by validations that are more related to real-world uncertainties. The car model chosen for this design project is the Mercedes AMG C63s and its specialization is listed below [1]:

- Mass: m=1505 kg
- Drag Coefficient: c_D=0.24
- Cross-sectional area: A=1.9m^2
- density of air: ρ=1.225kgm^{-3}
- vehicle length: $4.75m$

II. Longitudinal Controller

A. Linearization

The governing equation is:

$$m\dot{v} + \frac{1}{2}A\rho c_D v^2 = u \tag{1}$$

To obtain three pairs of equilibrium (v_e,u_e), $\dot{v}$, the rate of change of velocity is zero, and is rearranged into:

$$\dot{v} = \frac{u - \frac{1}{2}A\rho c_D v^2}{m}$$

Therefore, 3 sets of equilibrium are determined from this, which are ($v_{e1} = 20$,$u_{e1} = 111.72$),($v_{e2} = 40$,$u_{e2} = 446.88$),($v_{e1} = 60$,$u_{e3} = 1005.48$). The first two speeds are typical for the application of cruise control while the last one is much higher than most speed limits. So $v = 60m/s$ is chosen only for test and validation purposes. The next step is to linearize (1), it's desired to transform into partial form, where $\partial_v(t) = v(t) - v_e$, $\partial_u = u(t) - u_e$:

$$\dot{\partial_v}(t) + a\partial_v(t) = \partial_u(t)$$

Let $B = \frac{1}{2}A\rho c_D$ for simplification and$f(v) = v^2$, so $f(v) = f(v_e) + \partial f$. And then in partial form, (1) is transformed into:

$$m\dot{\delta_v} + B(f(v_e) + \delta f) = u_e + \delta u$$

At the equilibrium, all the partial term is zero, and hence $Bf(v_e) = ue$ is observed. 1^{st} order Taylor expansion is used:

$$f(v) = f(v_e) + \frac{\delta f(v)}{\delta v}|_{v=v_e}(v - v_e)$$

where $\frac{\delta f(v)}{\delta v}|_{v=v_e} = 2v_e$. Based on the above, It is further simplified into:

$$m\dot{v} + 2v_e B(v - v_e) = u - u_e \tag{2}$$

Rearrange (2) to obtain linearized (1):

$$m\dot{v} + 2v_e Bv = u - u_e + 2v_e v_e \tag{3}$$

Rearrange (3):

$$\dot{v} = \frac{-2Bv_e}{m}v + \frac{2v_e^2 B}{m} + \frac{u - u_e}{m} \tag{4}$$

The system diagram in Simulink is shown in figure 1, which is used to plot the following trajectories. Different initial conditions $v(0) = v_e, 2v_e, -2v_e$ are chosen to test the accuracy of the linearized equation. And each u is accordingly the u_e for their corresponding v_e.

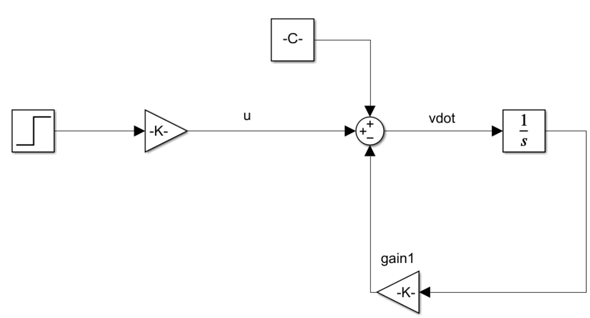

Fig. 1. Simulation for part a

The plots represent trajectories of the chosen equilibrium velocities. Regardless of their initial velocities, by setting their input u to u_e, each indicates a convergence to its equilibrium velocity. However, the trend is that it takes less time to converge if the equilibrium speed increases. For $v_e = 20m/s$, it takes around 800s to converge while it takes only around 300s if $v_e = 60m/s$. These data seem physically irrational, as a Mercedes AMG C63s could achieve neither $120m/s$ nor take more than 10 minutes to decelerate from $40m/s$ to $20m/s$. Therefore, a controller is required to modify this.

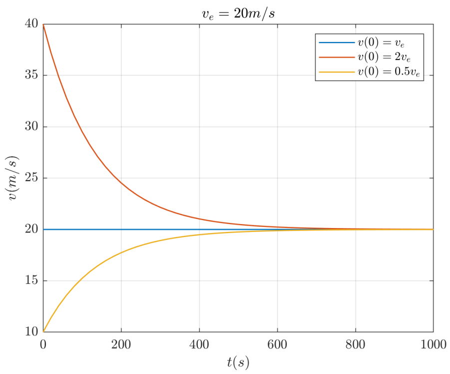

Fig. 2. Trajectories at $v_e = 20m/s$

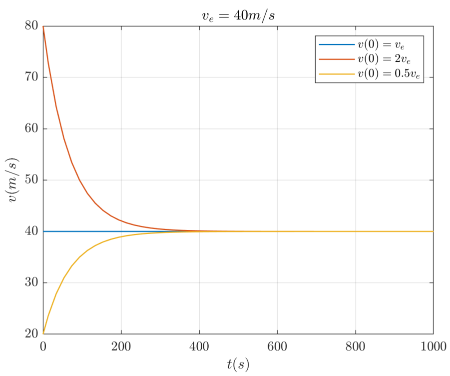

Fig. 3. Trajectories at $v_e = 40m/s$

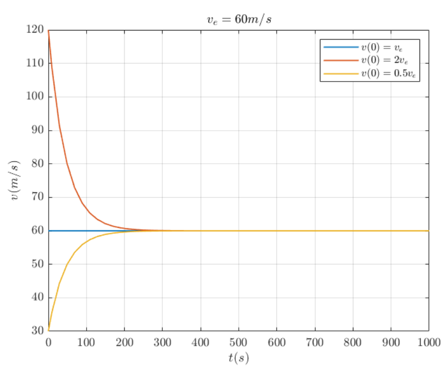

Fig. 4. Trajectories at $v_e = 60m/s$

B. Controller Design

Based on the complexity of the linearized system and stability required for the performance of the vehicle, the proportional controller is chosen for this case, where input u is expressed in the form $u(t) = K_p(r - v(t))$. Term r refers to a reference speed, which is usually the desired speed after implementing the control system. If this proportional controller is applied, equation (4) is changed into:

$$\dot{v} = -\frac{2v_e B + K_p}{m}v + \frac{K_p r}{m} - \frac{v_e}{m} + \frac{2v_e^2 B}{m} \tag{5}$$

The equilibrium point chosen here is ($v_{e2} = 20$,$u_{e2} = 111.72$). Figure 5 indicates the simulation of this system, where $K = \frac{2v_e B + K_p}{m}$ and $C = \frac{2v_e^2 B + K_p r - v_e}{m}$.

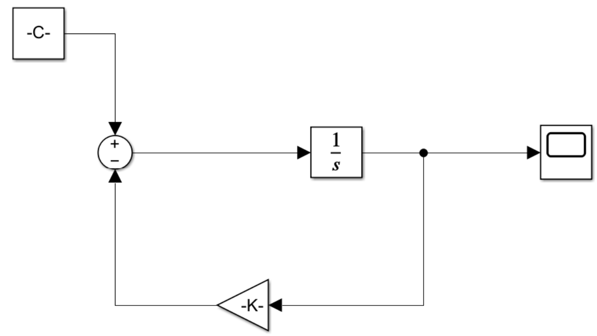

Fig. 5. Simulation using proportional control

The simulation for this proportional controller is shown in the above figure. For the gain of the proportional controller, it's discovered that the higher the value of K_p, the stronger correction it will bring to the system, and hence the faster the control process is. The maximum comfortable acceleration for passengers in a vehicle is around $0.5g$ to $0.7g$, where g is the gravitational acceleration [2]. This will be an important reference for the choice of K_p. Because of this, to control moderate speed, the settling time t_s between 3 to 5 seconds is desired, where $t_s \approx 4/a$. Variable a here is the coefficient of v term:

$$a = \frac{2v_e B + K_p}{m}$$

Rearrange equation (5):

$$\dot{v} + \frac{2v_e B + K_p}{m}v = \frac{K_p r}{m} - \frac{v_e}{m} + \frac{2v_e^2 B}{m}$$

Therefore,

$$K_p = \frac{4m}{t_s} - 2v_e B$$

By substituting the boundary conditions $t_s = 4, 6$, it is expressed as:

$$\frac{4m}{6} - 2v_e B \leq Kp \leq \frac{4m}{4} - 2v_e B \tag{6}$$

The value of K_p can vary within this range and in this case, $K_p = 1500$ is chosen, as a compromise to the performance of the car from human comfort design. Different initial velocities are chosen to test the effectiveness of the proportional control on this linear model, as shown on the next page. Regardless of v_0, it converges toward its equilibrium velocity. However, it is observed that these velocities don't precisely converge to $20m/s$, but $19.9261m/s$, with a steady state error of 0.3695%. This will be further discussed in the following section.

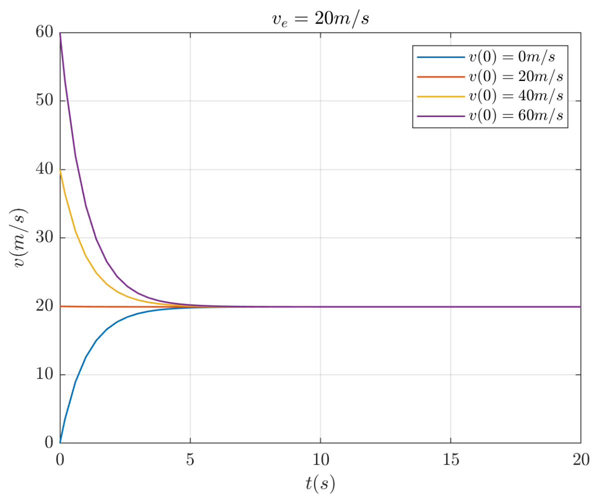

Fig. 6. Trajectories when $v_e = 20m/s$ using proportional control

C. Validation

1) Equation derivation: To derive the equation, Newton's second law is used,

$$\sum F = ma = m\dot{v} \tag{7}$$

A free-body diagram considering disturbance is shown below, where $F_{drag} = \frac{1}{2}\rho C_d A v^2$, u is the system input.

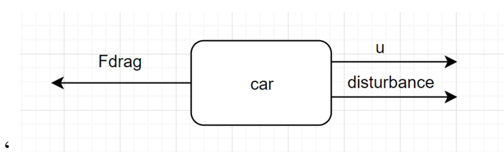

Fig. 7. Free-body diagram of the car under disturbance

Therefore, equation (6) is equivalent to:

$$m\dot{v} = u + d - \frac{1}{2}\rho C_d A v^2 \tag{8}$$

The desired equation is obtained by moving the drag force component to the left:

$$m\dot{v} + \frac{1}{2}\rho C_d A v^2 = u + d - \frac{1}{2}\rho C_d A v^2 \tag{9}$$

It is given that the vehicle encounters a sudden transition from flat ground to an uphill slope of 8% grade, which is 4.57 degree in equivalent. If the sine component of the weight is considered the only disturbance, then:

$$d = -mg\sin(4.57)$$

In addition, the proportional controller, $u = K_p(r - v)$, is substituted into the system as required.

$$m\dot{v} = K_p(r - v) - mg\sin(4.57) - \frac{1}{2}\rho C_d A v^2 \tag{10}$$

Its simulation diagram is drawn based on the above equation, where the step input is the disturbance component. If the vehicle starts to climb uphill at $t = 2s$, then it is when disturbance starts to involve.

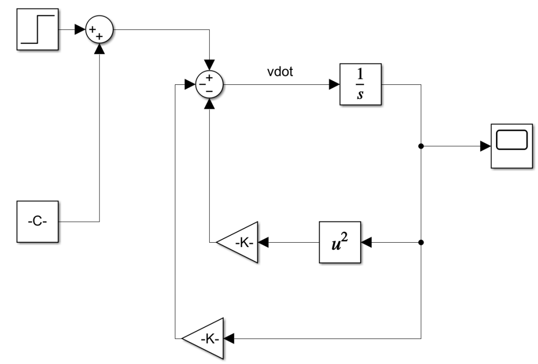

Fig. 8. Simulation with disturbance

Figure 9 is the result after implementing the linear controller into the non-linear system. As the disturbance is set starting from $t = 2s$, it is noticed that there is a slight increase in velocity. This is more obvious for $v(0) == 20m/s$, as the previous system without any disturbance remains at a horizontal line. The disturbance is accountable for this increase. In addition, all these initial velocities converge toward a speed of $20.705m/s$, which is above $20m/s$, the reference speed. This means disturbance here inevitably adds error to this proportional controller. The steady-state error here is 3.5%, which is significantly higher than the system without disturbance.To further investigate the steady-state error, $v_ref = 0m/$ is chosen and plotted.
There is a similar trend in fig 9 that the initial speed at $0m/s$ increases after the intervention of disturbance when $t = 2s$. The steady-state speed arrives at a slightly higher value ($0.7857m/s$) than desired. In reality, if this command is sent to stop the vehicle, with this error, it will still move forward in $2.83km/h$ approximately. It is a minor system error

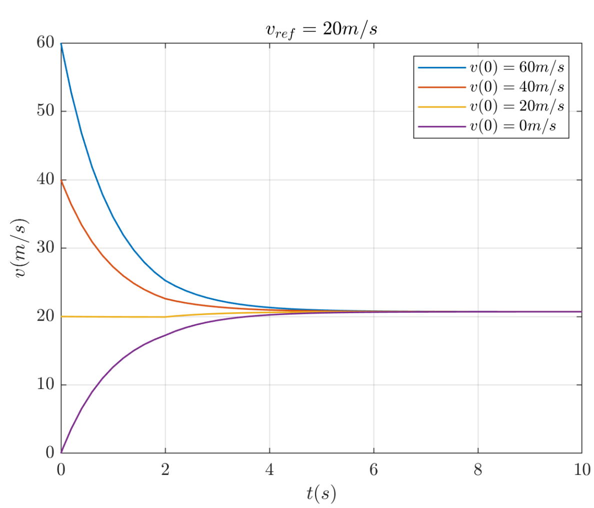

Fig. 9. Trajectories when $v_{ref} = 20m/s$ with disturbance

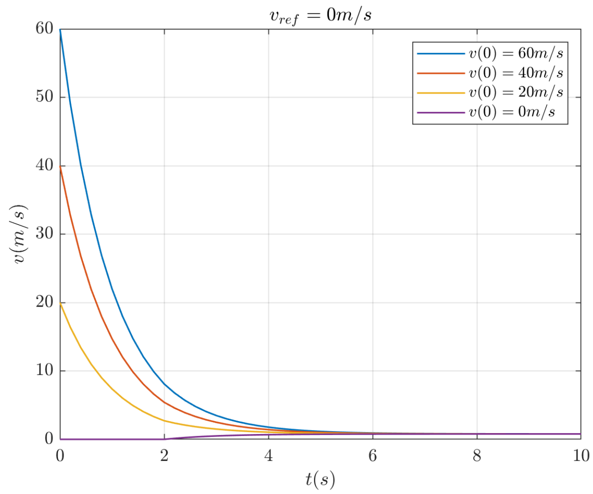

Fig. 10. Trajectories when $v_{ref} = 0m/s$ with disturbance

if the vehicle is run at other speeds. However, in this case, it will be significant as human intervention is required such as stepping on the brake. Mathematically, the steady-state value can be evaluated using the following equation:

$$v(\infty) = \frac{K_p r}{K_p + a} + \frac{d}{K_p + a} \tag{11}$$

The second term would be insignificant if K_p is large enough. However, there is an upper boundary limit for the sake of human comfort. hence the second term will bring a significant error to the system. In this case, there are several advantages found in using a proportional controller. Firstly, it features simplicity, that is easy to understand and implement. Other than that, it ensures stability as there is no oscillation or overshoot observed. However, it may not be suitable for achieving high accuracy especially if there is a disturbance encountered by the system. This will inevitably increase the steady-state error. The error with disturbance (3.5%) seems acceptable. However, in reality, there is numerous disturbance, which will lead to a tremendous steady-state error. To solve this, a more precise controller like PI or PD is required.

III. Lateral Control

Other than cruise speed, lane changing in automotives can also be controlled using different types of controllers. As shown below, given an input δf, which is the steering wheel angle, it will change lanes to desired lateral displacement.

A. Linearization

Given that $v(t) = v_o$, and using small angle approximation, transformations have been made:

$$\begin{aligned}\dot{y} &= v\sin(\psi+\beta),\\ &= v(\sin(\psi)\cos(\beta)+\cos(\psi)\sin(\beta)),\\ &= v_0(\psi+\beta)\end{aligned}$$

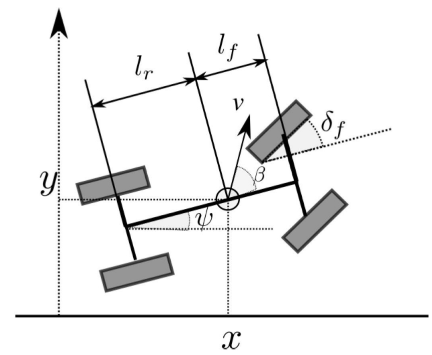

Fig. 11. Lane changing mechanism in vehicles

Assume $l_f = l_r$ and after differentiation:

$$\ddot{y} = v_0(\dot{\psi} + \dot{\beta}) \tag{12}$$

$$\tan(\beta) = \frac{l_r}{l_r + l_f}\tan(\delta f) = \frac{1}{2}\tan(\delta f)$$

For $l = l_r + l_f$ and using small angle approximation, equation 12 is changed into:

$$\beta = 0.5\delta f \tag{13}$$

Therefore:

$$\dot{\psi} = \frac{v_0}{l_r}\beta = \frac{v_0}{2l_r}\delta f \tag{14}$$

Substitute eqn(14) and derivative of eqn(13) into eqn(12):

$$\ddot{y} = \frac{v_o^2}{2l_r}\delta f + \frac{v_o}{2}\dot{\delta f} \tag{15}$$

To get transfer function, eqn(15) is rearranged into:

$$s^2 Y(s) = \frac{v_o^2}{2l_r}\delta F(s) + \frac{v_o}{2}s\delta F(s)$$

Hence:

$$G(s) = \frac{Y(s)}{\delta F(s)} = \frac{\frac{v_o^2}{2l_r} + \frac{v_o}{2}s}{s^2} = \frac{As + B}{s^2}$$

where $A = \frac{v_o}{2}$ and $B = \frac{v_o^2}{2l_r}$

B. Controller Design

Previously a proportional controller is chosen and there's a lack of accuracy to get desired steady-state output. Accordingly, a PD controller is chosen for this task, where δf and$\dot{\delta f}$ are expressed as:

$$\delta f = K_p(r - y) + K_d(\dot{r} - \dot{y}) \tag{16}$$

$$\dot{\delta f} = K_p(\dot{r} - \dot{y}) + K_d(\ddot{r} - \ddot{y}) \tag{17}$$

Substitute eqn(16) and eqn(17) into eqn(15) with $\dot{r} = \ddot{r} = 0$, as the reference point is assumed to be constant here.

$$\ddot{y} + \frac{(K_d B + K_p A)}{K_d A + 1}\dot{y} + \frac{K_p B}{K_d A + 1}y = \frac{K_p B}{K_d A + 1}r \tag{18}$$

This is in the same form as:

$$\ddot{x} + 2\omega_n \zeta \dot{x} + \omega_n^2 x = cu$$

Hence, ω_n, ζ can be expressed by known parameters:

$$\omega_n = \sqrt{\frac{K_p B}{K_d A + 1}} \tag{19}$$

$$\zeta = \frac{(K_d B + K_p A)}{(K_d A + 1) 2\omega_n} \tag{20}$$

During lane changing, the time taken must be reasonably short but in a steady mode. So the rise time is set to $5s$ based on this reason. Also, accuracy in changing lanes is crucial. This means the overshoot is not desired. Because of this, $\zeta = 0.75$ is chosen which aims to avoid overshoot that's larger than 5% ($\zeta > 0.7$). If the initial speed v_0 is set to $20m/s$, then $A = 10$ and $B = \frac{1600}{19}$.

$$t_r = \frac{1.8}{\omega_n} = \frac{1.8}{\sqrt{\frac{K_p B}{K_d A+1}}} = 5 \tag{21}$$

Combining eqn(21), eqn(20) and eqn(19), the value of K_p and K_d are calculated, where $K_p = 0.00163$ and $K_d = 0.00584$.

$$t_s = 3.93 * 2/a = 14.56$$

$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 0.028$$

Variable a here is the coefficient of $\dot{x}.M_p$ is the maximum overshoot and it's 2.8% in this case. For instance, if the lateral displacement of a lane change is $1m$, then the overshoot is only $2.8cm$, which is relatively accurate.

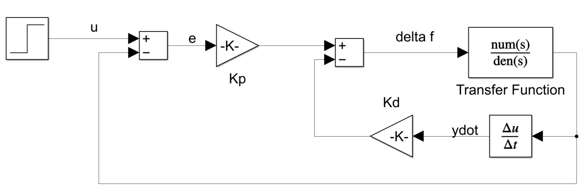

Fig. 12. Simulation diagram using PD controller

The figure above indicates the system diagram. The previously derived transfer function is used to simulate the proportional differential controller. The step input here, the reference value, is set to 3.5, which is the standard Australian lane width.

As shown in figure 13, the curve smoothly moves towards the reference displacement with a minor overshoot and then converges toward the steady state value. The steady-state error here is 0.003%, which is pretty accurate. This is only a single case and more validations are conducted in the following section.

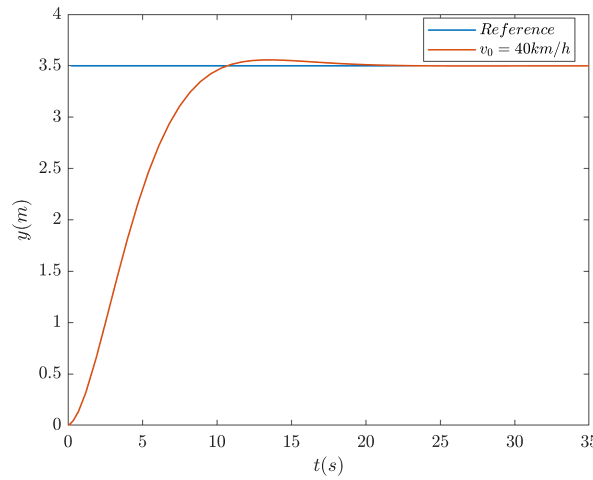

Fig. 13. Trajectory of the lane change when $v_0 = 20m/s$ and $r = 3.5m$

C. Validations

To further investigate the accuracy of this Proportional controller, different speeds are chosen ($v_0 = 40, 60, 80km/h$). As A and B are expressed in terms of v_0, there will be a change in the transfer function as well as the value of K_p and K_d. This is because both K_p and K_d are a function of v_0. Accordingly, there is a change in value to achieve the same response characteristics. The followings are the modified transfer function respectively:

$$G(s) = \frac{\frac{v_{40}}{2}s + \frac{v_{40}^2}{2l_r}}{s^2} \tag{22}$$

$$G(s) = \frac{\frac{v_{60}}{2}s + \frac{v_{60}^2}{2l_r}}{s^2} \tag{23}$$

$$G(s) = \frac{\frac{v_{80}}{2}s + \frac{v_{80}^2}{2l_r}}{s^2} \tag{24}$$

The trajectories shown on the next page are consistent with the previous discussion. There are minor changes in terms of the trajectory shape for different v_0, as the rise time and damping ratio are fixed. Table 1 shows more detail that relates to the response characteristic. Also, the steady-state error for each v_0 is 0.003, which is again negligible. Hence, it validates the accuracy of the chosen PD controllers. In general, the accuracy of a PD controller can be reduced by increasing K_p. This is because higher K_p means The system is more sensitive to the error and results in a larger correction for each error given. Other than that, K_d is related to the rate of change of the error term. By reducing it, the system will be less sensitive to the error change rate and therefore, result in a smoother settlement.

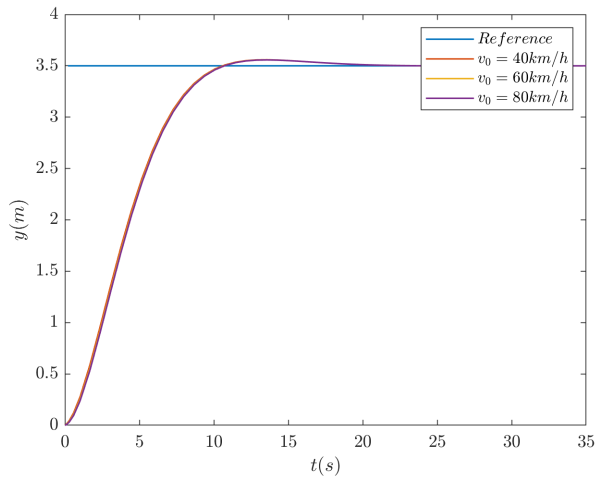

Fig. 14. Trajectory of the lane change for different v_0 and $r = 3.5m$

TABLE I
CONTROLLER SPECIFICATION

Velocity	K_d	K_p	Rise time(s)	overshoot(%)
$40km/h$	0.02213	0.0056	6.4654	1.6790
$60km/h$	0.0096	0.0024	6.4690	1.6545
$80km/h$	0.0054	0.0013	6.4680	1.6421

The table is a summary of the specifications of each PD controller. Values of K_p and K_d are decided from the same design requirement, where rise time is set to $5s$ and overshoot is less than 5% ($\zeta = 0.75$). This table shows a decrease in both K_p and K_d as the speed increase. This means at a higher speed, the vehicle will have more dynamics and the controller gains required will become less. The difference in rise time and overshoot between each v_0 are negligible and are consistent with the previous discussion. However, the rise time was set to $5s$, whereas the actual ones are around $6.5s$. The error may come from the equation of how t_r is expressed, where

$$t_r \approx \frac{1.8}{\omega_n}$$

Other than that, there is a slight increase in the overshoot as the speed increase. This can be explained by the fact that a higher speed can provide sufficient correction power. Next v_0 of the same magnitude but in opposite sign are tested while the other parameters remain the same.

These two figures compare trajectories of different v_0 in different signs but the same magnitude with respect to the reference value. It's found that if v_0 is negative, there is a negative displacement at the start of the lane-changing process, which means the vehicle will move in the opposite direction to the set one. More details are listed in table 2. In both cases, the settling time is larger if the sign of the velocity is positive. The reason behind that has to do with the proportional and differential gain of the system. As mentioned before, K_p is responsible for the error term, while K_d is used to deal with the rate of change in the error term. For a given negative v_0,

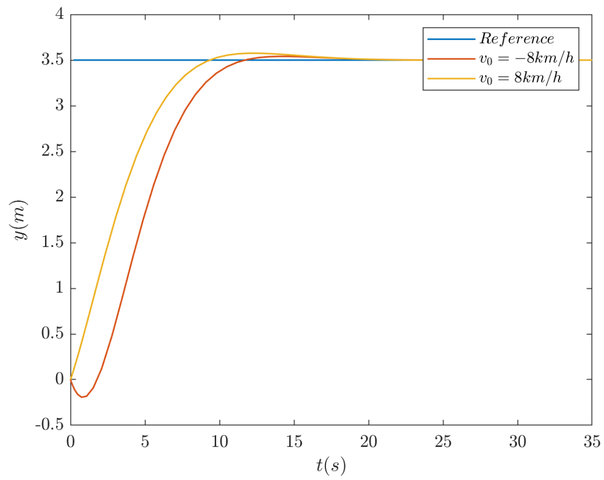

Fig. 15. Trajectories of $v_0 = 8km/h$ and $v_0 = -8km/h$

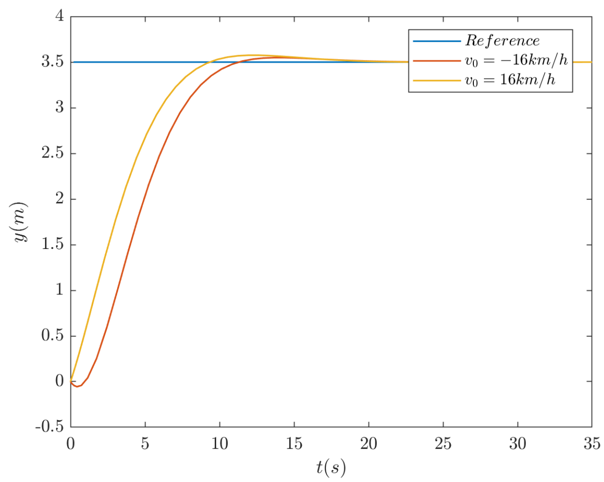

Fig. 16. Trajectories of $v_0 = 16km/h$ and $v_0 = -16km/h$

the error term will also start with a negative value, which is smaller than the original one (when v_0 is in the positive sign). As a result, the action from the proportional controller will be stronger and hence reduce the error quicker. This can be explained mathematically by the term $K_p(r - y)$. If y is negative, the magnitude of this term will be larger, which provides a stronger correction to the system. As a result, the overshoot and settling time for the same magnitude but with a negative sign will be smaller. As the speed in negative convention increases, the undershoot of the system is also reduced by a stronger correction from the control system. These conclusions are consistent with the data shown in the table below.

A closer look is taken to investigate the initial kick. For the input u in this case, it is expressed as $u = A\delta\dot{f} + B\delta f$, where $A = \frac{v_o}{2}$ and $B = \frac{v_o^2}{2l_r}$. If v_o is a negative value, then A becomes negative and B is always positive. That's the reason why there

TABLE II
COMPARISON BETWEEN POSITIVE AND NEGATIVE v_0 WITH THE SAME MAGNITUDE

Velocity	$t_s(s)$	$t_r(s)$	Overshoot(%)	Undershoot(%)
$-8km/h$	10.53	5.99	1.16	5.51
$8km/h$	13.45	6.04	2.15	0
$-16km/h$	10.21	6.27	1.40	1.59
$16km/h$	13.45	6.04	2.15	0

is an initial kick in the opposite direction to the steady-state response. In reality, the 'kick' at the start of the trajectories refers to an opposite displacement of the controlled vehicle, compared to the desired direction. This is caused by system zero, where the controller fails to respond to the system input. This is undesired and difficult to control due to the system's nature. Hence, it's better to start at a positive speed for the lane change. Otherwise, passengers inside the vehicle would enjoy less comfort, and potentially, it might even impose a threat to passenger safety in some extreme cases.

D. Discussion and Conclusions

In this report, cruise control and lane changing using different controllers are discussed and validated. For the first part, a proportional controller is used for the cruise control. A key feature of this controller is that it will send a correction back to the system according to the error term at the current time. The higher the gain K_p, the stronger correction it will provide to the system. However, compromises are taken from vehicle performance to ensure passenger comfort. For the first scenario, the steady-state error is insignificant. Until a disturbance is added to the system, this proportional controller is no longer accurate as it used to be (3.5% of steady-state error). The disturbance chosen in this case is only the $\sin$ component of the vehicle weight for simplicity. However, in reality, there will be more disturbance that will affect the controller performance. And it will be more reliable if more disturbance are tested.

Moving on to the second part, lane changing, another controller is required to achieve this function. This process requires high accuracy, especially at high speed. Based on previous understanding of proportional controllers from the first part, it's no longer an ideal choice and hence a PD controller is chosen for this course. Compared to the P controller, there is an additional differential gain to the system. This means the rate of change of the error term will also help to add a correction to the system. How strong it is depends on the gain K_d. The value of these two gains is derived from the design specifications: $t_r = 5s$ and $\zeta = 0.75$. These are determined from both human comfort and desired accuracy. After validation, it shows that the chosen PD controller works well to achieve the desired requirement, and the error terms in all tests are negligible. Stability and accurate response are accredited to PD controllers. However, it adds extra complexity to the system as there is one more gain K_d that needs to be tuned properly.

During lane changing, if the initial velocity is in a negative sign, there is an inevitable displacement opposite to its desired lane-changing direction. This is because if velocity is a negative value, the coefficient of $\dot{\delta f}$ is negative. This negative correction will send back to current system, which causes the 'initial kick' in the opposite direction, shown in each trajectory. Even the settling time and overshoot are reduced in this case, it's still undesired due to the passenger comfort and safety aspects.

To summarize, this report provides two application of controllers and addresses their validity and accuracy. A proportional differential controller can provide more responsive correction than just using proportional controller. Moving on to more complex system, a integral component may be added to the controller system to ensure accuracy.

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