Crop Insurance
Pricing using
Probability
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Crop Insurance Pricing using Probability Distributions and Copula Models

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- From 2007 to 2016, the federal crop insurance title had the second-largest outlays in the farm bill after nutrition. The total net cost of the program for crop years 2007-2016 was about $72 billion. (Rosa, Isabel)

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Why is Crop Insurance Important?

- Financially protects farmers from loss of crop and revenue
- Growing population: more people = more demand for food
- All consumers benefit from a secure agriculture industry

Causes of Loss

- Weather: rain, temperature, length of growing season
- Bacteria, viruses, pests
- Implies that insurance amounts may differ between region, and commodity type

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What is an Indemnity?

- A safeguard against loss
- In terms of US crop insurance: a payment made when crop yields underperform
- USDA RMA: authorizes 15 private companies to provide insurance

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The Goal

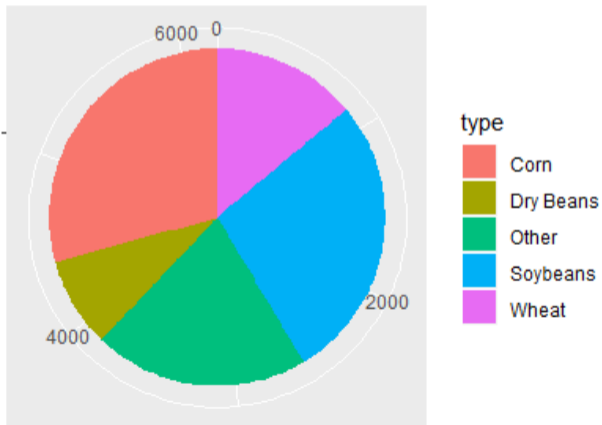- To predict indemnity amounts for specific farms based on region, and commodity grown
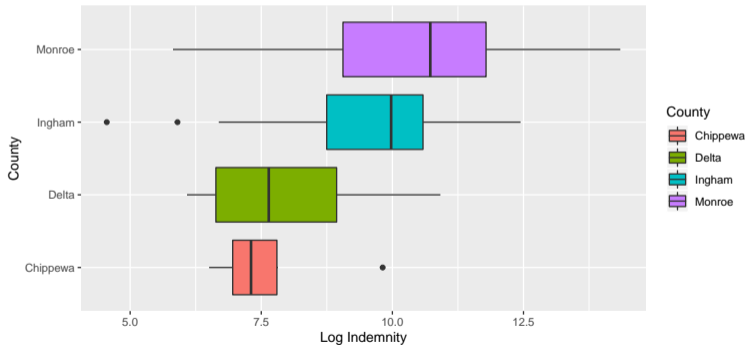
Pie Chart for Commodity

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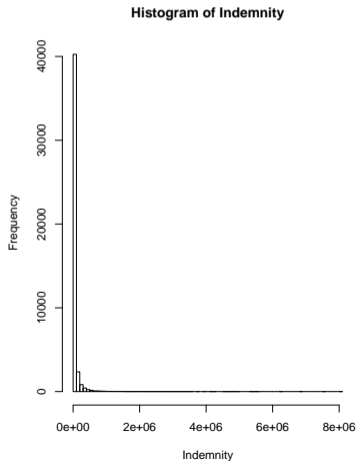
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Log indemnities by county

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Histogram of Indemnity

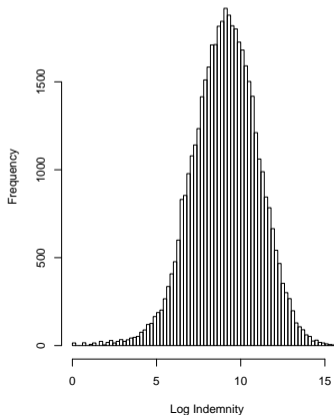**Histogram of Indemnity**

Histogram of Log Indemnity

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What do we want to do?

Determine the premium

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Gamma distribution

The density for the Gamma distribution is:

$$f(x, \theta, \alpha) = \frac{(\frac{x}{\theta})^{\alpha}}{x\Gamma(\alpha)} e^{(-\frac{x}{\theta})}$$

Statistic indicators

$$E(x) = \alpha\theta \quad Var(x) = \alpha(\theta)^2$$

Pareto Distribution

The density for Pareto Distribution:

$$f(x, \theta, \alpha) = \frac{\alpha \theta^{\alpha}}{(x + \theta)^{\alpha+1}}$$

Statistic indicators

$$E(x) = \frac{\theta}{\alpha - 1} \quad Var(X) = \frac{\alpha \theta^2}{(\alpha - 1)^2 (\alpha - 2)}$$

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Weibull Distribution

The density for Weibull Distribution:

$$f(x, \theta, \alpha) = (\frac{\alpha}{\theta})(\frac{x}{\alpha})^{\alpha-1} e^{-(\frac{x}{\theta})^\alpha}$$

Statistic indicators

$$E(x) = \theta\Gamma(1 + \frac{1}{\alpha}) \quad Var(X) = \theta^2(\Gamma(1 + \frac{2}{\alpha}) - [\Gamma(1 + \frac{1}{\alpha})]^2)$$

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Sklar's theorem

Sklar's theorem states that an m-dimensional copula is a function C from the unit m-cube $[0,1]^m$ to the unit interval $[0,1]$ which satisfies the following conditions:

(1) $C(1,...,1,a_n,1,..,1) = a_n$ for every $n \leq m$ and all a_n in $[0,1]$;

(2) $C(a_1,...,a_m) = 0$ if $a_n = 0$ for any $n \leq m$;

(3) C is m-increasing

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Dependence parameter

Copulas allow researchers to study the dependence between two separate but related issues.

$$\boldsymbol{F}(\boldsymbol{y}_1,...,\boldsymbol{y}_m) = C(\boldsymbol{F}_1(y_1),...,\boldsymbol{F}_m(y_m); \boldsymbol{\theta})$$

where θ is a parameter which measures dependence between the marginals.

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Product copula

The simplest copula, the product copula, has the form

$$C(\boldsymbol{u}_1, \boldsymbol{u}_2) = \boldsymbol{u}_1 \boldsymbol{u}_2,$$

where $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ take values in the unit interval of the real line.

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Farlie-Gumbel-Morgenstern copula

The FGM copula takes the form

$$C(u_1, u_2; \theta) = u_1 u_2 (1 + \theta(1 - u_1)(1 - u_2))$$

The FGM copula is a perturbation of the product copula, if the dependence parameter θ equals zero, then the FGM collapses to independence.

Parameter matrix

Assume the response vector is y. Then we imagine there is an $d \times d$ dimensional parameter matrix Σ, such that

$$\sigma_{i,j} = \begin{cases} 1 & \text{if } i = j \\ \rho & \text{if } s_i = s_j \\ 0 & \text{otherwise} \end{cases}$$

where s_i is the state code for the ith county, and s_j is the state code for the jth county.

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Gaussian copula

The multivariate Gaussian copula of dimension d is defined by

$$C(\boldsymbol{u}) = \Phi_d(\Phi^{-1}(u_1), \ldots, \Phi^{-1}(u_d), \Sigma)$$

where Φ^{-1} is the inverse cumulative distribution function of the standard normal, and Φ_d is the joint cumulative distribution function of a multivariate normal distribution with mean vector zero, and covariance matrix Σ

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Thank you!