1. True/False (30 pts.)

Given two functions **f** and **g** such that **f(n)** is **O(g(n)),** state for each statements whether it is true or false. For full credit, explain your answer in details preferable with an example. A simple True or False without full explanation WILL NOT get credit.

(A) $log_2 f(n)$ is $O(log_2 g(n))$

(B) $2^{f(n)}$ is $O(g(n)^2)$

(C) $f(n)^2$ is $O(g(n)^2)$

2. Ascending Asymptotic Order (40 pts.)

Arrange the following functions in the order of slowest growing function to fastest growing function. For full credit, explain your answer in details preferable with an example.

$$\sqrt{n}, \; n\sqrt{lg(n)}, \; 2^{\sqrt{lg(n)}}, \; lg(n)^2$$

3. 12 days of Christmas (30 pts.)

There are a class of holiday songs such each each verse of the song consists of all the previous verses with one extra line added. One such song is 12 days of Christmas as follows:

On the first day of Christmas
My true lord gave to me
A patridge in a pear tree

On the first day of Christmas
My true lord gave to me
Two turtle doves
A patridge in a pear tree

On the first day of Christmas
My true lord gave to me
Three French Hens
Two turtle doves
A patridge in a pear tree

On the first day of Christmas
My true lord gave to me
Four calling birds
Three French Hens
Two turtle doves
A patridge in a pear tree

On the first day of Christmas
My true lord gave to me
Five gold rings
Four calling birds
Three French Hens
Two turtle doves
A patridge in a pear tree

....

There is something asymptotic in nature that can be analyzed through such songs. Suppose, for concreteness, each line has a length that is bounded by a constant **k** and suppose that the song when sung out loud runs for a total of **n** words.

Show how to encode such a song using a script that has a length **f(n)** for a function **f(n)** that grows as slowly as possible. For full credit, illustrate with a pseudo code and derive the asymptotic complexity of the algorithm.