

## 1. Most Profitable Restaurant (50 pts.)

Jill is considering opening a series of new fast food restaurants - Jill's Cafe along highway 101 between San Jose and San Francisco. All the  $n$  possible restaurants are along a straight line. The distance of these cafes from the start of highway in San Jose (Diridon Station) are in miles and in increasing order:  $d_1, d_2, d_3, \dots, d_n$ . Jill has following constraints:

- (A) At each location, Jill may open at most 1 restaurant. The expected profit from opening a restaurant at location  $i$  is  $p_i$  where  $p_i > 0$  and  $i = 1, 2, 3, \dots, n$ .
- (B) Any two restaurants should be at least  $k$  miles apart, where  $k$  is a positive integer.

Given these constraints, design an **efficient algorithm** to compute the maximum expected total profit. Also provide the time and space complexity of your algorithm.

## 2. Best Party (50 pts.)

It's Natalie's birthday and she decides to host the best party ever. She puts together a list of her  $n$  friends. She also knows which of her friends knows whom and she decides to use this info to design an awesome social party. Natalie decides it would be best to invite the maximum number of friends possible with following constraints. At the party, each person **should know at least 4 people** and they **should also NOT know at least 4 people**.

Design an **efficient algorithm** that takes the list of  $n$  people and the list of pairs who know each other and outputs the best choice of party invitees. Provide the time and space complexity of your algorithm.