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1. (A) Yes. $f(n) = n$ $\log_2 f(n) = \log_2 n$ $\log_2 g(n) = \log_2 (cn)$

$= \log_2 c + \log_2 n$

$\log_2 c + \log_2 n$ always greater than $\log_2 n$ for any n, because c is always greater than 1 and $\log_2 c$ always greater than 0

$\therefore$ $\log_2 f(n)$ is $O(\log_2 g(n))$

(B) No. example: $f(n) = n$, $g(n) = 2n$, $2^{f(n)} = 2^n$ $O(g(n)^2) = O(4n^2)$

when $n = 9$, $2^{f(n)} = 2^9 = 512$, $4n^2 = 4 \times 81 = 324$, $2^{f(n)} \geq g(n)^2$ when $n \geq 9$

this is an counterexample

(C) Yes, for example, $f(n) = n$. $f(n)^2 = n^2$ $g(n) = cn$ and $c > 1$

$g(n)^2 = c^2 n^2$ $c^2 > c$, $\boxed{\text{for any } n,\ c^2 n^2 > n^2}$, so $f(n)^2$ is $O(g(n)^2)$

2. solution 1:

$$\lim_{n\to\infty} \frac{n\sqrt{\lg(n)}}{\sqrt{n}} = \lim_{n\to\infty} \sqrt{\frac{n^2 \cdot \lg(n)}{n}} = \lim_{n\to\infty} \sqrt{n \cdot \lg(n)} = \infty$$

$\therefore$ $n\sqrt{\lg(n)}$ grows faster than $\sqrt{n}$

$$\lim_{n\to\infty} \frac{\sqrt{n}}{(\lg(n))^2} = \lim_{n\to\infty} \frac{\frac{1}{2} \cdot n^{-\frac{1}{2}}}{2 \cdot (\lg(n)) \cdot \frac{1}{n \cdot \ln(10)}} = \lim_{n\to\infty} \frac{n \cdot \ln(10) \cdot n^{-\frac{1}{2}}}{4\,(\lg(n))} = \lim_{n\to\infty} \frac{\sqrt{n}}{\lg(n)} = \infty$$

$\therefore$ $\sqrt{n}$ grows faster than $\lg(n)^2$

$2^{\sqrt{\lg(n)}}$ is an exponential expression, it grows the fastest among the 4 expressions

$\therefore$ the final order is $2^{\sqrt{\lg(n)}} > n\sqrt{\lg(n)} > \sqrt{n} > \lg(n)^2$

Solution 2: let $\lg n = a \Rightarrow n = a^{10}$

$2^{\sqrt{\lg(n)}} = 2^a$

$n \cdot \sqrt{\lg n} = a^{10} \cdot a^{0.5} = a^{10.5}$

$\sqrt{n} = a^5$

$\therefore$ growth rate: $2^a > a^{10.5} > a^5 > a^2$

$2^{\sqrt{\lg(n)}} > n\sqrt{\lg(n)} > \sqrt{n} > (\lg(n))^2$

3. Time complexity

$f(n) = k$

```
sing-song(k):
   index = 1
   song = empty line
 while index <= k:
    if index < 4:
      concatenate a new line to song
      print song
      print a new line
    else:
       insert a new line to the third line in song
       print song
       print a new line
    index++
```