Week 2: Boolean algebra (cont.)

Boolean algebra has many of the same rules as normal high school level algebra, and indeed the symbols used (+ for OR, * for AND, etc) reflect this. Just like algebra, there are Rules you must follow.

Note that the Rules are similar to Set Theory rules.

- In Set Theory, where
  OR ⇔ ∪ (set union)
  AND ⇔ ∩ (set intersection)
  NOT ⇔ ' (set complement)
- 1 is the Universal Set and 0 (zero) is the Empty Set and also denoted sometimes as { }

Boolean algebra rules

Commutative law	$A+B=B+A$
Associative law	$A+(B+C)=A+B+C=(A+B)+C$ $A.(B.C)=A.B.C=(A.B).C$
Distributive law	$A.(B+C)=A.B+A.C$
de Morgan's theorems	$\overline{A+B}=\overline{A}.\overline{B}$ $\overline{A.B}=\overline{A}+\overline{B}$
Identities	$1 + any = 1,\quad 0 + A = A$ $1 . A = A,\quad 0 . any = 0$ $A . A = A,\quad A + A = A$ $A + \overline{A} = 1,\quad A . \overline{A} = 0$ $A.B + \overline{A}.B = B$ $A + B.C = (A + B).(A + C)$

De Morgan's theorem

- One way in which Boolean algebra is different to other forms is the following
  – Most commonly used to change a term involving ORs into one involving ANDs, and vice versa
  – It works due to the symmetry of the OR

  and AND truth tables

$$\boxed{\begin{aligned}\overline{A+B}&=\overline{A}.\overline{B}\\ \overline{A.B}&=\overline{A}+\overline{B}\end{aligned}}$$

- One is basically the upside-down inverse of the other
  – You can get upside-down by complementing the operands A and B
  – and then simply map AND to OR

A	B	$\overline{A}$	$\overline{B}$	A.B	A+B
0	0	1	1	0	0
0	1	1	0	0	1
1	0	0	1	0	1
1	1	0	0	1	1

- de Morgan's Laws are also applied to Set Theory, where
  OR ⇔ ∪ (set union)
  AND ⇔ ∩ (set intersection)
  NOT ⇔ ' (set complement)