Time series analysis on the stock price of Tesla Inc.

Wenhao Pan (3034946058), Ruojia Zhang, Mengzhu Sun, Xiangxi Wang, Mingmao Sun

November 13, 2021

Contents

1 Abstract

2 Introduction

3 Data Description

4 Exploratory Data Analysis

To obtain a comprehensive understanding of the data, we conduct explanatory data analysis (EDA) first. Figure 1(a) is the time series plot of all the given time points. We observe that the stock prices of Tesla before 2020 are averagely and considerably lower than those after 2020. The significantly different scales of different parts of the time series make it hard to visually examine the trend and seasonality pattern of the time series. Moreover, since we are majorly interested in the recent activities of Tesla, we do not have to analyze all the available data. Therefore, for the sake of interest and convenience, we decide only to analyze the last 300 time points, which cover the period from `2020-08-26` to `2021-11-02` excluding weekends. Thus, whenever we use the word "data" in the following analysis, we implicitly mean the time series of the last three hundred time points.

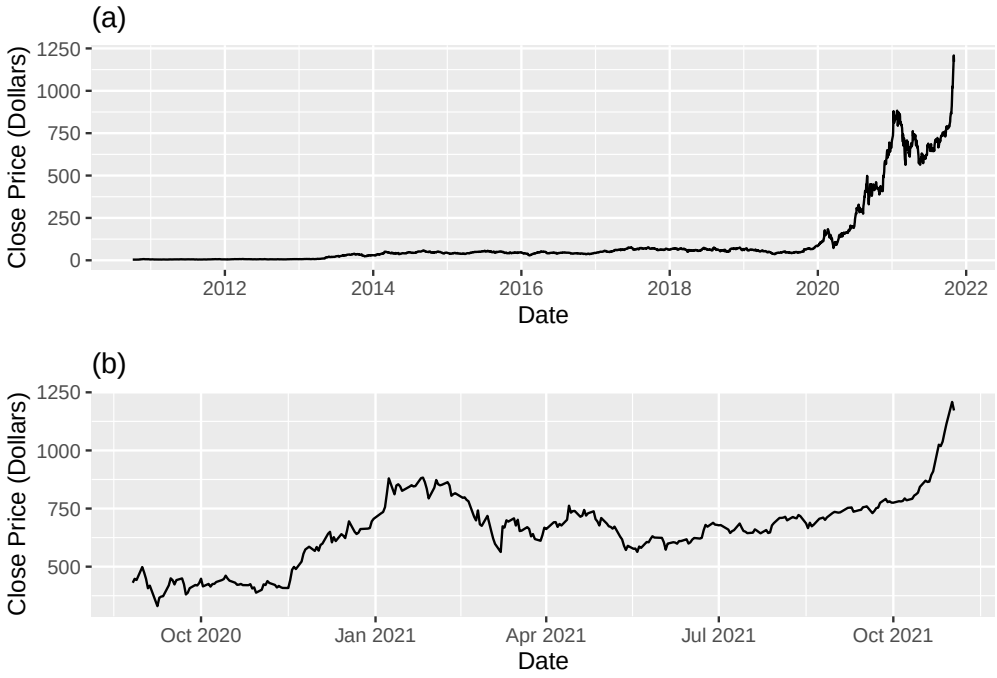


Figure 1: (a) Time series plot of all available trading days. (b) Time series plot of last 300 trading days

Figure 1(b) is the time series plot of the close prices of Tesla in the last three hundred trading days before and including `2021-11-02`. We first observe that our data is roughly homoscedastic based on Figure 1(b). To verify our observation, we try the square root and natural log transformations and see whether they effectively stabilize the variance of the time series. Their plots are below in Figure 2.

We can see that both transformations unnecessarily increase the variance of the time series before mid-November in 2020 and do not change the variance of other time series data. Although both transformations shorten the vertical distance between the maximum and minimum of the time series after Oct. 2021, the spike after Oct. 2021 is more like an increasing trend than a considerable fluctuation. In short, both transformations are redundant, and we do not need to use any variance stabilizing transformation.

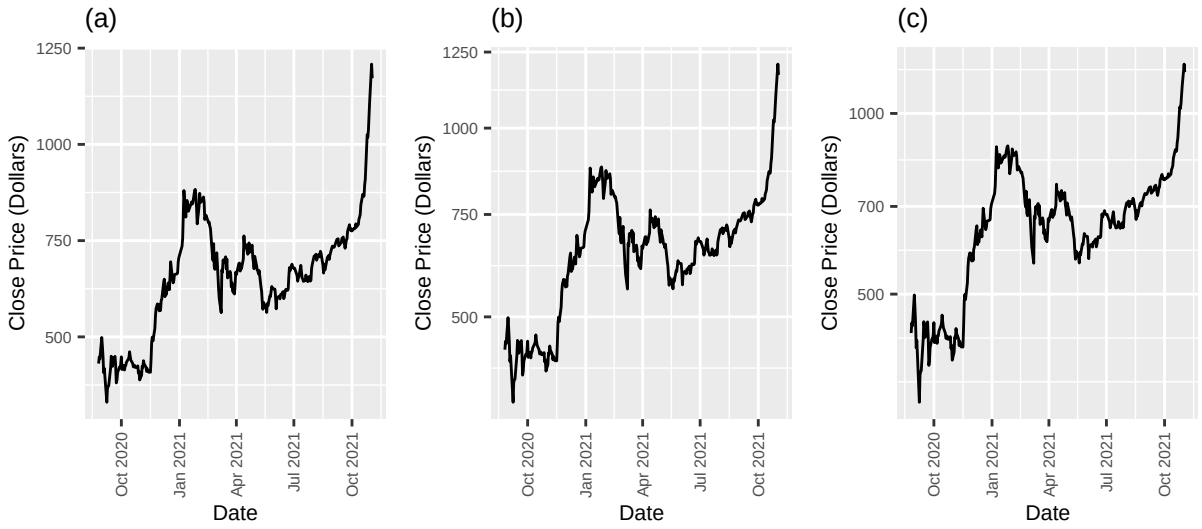


Figure 2: (a): Original time series. (b): Square root transfromed time seires. (c): Natural log transformed time series.

Back to Figure 1(b), intuitively, the data is not stationary because of a nonlinear and generally increasing trend. The trend first increases until around Feb. 2021 and then decreases until around Mid-May. 2021. Finally, the trend increases again until the end of the time series. Nonetheless, we do not observe an obvious or significant seasonality pattern. It matches the intuition since the granularity of our data is day, and the structure of stock price data is too complicated to have a seasonality pattern.

In conclusion, based on all the previous discussions in EDA, we decide to construct possible models on the original time series data, including only the last three hundred time points.

5 Model Construction

With a comprehensive understanding of our data, we start to experiment and construct different time series model. We choose and build two non-parametric signal models of the trend and seasonality in our data. We aim to make the residuals approximately weekly stationary. We do not consider any parametric trend model because we think the trend of the stock price data is too complicated to be modeled by a parametric model, such as a high-order polynomial. Certainly, we could use a 15 or 20 order polynomial, but it may overfit the training data and produce imprecise predictions. We do not consider a parametric seasonality model either because we do not find a clear seasonality pattern in our data by the EDA. Finally, based on each signal model, we provide two ARMA models or its extension, such as SARMA or ARIMA, to whiten the residuals of the signal model. Thus, we have four candidate models, and we will explain how we select a final model among them in the next section.

5.1 Non-parametric Signal Model: exponential smoothing

In this signal model, we choose exponential smoothing with weight $\alpha = 0.8$ and lag $k = 10$ and a seasonal differencing with period $d = 5$.

We experiment with different combinations of α and k with a careful consideration of overfitting issue. we choose $k = 10$ as the final value because we want to only use past two weeks, which are ten days in our data, to forecast. We choose $\alpha = 0.8$ as the final value because we think it best balances the smoothing effect and the capture of trend pattern among $(0, 1)$. Indeed, the smoothing line in Figure 4(a) fits the data in the way

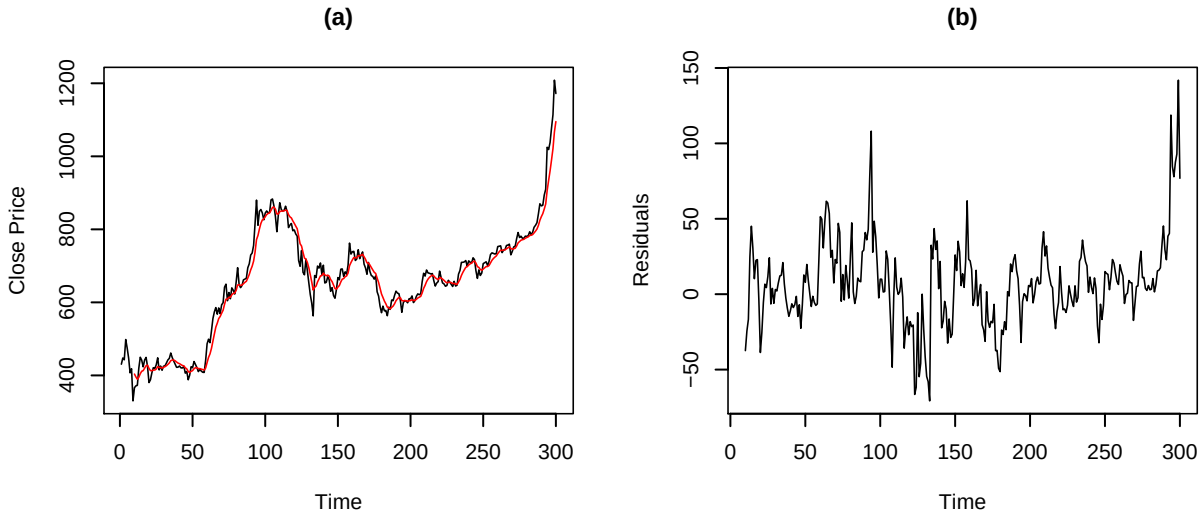


Figure 3: (a): Time series plot of the original data and fitted values. (b): The residual plot of exponential smoothing.

that we want. Note that we lose the first nine time points due to the computation process of the exponential smoothing.

However, the residual plot Figure 4(b) is fairly non-stationary, as it has cycling fluctuation pattern and still slightly nonlinear trend. It might be due to that we intentionally let exponential smoothing not fit the data perfectly. Next, We use the seasonal differencing with period $d = 5$, which is one week in our data, to further make the residuals more stationary.

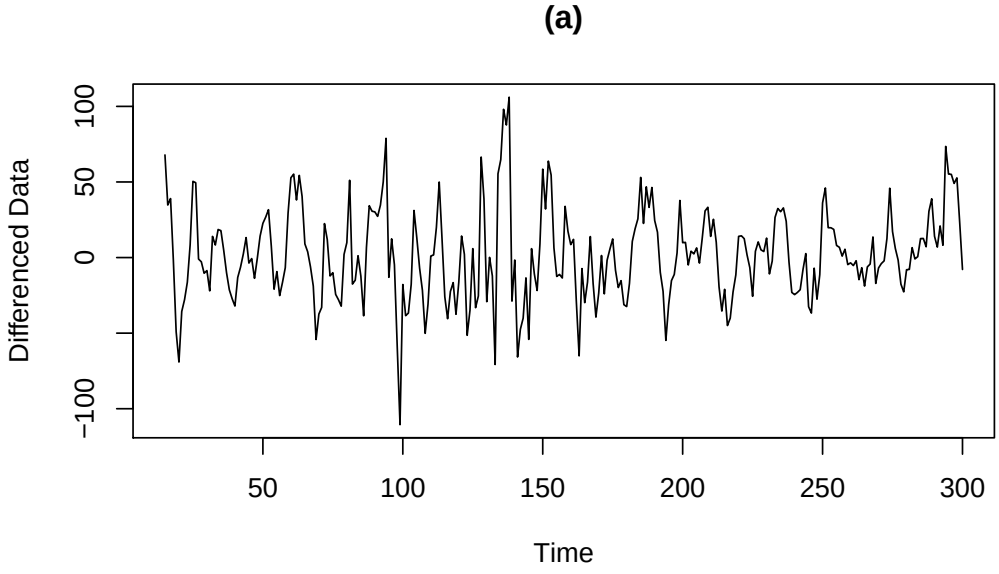


Figure 4: (a): Time series plot of the seasonal differenced (d = 5) residuals from the previous smoothing.

Indeed, now the differenced residuals become more stationary. There seems to be a contradiction that recalling in EDA, we claim that there is not a clear seasonality in our data. However, the effect of the seasonal differencing here implies a possible seasonality with period $d = 5$. We think it is because the seasonal differencing is actually removing the remaining trend left by the exponential smoothing instead of the seasonality. Nevertheless, We believe that the time series of the differenced residuals shown in Figure 5(a) is stationary enough for us to build ARMA models on it.

5.2 Non-parametric Signal Model: second-order differencing

In this model, we choose the second-order differencing to remove the trend. We observe that after the first-order differencing, there is still some trend pattern, such as the increasing one between 270 and 300, as shown by Figure TODO. This matches our previous analysis that the trend of our data is nonlinear in EDA. Thus, we take another differencing and acquire the second-order differencing data shown in Figure TODO.

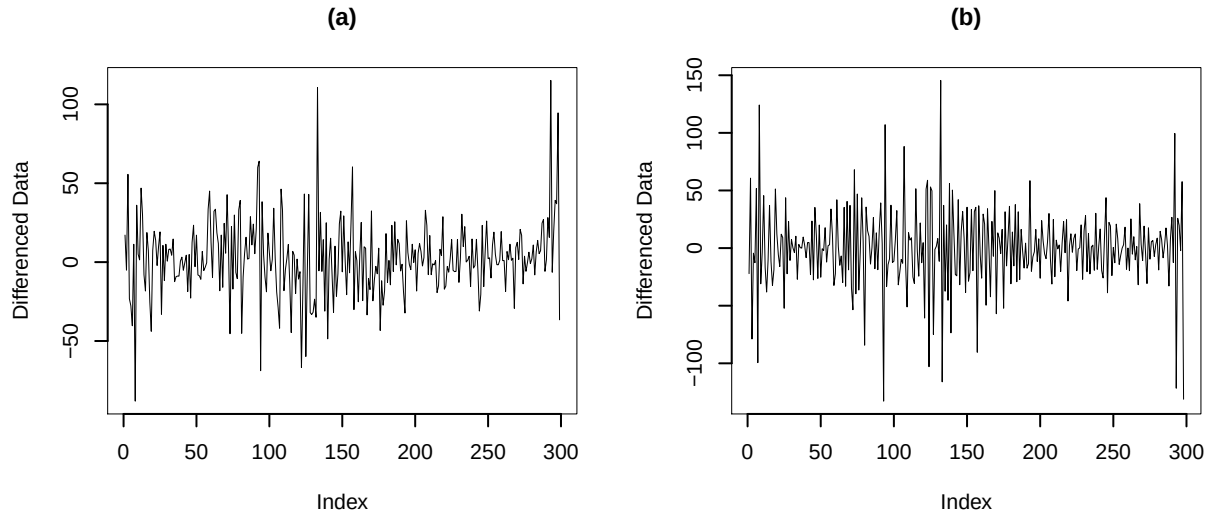


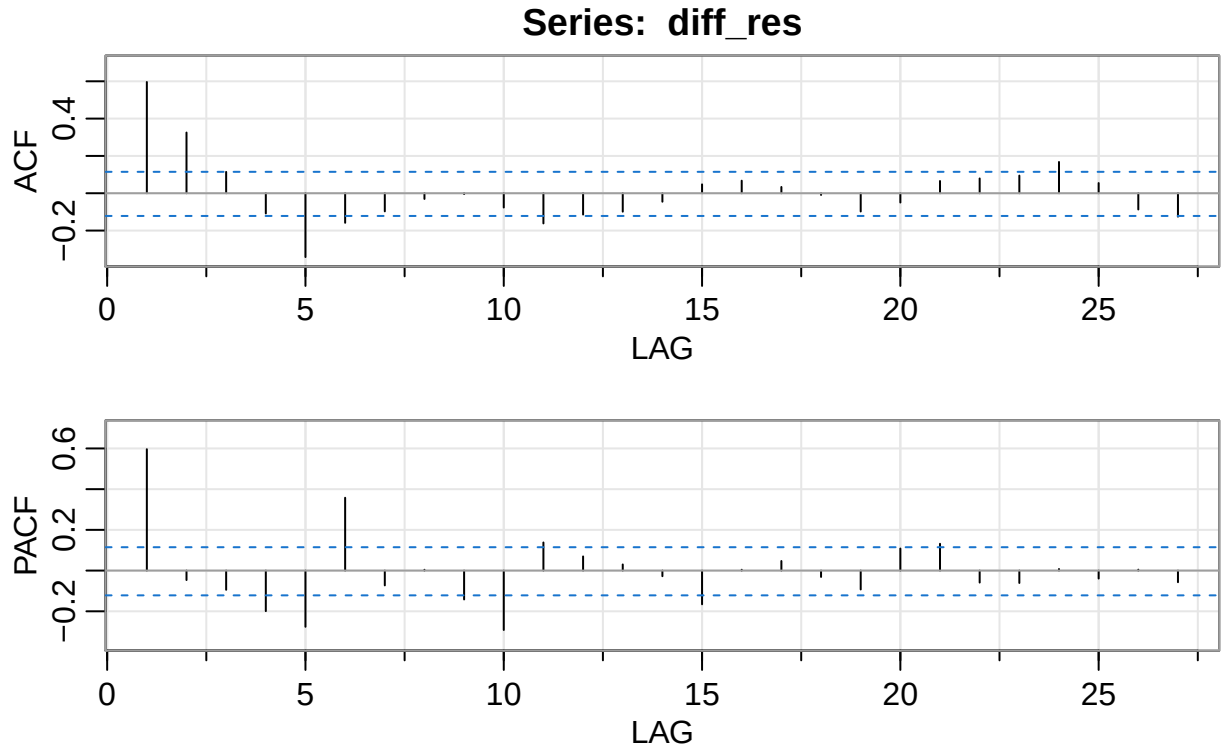
Figure 5: (a): The first-order differenced data. (b): The second-order differenced data.

The second-order differenced time series is more stationary than the first-order differenced time series. We can keep trying more higher-order differencings, but they may overfit our data. Therefore, we think the second-order differenced time series is already stationary enough for us to build ARMA model on it.

6 Model Comparision and Selection

6.1 [Exponential Smoothing + Seasonal Differencing] Model 2 (Ruojia Zhang)

Here we fit seasonal ARMA model of the seasonal differenced residuals after reducing the trend using exponential smoothing.




```
##      [,1]  [,2]  [,3]  [,4]  [,5]  [,6]  [,7]  [,8]  [,9] [,10] [,11] [,12]
## ACF   0.6  0.33  0.11 -0.11 -0.34 -0.16 -0.10 -0.03  0.00 -0.08 -0.16 -0.11
## PACF  0.6 -0.05 -0.09 -0.20 -0.28  0.36 -0.07  0.00 -0.14 -0.29  0.14  0.07
##      [,13] [,14] [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22] [,23] [,24]
## ACF  -0.10 -0.05  0.05  0.07  0.03 -0.01 -0.10 -0.05  0.07  0.08  0.09  0.17
## PACF  0.03 -0.03 -0.17  0.00  0.05 -0.03 -0.09  0.11  0.13 -0.06 -0.06  0.01
##      [,25] [,26] [,27]
## ACF   0.05 -0.09 -0.13
## PACF -0.04  0.00 -0.06
```

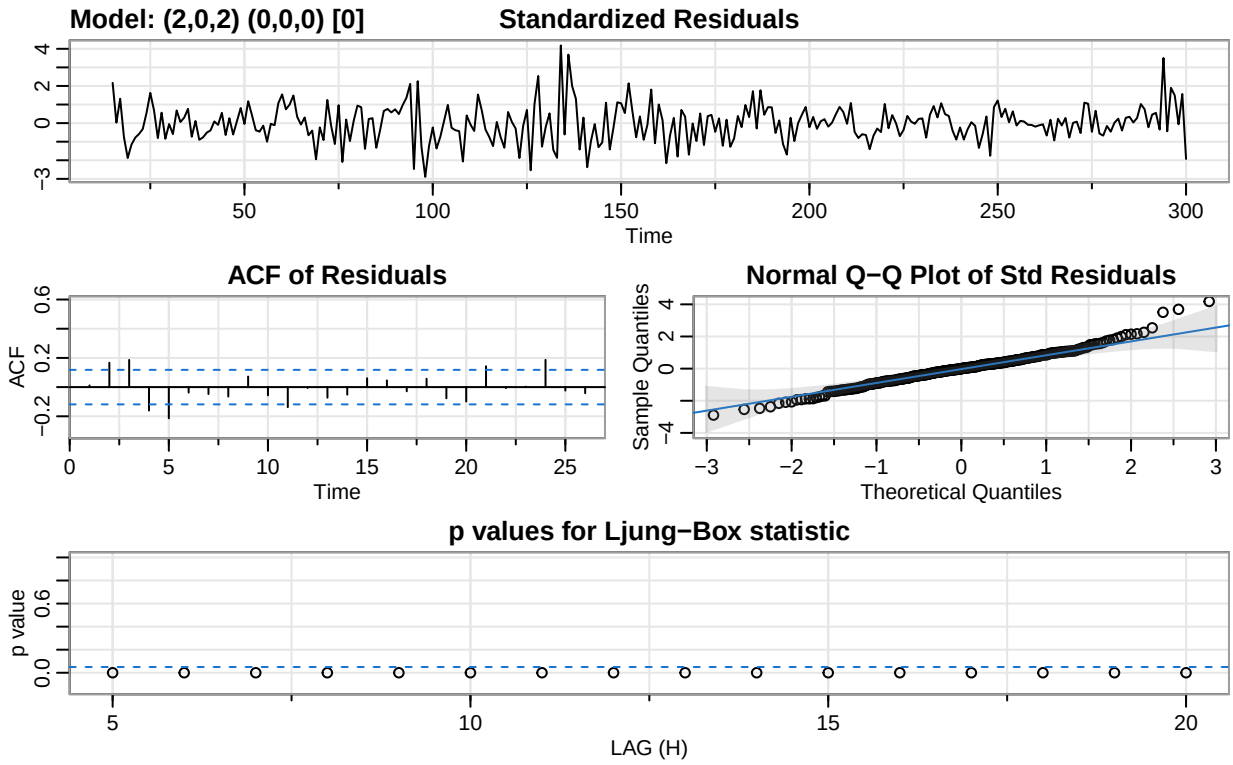
From the acf and pacf plots of the differenced residuals above we can observe negative spike at lag = 5 for both plots. Neither the acf or the pacf plot shows reasonable cutoff. So here we need to choose a model where p > 0 and q > 0.

First, using auto.arima() to see the model suggested by R:

```
## Series: diff_res
## ARIMA(2,0,2) with non-zero mean
##
## Coefficients:
##           ar1      ar2     ma1     ma2    mean
##       -0.7582  -0.0918  1.5843  0.9232  1.7195
## s.e.   0.0653   0.0714  0.0272  0.0304  2.5360
##
## sigma^2 estimated as 522.5:  log likelihood=-1299.87
## AIC=2611.75   AICc=2612.05   BIC=2633.68


## initial  value 3.407572
## iter   2 value 3.403434
```

```
## iter   3 value 3.346164
## iter   4 value 3.258636
## iter   5 value 3.191177
## iter   6 value 3.183256
## iter   7 value 3.183046
## iter   8 value 3.182992
## iter   9 value 3.182857
## iter  10 value 3.182711
## iter  11 value 3.181575
## iter  12 value 3.181352
## iter  13 value 3.179559
## iter  14 value 3.178037
## iter  15 value 3.172427
## iter  16 value 3.171845
## iter  17 value 3.165885
## iter  18 value 3.164774
## iter  19 value 3.163647
## iter  20 value 3.163294
## iter  21 value 3.162700
## iter  22 value 3.160894
## iter  23 value 3.154028
## iter  24 value 3.153765
## iter  25 value 3.151051
## iter  26 value 3.150299
## iter  27 value 3.149226
## iter  28 value 3.149077
## iter  29 value 3.148956
## iter  30 value 3.148868
## iter  31 value 3.148832
## iter  32 value 3.148827
## iter  33 value 3.148827
## iter  33 value 3.148827
## iter  33 value 3.148827
## final  value 3.148827
## converged
## initial  value 3.140662
## iter   2 value 3.139733
## iter   3 value 3.132547
## iter   4 value 3.129600
## iter   5 value 3.128885
## iter   6 value 3.127033
## iter   7 value 3.126798
## iter   8 value 3.126665
## iter   9 value 3.126656
## iter  10 value 3.126635
## iter  11 value 3.126344
## iter  12 value 3.126119
## iter  13 value 3.126091
## iter  14 value 3.126077
## iter  15 value 3.126074
## iter  16 value 3.126073
## iter  17 value 3.126073
## iter  17 value 3.126073
## iter  17 value 3.126073
```

```
## final  value 3.126073
## converged
```




```
## [1] 9.131981
```

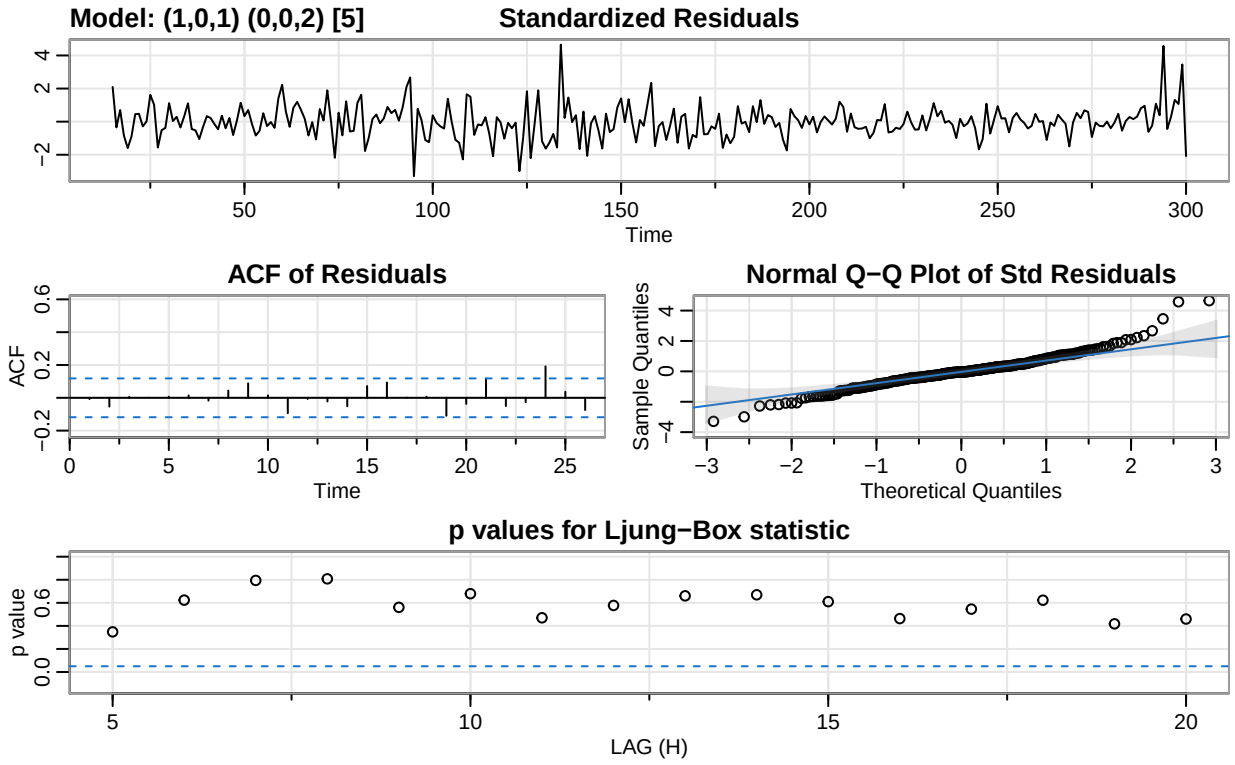
```
## [1] 9.13273
```

```
## [1] 9.20868
```

From the performance of the model recommended by R presented above, we can see that the model does not pass the Ljung-Box test (all the p-values are below the blue band). Combined with my observations from the acf and pacf plots, I adjusted the model parameters and tried the lower the AIC, AICc, and BIC values of the model at the same time. In the end, keeping other parameters unchanged, I reduced the values for p and q from 2 to 1, and chose S = 5 and Q = 2, and the model presented p-values lying far beyond the blue band in the Ljung-Box statistic plot with relatively lower AIC, AICc, and BIC values than before:

```
## initial  value 3.407921
## iter   2 value 3.268990
## iter   3 value 2.974443
## iter   4 value 2.966190
## iter   5 value 2.957344
## iter   6 value 2.954149
## iter   7 value 2.948214
## iter   8 value 2.946849
## iter   9 value 2.946355
## iter  10 value 2.946259
## iter  11 value 2.946247
```

```
## iter  12 value 2.946246
## iter  12 value 2.946246
## final  value 2.946246
## converged
## initial  value 2.938108
## iter   2 value 2.934170
## iter   3 value 2.929293
## iter   4 value 2.927175
## iter   5 value 2.926083
## iter   6 value 2.925194
## iter   7 value 2.923708
## iter   8 value 2.922816
## iter   9 value 2.922277
## iter  10 value 2.922094
## iter  11 value 2.922050
## iter  12 value 2.922045
## iter  13 value 2.922045
## iter  14 value 2.922045
## iter  15 value 2.922045
## iter  16 value 2.922045
## iter  16 value 2.922045
## iter  16 value 2.922045
## final  value 2.922045
## converged
```



From the acf plot of the residuals, we can see that most of the sample residuals autocorrelations fit nicely inside the confidence band. From the normal q-q plot of the stanard residuals we can see that normal distribution is a convincing assumption. We also have large p-values that suggests weak groupwise residual autocorrelations in the plot for Ljung-Box statistic. The model seems like a good fit.

Then we report the AIC, AICc, BIC values for the model:

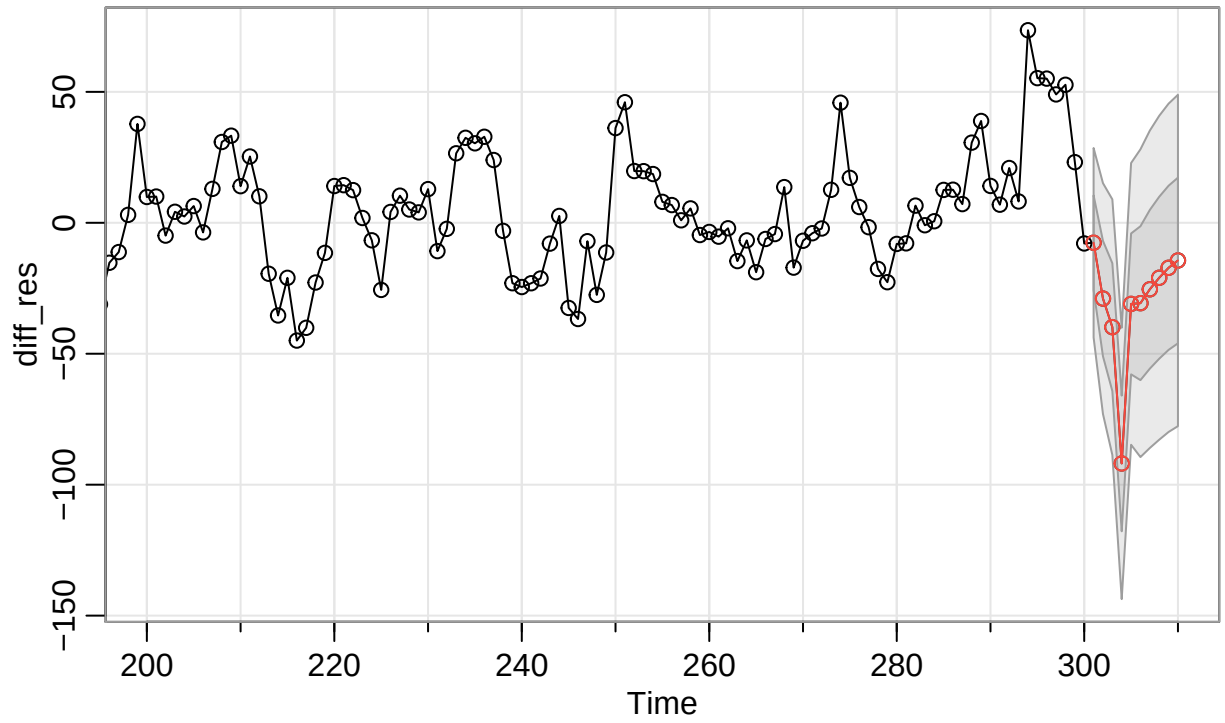
```
## [1] 8.723925
```

```
## [1] 8.724674
```

```
## [1] 8.800624
```

We then perform time-series cross validation. For this model, we generate forecasts for every 10 days (2 weeks) and compute the sum of squares of errors of the forecasts. Then average the sum of squares of errors of forecasts over the days considered, which generates the mse of this model.

```
##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 250       679.7153 649.2001 710.2304 633.0464 726.3842
## 251       679.7153 638.6613 720.7692 616.9287 742.5019
## 252       679.7153 630.3222 729.1084 604.1751 755.2555
## 253       679.7153 623.2004 736.2301 593.2833 766.1473
## 254       679.7153 616.8807 742.5498 583.6181 775.8124
## 255       679.7153 611.1410 748.2896 574.8400 784.5906
## 256       679.7153 605.8459 753.5847 566.7418 792.6887
## 257       679.7153 600.9058 758.5248 559.1866 800.2440
## 258       679.7153 596.2576 763.1730 552.0777 807.3528
## 259       679.7153 591.8549 767.5756 545.3445 814.0861
```




```
##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 251       703.6415 673.1112 734.1718 656.9494 750.3336
## 252       703.6415 662.5672 744.7159 640.8237 766.4593
## 253       703.6415 654.2239 753.0592 628.0638 779.2193
## 254       703.6415 647.0986 760.1845 617.1665 790.1165
```

```
## 255       703.6415 640.7757 766.5073 607.4966 799.7864
## 256       703.6415 635.0332 772.2499 598.7141 808.5690
## 257       703.6415 629.7354 777.5476 590.6119 816.6711
## 258       703.6415 624.7929 782.4902 583.0529 824.2301
## 259       703.6415 620.1423 787.1407 575.9406 831.3425
## 260       703.6415 615.7375 791.5455 569.2039 838.0791

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 252       708.0051 677.5336 738.4766 661.4030 754.6073
## 253       708.0051 667.0099 749.0004 645.3084 770.7019
## 254       708.0051 658.6827 757.3276 632.5730 783.4373
## 255       708.0051 651.5711 764.4391 621.6968 794.3135
## 256       708.0051 645.2605 770.7498 612.0455 803.9648
## 257       708.0051 639.5290 776.4813 603.2799 812.7304
## 258       708.0051 634.2415 781.7688 595.1933 820.8170
## 259       708.0051 629.3084 786.7019 587.6489 828.3614
## 260       708.0051 624.6668 791.3434 580.5502 835.4601
## 261       708.0051 620.2705 795.7398 573.8266 842.1837

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 253       710.8805 680.4689 741.2921 664.3700 757.3910
## 254       710.8805 669.9659 751.7951 648.3070 773.4540
## 255       710.8805 661.6551 760.1060 635.5967 786.1644
## 256       710.8805 654.5575 767.2036 624.7418 797.0192
## 257       710.8805 648.2592 773.5018 615.1095 806.6516
## 258       710.8805 642.5390 779.2221 606.3611 815.3999
## 259       710.8805 637.2618 784.4992 598.2905 823.4706
## 260       710.8805 632.3385 789.4226 590.7608 831.0002
## 261       710.8805 627.7061 794.0550 583.6762 838.0849
## 262       710.8805 623.3184 798.4427 576.9657 844.7953

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 254        702.132 671.7709 732.4932 655.6987 748.5654
## 255        702.132 661.2853 742.9788 639.6623 764.6017
## 256        702.132 652.9882 751.2758 626.9731 777.2910
## 257        702.132 645.9024 758.3616 616.1363 788.1278
## 258        702.132 639.6146 764.6494 606.5199 797.7442
## 259        702.132 633.9039 770.3602 597.7860 806.4780
## 260        702.132 628.6355 775.6286 589.7288 814.5353
## 261        702.132 623.7203 780.5437 582.2117 822.0524
## 262        702.132 619.0956 785.1685 575.1387 829.1253
## 263        702.132 614.7151 789.5489 568.4394 835.8246

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 255       710.9412 680.6300 741.2523 664.5843 757.2981
## 256       710.9412 670.1617 751.7207 648.5744 773.3080
## 257       710.9412 661.8783 760.0041 635.9060 785.9764
## 258       710.9412 654.8042 767.0782 625.0870 796.7953
## 259       710.9412 648.5267 773.3556 615.4865 806.3959
## 260       710.9412 642.8254 779.0570 606.7670 815.1153
## 261       710.9412 637.5657 784.3167 598.7230 823.1593
## 262       710.9412 632.6586 789.2238 591.2183 830.6641
## 263       710.9412 628.0415 793.8409 584.1570 837.7254
## 264       710.9412 623.6683 798.2141 577.4688 844.4136
```

```
##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 256       728.9131 698.6192 759.2070 682.5825 775.2437
## 257       728.9131 688.1568 769.6694 666.5817 791.2445
## 258       728.9131 679.8781 777.9481 653.9205 803.9057
## 259       728.9131 672.8079 785.0182 643.1077 814.7185
## 260       728.9131 666.5341 791.2921 633.5126 824.3136
## 261       728.9131 660.8359 796.9902 624.7981 833.0281
## 262       728.9131 655.5793 802.2469 616.7587 841.0675
## 263       728.9131 650.6749 807.1512 609.2582 848.5680
## 264       728.9131 646.0604 811.7658 602.2009 855.6253
## 265       728.9131 641.6897 816.1365 595.5164 862.3097

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 257       735.0393 704.8001 765.2785 688.7924 781.2861
## 258       735.0393 694.3566 775.7220 672.8205 797.2581
## 259       735.0393 686.0929 783.9857 660.1822 809.8964
## 260       735.0393 679.0355 791.0431 649.3889 820.6897
## 261       735.0393 672.7730 797.3056 639.8112 830.2674
## 262       735.0393 667.0851 802.9934 631.1124 838.9662
## 263       735.0393 661.8379 808.2406 623.0875 846.9911
## 264       735.0393 656.9425 813.1361 615.6006 854.4780
## 265       735.0393 652.3363 817.7422 608.5560 861.5225
## 266       735.0393 647.9735 822.1051 601.8837 868.1949

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 258        734.185 704.0050 764.3649 688.0287 780.3412
## 259        734.185 693.5820 774.7879 672.0881 796.2818
## 260        734.185 685.3345 783.0355 659.4746 808.8953
## 261        734.185 678.2909 790.0790 648.7024 819.6675
## 262        734.185 672.0407 796.3292 639.1435 829.2265
## 263        734.185 666.3640 802.0059 630.4617 837.9082
## 264        734.185 661.1271 807.2428 622.4525 845.9174
## 265        734.185 656.2412 812.1287 614.9803 853.3896
## 266        734.185 651.6441 816.7258 607.9496 860.4204
## 267        734.185 647.2898 821.0801 601.2903 867.0796

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 259       732.5695 702.4482 762.6908 686.5030 778.6360
## 260       732.5695 692.0455 773.0935 670.5933 794.5457
## 261       732.5695 683.8140 781.3251 658.0043 807.1347
## 262       732.5695 676.7841 788.3549 647.2531 817.8859
## 263       732.5695 670.5460 794.5930 637.7127 827.4263
## 264       732.5695 664.8804 800.2587 629.0479 836.0911
## 265       732.5695 659.6536 805.4854 621.0543 844.0847
## 266       732.5695 654.7773 810.3618 613.5965 851.5425
## 267       732.5695 650.1890 814.9500 606.5795 858.5595
## 268       732.5695 645.8432 819.2958 599.9331 865.2059

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 260         733.47 703.4072 763.5327 687.4930 779.4469
## 261         733.47 693.0247 773.9152 671.6143 795.3256
## 262         733.47 684.8092 782.1307 659.0497 807.8902
## 263         733.47 677.7930 789.1469 648.3194 818.6205
```

```
## 264         733.47 671.5670 795.3729 638.7976 828.1423
## 265         733.47 665.9124 801.0275 630.1496 836.7904
## 266         733.47 660.6958 806.2441 622.1715 844.7684
## 267         733.47 655.8289 811.1110 614.7283 852.2117
## 268         733.47 651.2496 815.6903 607.7248 859.2151
## 269         733.47 646.9123 820.0276 601.0914 865.8485

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 261        750.975 720.9305 781.0195 705.0259 796.9241
## 262        750.975 710.5542 791.3957 689.1568 812.7932
## 263        750.975 702.3437 799.6063 676.5998 825.3501
## 264        750.975 695.3318 806.6182 665.8760 836.0739
## 265        750.975 689.1095 812.8404 656.3600 845.5900
## 266        750.975 683.4583 818.4916 647.7172 854.2328
## 267        750.975 678.2449 823.7050 639.7439 862.2060
## 268        750.975 673.3810 828.5690 632.3052 869.6447
## 269        750.975 668.8045 833.1455 625.3060 876.6439
## 270        750.975 664.4697 837.4802 618.6766 883.2733

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 262       753.5805 723.5932 783.5678 707.7188 799.4422
## 263       753.5805 713.2367 793.9243 691.8799 815.2811
## 264       753.5805 705.0417 802.1192 679.3469 827.8141
## 265       753.5805 698.0432 809.1178 668.6435 838.5175
## 266       753.5805 691.8328 815.3282 659.1455 848.0155
## 267       753.5805 686.1923 820.9687 650.5192 856.6418
## 268       753.5805 680.9888 826.1722 642.5611 864.5999
## 269       753.5805 676.1342 831.0268 635.1365 872.0244
## 270       753.5805 671.5663 835.5946 628.1507 879.0103
## 271       753.5805 667.2399 839.9211 621.5339 885.6271

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 263        754.732 724.8022 784.6618 708.9584 800.5057
## 264        754.732 714.4656 794.9985 693.1499 816.3142
## 265        754.732 706.2864 803.1776 680.6409 828.8232
## 266        754.732 699.3013 810.1628 669.9580 839.5061
## 267        754.732 693.1028 816.3612 660.4783 848.9858
## 268        754.732 687.4732 821.9909 651.8685 857.5956
## 269        754.732 682.2797 827.1844 643.9257 865.5383
## 270        754.732 677.4343 832.0297 636.5154 872.9487
## 271        754.732 672.8753 836.5888 629.5429 879.9211
## 272        754.732 668.5571 840.9070 622.9388 886.5252

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 264       738.1162 708.2079 768.0245 692.3755 783.8570
## 265       738.1162 697.8787 778.3537 676.5783 799.6541
## 266       738.1162 689.7054 786.5270 664.0783 812.1541
## 267       738.1162 682.7253 793.5071 653.4031 822.8293
## 268       738.1162 676.5313 799.7011 643.9302 832.3022
## 269       738.1162 670.9057 805.3267 635.3266 840.9058
## 270       738.1162 665.7159 810.5165 627.3896 848.8429
## 271       738.1162 660.8741 815.3584 619.9846 856.2479
## 272       738.1162 656.3183 819.9141 613.0171 863.2153
## 273       738.1162 652.0032 824.2292 606.4178 869.8147
```

```
##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 265       742.5116 712.6580 772.3653 696.8544 788.1688
## 266       742.5116 702.3476 782.6756 681.0861 803.9371
## 267       742.5116 694.1893 790.8340 668.6090 816.4143
## 268       742.5116 687.2219 797.8014 657.9533 827.0700
## 269       742.5116 681.0392 803.9841 648.4977 836.5256
## 270       742.5116 675.4239 809.5994 639.9098 845.1135
## 271       742.5116 670.2436 814.7797 631.9872 853.0360
## 272       742.5116 665.4106 819.6127 624.5957 860.4275
## 273       742.5116 660.8631 824.1601 617.6410 867.3822
## 274       742.5116 656.5559 828.4673 611.0537 873.9695

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 266       744.2922 714.4949 774.0894 698.7212 789.8631
## 267       744.2922 704.2041 784.3803 682.9827 805.6016
## 268       744.2922 696.0611 792.5232 670.5291 818.0552
## 269       744.2922 689.1069 799.4774 659.8936 828.6908
## 270       744.2922 682.9359 805.6484 650.4558 838.1285
## 271       744.2922 677.3312 811.2531 641.8842 846.7002
## 272       744.2922 672.1606 816.4237 633.9765 854.6078
## 273       744.2922 667.3368 821.2476 626.5990 861.9853
## 274       744.2922 662.7979 825.7864 619.6574 868.9269
## 275       744.2922 658.4988 830.0855 613.0826 875.5017

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 267       754.6762 724.9215 784.4309 709.1704 800.1821
## 268       754.6762 714.6454 794.7071 693.4544 815.8981
## 269       754.6762 706.5141 802.8384 681.0186 828.3339
## 270       754.6762 699.5698 809.7827 670.3982 838.9543
## 271       754.6762 693.4076 815.9449 660.9739 848.3785
## 272       754.6762 687.8109 821.5416 652.4145 856.9379
## 273       754.6762 682.6478 826.7047 644.5182 864.8343
## 274       754.6762 677.8308 831.5217 637.1512 872.2012
## 275       754.6762 673.2984 836.0541 630.2196 879.1329
## 276       754.6762 669.0055 840.3470 623.6541 885.6983

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 268       756.7586 727.0596 786.4577 711.3378 802.1794
## 269       756.7586 716.8026 796.7146 695.6512 817.8660
## 270       756.7586 708.6865 804.8307 683.2387 830.2786
## 271       756.7586 701.7552 811.7620 672.6382 840.8791
## 272       756.7586 695.6045 817.9127 663.2315 850.2857
## 273       756.7586 690.0183 823.4989 654.6881 858.8291
## 274       756.7586 684.8648 828.6524 646.8066 866.7107
## 275       756.7586 680.0568 833.4604 639.4534 874.0639
## 276       756.7586 675.5329 837.9843 632.5347 880.9826
## 277       756.7586 671.2480 842.2692 625.9815 887.5357

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 269       759.2169 729.5729 788.8608 713.8803 804.5534
## 270       759.2169 719.3350 799.0987 698.2228 820.2109
## 271       759.2169 711.2339 807.1998 685.8333 832.6004
## 272       759.2169 704.3155 814.1182 675.2525 843.1812
```

```
## 273       759.2169 698.1762 820.2575 665.8633 852.5704
## 274       759.2169 692.6004 825.8333 657.3357 861.0980
## 275       759.2169 687.4565 830.9772 649.4688 868.9649
## 276       759.2169 682.6574 835.7763 642.1292 876.3045
## 277       759.2169 678.1419 840.2918 635.2234 883.2103
## 278       759.2169 673.8649 844.5688 628.6824 889.7513

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 270       733.0747 703.3987 762.7506 687.6892 778.4601
## 271       733.0747 693.1498 772.9996 672.0148 794.1346
## 272       733.0747 685.0399 781.1094 659.6119 806.5374
## 273       733.0747 678.1140 788.0353 649.0196 817.1297
## 274       733.0747 671.9681 794.1812 639.6203 826.5290
## 275       733.0747 666.3863 799.7631 631.0835 835.0658
## 276       733.0747 661.2368 804.9125 623.2081 842.9412
## 277       733.0747 656.4325 809.7168 615.8606 850.2887
## 278       733.0747 651.9122 814.2372 608.9473 857.2020
## 279       733.0747 647.6306 818.5187 602.3992 863.7501

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 271       738.7495 709.1248 768.3741 693.4425 784.0565
## 272       738.7495 698.8936 778.6054 677.7952 799.7038
## 273       738.7495 690.7978 786.7012 665.4137 812.0852
## 274       738.7495 683.8839 793.6151 654.8398 822.6592
## 275       738.7495 677.7486 799.7504 645.4567 832.0423
## 276       738.7495 672.1764 805.3226 636.9347 840.5643
## 277       738.7495 667.0358 810.4632 629.0729 848.4261
## 278       738.7495 662.2398 815.2591 621.7381 855.7609
## 279       738.7495 657.7273 819.7717 614.8367 862.6622
## 280       738.7495 653.4531 824.0458 608.3000 869.1990

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 272       750.6209 721.0335 780.2084 705.3708 795.8711
## 273       750.6209 710.8151 790.4268 689.7431 811.4988
## 274       750.6209 702.7294 798.5125 677.3772 823.8647
## 275       750.6209 695.8242 805.4177 666.8165 834.4254
## 276       750.6209 689.6966 811.5453 657.4452 843.7967
## 277       750.6209 684.1313 817.1106 648.9339 852.3080
## 278       750.6209 678.9972 822.2447 641.0819 860.1600
## 279       750.6209 674.2073 827.0346 633.7563 867.4856
## 280       750.6209 669.7004 831.5415 626.8636 874.3783
## 281       750.6209 665.4316 835.8103 620.3351 880.9068

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 273       753.3381 723.8045 782.8717 708.1704 798.5058
## 274       753.3381 713.6047 793.0715 692.5712 814.1051
## 275       753.3381 705.5338 801.1424 680.2278 826.4485
## 276       753.3381 698.6411 808.0351 669.6863 836.9899
## 277       753.3381 692.5247 814.1515 660.3321 846.3441
## 278       753.3381 686.9696 819.7066 651.8363 854.8399
## 279       753.3381 681.8449 824.8314 643.9986 862.6776
## 280       753.3381 677.0637 829.6126 636.6864 869.9898
## 281       753.3381 672.5650 834.1113 629.8062 876.8700
## 282       753.3381 668.3039 838.3723 623.2896 883.3866
```

```
##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 274       772.2848 742.7603 801.8094 727.1309 817.4388
## 275       772.2848 732.5636 812.0061 711.5364 833.0332
## 276       772.2848 724.4951 820.0745 699.1968 845.3728
## 277       772.2848 717.6046 826.9651 688.6586 855.9111
## 278       772.2848 711.4900 833.0796 679.3072 865.2625
## 279       772.2848 705.9366 838.6330 670.8140 873.7557
## 280       772.2848 700.8134 843.7562 662.9787 881.5909
## 281       772.2848 696.0337 848.5360 655.6687 888.9009
## 282       772.2848 691.5364 853.0333 648.7907 895.7790
## 283       772.2848 687.2766 857.2930 642.2760 902.2936

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 275       789.4525 759.9450 818.9600 744.3246 834.5803
## 276       789.4525 749.7542 829.1507 728.7392 850.1657
## 277       789.4525 741.6904 837.2145 716.4067 862.4982
## 278       789.4525 734.8038 844.1011 705.8746 873.0304
## 279       789.4525 728.6928 850.2121 696.5286 882.3763
## 280       789.4525 723.1426 855.7623 688.0403 890.8646
## 281       789.4525 718.0224 860.8826 680.2096 898.6954
## 282       789.4525 713.2454 865.6595 672.9038 906.0011
## 283       789.4525 708.7507 870.1543 666.0297 912.8752
## 284       789.4525 704.4934 874.4115 659.5188 919.3861

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 276       778.7492 749.2814 808.2171 733.6821 823.8164
## 277       778.7492 739.1043 818.3942 718.1176 839.3809
## 278       778.7492 731.0514 826.4471 705.8016 851.6968
## 279       778.7492 724.1740 833.3245 695.2837 862.2148
## 280       778.7492 718.0712 839.4272 685.9502 871.5482
## 281       778.7492 712.5285 844.9700 677.4734 880.0251
## 282       778.7492 707.4151 850.0833 669.6532 887.8453
## 283       778.7492 702.6446 854.8539 662.3572 895.1413
## 284       778.7492 698.1559 859.3426 655.4924 902.0061
## 285       778.7492 693.9044 863.5941 648.9902 908.5083

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 277       781.0539 751.6392 810.4686 736.0680 826.0398
## 278       781.0539 741.4805 820.6273 720.5316 841.5762
## 279       781.0539 733.4421 828.6658 708.2379 853.8699
## 280       781.0539 726.5772 835.5307 697.7389 864.3689
## 281       781.0539 720.4854 841.6225 688.4223 873.6855
## 282       781.0539 714.9526 847.1552 679.9607 882.1471
## 283       781.0539 709.8485 852.2593 672.1546 889.9532
## 284       781.0539 705.0865 857.0213 664.8718 897.2360
## 285       781.0539 700.6060 861.5019 658.0194 904.0885
## 286       781.0539 696.3621 865.7458 651.5289 910.5789

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 278       776.0374 746.6731 805.4017 731.1285 820.9462
## 279       776.0374 736.5317 815.5430 715.6187 836.4561
## 280       776.0374 728.5071 823.5677 703.3461 848.7287
## 281       776.0374 721.6539 830.4209 692.8650 859.2097
```

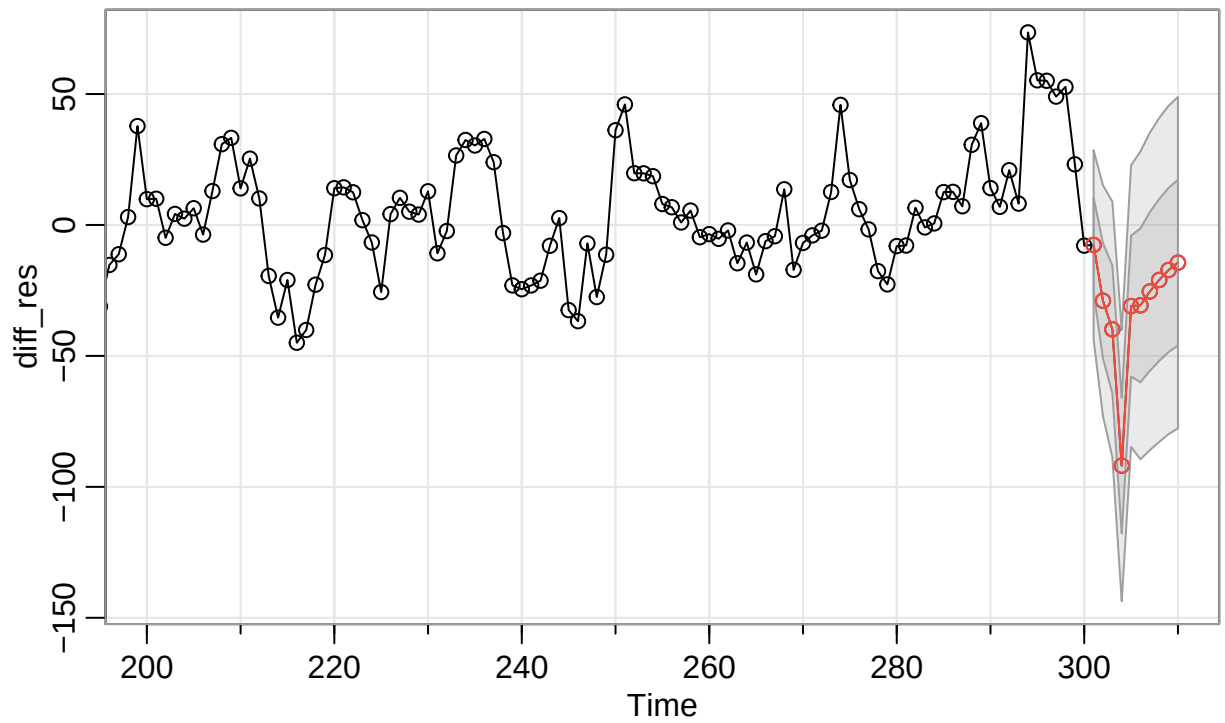
```
## 282       776.0374 715.5725 836.5022 683.5644 868.5104
## 283       776.0374 710.0493 842.0255 675.1173 876.9575
## 284       776.0374 704.9539 847.1209 667.3246 884.7502
## 285       776.0374 700.2001 851.8747 660.0542 892.0205
## 286       776.0374 695.7272 856.3476 653.2135 898.8612
## 287       776.0374 691.4906 860.5842 646.7342 905.3406

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 279       775.3017 745.9906 804.6129 730.4742 820.1292
## 280       775.3017 735.8676 814.7358 714.9924 835.6110
## 281       775.3017 727.8575 822.7459 702.7420 847.8614
## 282       775.3017 721.0167 829.5867 692.2800 858.3234
## 283       775.3017 714.9464 835.6570 682.9962 867.6072
## 284       775.3017 709.4331 841.1703 674.5644 876.0390
## 285       775.3017 704.3469 846.2565 666.7858 883.8177
## 286       775.3017 699.6018 851.0017 659.5286 891.0748
## 287       775.3017 695.1369 855.4665 652.7003 897.9031
## 288       775.3017 690.9080 859.6954 646.2327 904.3707

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 280       780.9072 751.6451 810.1693 736.1547 825.6597
## 281       780.9072 741.5390 820.2753 720.6988 841.1156
## 282       780.9072 733.5423 828.2721 708.4689 853.3455
## 283       780.9072 726.7130 835.1014 698.0243 863.7901
## 284       780.9072 720.6528 841.1616 688.7561 873.0583
## 285       780.9072 715.1488 846.6656 680.3384 881.4760
## 286       780.9072 710.0711 851.7433 672.5728 889.2416
## 287       780.9072 705.3339 856.4805 665.3277 896.4867
## 288       780.9072 700.8765 860.9379 658.5108 903.3036
## 289       780.9072 696.6547 865.1597 652.0541 909.7603

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 281       780.6217 751.4123 809.8312 735.9497 825.2937
## 282       780.6217 741.3244 819.9190 720.5217 840.7218
## 283       780.6217 733.3421 827.9014 708.3138 852.9297
## 284       780.6217 726.5251 834.7184 697.8880 863.3554
## 285       780.6217 720.4758 840.7677 688.6365 872.6070
## 286       780.6217 714.9817 846.2618 680.2339 881.0096
## 287       780.6217 709.9132 851.3303 672.4823 888.7612
## 288       780.6217 705.1844 856.0591 665.2503 895.9932
## 289       780.6217 700.7351 860.5084 658.4457 902.7978
## 290       780.6217 696.5208 864.7226 652.0005 909.2430

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 282       782.5372 753.3797 811.6947 737.9446 827.1297
## 283       782.5372 743.3098 821.7646 722.5440 842.5303
## 284       782.5372 735.3416 829.7327 710.3578 854.7166
## 285       782.5372 728.5367 836.5376 699.9506 865.1238
## 286       782.5372 722.4982 842.5762 690.7155 874.3589
## 287       782.5372 717.0138 848.0605 682.3278 882.7465
## 288       782.5372 711.9543 853.1200 674.5900 890.4844
## 289       782.5372 707.2340 857.8404 667.3709 897.7035
## 290       782.5372 702.7926 862.2818 660.5783 904.4960
## 291       782.5372 698.5858 866.4885 654.1446 910.9297
```

```
##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 283       792.5027 763.3850 821.6205 747.9709 837.0345
## 284       792.5027 753.3288 831.6766 732.5914 852.4141
## 285       792.5027 745.3715 839.6339 720.4218 864.5837
## 286       792.5027 738.5759 846.4295 710.0287 874.9767
## 287       792.5027 732.5456 852.4598 700.8062 884.1992
## 288       792.5027 727.0687 857.9367 692.4300 892.5754
## 289       792.5027 722.0161 862.9893 684.7027 900.3027
## 290       792.5027 717.3022 867.7032 677.4935 907.5119
## 291       792.5027 712.8669 872.1386 670.7102 914.2952
## 292       792.5027 708.6658 876.3396 664.2853 920.7201

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 284       786.1913 757.1204 815.2621 741.7313 830.6513
## 285       786.1913 747.0804 825.3021 726.3764 846.0061
## 286       786.1913 739.1360 833.2465 714.2264 858.1561
## 287       786.1913 732.3513 840.0312 703.8502 868.5324
## 288       786.1913 726.3307 846.0518 694.6425 877.7400
## 289       786.1913 720.8627 851.5198 686.2798 886.1027
## 290       786.1913 715.8182 856.5643 678.5650 893.8175
## 291       786.1913 711.1119 861.2706 671.3673 901.0152
## 292       786.1913 706.6837 865.6988 664.5950 907.7876
## 293       786.1913 702.4895 869.8931 658.1804 914.2021

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 285       791.3651 762.3426 820.3877 746.9789 835.7513
## 286       791.3651 752.3193 830.4110 731.6496 851.0806
## 287       791.3651 744.3880 838.3423 719.5198 863.2104
## 288       791.3651 737.6146 845.1157 709.1608 873.5695
## 289       791.3651 731.6040 851.1262 699.9684 882.7619
## 290       791.3651 726.1450 856.5852 691.6196 891.1107
## 291       791.3651 721.1089 861.6213 683.9176 898.8127
## 292       791.3651 716.4105 866.3198 676.7319 905.9984
## 293       791.3651 711.9896 870.7406 669.9707 912.7595
## 294       791.3651 707.8023 874.9279 663.5668 919.1634

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 286       804.2845 775.2926 833.2764 759.9452 848.6237
## 287       804.2845 765.2799 843.2891 744.6321 863.9368
## 288       804.2845 757.3570 851.2119 732.5151 876.0538
## 289       804.2845 750.5908 857.9782 722.1671 886.4019
## 290       804.2845 744.5866 863.9824 712.9844 895.5846
## 291       804.2845 739.1334 869.4356 704.6444 903.9245
## 292       804.2845 734.1026 874.4664 696.9505 911.6184
## 293       804.2845 729.4091 879.1599 689.7724 918.7965
## 294       804.2845 724.9929 883.5761 683.0185 925.5505
## 295       804.2845 720.8100 887.7589 676.6213 931.9476

##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 287       810.4005 781.4551 839.3459 766.1323 854.6686
## 288       810.4005 771.4584 849.3425 750.8437 869.9572
## 289       810.4005 763.5482 857.2527 738.7462 882.0548
## 290       810.4005 756.7928 864.0081 728.4147 892.3863
```

```
## 291       810.4005 750.7982 870.0027 719.2467 901.5542
## 292       810.4005 745.3538 875.4472 710.9201 909.8808
## 293       810.4005 740.3311 880.4699 703.2386 917.5624
## 294       810.4005 735.6451 885.1558 696.0720 924.7290
## 295       810.4005 731.2360 889.5649 689.3288 931.4721
## 296       810.4005 727.0598 893.7411 682.9420 937.8590
```

```
##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 288       817.5281 788.6272 846.4289 773.3281 861.7281
## 289       817.5281 778.6460 856.4101 758.0630 876.9931
## 290       817.5281 770.7480 864.3081 745.9841 889.0720
## 291       817.5281 764.0030 871.0531 735.6685 899.3876
## 292       817.5281 758.0176 877.0385 726.5147 908.5414
## 293       817.5281 752.5815 882.4746 718.2009 916.8552
## 294       817.5281 747.5666 887.4895 710.5312 924.5249
## 295       817.5281 742.8878 892.1683 703.3756 931.6805
## 296       817.5281 738.4855 896.5706 696.6429 938.4132
## 297       817.5281 734.3157 900.7404 690.2658 944.7903
```

```
##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 289       840.4798 811.5649 869.3947 796.2583 884.7014
## 290       840.4798 801.5788 879.3809 780.9858 899.9738
## 291       840.4798 793.6770 887.2827 768.9010 912.0586
## 292       840.4798 786.9287 894.0310 758.5804 922.3793
## 293       840.4798 780.9404 900.0193 749.4221 931.5376
## 294       840.4798 775.5017 905.4580 741.1043 939.8554
## 295       840.4798 770.4842 910.4754 733.4308 947.5288
## 296       840.4798 765.8032 915.1565 726.2718 954.6879
## 297       840.4798 761.3987 919.5609 719.5357 961.4239
## 298       840.4798 757.2270 923.7327 713.1556 967.8041
```



```
##     Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 290        867.147 838.1956 896.0984 822.8696 911.4243
## 291        867.147 828.1969 906.0971 807.5779 926.7160
## 292        867.147 820.2851 914.0089 795.4778 938.8161
## 293        867.147 813.5282 920.7657 785.1442 949.1497
## 294        867.147 807.5324 926.7615 775.9744 958.3196
## 295        867.147 802.0868 932.2071 767.6460 966.6479
## 296        867.147 797.0631 937.2309 759.9629 974.3310
## 297        867.147 792.3761 941.9178 752.7948 981.4991
## 298        867.147 787.9661 946.3278 746.0503 988.2436
## 299        867.147 783.7891 950.5049 739.6621 994.6319

## cross validation error 69985112
```

7 Final Model

7.1 Model interpretation

7.2 Prediction

8 Conclusion