

# Math 104 Worksheet 14

UC Berkeley, Summer 2021

Wednesday, July 28

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Let  $(X, d)$  be a metric space, and let  $E \subseteq X$ .

Exercise 1. Show that  $D(f, g) := \sup\{|f(x) - g(x)| : x \in E\}$  defines a metric on the space of bounded real-valued functions on  $E$ ,  $B(X) := \{f : E \rightarrow \mathbb{R} : f \text{ is bounded}\}$ .

Exercise 2. What does it mean for a sequence of functions in  $B(X)$  to converge?

Exercise 3. Formulate a definition for a sequence of functions  $(f_n)$  on  $E$  to be **uniformly Cauchy**.

Exercise 4. Formulate a definition for a series of functions  $\sum_{n=1}^{\infty} g_n$  on  $E$  to be **uniformly convergent on E**.

Exercise 5. Formulate a definition for a series  $\sum_{n=1}^{\infty} g_n$  to satisfy the **uniform Cauchy criterion**.