Math 104 Worksheet 17

UC Berkeley, Summer 2021

Wednesday, August 4

Consider the function

$$f(x) = \begin{cases} e^{-1/x} & \text{if } x > 0, \\ 0 & \text{if } x \leq 0. \end{cases}$$

Exercise 1. Show that $f'(0) = 0$. (*Hint:* Consider the left and right limits separately.)

Exercise 2. Show by induction that for $x > 0$, $f^{(n)}(x)$ has the form

$$q_n\left(\frac{1}{x}\right)e^{-1/x}$$

where $q_n(t)$ is a polynomial in t.

Exercise 3. Show by induction that $f^{(n)}(0) = 0$ for all n.
(Therefore, $T^{f,0}(x) \equiv 0$, so $f(x) \neq T^{f,0}(x)$ for all $x > 0$.)