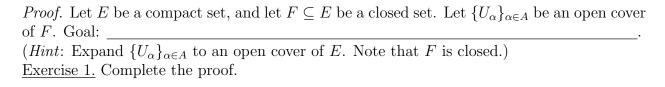
Math 104 Worksheet 7 UC Berkeley, Summer 2021 Thursday, July 8

Let (X, d) be a metric space.

Theorem. Closed subsets of compact sets are compact.



Theorem. Every sequence in a compact set has a convergent subsequence.

Proof. Let E be a compact set, and let (x_n) be a sequence of points in E. Consider the set $S = \{x_n : n \in \mathbb{N}\}.$

Exercise 2. Explain why if S is finite, then (x_n) has a convergent subsequence.

If S is infinite, then it suffices to show that S has a limit point in E. Why?

Suppose (for contradiction) that no point in E is a limit point of S.

Exercise 3. Construct an open cover of E which has no finite subcover. This would imply that E is not compact, which is a contradiction.