Math 104 Midterm Exam

Wenhao Pan

TOTAL POINTS

25 / 30

QUESTION 1

Problem 1 10 pts

1.1 a **2 / 2**

✓ **+ 2 pts** **Correct**

+ 0 pts Incorrect

1.2 b **2 / 2**

✓ **+ 2 pts** **Correct**

+ 0 pts Incorrect

1.3 c **2 / 2**

✓ **+ 2 pts** **Correct**

+ 1 pts Possibly the correct solution in mind but written ambiguously or incorrectly

+ 0 pts Incorrect

1.4 d **2 / 2**

✓ **+ 2 pts** **Correct**

+ 0 pts Incorrect

1.5 e **2 / 2**

✓ **+ 2 pts** **Correct**

+ 1 pts Almost correct

+ 0 pts Incorrect

QUESTION 2

2 Problem 2 **3 / 5**

+ 5 pts Correct

+ 4 pts Almost correct

✓ **+ 3 pts** **Showed that 0 is a lower bound, but not that it is the greatest lower bound.**

+ 2 pts Some valid points

+ 2 pts Minimal progress

+ 3 pts Did not apply the correction to the typo and proved that the statement as written is false

+ 2 pts Did not apply the correction to the typo and made no progress

QUESTION 3

3 Problem 3 **5 / 5**

✓ **+ 5 pts** **Correct**

+ 4 pts Almost correct

+ 3 pts Some relevant ideas

+ 2 pts A relevant idea or two

+ 0 pts Minimal progress

QUESTION 4

4 Problem 4 **4 / 5**

+ 5 pts Correct

✓ **+ 4 pts** **Almost correct**

+ 3 pts Some relevant ideas

+ 2 pts A relevant idea or two

+ 0 pts Minimal progress

QUESTION 5

5 Problem 5 **3 / 5**

+ 5 pts Correct

✓ **+ 3 pts** **Correct construction of open cover**

+ 2 pts Some relevant ideas

+ 0 pts Minimal progress

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b) all s_{n_k} increasing . $\sup L = \infty$. $\inf L = 104$

$(104-1, 106, 104-\frac{1}{2}, 106, 104-\frac{1}{3}, 107, \dots)$

All odd terms are $104-\frac{1}{n}$, so $\lim 104-\frac{1}{n} = 104 \Rightarrow \liminf s_n = 104$

All even terms are $104+n$, so $\lim 104+n = +\infty \Rightarrow \limsup s_n = \infty$

c) $\{x \in \mathbb{Z} : 0 \le x < \infty\} \cup \{+\infty\}$

d) (i) is True

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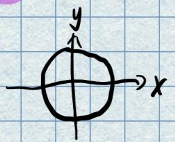
e) 

$\{B_{1.1}(0) \setminus \overline{B}_r(0)\}_{r \in (0, 1.1)}$

closed ball

2. ① $x > y \Rightarrow x - y > 0$

$\because x-y+z > z+0 = z > 0$

$\therefore x-y+z > 0 \Rightarrow 0 \le \inf\{x-y+z : x>y>z>0\}$

$\inf\{x-y+z : x>y>z>0\} = 0$

② Show $\forall \varepsilon > 0$, $\inf\{x-y+z : x>y>z>0\} = m \le \varepsilon$, then $m \le 0$ $\quad -m < \varepsilon -$

Let $\varepsilon > 0$, $\exists x, y, z$ $\quad x-y+z < m+\varepsilon \Rightarrow$

$\Rightarrow$

$x-y>0$

$x-y+z>0$

Thus $\inf\{x-y+z : x>y>z>0\} \le 0$. Combine with ① we have $\inf\{x-y+z : x>y>z>0\} \le 0$

3. $\forall \varepsilon > 0$ $\exists N \in \mathbb{N}$ $\quad n \ge N \Rightarrow ||s_n| - s| < \varepsilon$

$\Rightarrow s-\varepsilon < |s_n| < s+\varepsilon$

$\lim |s_n| = s \Rightarrow \limsup |s_n| = \liminf |s_n| = s$

① Suppose all but finitely many terms are negative, then $\exists N \in \mathbb{N}$, $n \ge N \Rightarrow s_n > 0$. Consider all these positive terms a subsequence (s_{n_k})

Then $\sup\{|s_{n_k}| : k \ge K\} = \sup\{s_{n_k} : k \ge K\}$ and $\inf\{|s_{n_k}| : k \ge K\} = \inf\{s_{n_k} : k \ge K\}$

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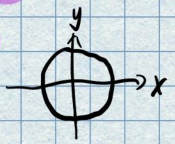
e) 

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$\uparrow$

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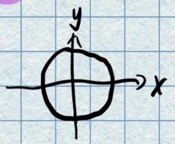
e) 

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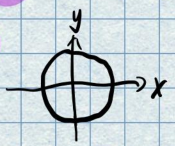
e) 

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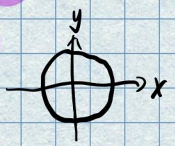
e) 

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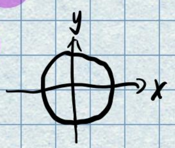
e) 

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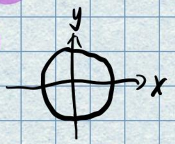

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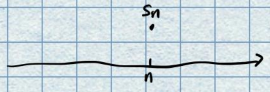
$V_n = \sup\{S_m : m \geq n\}$.

Take $m = n$. $S_m = S_n \leq V_n \Rightarrow 0 \leq V_n - S_n \Rightarrow 0 \leq \inf\{V_n - S_n : n \in \mathbb{N}\}$

If we can show $\forall \varepsilon > 0$ $\inf\{V_n - S_n : n \geq N\} \leq \varepsilon$, then $\inf\{V_n - S_n : n \geq N\} \leq 0$

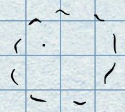
Let $\varepsilon > 0$. $\exists m \geq N$ $V_n - \varepsilon < S_m \Rightarrow V_n - S_m < \varepsilon$



5. (X_n) is Cauchy. $\forall \varepsilon > 0$, $\exists N \in \mathbb{N}$, $m, n \geq N \Rightarrow |X_m - X_n| < \varepsilon$

$\forall x \in E$, $X_n \not\to X$

i.e. $\forall x \in E$, $\exists \varepsilon_x > 0$ s.t. $\forall N \in \mathbb{N}$, $\exists n \geq N$ s.t. $x_n \in E$ and $d(x_n, x) \geq \varepsilon$.



In other words. $\forall x \in E$, there exists $x' \in E$ s.t. $x' \notin B_{\varepsilon_x}(X)$

Now, obviously $\mathcal{U} = \{B_{\varepsilon_x}(X)\}_{x \in E}$ is an open cover of E without finite subcover.

4 Problem 4 **4 / 5**

- \+ **5 pts** Correct
- ✓ + **4 pts** **Almost correct**
- \+ **3 pts** Some relevant ideas
- \+ **2 pts** A relevant idea or two
- \+ **0 pts** Minimal progress

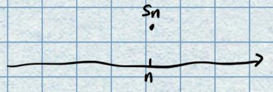
4.

$V_n = \sup\{S_m : m \geq n\}$.

Take $m = n$, $S_m = S_n \leq V_n \Rightarrow 0 \leq V_n - S_n \Rightarrow 0 \leq \inf\{V_n - S_n : n \in \mathbb{N}\}$

If we can show $\forall \varepsilon > 0$ $\inf\{V_n - S_n : n \geq N\} \leq \varepsilon$, then $\inf\{V_n - S_n : n \geq N\} \leq 0$

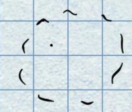
Let $\varepsilon > 0$. $\exists m \geq N$ $V_n - \varepsilon < S_m$ $\Rightarrow V_n - S_m < \varepsilon$



5. (X_n) is Cauchy. $\forall \varepsilon > 0$, $\exists N \in \mathbb{N}$, $m, n \geq N \Rightarrow |X_m - X_n| < \varepsilon$

$\forall x \in E$, $X_n \not\to x$

i.e. $\forall x \in E$, $\exists \varepsilon_x > 0$ s.t. $\forall N \in \mathbb{N}$, $\exists n \geq N$ s.t. $x_n \in E$ and $d(x_n, x) \geq \varepsilon$.



In other words, $\forall x \in E$, there exists $x' \in E$ s.t. $x' \notin B_{\varepsilon_x}(x)$

Now, obviously $\mathcal{U} = \{B_{\varepsilon_x}(x)\}_{x \in E}$ is an open cover of E without finite subcover.

5 Problem 5 **3 / 5**

- \+ **5 pts** Correct
- ✓ + **3 pts** **Correct construction of open cover**
- \+ **2 pts** Some relevant ideas
- \+ **0 pts** Minimal progress