Math 104 Worksheet 11

UC Berkeley, Summer 2021

Wednesday, July 21

Exercise 1. (a) Prove that the function $f(x) = |x|$ is continuous on $\mathbb{R}$ using the $\varepsilon - \delta$ definition of continuity. (Hint: Recall the reverse triangle inequality, $||a| - |b|| \leq |a - b|$ for any $a, b \in \mathbb{R}$.)

(b) Prove that if g is continuous at x_0, then the function $|g|$ is continuous at x_0.

Exercise 2. (a) Prove that $\max(a, b) = \frac{1}{2}(a + b) + \frac{1}{2}|a - b|$ for any $a, b \in \mathbb{R}$.

(b) Prove that $\min(a, b) = -\max(-a, -b)$ for any $a, b \in \mathbb{R}$.

(c) Prove that if f and g are continuous at x_0, then the functions $\max(f, g)$ and $\min(f, g)$ are continuous at x_0.