Math 104 Worksheet 9

UC Berkeley, Summer 2021

Tuesday, July 13

Exercise 1. Justify the following facts about the Cantor set $\mathcal{C}$.

(a) $\mathcal{C}$ is compact.

(b) $\mathcal{C}$ does not contain any intervals.

(c) $\mathcal{C}$ does not have any interior points.

(d) Every point in $\mathcal{C}$ is a limit point of $\mathcal{C}$.

(e) Every point in $\mathcal{C}$ is a limit point of $\mathcal{C}^c$.

Exercise 2. Prove that any open set in $\mathbb{R}$ is an at most countable disjoint union of open intervals. (*Hint:* If $U \subseteq \mathbb{R}$ is open, for any $x \in U$, any interval containing x can be expanded to the largest interval in U containing x.)