Math 104 Worksheet 4

UC Berkeley, Summer 2021
Tuesday, June 29

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1. Let (s_n) be a sequence of nonnegative real numbers which converges to s.
(a) Show that $s \geq 0$. (*Hint*: Argue by contradiction.)

(b) Show that $\sqrt{s_n} \to \sqrt{s}$. (*Hint*: Consider the cases $s = 0$ and $s > 0$ separately. For the case $s > 0$, observe that $|\sqrt{s_n} - \sqrt{s}| = \frac{|s_n - s|}{\sqrt{s_n} + \sqrt{s}} \leq \frac{|s_n - s|}{\sqrt{s}}$.)

2. (Theorem 9.9) Let (s_n) and (t_n) be sequences such that $\lim s_n = \infty$ and (t_n) converges to $t > 0$. Then $\lim s_n t_n = \infty$.

Proof. Let $M > 0$. Goal: Show that there exists $N \in \mathbb{N}$ such that ...

First, since $t_n \to t > 0$, we can find $N_1 \in \mathbb{N}$ such that $|t_n - t| < \frac{t}{2}$ for all $n \geq N_1$. Then $t_n \geq \frac{t}{2}$ for all $n \geq N_1$. Now since $s_n \to \infty$, there exists $N_2 \in \mathbb{N}$ such that

$$s_n > \underline{\hspace{3cm}}$$

for all $n \geq N_2$. Set $N = \max(N_1, N_2)$. Then for $n \geq N$,

$$s_n t_n > \underline{\hspace{3cm}} = M.$$

$\square$

3. Give an example of ...

1. a sequence (s_n) of rational numbers which converges to an irrational number.

2. a sequence (s_n) of irrational numbers which converges to a rational number.

3. a divergent sequence (s_n) such that $(|s_n|)$ converges.

4. a sequence (s_n) of nonzero real numbers which converges to 0 such that the sequence $(1/s_n)$ does not have a limit.

5. two divergent sequences (s_n) and (t_n) such that the sequence $(s_n + t_n)$ converges.

6. a sequence (s_n) of nonzero real numbers and a divergent sequence (t_n) such that the sequence $(s_n t_n)$ converges.

7. two convergent sequences (s_n) and (t_n) such that $s_n < t_n$ for all n and $\lim s_n = \lim t_n$.

8. a divergent sequence (s_n) of positive real numbers such that $\lim|s_{n+1}/s_n| = 1$. (cf. Homework 2 Problem 8)

9. a bounded divergent sequence (s_n) such that $|s_n|$ is strictly increasing.

10. a divergent sequence (s_n) such that $|s_{n+1} - s_n| < \frac{1}{n}$ for all n.