Math 104 Worksheet 1

UC Berkeley, Summer 2021
Monday, June 21

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1. Prove the following consequences of the field properties. Continue from the first step(s) that have been provided for some of the problems.

(a) If $a + c = b + c$, then $a = b$.

Proof. Suppose $a + c = b + c$. Then

$$a = a + 0 = a + (c + (-c)) = \dots$$

$\square$

(b) $a \cdot 0 = 0$ for all a.

Proof. $0 + a \cdot 0 = a \cdot 0 = a \cdot (0 + 0) = \dots$

(The result should follow by applying assertion (a).)

$\square$

(c) $(-a)b = -ab$ for all a, b.

Proof. $ab + (-a)b = \dots$

(The result should follow by applying assertion (a).)

$\square$

(d) $-(-a) = a$ for all a.

Proof.

$\square$

(e) $(-a)(-b) = ab$ for all a, b.

Proof. (Use assertions (c) and (d).)

$\square$

(f) If $ab = 0$, then $a = 0$ or $b = 0$ for all a, b.

Proof. Suppose that a and b are both nonzero. (Show that $ab \neq 0$ by using assertion (b).)

$\square$

2. Prove the following consequences of the properties of an ordered field. Continue from the first step(s) that have been provided for some of the problems.

(a) If $a \leq b$, then $-b \leq -a$.

Proof. Suppose that $a \leq b$. Then

$$-b = (a + (-a)) + (-b) = a + ((-a) + (-b)) \leq \dots$$

$\square$

(b) If $0 \leq a$ and $0 \leq b$, then $0 \leq ab$.

Proof. $0 = 0 \cdot b \leq \dots$

$\square$

(c) $0 \leq a^2$ for all a.

Proof. (Consider the two cases $0 \leq a$ and $a \leq 0$.)

$\square$

(d) $0 < 1$.

Proof. (First justify $0 \leq 1$, then justify $0 \neq 1$.)

$\square$

(e) If $0 < a$, then $0 < a^{-1}$.

Proof. Let $0 < a$, and suppose that $a^{-1} \leq 0$. Then $0 \leq -a^{-1}$ by assertion (a). (Show that $0 \leq -1$, a contradiction.)

$\square$

(f) If $0 < a < b$, then $0 < b^{-1} < a^{-1}$.

Proof.

$\square$