Design Theory for Relational Databases

CSC343 Introduction to Databases

Introduction

- There are always many different schemas for a given set of data, e.g., we could combine or divide tables.

- **How do we pick a schema?**
  - **➔ Which is better?**
    - **→What does "better" mean?**
      - There are principles to guide us.

Database Design Theory

- It allows us to improve a schema systematically.

- General idea:
  - Express constraints on the relationships between attributes
  - Use these to decompose the relations

- Ultimately, get a schema that is in a "normal form" that guarantees good properties.

  ➔ "normal" in the sense of conforming to a standard.

- The process of converting a schema to a normal form is called normalization.

Part I:
Functional Dependency Theory

A poorly designed table

part	manufacturer	manAddress	seller	sellerAddress	price
1983	Hammers 'R Us	99 Pinecrest	ABC	1229 Bloor W	5.59
8624	Lee Valley	102 Vaughn	ABC	1229 Bloor W	23.99
9141	Hammers 'R Us	99 Pinecrest	ABC	1229 Bloor W	12.50
1983	Hammers 'R Us	99 Pinecrest	Walmart	5289 St Clair W	4.99

- In any domain, there may be relationships between attribute values.

- Perhaps:
  - Every part has 1 manufacturer
  - Every manufacturer has 1 address
  - Every seller has 1 address

- If so, this table will have redundant data.

Principle: Avoid redundancy

Redundant data can lead to anomalies.

part	manufacturer	manAddress	seller	sellerAddress	price
1983	Hammers 'R Us	99 Pinecrest	ABC	1229 Bloor W	5.59
8624	Lee Valley	102 Vaughn	ABC	1229 Bloor W	23.99
9141	Hammers 'R Us	99 Pinecrest	ABC	1229 Bloor W	12.50
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- Update anomaly: if Hammers 'R Us moves and we update only one tuple, the data is inconsistent.

- Deletion anomaly: If ABC stops selling part 8624 and Lee Valley makes only that one part, we lose track of its address.

Definition of FD

- Suppose R is a relation, and X and Y are subsets of the attributes of R.

- $X \rightarrow Y$ asserts that:
  - If two tuples agree on all the attributes in set X, they must also agree on all the attributes in set Y.

- We say that "$X \rightarrow Y$ holds in R", or "X functionally determines Y."

- An FD constrains what can go in a relation.

Formally…

$A \rightarrow B$ means:

$\forall$ tuples t_1, t_2,

$(t_1[A] = t_2[A]) \Rightarrow (t_1[B] = t_2[B])$

Or equivalently:

$\neg \exists$ tuples t_1, t_2 such that

$(t_1[A] = t_2[A]) \wedge (t_1[B] \neq t_2[B])$

Generalization to multiple attributes

$A_1A_2 \ldots A_m \rightarrow B_1B_2 \ldots B_n$ means:

$\forall$ tuples t_1, t_2,

$$(t_1[A_1] = t_2[A_1] \wedge \ldots \wedge t_1[A_m] = t_2[A_m]) \Rightarrow$$
$$(t_1[B_1] = t_2[B_1] \wedge \ldots \wedge t_1[B_n] = t_2[B_n])$$

Or equivalently:

$\neg \exists$ tuples t_1, t_2 such that

$$(t_1[A_1] = t_2[A_1] \wedge \ldots \wedge t_1[A_m] = t_2[A_m]) \wedge$$
$$\neg (t_1[B_1] = t_2[B_1] \wedge \ldots \wedge t_1[B_n] = t_2[B_n])$$

Why "functional dependency"?

- "dependency" because the value of Y depends on the value of X
- "functional" because there is a mathematical function (mapping) that given a value for X, gives a unique value for Y

Splitting rules for FDs

- We can split the RHS of an FD and get multiple, equivalent FDs.

- Can we split the LHS of an FD and get multiple, equivalent FDs?

Coincidence or FD?

- An FD is an assertion about every instance of a relation.
- You can’t know it holds just by looking at one instance.
- You must use knowledge of the domain to determine whether an FD holds.

FDs are closely related to keys

- Suppose K is a set of attributes for relation R.

- Recall definition of superkey:
  - a set of attributes for which no two rows can have the same values.

- A claim about FDs:

  K is a superkey for R

  iff

  K functionally determines all of R.

FDs are a generalization of keys

- key:
  $X \rightarrow R$ ← Every attribute

- Functional dependency:

  $X \rightarrow Y$ ← Not necessarily every attribute

Y is a set of attributes, S is a set of FDs.
Return the closure of Y under S.

Attribute_closure(Y, S):

Initialize Y^+ to Y

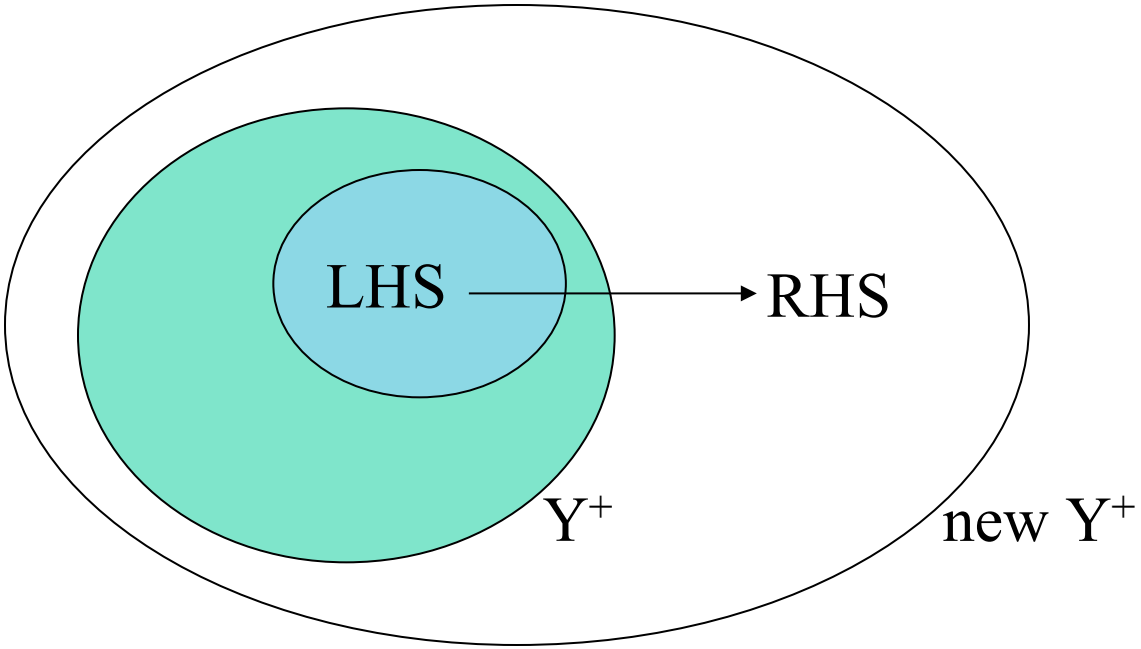
Repeat until no more changes occur:

If there is an FD, LHS → RHS in S s.t. LHS $\in Y^+$:

Add RHS to Y^+

Return Y^+

Visualizing attribute closure



If LHS is in Y^+ and LHS $\rightarrow$ RHS holds, we can add RHS to Y^+

S is a set of FDs;
LHS → RHS is a single FD.
Return true iff LHS → RHS follows from S.

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FD_follows(S, LHS → RHS):

  Y+ = Attribute_closure(LHS, S)

  return (RHS is in Y+)
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Inferring FDs

- Given a set of FDs, we can often infer further FDs.
- This will be handy when we apply FDs to the problem of database design.
- Big task: given a set of FDs, infer every other FD that must also hold.
- Simpler task: given a set of FDs, check whether a given FD must also hold.

Examples

- If $A \rightarrow B$ and $B \rightarrow C$ hold,
  must $A \rightarrow C$ hold?

- If $A \rightarrow H$, $C \rightarrow F$, and $F\ G \rightarrow A\ D$ hold,
  must $F\ A \rightarrow D$ hold?
  must $C\ G \rightarrow F\ H$ hold?

- If $H \rightarrow GD$, $HD \rightarrow CE$, and $BD \rightarrow A$ hold,
  must $EH \rightarrow C$ hold?

- Note: we are not generating new FDs, but testing a specific, possible, one.

Method 1: Prove that an FD holds using first principles

- You can prove an FD by referring back to
  - the FDs that you know hold, and
  - the definition of a functional dependency.

- … but the Closure Test is easier.

Method 2: Prove that an FD holds using the Closure Test

- Assume you know the values of the LHS attributes, and figure out everything else that is determined.
- If it includes the RHS attributes, then you know that LHS $\rightarrow$ RHS
- This is called the closure test.

Equivalent sets of FDs

- When we write a set of FDs, we mean that **all of them hold**.
- We rewrite sets of FDs in equivalent ways.
- When we say that a set of FDs S_1 is equivalent to a set of FDs S_2 we mean that:
  - S_1 holds in a relation iff S_2 does.

Projecting FDs

- Later, we will learn how to **normalize** a schema by decomposing relations.
  ➔ This is the whole point of this theory!

- We will need to know what FDs hold in the new, smaller, relations.

  - We must **project** our FDs onto the attributes of our new relations.

Example

R(A1, ..., An) Set of attributes: A

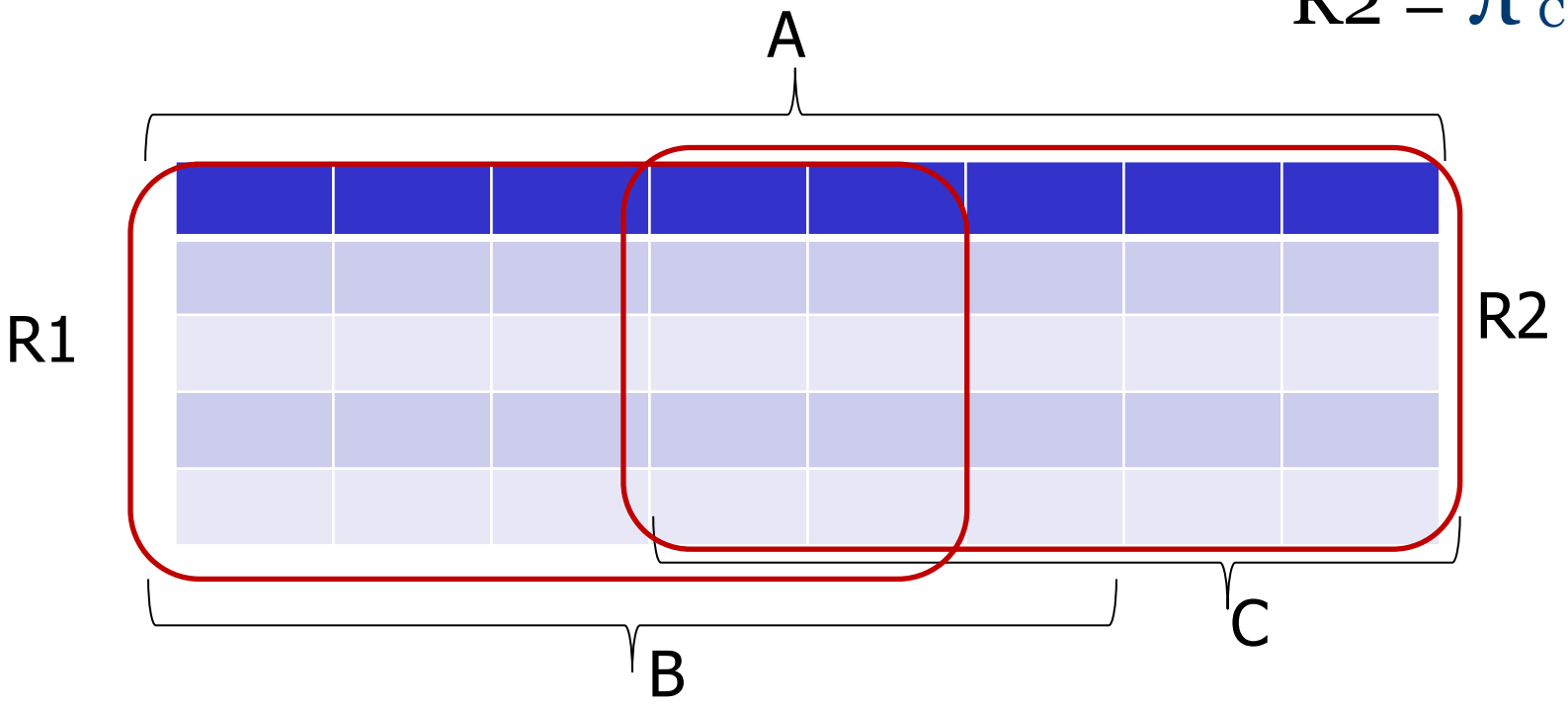
Decompose into:

- R1(B1, ..., Bk) Set of attributes: B
- R2(C1, ..., Cm) Set of attributes: C

$B \cup C = A$, $R1 \bowtie R2 = R$

$$R1 = \pi_B(R)$$

$$R2 = \pi_C(R)$$



S is a set of FDs; L is a set of attributes.

Return the projection of S onto L: all FDs that follow from S and involve only attributes from L.

Project(S, L):

Initialize T to {}.

For each subset X of L:
Compute X^+ Close X and see what we get.

For every attribute A in X^+:
If A is in L:
add $X \rightarrow A$ to T $X \rightarrow A$ is only relevant if A is in L (we know X is!).

Return T.

A few optimizations

- No need to add $X \rightarrow A$ if A is in X itself.
  → It's a trivial FD.

- These subsets of X won't yield anything, so no need to compute their closures:
  - the empty set
  - the set of all attributes

An important optimization

- If we find X^+ = all attributes, we can ignore any superset of X.
  - It can only give use "weaker" FDs (with more on the LHS).

- This is a big time saver!

Projection is expensive

- Even with these optimizations, projection is still expensive.

- Suppose R_1 has n attributes. How many subsets of R_1 are there?

Minimal Basis

- We saw earlier that we can often rewrite sets of FDs in equivalent ways.

- **Example:**

  $S_1 = \{A \rightarrow BC\}$ is equivalent to $S_2 = \{A \rightarrow B, A \rightarrow C\}$.

- Given a set of FDs S, we may want to find a minimal basis: a set of FDs that is equivalent to S, but has
  - no redundant FDs, and
  - no FDs with unnecessary attributes on the LHS.

S is a set of FDs. Return a minimal basis for S.

Minimal_basis(S):

1. Split the RHS of each FD

2. For each FD $X \rightarrow Y$ where $|X| \geq 2$:
   If we can remove an attribute from X and get an FD that follows from S:
   We do so! (It's a stronger FD.)

3. For each FD f :
   If S – {f } implies f :
   Remove f from S.

Some comments on minimal basis

- Often there are multiple possible results.
  - ➔Depends on the order in which you consider the possible simplifications.

- After we identify a redundant FD, we must not use it when computing subsequent closures.

… and less intuitive

- When we are computing closures to decide whether the LHS of an FD

  $X \rightarrow Y$

  can be simplified, continue to use that FD.

- You must do (2) and (3) in that order.

Part II:

Using FD Theory to do Database Design

Recall that poorly designed table?

part	manufacturer	manAddress	seller	sellerAddress	price
1983	Hammers 'R Us	99 Pinecrest	ABC	1229 Bloor W	5.59
8624	Lee Valley	102 Vaughn	ABC	1229 Bloor W	23.99
9141	Hammers 'R Us	99 Pinecrest	ABC	1229 Bloor W	12.50
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- We can now express the relationships as FDs:
  - part → manufacturer
  - manufacturer → address
  - seller → address

- The FDs tell us there can be redundancy, thus the design is bad.

- That's why we care about FDs.

Decomposition

- To improve a badly-designed schema $R(A_1, \ldots A_n)$, we will decompose it into smaller relations
  - $R1(B_1, \ldots B_j)$ and $R2(C_1, \ldots C_k)$ such that:

- $R1 = \pi_{B1, \ldots Bj}(R)$
- $R2 = \pi_{C1, \ldots Ck}(R)$
- $\{B_1, \ldots B_j\} \cup \{C_1, \ldots C_k\} = \{A_1, \ldots A_n\}$
- $R1 \bowtie R2 = R$

$R(A_1, \ldots A_n)$ Set of attributes: A

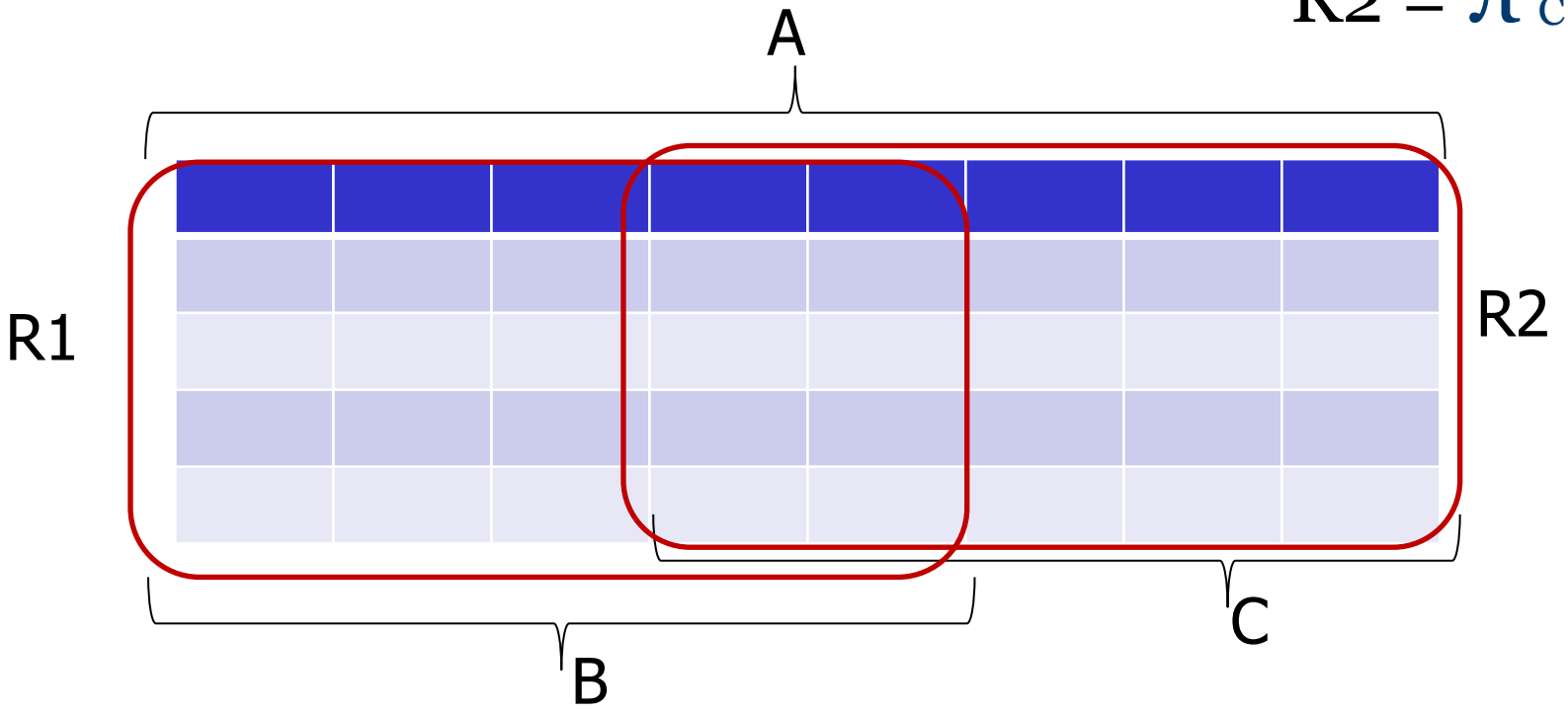
Decompose into:

- $R1(B_1, \ldots B_j)$ Set of attributes: B, and
- $R2(C_1, \ldots C_k)$ Set of attributes: C

$B \cup C = A$, $R1 \bowtie R2 = R$

$$R1 = \pi_B(R)$$

$$R2 = \pi_C(R)$$



But which decomposition?

- Decomposition can definitely improve a schema.
- But which decomposition?
  → There are many possibilities.
- And how can we be sure a new schema doesn't exhibit other anomalies?
- Boyce-Codd Normal Form guarantees it.

Boyce-Codd Normal Form

- We say a relation R is in BCNF if for every nontrivial FD $X \rightarrow Y$ that holds in R, X is a superkey.
  - Remember: nontrivial means Y is not contained in X.
  - Remember: a superkey doesn't have to be minimal.

- X is a superkey $\leftarrow\rightarrow$ X^+ contains all attributes!

Intuition

- In other words, BCNF requires that:

  ➔ Only things that functionally determine **everything** can functionally determine **anything**.

- Note:
  - FDs are not the problem. They are facts!
  - The schema (in the context of the FDs) is the problem.

R is a relation; F is a set of FDs.
Return the BCNF decomposition of R, given these FDs.

BCNF_decomposition(R, F):

If an FD $X \rightarrow Y$ in F violates BCNF

Compute X^+

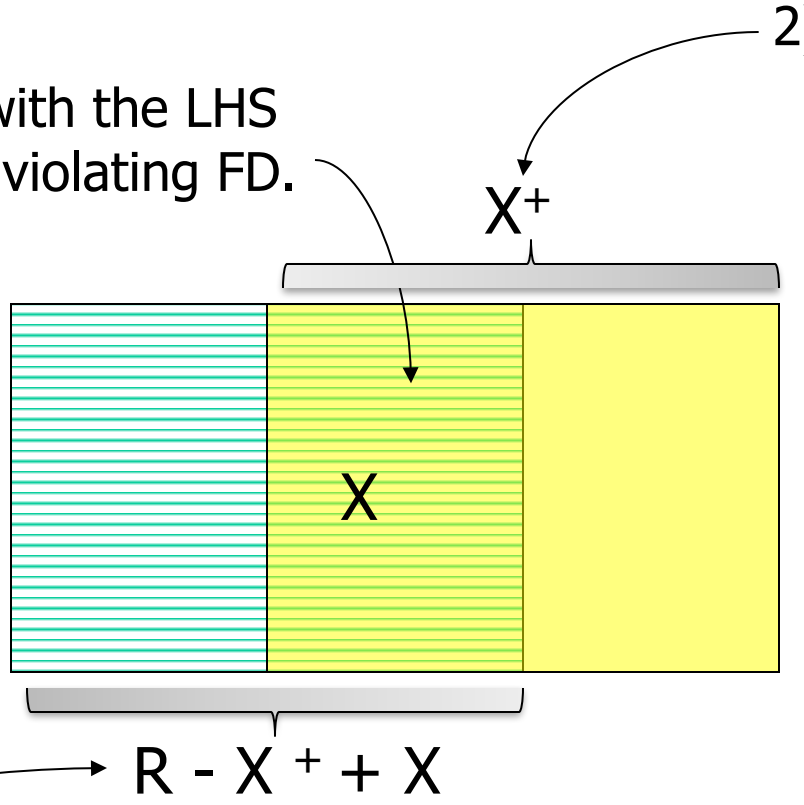
Replace R by two relations with schemas:

$$R_1 = X^+$$
$$R_2 = R - (X^+ - X)$$

Project the FD's F onto R_1 and R_2.

Recursively decompose R_1 and R_2 into BCNF.

Decomposition Picture



3) Everything except the new stuff is the other new relation.
X is in both new relations to make a connection between them.

Some comments on BCNF decomp

- If more than one FD violates BCNF, you may decompose based on any one of them.
  - So there may be multiple results possible.

- The new relations we create may not be in BCNF. We must recurse.
  - We only keep the relations at the "leaves".

- How does the decomposition step help?

Speed-ups for BCNF decomposition

- Don’t need to know any keys.
  - Only superkeys matter.

- And don’t need to know all superkeys.
  - Only need to check whether the LHS of each FD is a superkey.
  - Use the closure test (simple and fast!).

BCNF

- Every attribute depends on:
  - The key
  - The whole key
  - And nothing but the key…

More speed-ups

- When projecting FDs onto a new relation, check each new FD:
  - Does the new relation violate BCNF because of this FD?

- If so, abort the projection.
  - You are about to discard this relation anyway (and decompose further).

Properties of Decompositions

What we want from a decomposition

1. No anomalies.

2. Lossless Join : It should be possible to
   a) project the original relations onto the decomposed schema
   b) then reconstruct the original by joining.
      We should get back exactly the original tuples.

3. Dependency Preservation :
   All the original FD's should be satisfied.

What is lost in a "lossy" join?

- For any decomposition, it is the case that:
  - $r \subseteq r_1 \bowtie \ldots \bowtie r_n$
  - I.e., we will get back every tuple.
- But it may not be the case that:
  - $r \supseteq r_1 \bowtie \ldots \bowtie r$
  - I.e., we can get spurious tuples.
- [Exercise]

What BCNF decomposition offers

1. No anomalies : ✓ (Due to no redundancy)
2. Lossless Join : ✓
3. Dependency Preservation : ✗

The BCNF property does not guarantee lossless join

- If you use the BCNF decomposition algorithm, a lossless join is guaranteed.

- If you generate a decomposition some other way
  - you have to check to make sure you have a lossless join

➔ We'll learn an algorithm for this check later.

Preservation of dependencies

- BCNF decomposition does not guarantee preservation of dependencies.

- I.e., in the schema that results, it may be possible to create an instance that:
  - satisfies all the FDs in the final schema, but violates one of the original FDs.

- Why? Because the algorithm goes too far — breaks relations down too much.

3NF is less strict than BCNF

- 3rd Normal Form (3NF) modifies the BCNF condition to be less strict.
- An attribute is prime if it is a member of any key.
- $X \rightarrow A$ violates 3NF iff
  X is not a superkey and A is not prime.
- **It's ok if X is not a superkey as long as A is prime.**
- [Exercise]

F is a set of FDs; L is a set of attributes.
Synthesize and return a schema in 3rd Normal Form.

3NF_synthesis(F, L):

Construct a minimal basis M for F.

For each FD X $\rightarrow$Y in M

Define a new relation with schema X $\cup$ Y.

If no relation is a superkey for L

Add a relation whose schema is some key.

[Example]

3NF synthesis doesn't "go too far"

- BCNF decomposition doesn't stop decomposing until in all relations:
  - if $X \rightarrow A$ then X is a superkey.

- 3NF generates relations where:
  - $X \rightarrow A$ and yet X is not a superkey, but A is at least prime.

- [Example]

What a 3NF decomposition offers

1. No anomalies : ✗
2. Lossless Join : ✓
3. Dependency Preservation : ✓

- Neither BCNF nor 3NF can guarantee all three! We must be satisfied with 2 of 3.
- Decompose too far ⇒ can't enforce all FDs.
- Not far enough ⇒ can have redundancy.
- We consider a schema "good" if it is in either BCNF or 3NF.

How can we get anomalies?

- 3NF synthesis guarantees that the resulting schema will be in 3rd normal form.
- This allows FDs with a non-superkey on the LHS.
- This allows redundancy, and thus anomalies.

How do we know…?

… that the algorithm guarantees:

- 3NF: A property of minimal bases [see the textbook for more]
- Preservation of dependencies: Each FD from a minimal basis is contained in a relation, thus preserved.
- Lossless join

“Synthesis” vs “decomposition”

- 3NF synthesis:

  ➔ We "build up" the relations in the schema.

- BCNF decomposition:

  ➔ We start with a bad relation schema and break it down.

Testing for a Lossless Join

- If we project R onto R_1, R_2,..., R_k , can we recover R by rejoining?

➔We will get all of R.

  - Any tuple in R can be recovered from its projected fragments. This is guaranteed.

- But will we get only R?
  - Can we get a tuple we didn't have in R? This part we must check.

When we don't need to test for lossless Join

- Both BCNF decomposition and 3NF synthesis guarantee lossless join.
- We don't need to test for lossless join if the schema was generated via BCNF decomposition or 3NF synthesis.
- But merely satisfying BCNF or 3NF does not guarantee a lossless join!

The Chase Test

- Suppose tuple t appears in the join.
- Then t is the join of projections of some tuples of R, one for each R_i of the decomposition.
- Can we use the given FD's to show that one of these tuples must be t ?

Setup for the Chase Test

- Start by assuming t = abc… .
- For each i, there is a tuple s_i of R that has a, b, c,… in the attributes of R_i.
- s_i can have any values in other attributes.
- We'll use the same letter as in t, but with a subscript, for these components.

The algorithm

1. If two rows agree in the left side of a FD, make their right sides agree too.

2. Always replace a subscripted symbol by the corresponding unsubscripted one, if possible.

3. If we ever get a completely unsubscripted row, we know any tuple in the project-join is in the original (i.e., the join is lossless).

4. Otherwise, the final tableau is a counterexample (i.e., the join is lossy).

5. [Exercise]