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CSC 373 Tutorial Exercises for Week 8 Winter 2019
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1. "Duckwheat" is produced in Kansas and Mexico and consumed in New York and California. Each month, Kansas produces 15 "shnupells" of duckwheat and Mexico, 8, while New York consumes 10 shnupells and California, 13. The monthly transportation costs per shnupell are $4 from Mexico to New York, $1 from Mexico to California, $2 from Kansas to New York, and $3 from Kansas to California.

   Determine the monthly amount of duckwheat to be transported from each producer to each consumer, in order to minimize the total cost of transportation.

2. Consider a set of mobile computing clients in a certain town who each need to be connected to one of several possible "base stations". We'll suppose there are n clients, with the position of each client specified by its (x,y) coordinates in the plane. There are also m base stations, each of whose position is specified by (x,y) coordinates as well.

   We wish to connect each client to exactly one base station. Our choice of connections is constrained in the following ways. There is a "range parameter" r -- a client can only be connected to a base station that is within distance r. There is also a "load parameter" L -- no more than L clients can be connected to any single base station.

   Show how to represent this problem as a linear program, and how to solve it using linear programming algorithms. Justify carefully that your solution is correct. Can you guarantee that your algorithm runs in polytime?

3. Show that the following UNARY-PRIMES decision problem belongs to P.
   Input: 1^n (i.e., a string of '1's of length n).
   Question: Is n prime?

4. Show that the following TRIANGLE decision problem belongs to P.
   Input: An undirected graph G = (V,E).
   Question: Does G contain a "triangle", i.e., a subset of three vertices with all edges between them present in the graph?

NOTE: The next two problems will make sense after Monday's lecture. Don't worry if you're not quite sure yet how to answer them!

5. Show that the following CLIQUE decision problem belongs to NP.
   Input: An undirected graph G = (V,E) and a positive integer k.
   Question: Does G contain a k-clique, i.e., a subset of k vertices with all edges between them present in the graph?

6. Show that the following SUBSET-SUM decision problem belongs to NP.
   Input: A set of positive integers S and a positive integer t.
   Question: Is there some subset of S whose sum is exactly t?