Worth: 7.5% (*best four of the five assignments*) **Due:** *before* 6:00pm on **Fri. 8 March**

Required filename for MarkUs submission: `a3.pdf`

Remember to write the *full name* and *MarkUs username* of *every* group member (up to three) prominently on your submission.

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*Please read and understand the policy on Collaboration given on the Course Information Sheet. Then, to protect yourself, list on the front of your submission **every** source of information you used to complete this homework (other than your own lecture and tutorial notes). For example, indicate clearly the **name** of every student from another group with whom you had discussions, the **title and sections** of every textbook you consulted (including the course textbook), the **source** of every web document you used (including documents from the course webpage), etc.*

*For each question, please write up detailed answers carefully. Make sure that you use notation and terminology correctly, and that you explain and justify what you are doing. Marks **will** be deducted for incorrect or ambiguous use of notation and terminology, and for making incorrect, unjustified, ambiguous, or vague claims in your solutions.*

For every problem below, give a Network Flow solution. Please write your algorithms in pseudo-code and include a high-level English description (either separately or as comments throughout your algorithm). Also, remember to justify the correctness of your solutions (this is important!) and to analyze their complexity.

1. **Matrix Puzzle**

   You want to fill a matrix of n rows and m columns with non-negative integers so that the sum of each row or each column corresponds to a predetermined number. Each matrix entry also has an upper bound constraint. Specifically, given **non-negative** integers $r_1, \dots, r_n$, $c_1, \dots, c_m$, and $b_{1,1}, \dots, b_{n,m}$, find integers $a_{1,1}, \dots, a_{n,m}$ such that:

   (a) $0 \leqslant a_{i,j} \leqslant b_{i,j}$ for all $1 \leqslant i \leqslant n$, $1 \leqslant j \leqslant m$;

   (b) $r_i = \sum_{k=1}^{m} a_{i,k}$ for all $1 \leqslant i \leqslant n$;

   (c) $c_j = \sum_{k=1}^{n} a_{k,j}$ for all $1 \leqslant j \leqslant m$.

2. **Knight Coexistence**

   You want to place knights on an $n \times n$ chessboard. The chessboard has n^2 positions from $(0,0)$ to $(n-1, n-1)$. A knight at position (x,y) is able to attack up to eight different positions: $(x+2, y+1)$, $(x+2, y-1)$, $(x+1, y+2)$, $(x+1, y-2)$, $(x-1, y+2)$, $(x-1, y-2)$, $(x-2, y+1)$, and $(x-2, y-1)$—for each position that lies on the board, of course. Moreover, some positions are blocked so that you cannot place knights on them. Given n and a list of all blocked positions, find a way to place as many knights as possible on the chessboard so that the placed knights cannot attack each other.

One or two more problems will follow, by the end of Reading Week at the latest...