Worth: 7.5% (*best four of the five assignments*) **Due:** *before* 6:00pm on **Fri. 8 Feb.**

Required filename for MarkUs submission: `a2.pdf`

Remember to write the *full name* and *MarkUs username* of *every* group member (up to three) prominently on your submission.

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*Please read and understand the policy on Collaboration given on the Course Information Sheet. Then, to protect yourself, list on the front of your submission **every** source of information you used to complete this homework (other than your own lecture and tutorial notes). For example, indicate clearly the **name** of every student from another group with whom you had discussions, the **title and sections** of every textbook you consulted (including the course textbook), the **source** of every web document you used (including documents from the course webpage), etc.*

*For each question, please write up detailed answers carefully. Make sure that you use notation and terminology correctly, and that you explain and justify what you are doing. Marks **will** be deducted for incorrect or ambiguous use of notation and terminology, and for making incorrect, unjustified, ambiguous, or vague claims in your solutions.*

For every problem, give a Dynamic Programming solution, following the steps outlined in class. Don't forget to justify that your recurrence relation is correct based on the recursive structure of the problem, and to analyze the time complexity of your final algorithm.

1. **Riding a bicycle in Toronto**

   Suppose the streets in some section of Toronto are modelled as a simple grid, and you want to travel from the lower-left corner (coordinate $(0,0)$) to the upper-right corner (coordinate (m,n)). Further, to simplify the problem, suppose that every street is one-way so you can only travel right or up (never left and never down).

   But riding a bicycle in Toronto is dangerous! So every street segment has a probability that riding on that segment will result in an accident. Let $p_{i,j}$ be the probability that riding right from coordinate $(i-1,j)$ to (i,j) results in an accident, and $q_{i,j}$ be the probability that riding up from coordinate $(i,j-1)$ to (i,j) results in an accident. As usual, all probabilities are real numbers in the interval $[0,1]$. The probability that you have **no** accident on a path is simply the *product* of the probabilities that you have no accident on each street segment of the path.

   Your goal is to find a path from $(0,0)$ to (m,n) with the maximum probability of having no accident.

2. **Study group lunch**

   Suppose you want to buy B burgers, C fries, and D drinks at your favourite fast-food place, for a study group you're organizing (this question is most certainly not about good eating habits)! The unit price of each item is p_b (for burgers), p_c (for fries), and p_d (for drinks). You also own n coupons $N_1,\dots,N_n$. Each coupon N_i is a combo that offers you b_i burgers, c_i fries, and d_i drinks at price p_i. Each coupon can only be used at most once. NOTE: **All** values are natural numbers, which means some of them could be equal to zero (e.g., it's possible for a coupon to offer a deal that includes only drinks and fries, but no burger; it's also possible for a coupon to offer more burgers, or fries, or drinks than you need).

   What is the minimum amount of money you need to pay to get at least B burgers, C fries, and D drinks? It's okay if you end up with more food (or drinks) than you need.

3. Homework planning

You have n assignments $A_1, \ldots, A_n$ where $A_i = (p_i, t_i)$, $p_i \in \mathbb{Z}^+$ is the number of points earned if you complete the assignment, and $t_i \in \mathbb{R}^+$ is the time required to complete the assignment. You also have a total time $T \in \mathbb{R}^+$ available to complete all the assignments.

Find a subset of assignments that you can complete within your time limit and that maximizes the total number of points obtained. (NOTE: you cannot use arbitrary real numbers as array indices.)