

Recall that a path in a graph is simple iff it contains no repeated vertex or edge. The definition of simple cycle is similar (except, of course, that the first and last vertex are the same). Consider the following decision problems.

s-t Hamiltonian Path (st-HP):

Input: Graph  $G = (V, E)$  (directed or undirected),  
vertices  $s, t$  in  $V$ .

Output: Does  $G$  contain some simple path from  $s$  to  $t$  that includes every vertex?

Hamiltonian Path (HP):

Input: Graph  $G = (V, E)$  (directed or undirected).

Output: Does  $G$  contain some simple path that includes every vertex?

Hamiltonian Cycle (HC):

Input: Graph  $G = (V, E)$  (directed or undirected).

Output: Does  $G$  contain some simple cycle that includes every vertex?

The textbook proves that HC (which they call Rudrata Cycle) is NP-complete, on pp.256-260.

Prove that HP and st-HP are also both NP-complete. You may (and should) make use of the fact that HC is NP-complete.

You may prove the NP-completeness of both problems in any order, and it is acceptable to use one problem in the proof of the other -- so give some thought to which problem you want to prove NP-complete first.