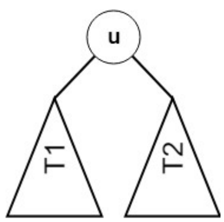
3. AVL Trees

2018年9月25日　　17:18

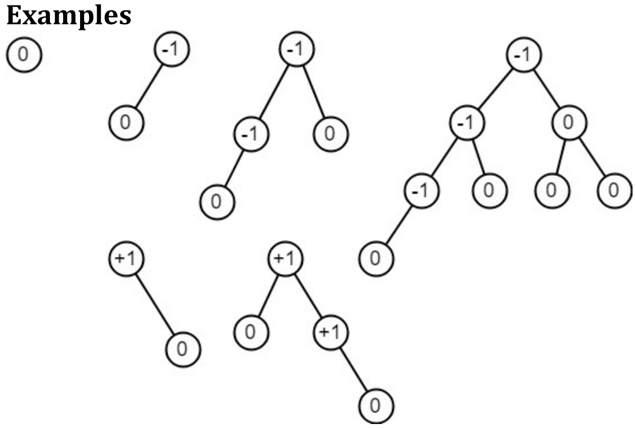
Dictionary (ADT) set of objects with keys
Operations $insert(e), delete(k), find(k)$
Data structure BST, AVL Tree, Red Black Tree, 2-3 Tree, Hash Tables

Problems with BST insert, delete, find in a BST runs in WC $O(log\ n)$ while a BST can be unbalanced

AVT Tree
Height $height(u)$ = length of the longest path from u to leaf.
$height(T) = height(root)$
define base cases
$height(one\text{-}node\text{-}BST) = 0$
$height(\emptyset) = -1$
Balance Factor $BF(u) = height(T_2) - height(T_1)$



AVT Tree a BST in which every node u, has $BF(u) \in \{-1,0,1\}$
Examples



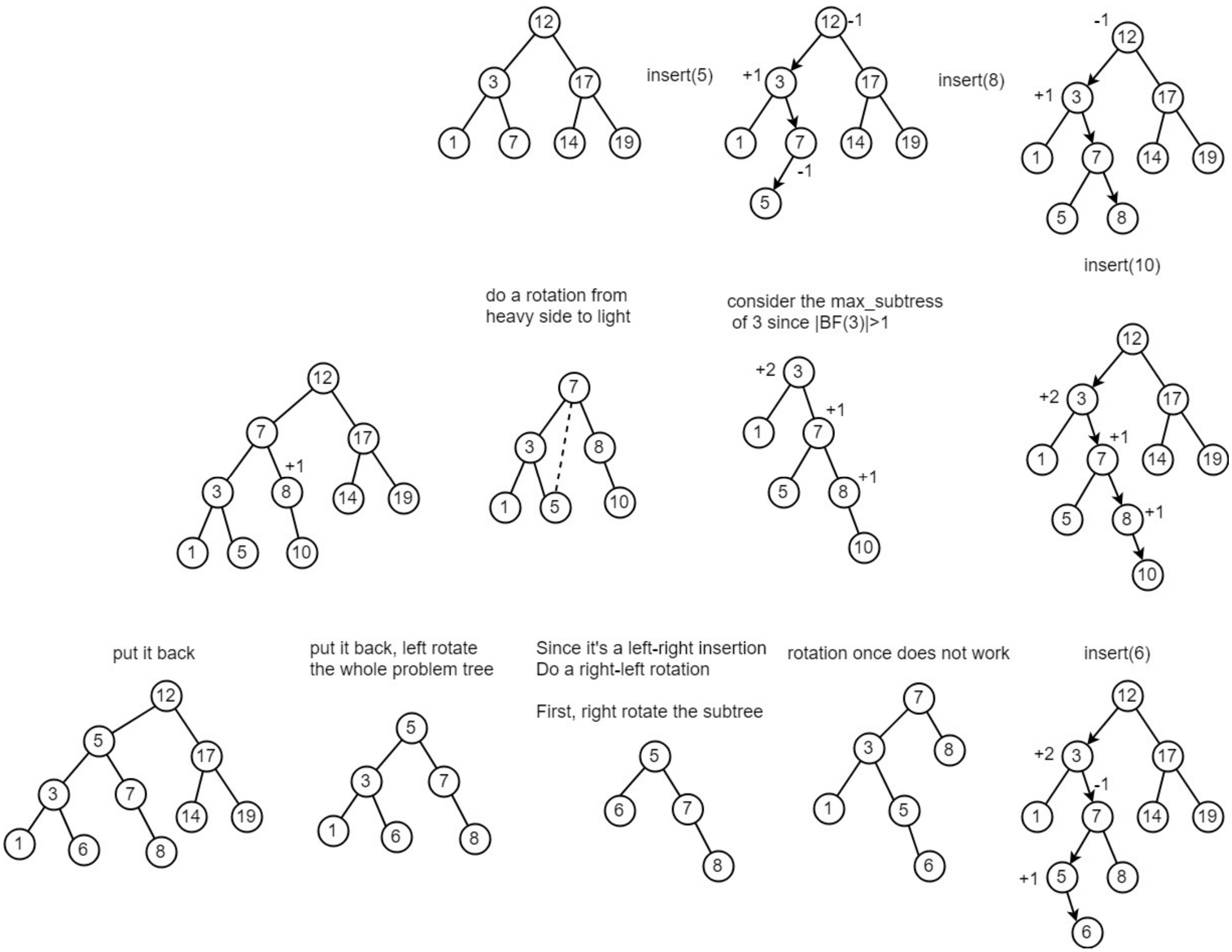
$\boldsymbol{f(h)}$ the smallest #nodes in a AVL tree of height h
Claim $f(h) = f(h-1) + f(h-2) + 1$
Proof the left subtree and the right subtree can defer height by 1, hence one tree to increase height, another not. And adding one more node for root.

Note Fibonacci series $F(0) = 0, F(1) = 1, F(x) = F(x-1) + F(x-2)$
$f(0) = 1, f(1) = 2, f(x) = f(x-1) + f(x-2) + 1$
$
Claim $f(h) = F(h+3) - 1$
Proof $f(0) = F(3) - 1 = 2 - 1 = 1, f(1) = F(4) - 1 = 3 - 1 = 2$
$f(h) = f(h-2) + f(h-1) + 1 = F(h+1) - 1 + F(h+2) - 1 + 1 = F(h+3) - 1$

Since $F(n) \geq \frac{\phi^h}{\sqrt{5}-1}, \phi = \frac{1+\sqrt{5}}{2}, \#nodes = \ n \geq f(h) \geq \frac{\phi^h}{\sqrt{5}-1}$

$\Rightarrow h \leq \log_{\phi} \sqrt{5}(n+2) - 3 \in \Theta(\log n)$

Insert(e)



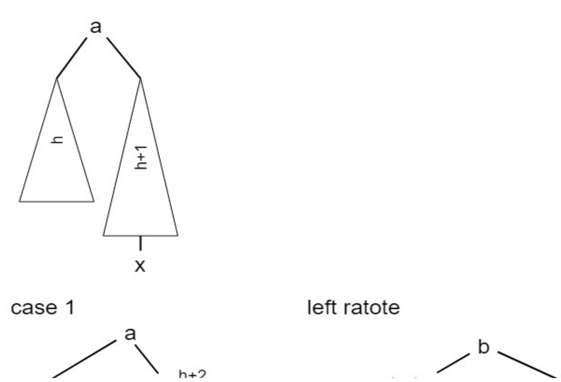
General procedure

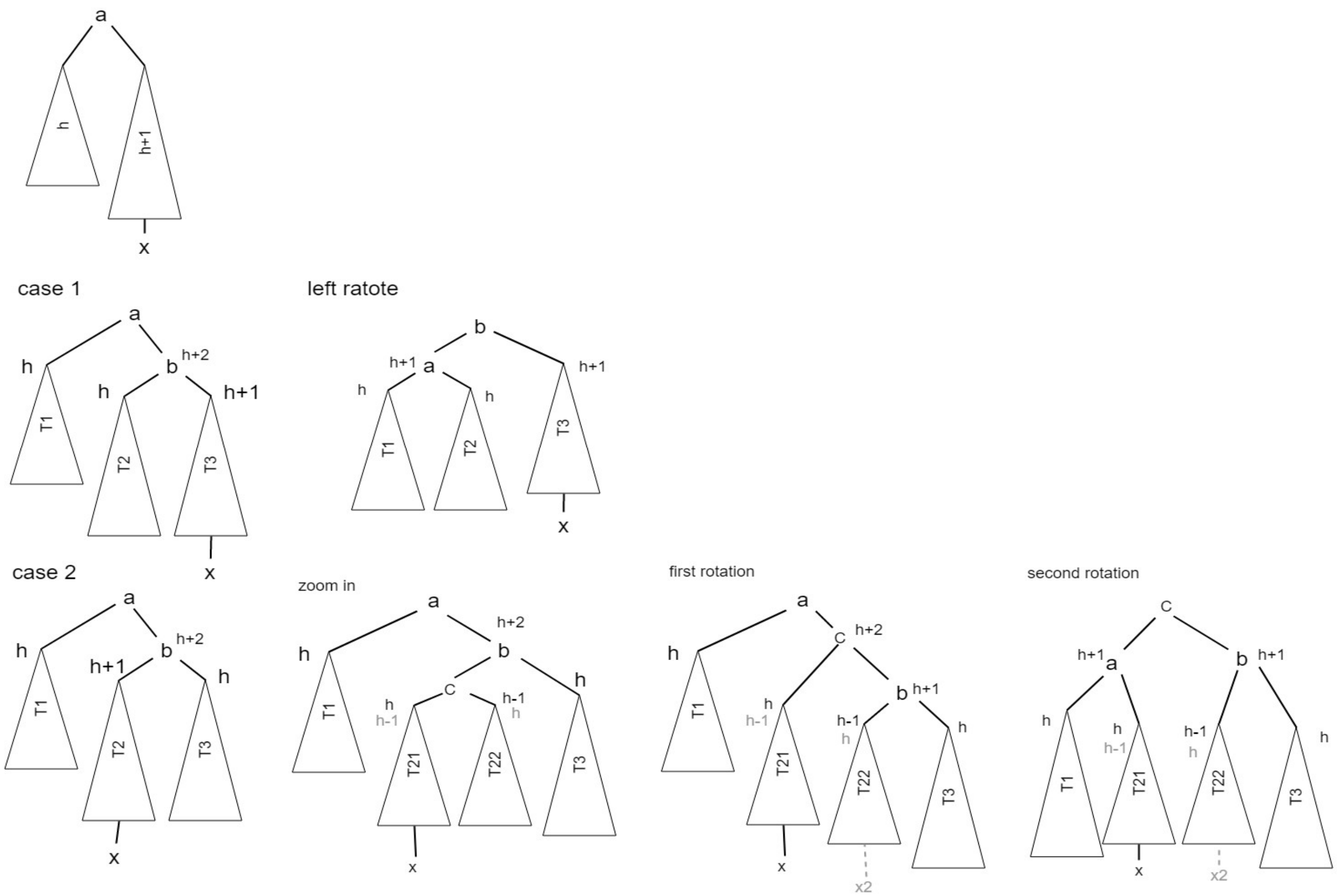
1. Insert as in any BST
2. Go up through the path from the new node to the root and update BFs
3. Look at the BFs
    a. If BF goes from $\pm 1 \rightarrow 0$, stop
    b. If $|BF| > 1$, stop and do rotations

Claim $BF(b) = 0 \text{ } before \text{ } insertion \Leftrightarrow \text{ } height(T_2) = height(T_3)$

Proof Suppose $BF(b) = 1$, then inserting x will cause $BF(b) = 2$, then go to 3b before reach *node a*

Suppose $BF(b) = -1$, then go to 3a, the program will stop





a
h
h+1
x
case 1
left ratote
a
b
h+2
h
h+1
T1
T2
T3
x
case 2
zoom in
first rotation
second rotation
C
T21
T22
h-1
x2