

1. Consider the network described by the table on the right. [This is tricky to draw in ASCII so I'll simply provide all of the information -- you should draw this for yourself, with node 'a' above, node 'd' below, and nodes 'b', 'c' in the middle (horizontally in between 's' and 't'; vertically in between 'a' and 'd').]
- | edge | capacity |
|--------|----------|
| s -> a | 10 |
| s -> b | 8 |
| s -> d | 5 |
| b -> a | 3 |
| b -> c | 10 |
| b -> d | 3 |
| a -> c | 3 |
| a -> t | 5 |
| d -> c | 3 |
| d -> t | 10 |
| c -> t | 8 |
- (a) Find a maximum flow in this network. (Hint: start with augmenting paths that are as "direct" as possible, i.e., using few edges).
- (b) Identify all forward and backward edges across cut $X_0 = (\{s, b, c, d\}, \{a, t\})$.
- (c) Compute the capacity of cut X_0 , and the flow across X_0 , based on the maximum flow you found in part (a).
- (d) Find a cut in the network above whose capacity is equal to the value of your maximum flow. Use the algorithm outlined in the proof of the Ford-Fulkerson theorem.
2. Explain carefully how to solve the maximum flow problem in a multi-source, multi-sink network -- one where there can be more than one source vertex s_1, \dots, s_k and more than one sink t_1, \dots, t_l . Give a detailed answer and justify that your solution is correct.