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CSC 373             Sample Solutions for Week  6 Tutorial         Winter 2019
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 1. (a) Augmenting path                 Residual capacity

        s -0/10-> a -0/5-> t            5

        (Indicating path in *original* network N with current flow and
        capacity. And remember: "augmenting f along path P" means to add the
        residual capacity to each forward edge, and to subtract it from each
        backward edge. In this case, set f(s,a) = 5, f(a,t) = 5.)

        s -0/5-> d -0/10-> t            5

        s -0/8-> b -0/10-> c -0/8-> t   8

        s -5/10-> a -0/3-> c <-8/10- b -0/3-> d -5/10-> t   3

        No more augmenting paths.

        Final flows:
                f(s,a) = 8        f(b,a) = 0        f(a,t) = 5
                f(s,b) = 8        f(b,c) = 5        f(d,c) = 0
                f(s,d) = 5        f(b,d) = 3        f(d,t) = 8
                                  f(a,c) = 3        f(c,t) = 8
        Flow value |f| = f(s,a) + f(s,b) + f(s,d) = 8 + 8 + 5 = 21.

    (b) Forward edges across X_0: (s,a), (b,a), (c,t), (d,t).
        Backward edges across X_0: (a,c).

    (c) Capacity of cut X_0:
            c(X_0) = c(s,a) + c(b,a) + c(c,t) + c(d,t)
                   = 10 + 3 + 8 + 10
                   = 31.
        Flow across cut X_0:
            f(X_0) = f(s,a) + f(b,a) + f(c,t) + f(d,t) - f(a,c)
                   = 8 + 0 + 8 + 8 - 3
                   = 21.

    (d) Start with X_1 = ({s}, {a,b,c,d,t}) and flow from part (a).
        Edge (s,a) crosses cut forward with residual capacity 2, so set
        X_1 = ({s,a}, {b,c,d,t}).
        All edges forward across cut have f = c: (s,b), (s,d), (a,c), (a,t).
        All edges backward across cut have f = 0: (b,a).
        Cut X_1 = ({s,a},{b,c,d,t}) has capacity
        c(X_1) = c(s,d) + c(s,b) + c(a,c) + c(a,t)
               = 5 + 8 + 3 + 5
               = 21
               = |f|.


 2. Add "super-source" s with edges (s,s_1), ..., (s,s_k) each of capacity
    oo; add "super-sink" t with edges (t_1,t), ..., (t_l,t) each of capacity
    oo. (Instead of using oo, can set capacity to sum of outgoing/incoming
    capacities).
    Max flow in resulting network N' = max flow in original network N:
      - any flow in original network can be extended to a flow in resulting
        network (for new edges from super-source to source, set flow equal
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to total flow out of source; for new edges from sink to super-sink, set flow equal to total flow into sink) -- hence, max flow in N' >= max flow in N;

- any flow in N' induces flow in N (flow out of every source and into every sink limited only by edges in original network because of "infinite" capacities on new edges) -- hence, max flow in N >= max flow in N'.