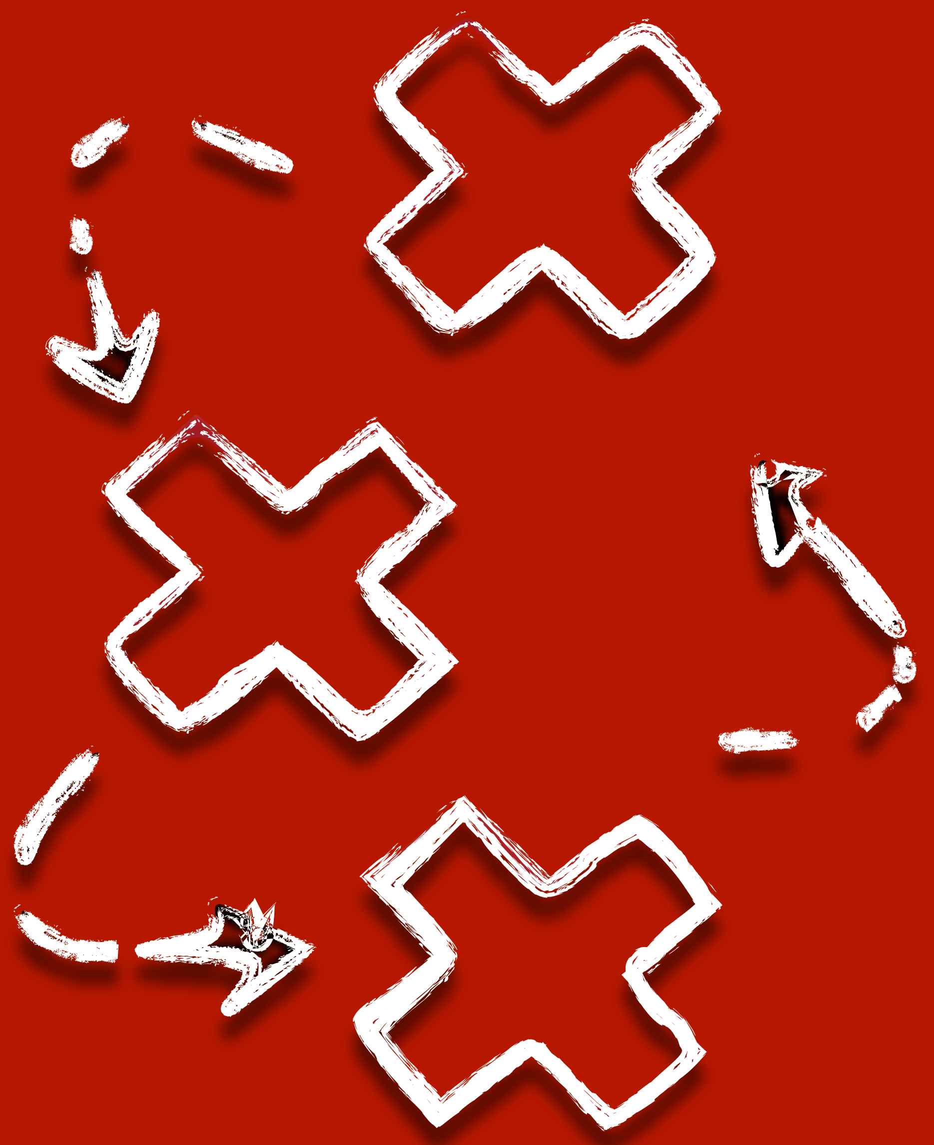


Causal Data Science

Lecture 3.0: Solutions to the previous quizzes

Lecturer: Sara Magliacane

UvA - Spring 2024

Exercise 1 : Conditional probabilities

- X is dice 1, Y is dice 2. If X <=3, then we fix Y=1, otherwise we cast dice 2.

$p(X,Y)$

Y \ X	1	2	3	4	5	6
6	0	0	0	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
5	0	0	0	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
4	0	0	0	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
3	0	0	0	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
2	0	0	0	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
1	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$

$p(X=6, Y=2) = 1/36$

1 Multiple choice 1 point

What is P(X=1| Y=2)?

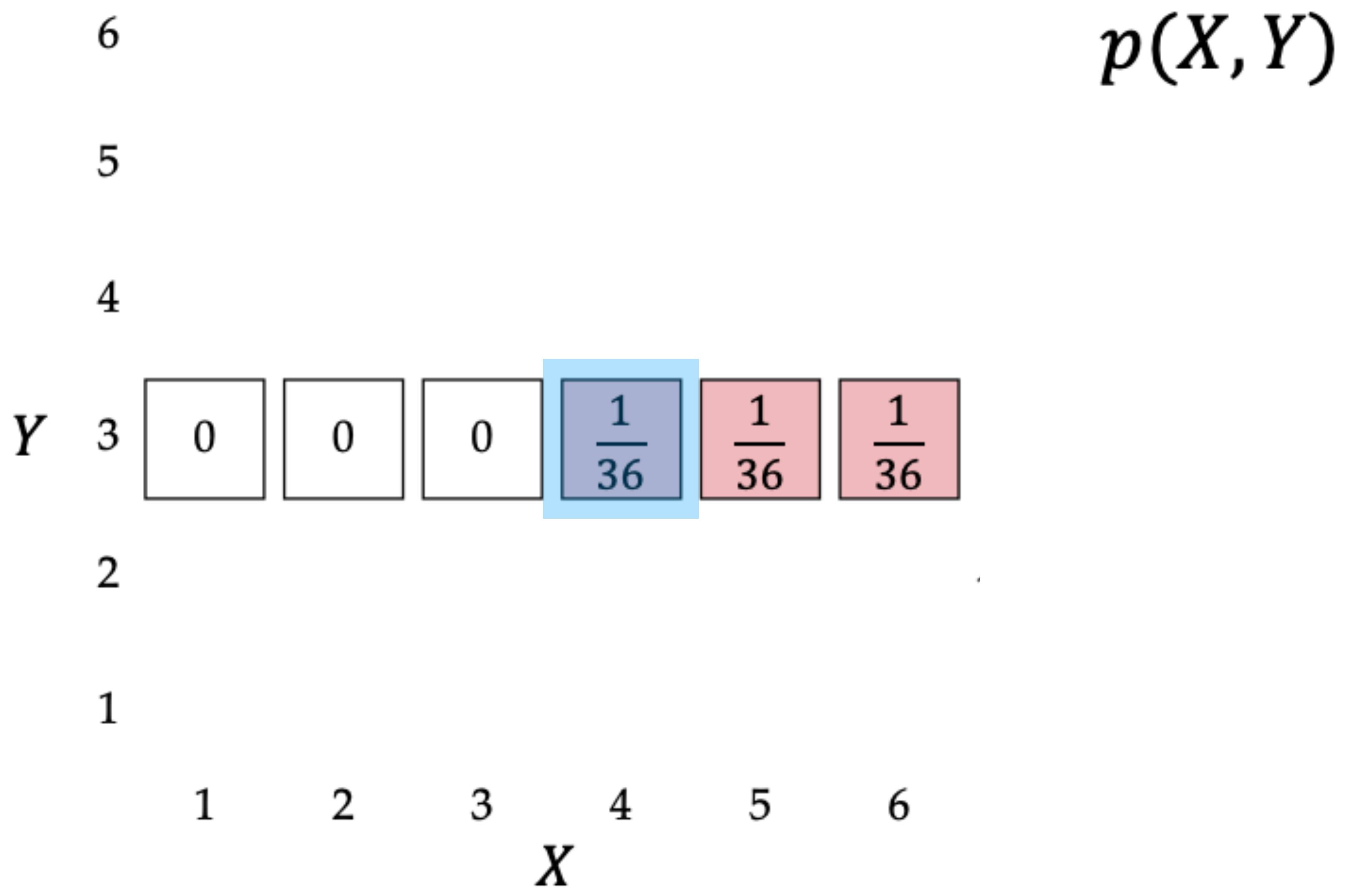
- 1/36
- 0
- 1/6
- 2

$$P(X = x \mid Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)}$$

$$P(X = 1 \mid Y = 2) = \frac{P(X = 1, Y = 2)}{P(Y = 2)} = \frac{0}{P(Y = 2)} = 0$$

Exercise 1 : Conditional probabilities

- X is dice 1, Y is dice 2. If X <=3, then we fix Y=1, otherwise we cast dice 2.



2 Multiple choice 1 point

What is P(X=4|Y=3)?

- 1/3
- 1/6
- 0
- 1/36

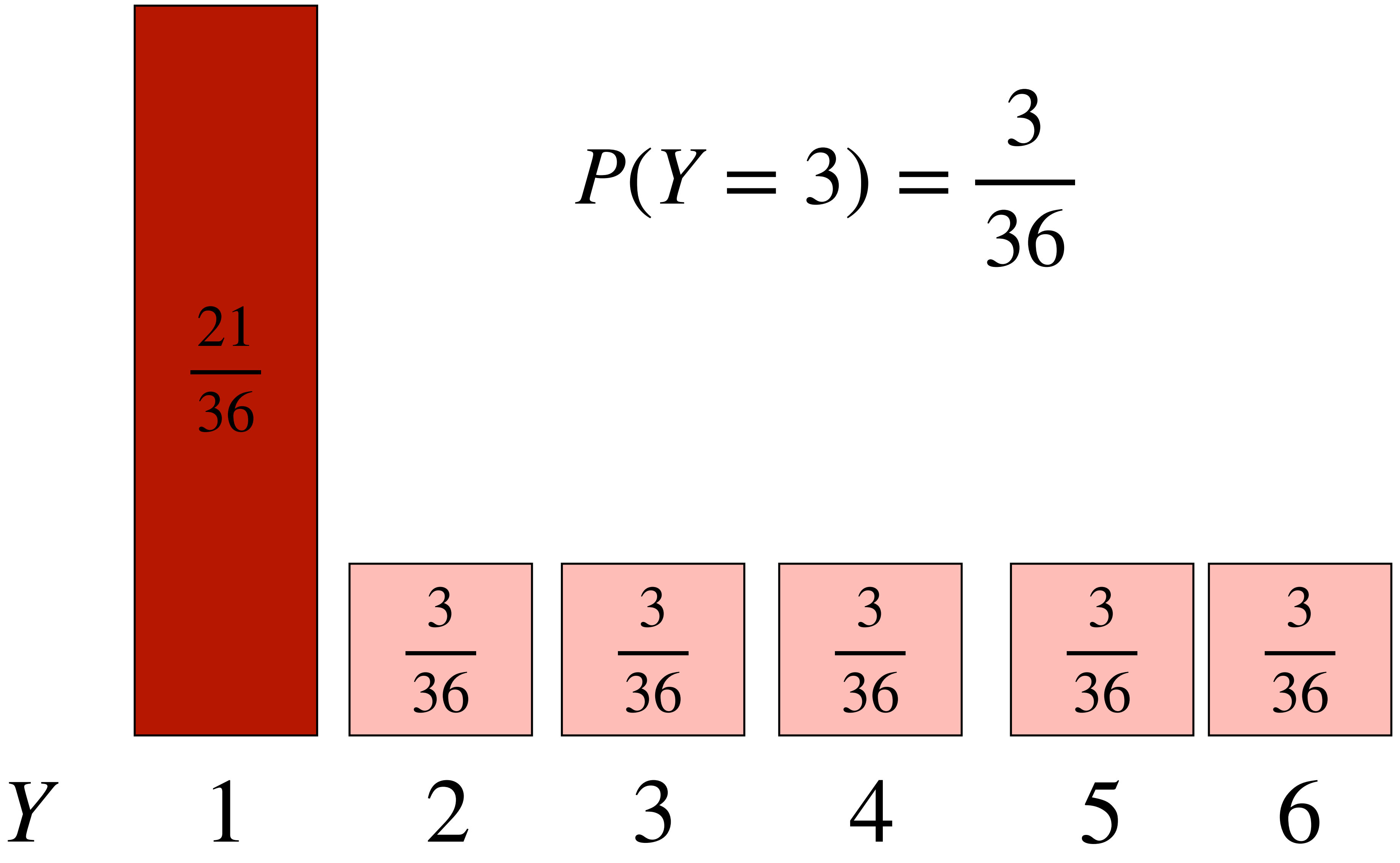
$$P(X=x \mid Y=y) = \frac{P(X=x, Y=y)}{P(Y=y)} \qquad P(X=4 \mid Y=3) = \frac{P(X=4, Y=3)}{P(Y=3)} = \frac{\frac{1}{36}}{P(Y=3)}$$

Marginal distributions

Y \ X	1	2	3	4	5	6
6	0	0	0	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
5	0	0	0	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
4	0	0	0	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
3	0	0	0	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
2	0	0	0	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
1	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$

$$P(Y = y) = \sum_{x \in \mathcal{X}} P(X = x, Y = y)$$

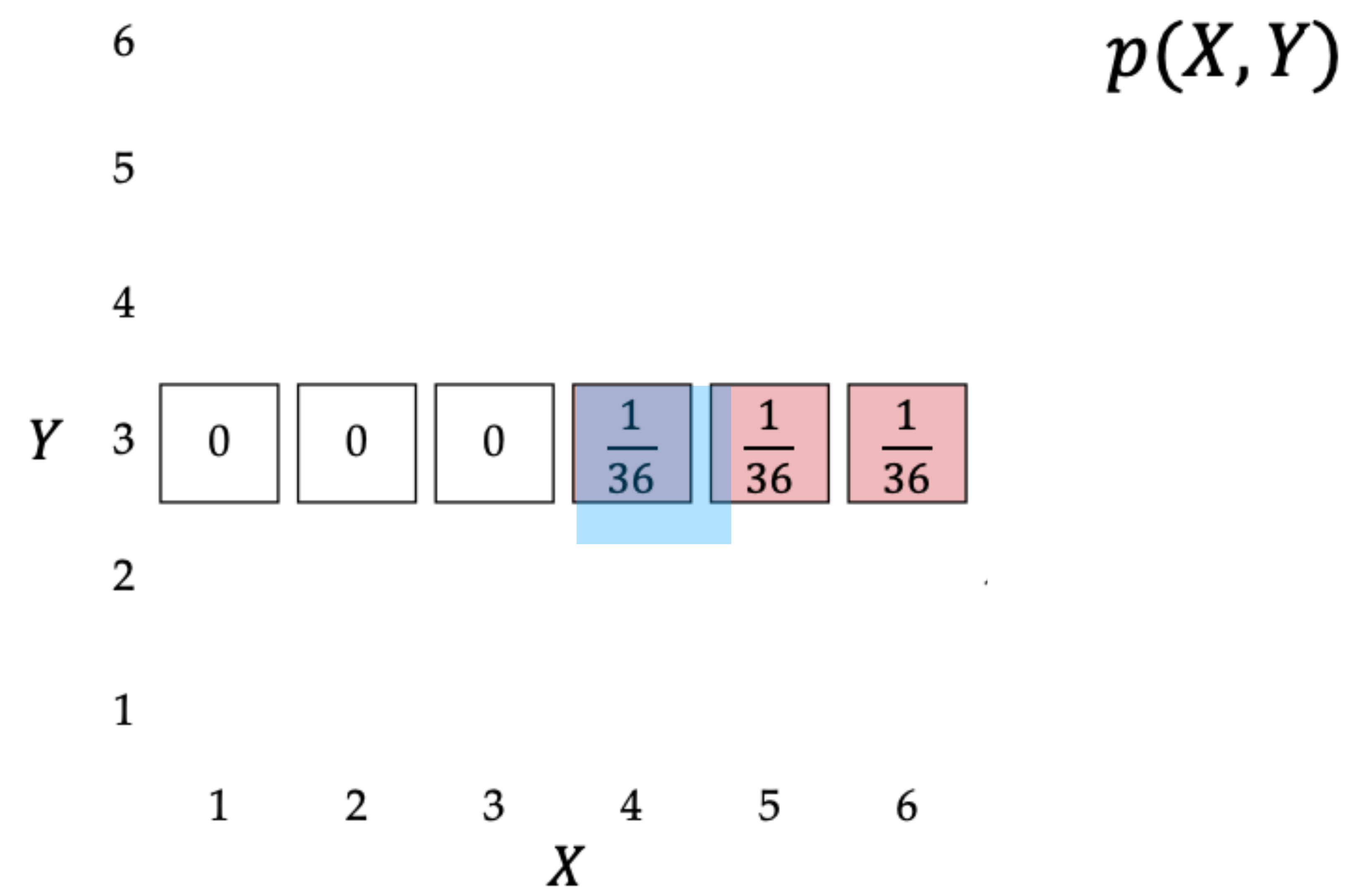
$$P(Y = 3) = \frac{3}{36}$$

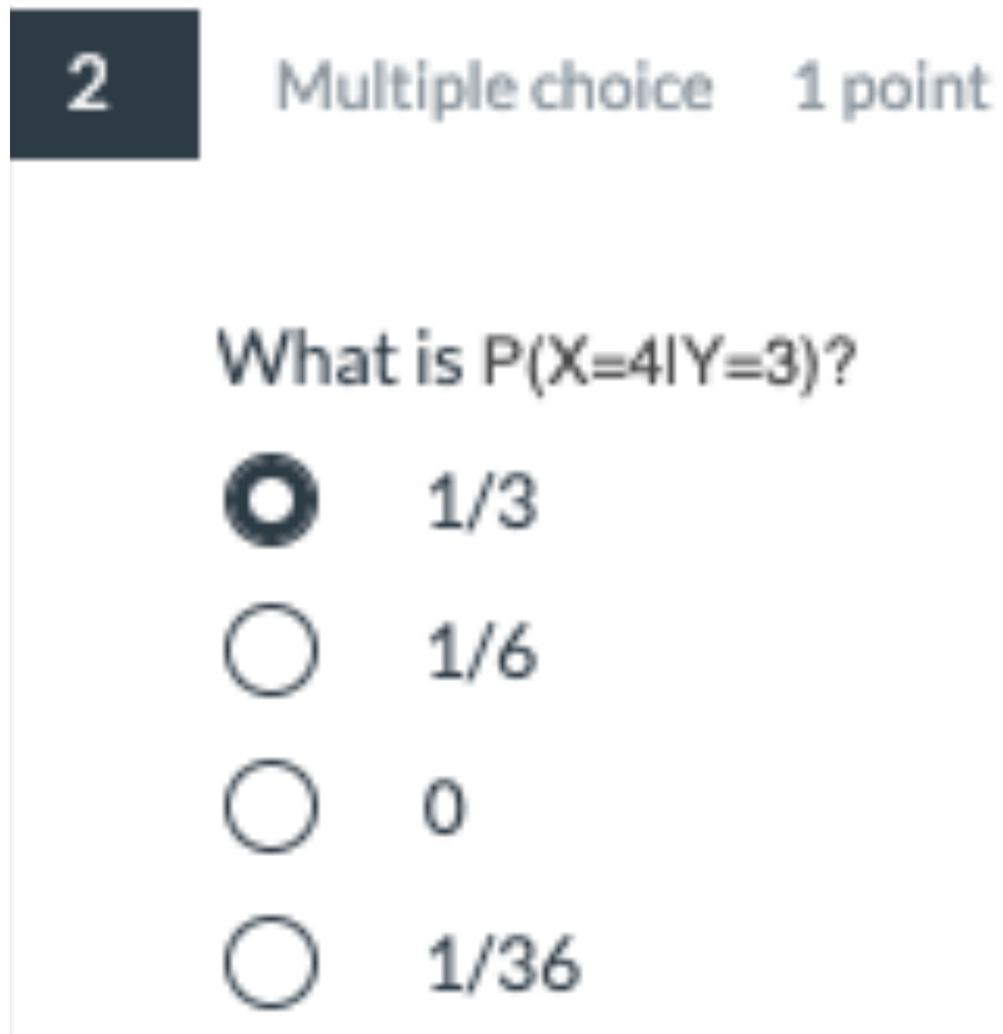


example by Fredrik Johansson

Exercise 1 : Conditional probabilities

- X is dice 1, Y is dice 2. If X <=3, then we fix Y=1, otherwise we cast dice 2.

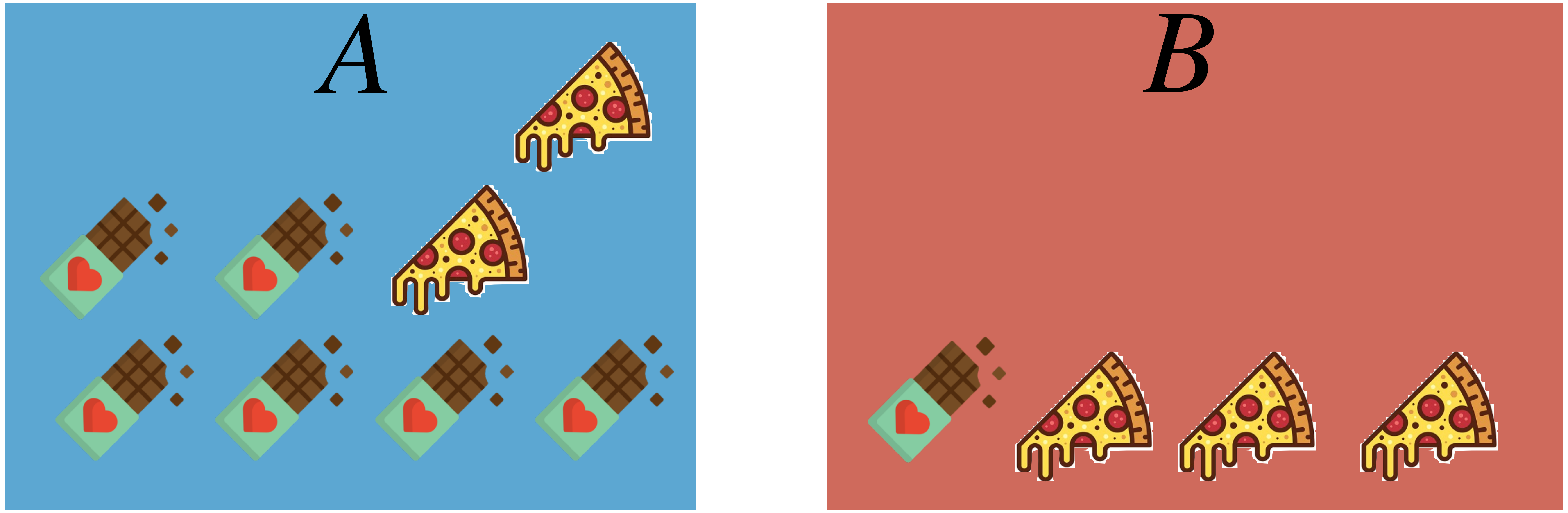




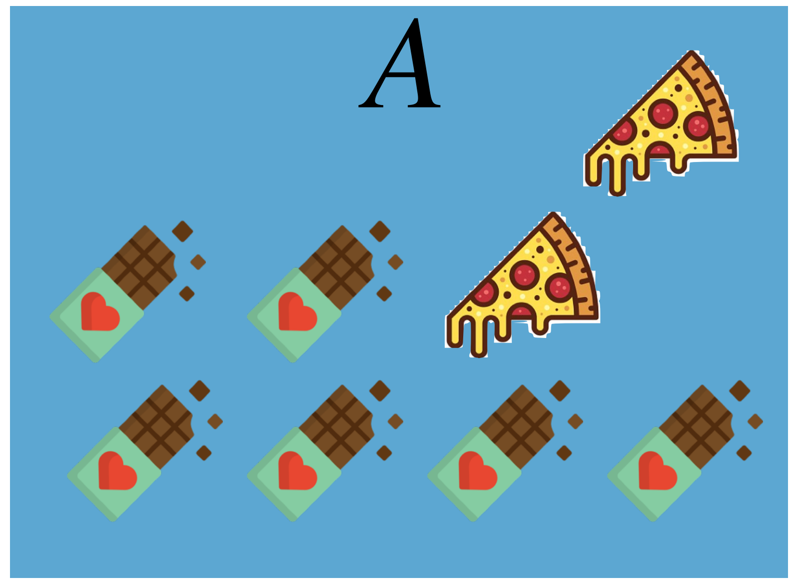
$$P(X = x \mid Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)}$$

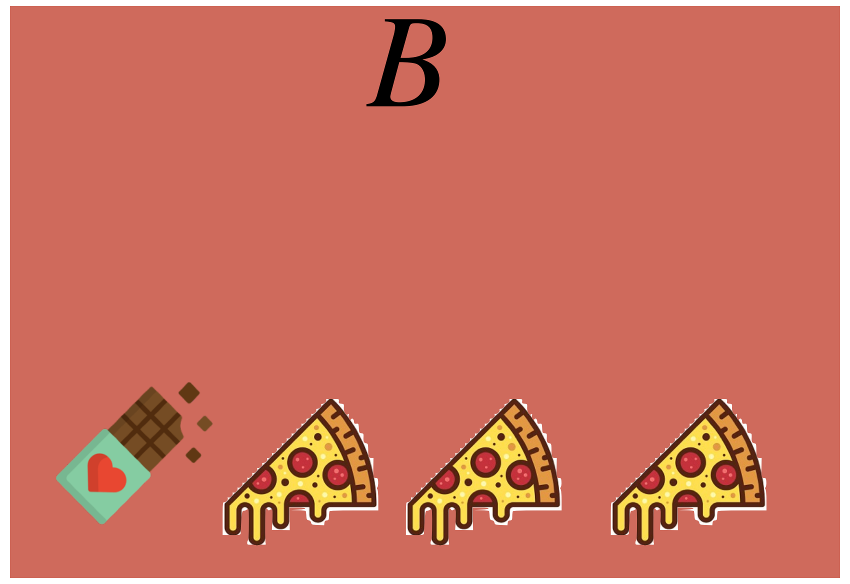
$$P(X = 4 \mid Y = 3) = \frac{P(X = 4, Y = 3)}{P(Y = 3)} = \frac{\frac{1}{36}}{\frac{3}{36}} = \frac{1}{3}$$

Exercise 2: Bayes theorem



Exercise 2: Bayes theorem





$$P(Y \mid X) = \frac{P(X, Y)}{P(X)} = \frac{P(X \mid Y)P(Y)}{P(X)}$$

1 Multiple choice 0.5 points Compute the conditional probability based on the image

Compute the conditional probability of P(item=pizza | box=A)

- 1/4
- 2/7
- 6/8
- 2/6

$$P(item = pizza \mid box = A) = \frac{2}{8}$$

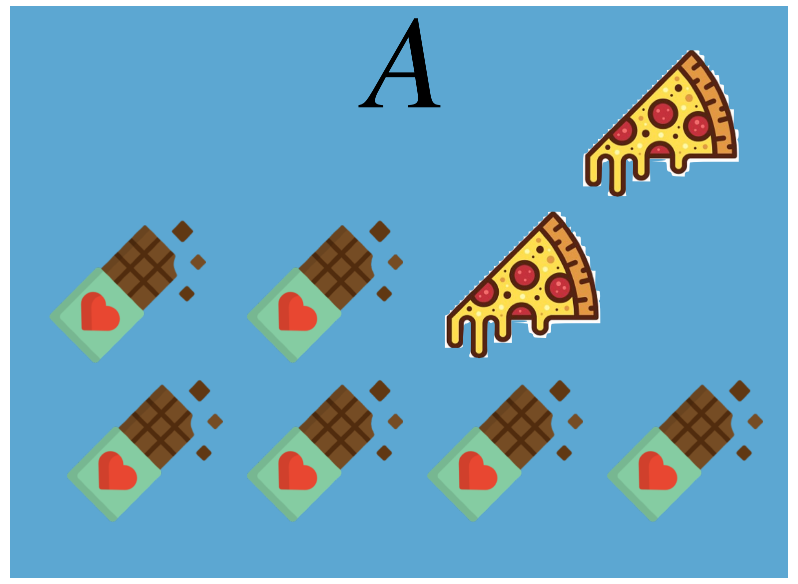
2 Multiple choice 0.5 points

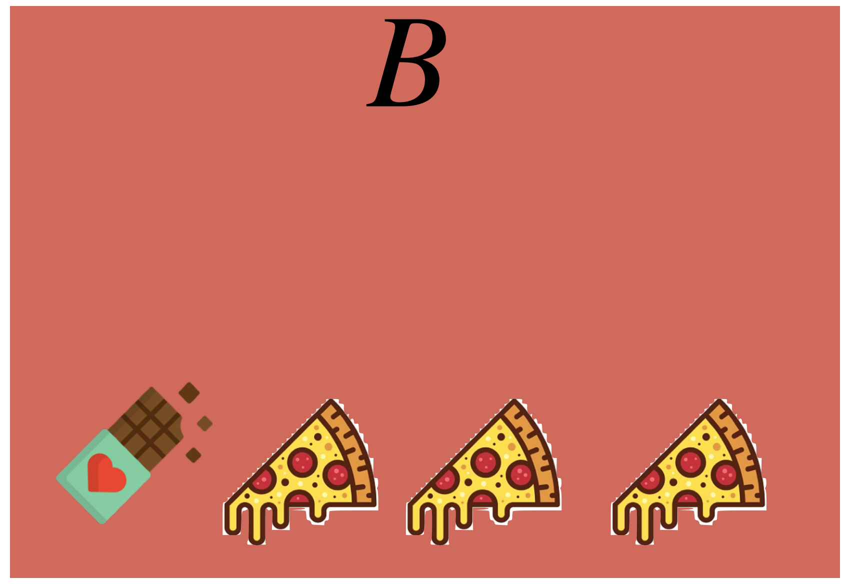
Compute the conditional probability of P(item = pizza | box = B)

- 3/4
- 1/4
- 1/3
- 1

$$P(item = pizza \mid box = A) = \frac{3}{4}$$

Exercise 2: Bayes theorem





$$P(Y|X) = \frac{P(X,Y)}{P(X)} = \frac{P(X|Y)P(Y)}{P(X)}$$

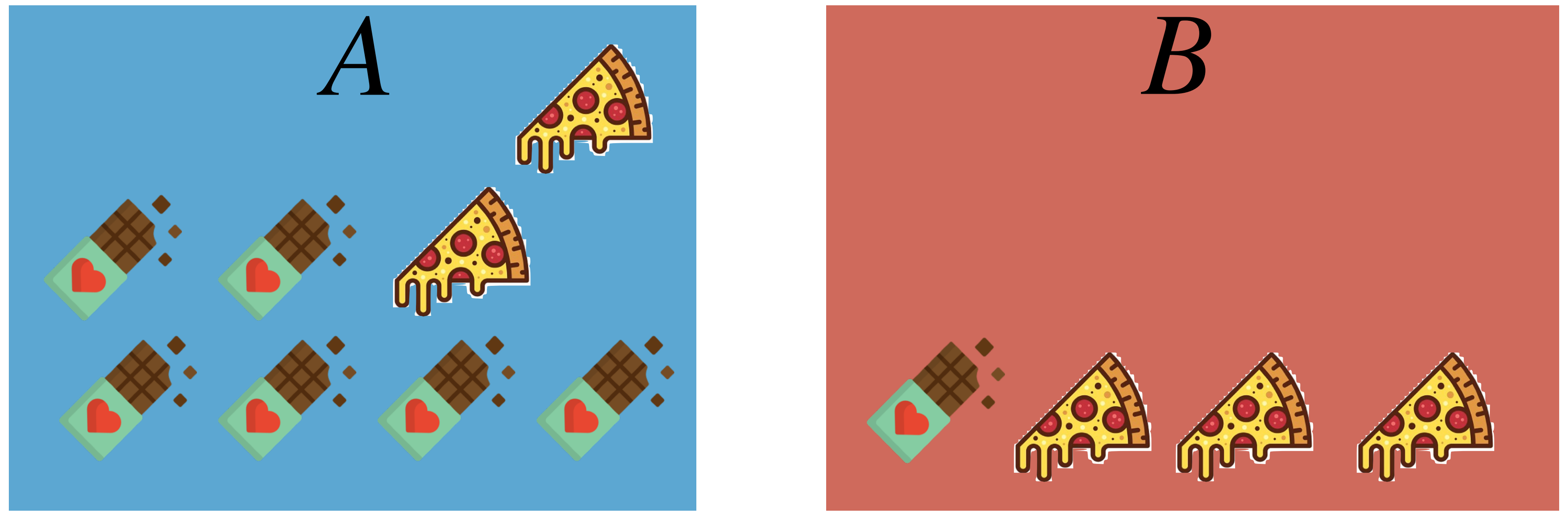
3 0.5 points

Given that the prior probability of picking box A is 0.4, i.e. P(box=A)=0.4, and P(box=B)=0.6, compute the marginal probability of picking a pizza, i.e. P(item="pizza")

- ○ 9/20
- ◉ 11/20
- ○ 3/4
- ○ 1/4

$$\begin{aligned} P(\text{item} = \text{pizza}) = P(\text{item} = \text{pizza}\,|\,\text{box} = \text{A}) \cdot P(\text{box} = \text{A}) \\ + P(\text{item} = \text{pizza}\,|\,\text{box} = \text{B}) \cdot P(\text{box} = \text{B}) \\ = \frac{2}{8} \cdot \frac{4}{10} + \frac{3}{4} \cdot \frac{6}{10} = \frac{2}{20} + \frac{9}{20} = \frac{11}{20} \end{aligned}$$

Exercise 2: Bayes theorem



$$P(Y|X) = \frac{P(X,Y)}{P(X)} = \frac{P(X|Y)P(Y)}{P(X)}$$

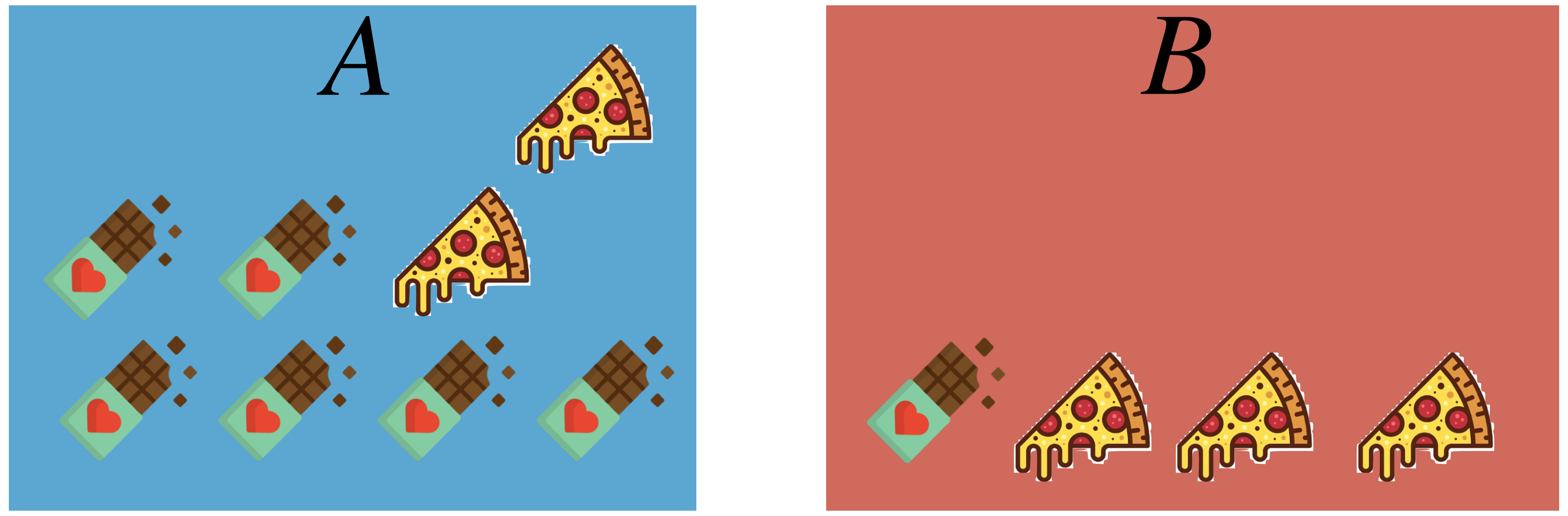
4 0.5 points

Similarly, for the same setting in which the prior probability of picking box A is 0.4, i.e. P(box=A)=0.4, and P(box=B)=0.6, compute the marginal probability of picking a **CHOCOLATE**, i.e. P(item="chocolate")

- (selected) 9/20
- 11/20
- 3/4
- 1/4

$$P(\text{item} = \text{chocolate}) = 1 - P(\text{item} = \text{pizza}) = 1 - \frac{11}{20} = \frac{9}{20}$$

Exercise 2: Bayes theorem



$$P(Y \mid X) = \frac{P(X, Y)}{P(X)} = \frac{P(X \mid Y)P(Y)}{P(X)}$$

5 1 point

We are still in the same setting, with the prior probability of picking box A is 0.4, i.e. P(box=A)=0.4, and P(box=B)=0.6. You picked a pizza, what is it the probability it came from box A. In other words, can you compute P(box=A | item=pizza)?

- ○ 7/12
- ○ 2/3
- ○ 1/3
- ○ 5/12
- ◉ 2/11

Q1 given

$$P(\text{box} = \text{A} \mid \text{item} = \text{pizza}) = \frac{P(\text{item} = \text{pizza} \mid \text{box} = \text{A}) \cdot P(\text{box} = \text{A})}{P(\text{item} = \text{pizza})}$$

Q3

$$\frac{\frac{2}{8} \cdot \frac{4}{10}}{\frac{11}{20}} = \frac{1}{10} \cdot \frac{20}{11}$$