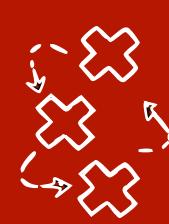


# Causal Data Science

## Lecture 3.2: Bayesian networks

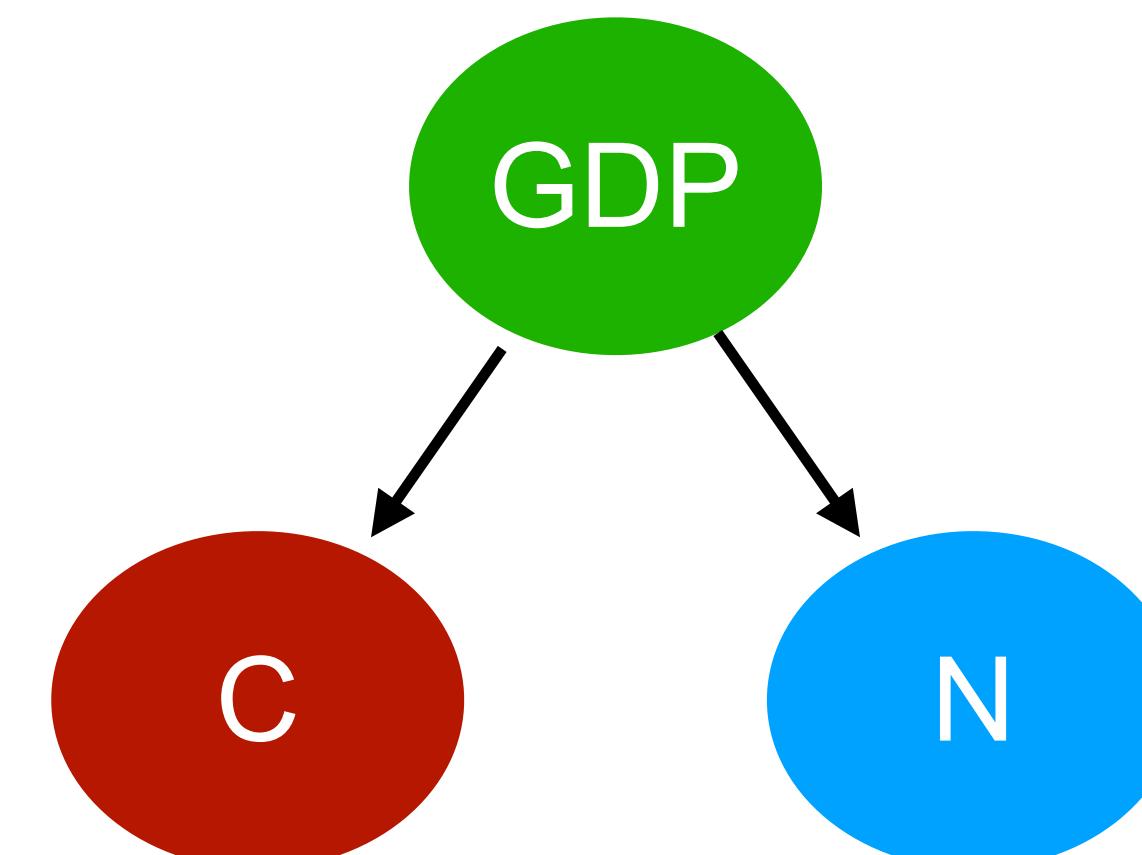
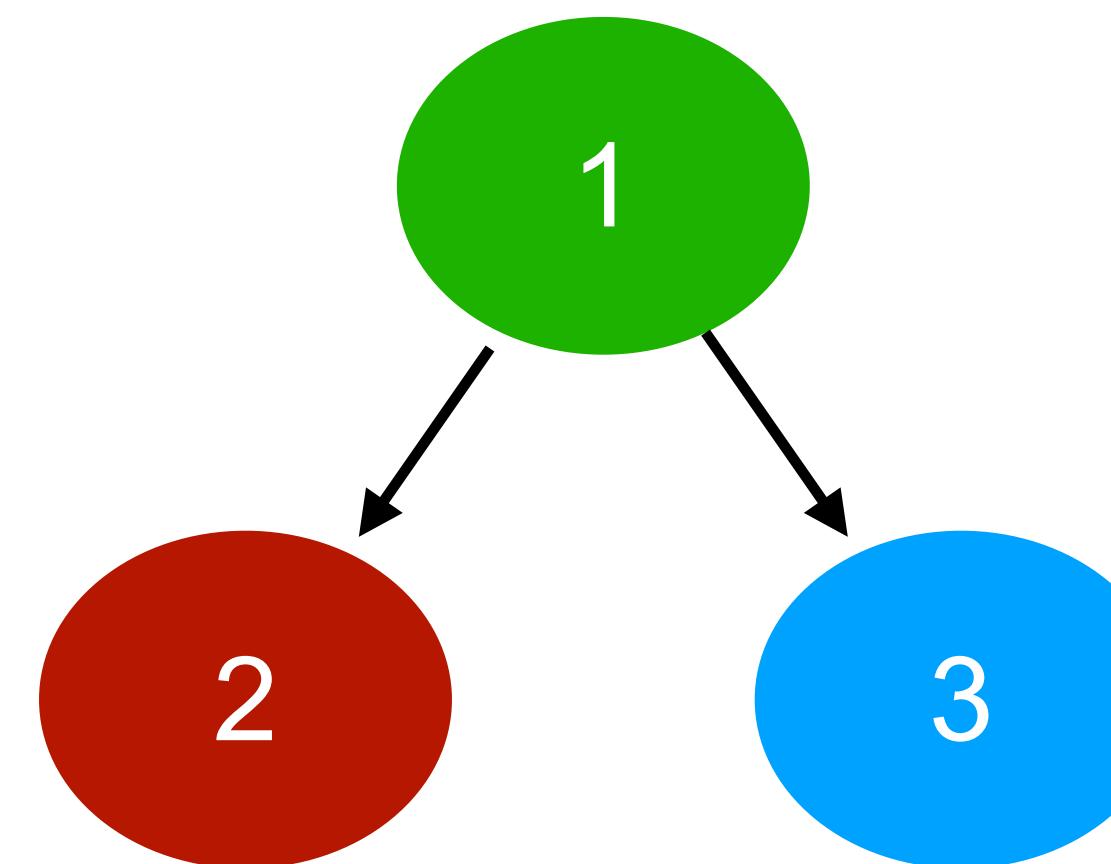
Lecturer: Sara Magliacane

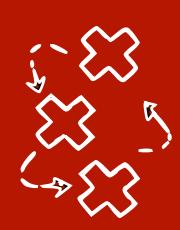
UvA - Spring 2024



# Today: graphs and random variables

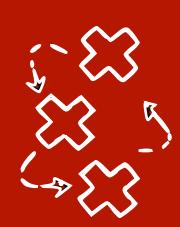
- We can represent a **(factorisation of)** joint probability as a **graph**
- **Each node  $i \in V$**  represents a **random variable  $X_i$** 
  - For  $A \subseteq V$ , we can define  $X_A := \{X_i : i \in A\}$
- **Edges** represent relationships between variables (*it will be clearer later*)





# Factorizing joint distributions

- A joint distribution can always be factorized in several ways by iterating the **chain rule**, where  $X, Y$  are disjoint sets of variables

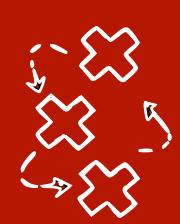


# Factorizing joint distributions

- A joint distribution can always be factorized in several ways by iterating the **chain rule**, where  $X, Y$  are disjoint sets of variables

$$P(X, Y) = P(X | Y)P(Y)$$

$$P(X = x | Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)}$$



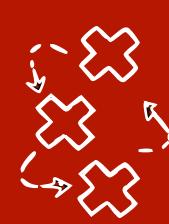
# Factorizing joint distributions

- A joint distribution can always be factorized in several ways by iterating the **chain rule**, where  $\mathbf{X}, \mathbf{Y}$  are disjoint sets of variables

$$P(\mathbf{X}, \mathbf{Y}) = P(\mathbf{X} \mid \mathbf{Y})P(\mathbf{Y})$$

- In general, given any **ordering** of the variables  $(X_1, \dots, X_p)$ , we can write:

$$P(X_1, \dots, X_p) = P(X_1)P(X_2 \mid X_1)\dots P(X_p \mid X_1, \dots, X_{p-1})$$



# Equivalent factorizations

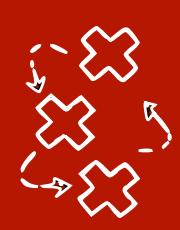
- Given any **ordering** of the variables  $(X_1, \dots, X_p)$  we can write:

$$P(X_1, \dots, X_p) = P(X_1)P(X_2 | X_1)\dots P(X_p | X_1, \dots, X_{p-1})$$

- For example  $P(X, Y, Z)$  can be equivalently factorized as:

- $P(X, Y, Z) = P(X)P(Y | X)P(Z | X, Y)$
- $P(X, Z, Y) = P(X)P(Z | X)P(Y | X, Z)$
- $P(Z, Y, X) = P(Z)P(Y | Z)P(X | Y, Z) \dots$

These are all the  
same!

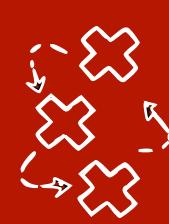


# From probability recap: Conditional independence

- $X$  is independent of  $Y$  **conditioned/given**  $Z$  iff

$$\forall x, y, z : P(X = x | Y = y, Z = z) = P(X = x | Z = z) \quad (\text{for } P(Z = z) > 0)$$

- We then write  $X \perp\!\!\!\perp Y | Z$ , otherwise  $X \not\perp\!\!\!\perp Y | Z$

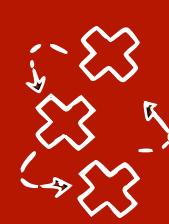


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- We then write  $X \perp\!\!\!\perp Y | Z$ , otherwise  $X \not\perp\!\!\!\perp Y | Z$
- Intuitively this means that **Y does not add any information** to predict  $X$  that isn't already offered by  $Z$
- $Z$  can be a **set of variables**, e.g.  $X \perp\!\!\!\perp Y | \{Z_1, Z_2\}$

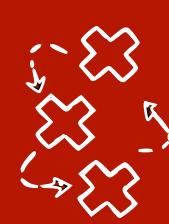


# Example: Conditional independence

- A fire alarm ( $A$ ) is triggered by smoke ( $S$ ), which is usually caused by fire ( $F$ )

$$A \perp\!\!\!\perp F$$

$$P(A = 1 | F = 1) \neq P(A = 1 | F = 0) \neq P(A = 1)$$



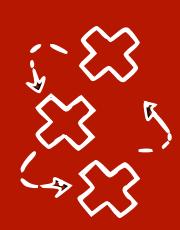
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- A fire alarm (A) is triggered by smoke (S), which is usually caused by fire (F)

$A \perp\!\!\!\perp F$

$$\begin{aligned} P(A = 1 | F = 1) &\neq P(A = 1 | F = 0) \neq P(A = 1) \\ P(A = 0 | F = 1) &\neq P(A = 0 | F = 0) \neq P(A = 0) \end{aligned}$$

In this case, we just need one of these to be true to have a dependence



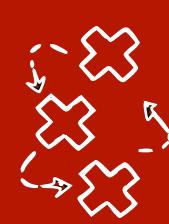
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$$A \perp\!\!\!\perp F | S \quad P(A = 1 | F = 1, S = 1) = P(A = 1 | F = 0, S = 1) = P(A = 1 | S = 1)$$



# Example: Conditional independence

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$$P(A = 1 | F = 1) \neq P(A = 1 | F = 0) \neq P(A = 1)$$

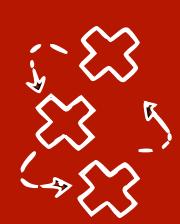
$$P(A = 0 | F = 1) \neq P(A = 0 | F = 0) \neq P(A = 0)$$

$$A \perp\!\!\!\perp F | S$$

$$P(A = 1 | F = 1, S = 1) = P(A = 1 | F = 0, S = 1) = P(A = 1 | S = 1)$$

Same for other combination of values of A and S

- In this example, all information about F that A can use is contained in S



# Example: Conditional independence

- A fire alarm ( $A$ ) is triggered by smoke ( $S$ ), which is usually caused by fire ( $F$ )

$$A \perp\!\!\!\perp F$$

$$P(A = 1 | F = 1) \neq P(A = 1 | F = 0) \neq P(A = 1)$$

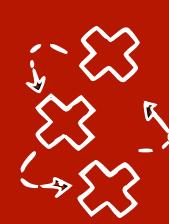
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$$A \perp\!\!\!\perp S | F$$

$$P(A = 1 | S = 1, F = 1) \neq P(A = 1 | S = 0, F = 1) \neq P(A = 1 | F = 1)$$



# Exploiting conditional independences

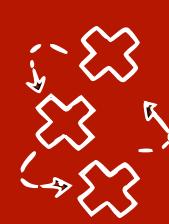
- Given any **ordering** of the variables  $(X_1, \dots, X_p)$  we can write:

$$P(X_1, \dots, X_p) = P(X_1)P(X_2 | X_1)\dots P(X_p | X_1, \dots, X_{p-1})$$

- We can **simplify** the factorisation by using **conditional independences**:

$$X_i \perp\!\!\!\perp X_j | X_Z \implies P(X_i | X_j, X_Z) = P(X_i | X_Z)$$

(special case  $X_i \perp\!\!\!\perp X_j \implies P(X_i | X_j) = P(X_i)$ )



# Exploiting conditional independences

- Given any **ordering** of the variables  $(X_1, \dots, X_p)$  we can write:

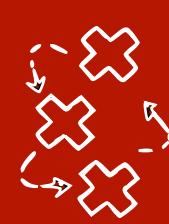
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$$\text{(special case } X_i \perp\!\!\!\perp X_j \implies P(X_i | X_j) = P(X_i))$$

Simpler  
factorisations  
mean less  
parameters to  
learn (less data)



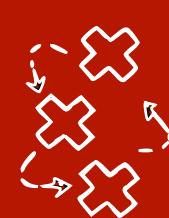
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- For example:  $X \perp\!\!\!\perp Y | Z$  (and symmetrically  $Y \perp\!\!\!\perp X | Z$ ):

- $P(X, Y, Z) = P(X)P(Y | X)P(Z | X, Y)$
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- $P(Z, Y, X) = P(Z)P(Y | Z)P(X | Y, Z) \dots$



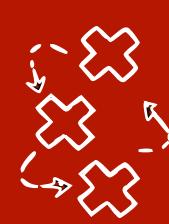
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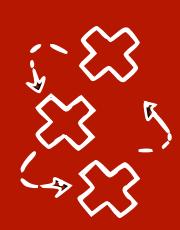
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- For example: if  $X \perp\!\!\!\perp Y | Z$  and  $X \perp\!\!\!\perp Z$

- $P(X, Y, Z) = P(X)P(Y | X)P(Z | X, Y) = P(X)P(Y | X)P(Z | Y)$
- $P(X, Z, Y) = P(X)P(Z | X)P(Y | X, Z) = P(X)P(Z)P(Y | Z)$
- $P(Z, Y, X) = P(Z)P(Y | Z)P(X | X, Z) = P(Z)P(Y | Z)P(X)$

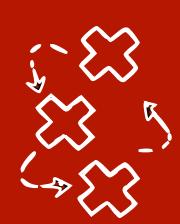


# Example: Fire-smoke-alarm

- A fire alarm (A) is triggered by smoke (S), which is usually caused by fire (F)

$$A \perp\!\!\!\perp F \quad A \perp\!\!\!\perp F \mid S \quad P(A, F, S) \neq P(A)P(F)P(S)$$

$$P(F, S, A) = P(F)P(S \mid F)P(A \mid S, F) = P(F)P(S \mid F)P(A \mid S)$$



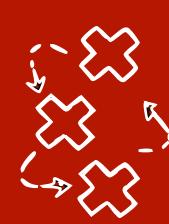
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$$P(A, F, S) = P(A)P(F \mid A)P(S \mid A, F)$$



# Example: Fire-smoke-alarm

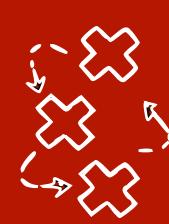
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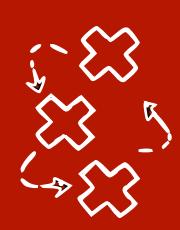
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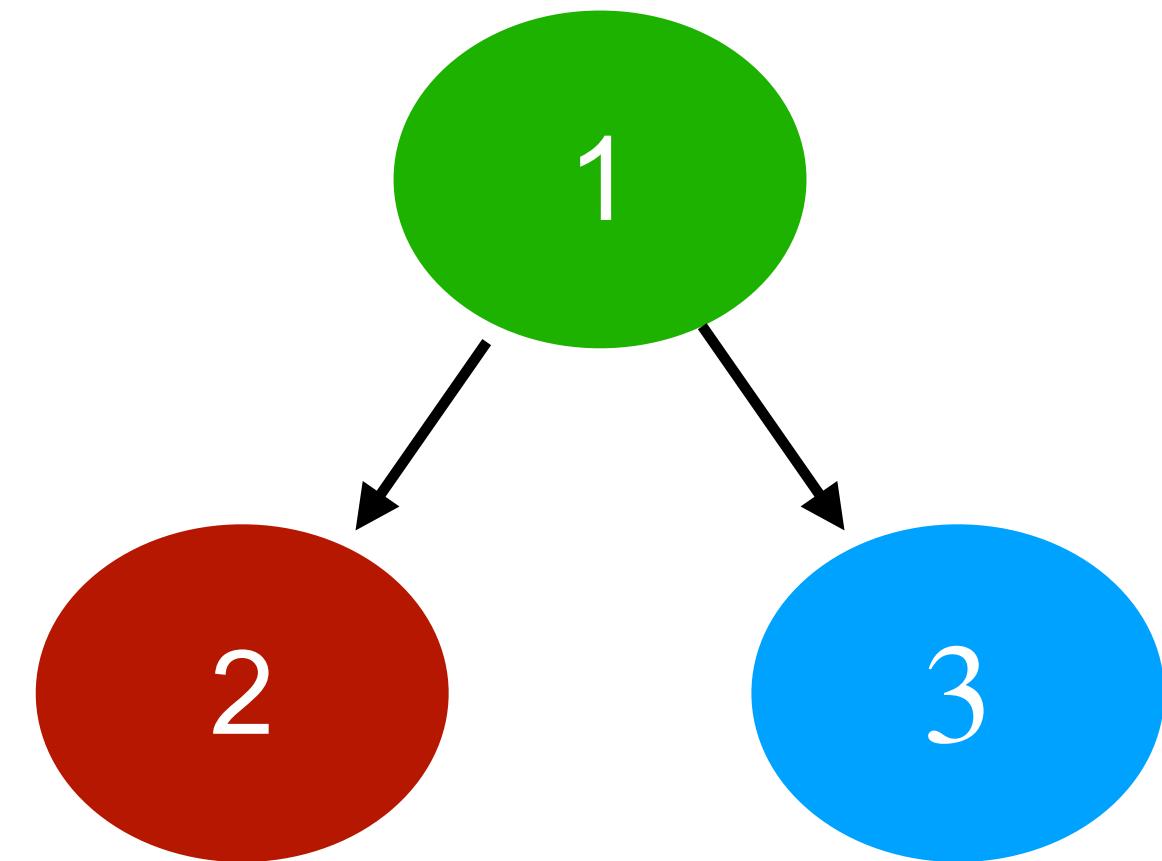


# Bayesian networks

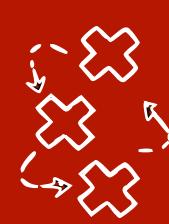
- We have a set of random variables  $X_1, \dots, X_p$  with joint  $p(X_1, \dots, X_p)$
- We have a DAG  $G$ , s.t. **each random variable  $X_i$  is represented by node  $i$**
- We then say  $p(X_1, \dots, X_p)$  **factorizes over  $G$**  if
$$p(X_1, \dots, X_p) = \prod_{i \in V} p(X_i | \mathbf{X}_{\text{Pa}(i)})$$
- A **Bayesian network** (BN) is the tuple  $(G, p)$  s.t.  $p$  **factorizes over  $G$**



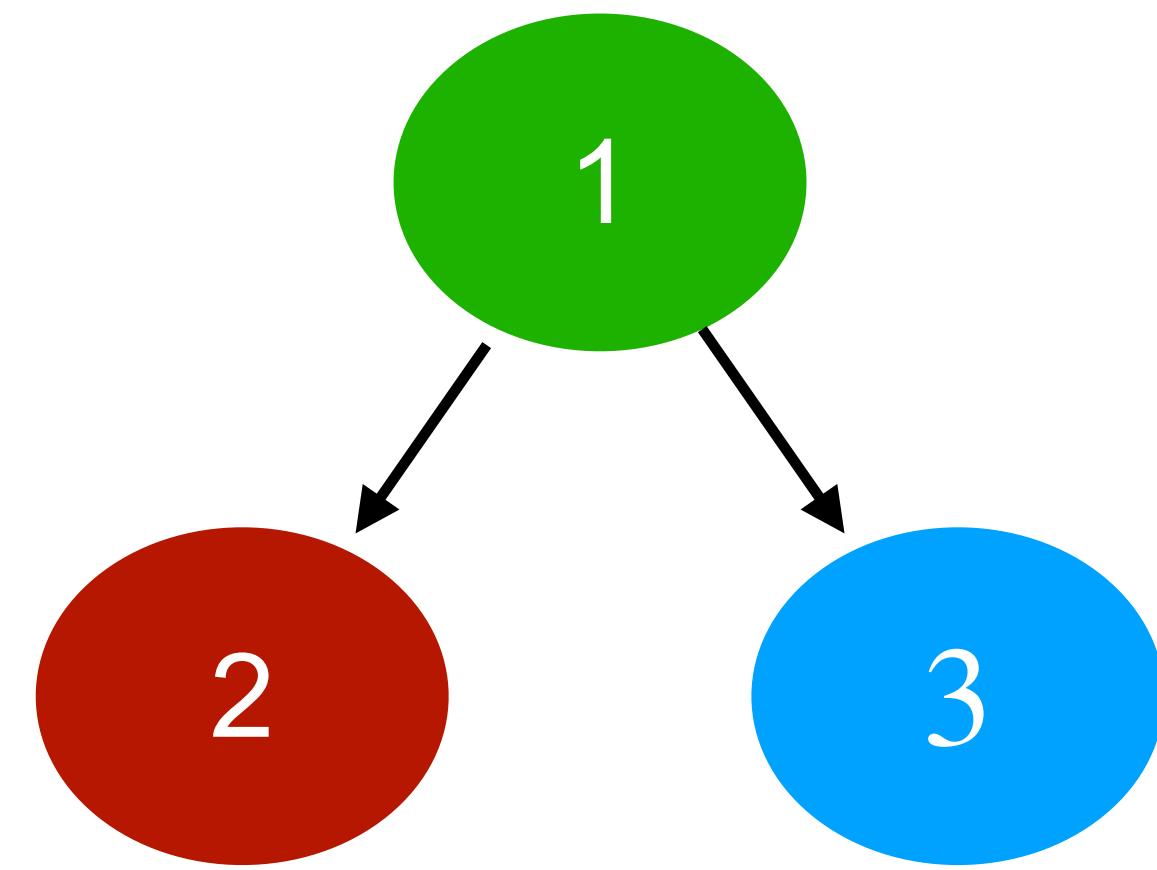
# Example Bayesian networks



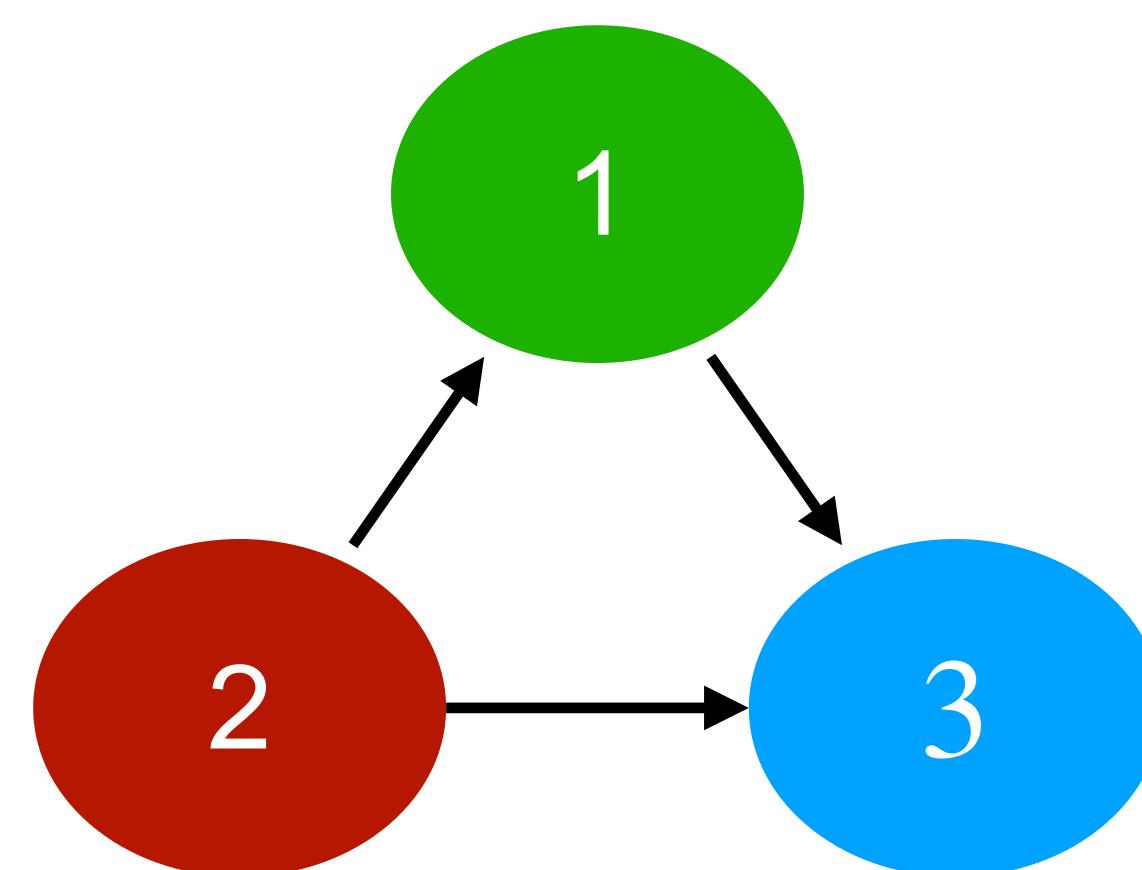
$$p(X_1, X_2, X_3) = p(X_1 | X_{\text{Pa}(1)})p(X_2 | X_{\text{Pa}(2)})p(X_3 | X_{\text{Pa}(3)})$$
$$p(X_1) \quad p(X_2 | X_1) \quad p(X_3 | X_1)$$



# Example Bayesian networks

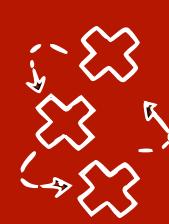


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$$p(X_1) \quad p(X_2 | X_1) \quad p(X_3 | X_1)$$



The DAG/factorization is not unique for a given joint distribution:  
(for example any fully connected graph factorizes any  $p$ )

$$p(X_1, X_2, X_3) = p(X_2)p(X_1 | X_2)p(X_3 | X_1, X_2)$$



# Equivalent factorizations

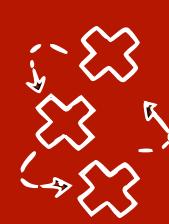
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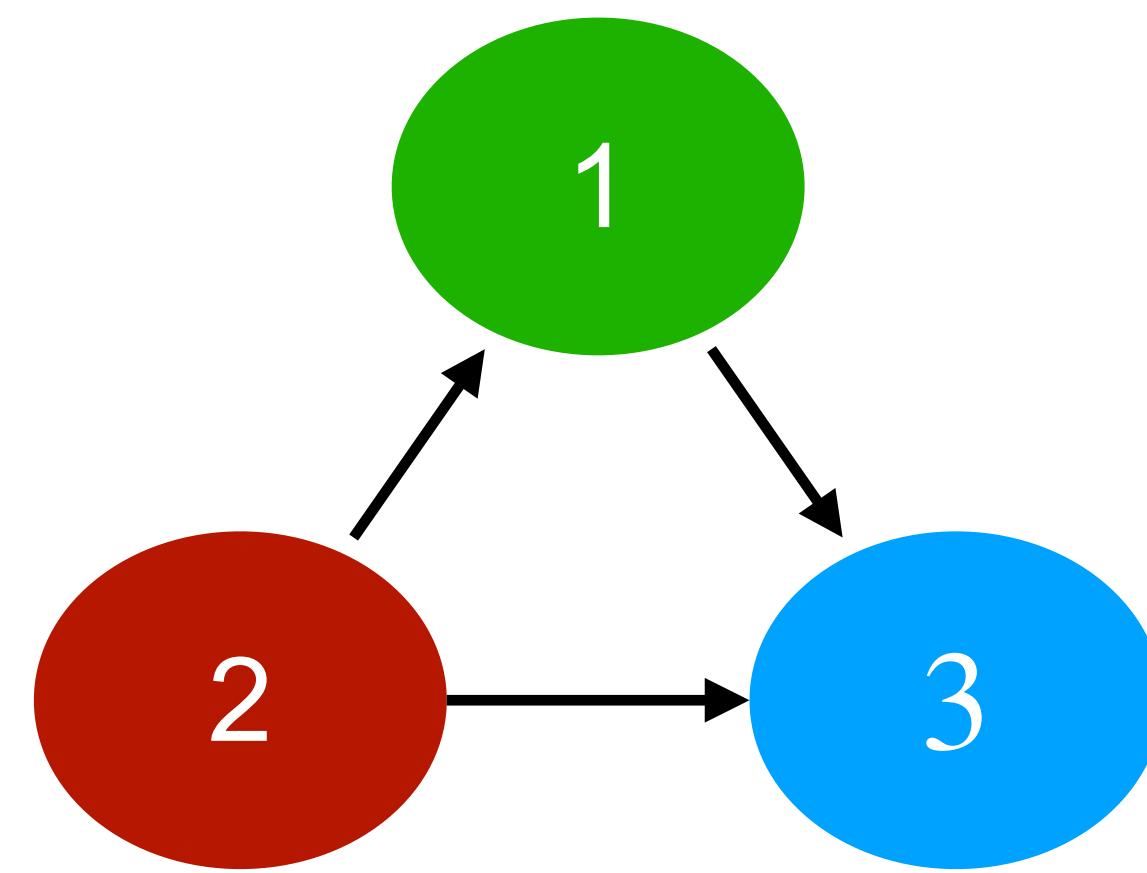
- For example  $P(X, Y, Z)$  can be equivalently factorized as:

- $P(X, Y, Z) = P(X)P(Y | X)P(Z | X, Y)$
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- $P(Z, Y, X) = P(Z)P(Y | Z)P(X | Y, Z) \dots$

Each of these factorizes according to a fully connected DAG



# Example Bayesian networks



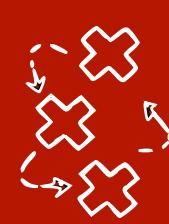
If we know  $X_3 \perp\!\!\!\perp X_2 | X_1$  then we can simplify the factorisation

$$p(X_1, X_2, X_3) = p(X_2)p(X_1 | X_2)p(X_3 | X_1, X_2)$$

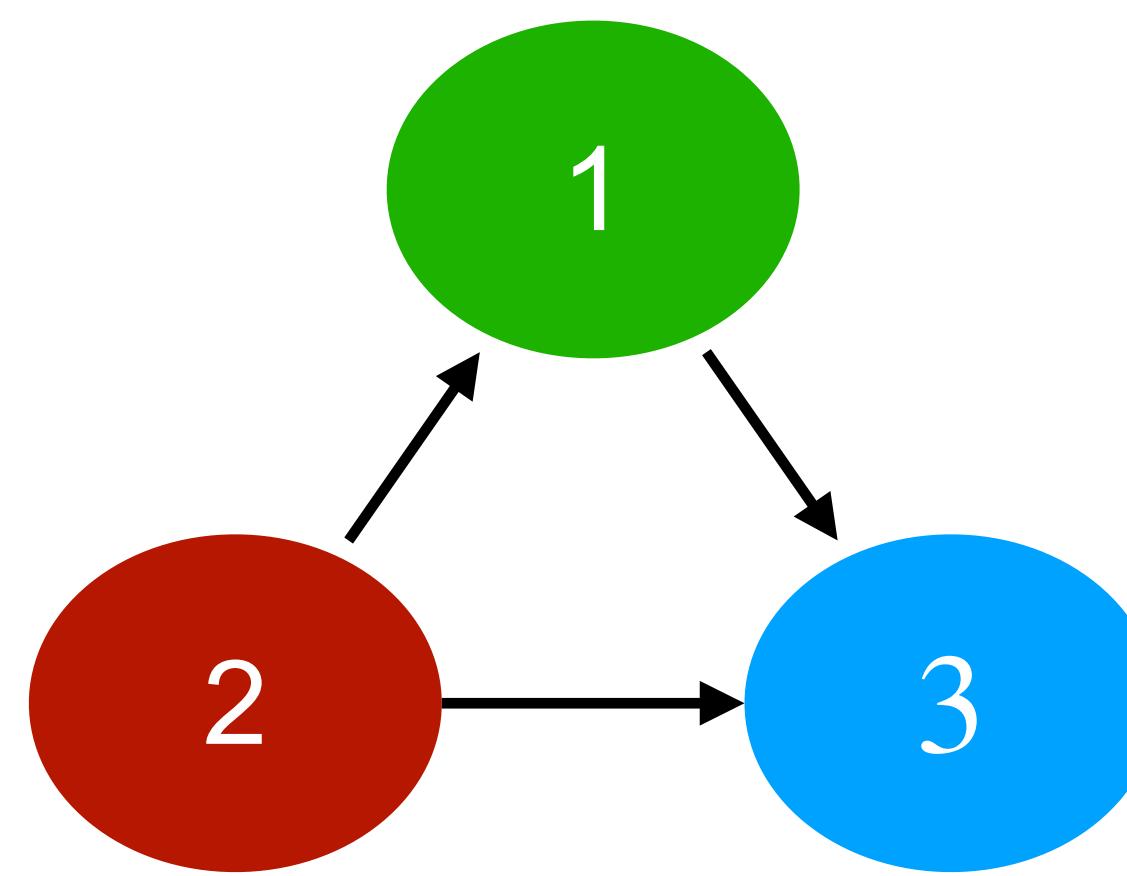
**Note:** The fully connected graph still **factorizes** according p, even if we can simplify factorisation, since this still holds

$$p(X_1 | X_{\text{Pa}(1)})p(X_2 | X_{\text{Pa}(2)})p(X_3 | X_{\text{Pa}(3)})$$

$$p(X_1 | X_2)p(X_2)p(X_3 | X_2, X_1) = p(X_1, X_2, X_3)$$



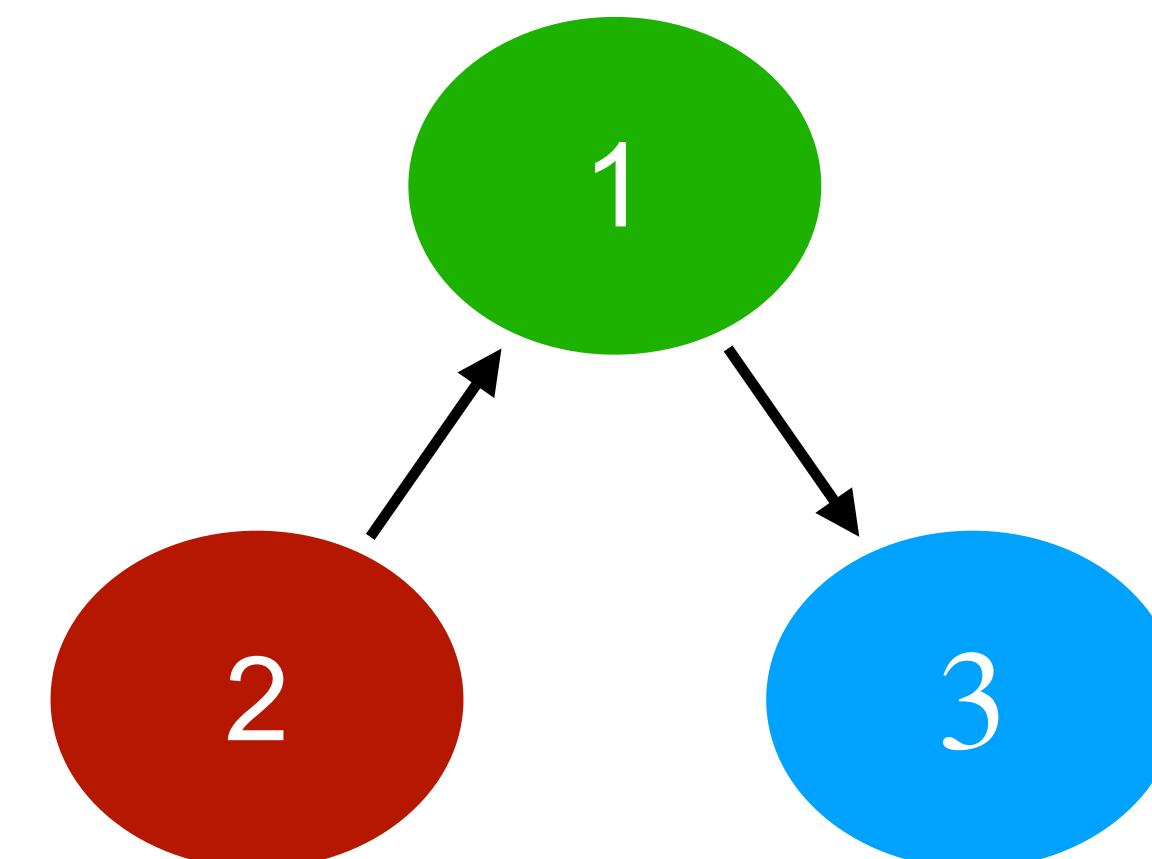
# Example Bayesian networks



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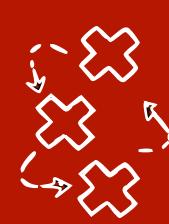
**Note:** The fully connected graph still **factorizes** according  $p$ , even if we can simplify factorisation, since this still holds



But if  $X_3 \perp\!\!\!\perp X_2 | X_1$  holds in  $p$ , then this DAG also factorizes accordingly

$$p(X_1 | X_{\text{Pa}(1)})p(X_2 | X_{\text{Pa}(2)})p(X_3 | X_{\text{Pa}(3)})$$

$$p(X_1 | X_2) \quad p(X_2) \quad p(X_3 | X_1)$$



# Why should we care about Bayesian networks?

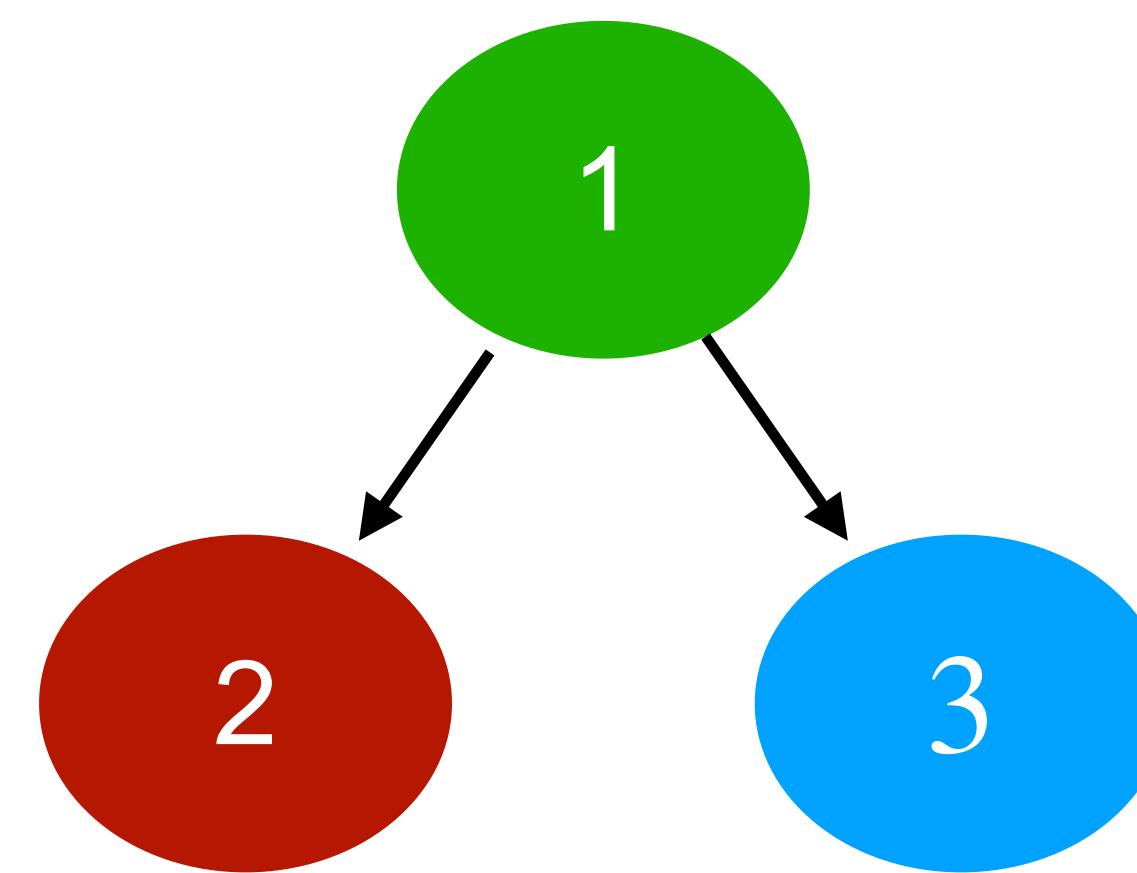
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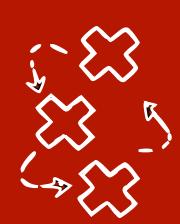
$$p(X_1, \dots, X_p) = \prod_{i \in V} p(X_i | \mathbf{X}_{\text{pa}(i)})$$

They can help simplify the factorisation

We can easily read conditional independences

They can represent causal models

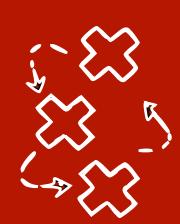




# Global Markov Property and faithfulness

- If  $(G, p)$  is a Bayesian network with a DAG  $G = (\mathbf{V}, \mathbf{E})$ , i.e. **p factorizes according to G**, then for any disjoint  $\mathbf{A}, \mathbf{B}, \mathbf{C} \subseteq \mathbf{V}$ :

$$\mathbf{A} \perp_d \mathbf{B} | \mathbf{C} \implies X_{\mathbf{A}} \perp\!\!\!\perp X_{\mathbf{B}} | X_{\mathbf{C}}$$

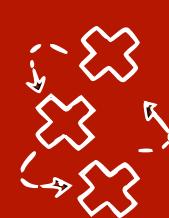


# Global Markov Property and faithfulness

- If  $(G, p)$  is a Bayesian network with a DAG  $G = (\mathbf{V}, \mathbf{E})$ , i.e. **p factorizes according to G**, then for any disjoint  $\mathbf{A}, \mathbf{B}, \mathbf{C} \subseteq \mathbf{V}$ :

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- **d-separations** that can be read purely from a graph imply **conditional independences** in the random variables and data generated by the graph



# Global Markov Property and faithfulness

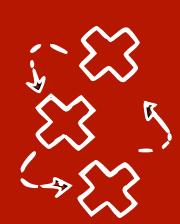
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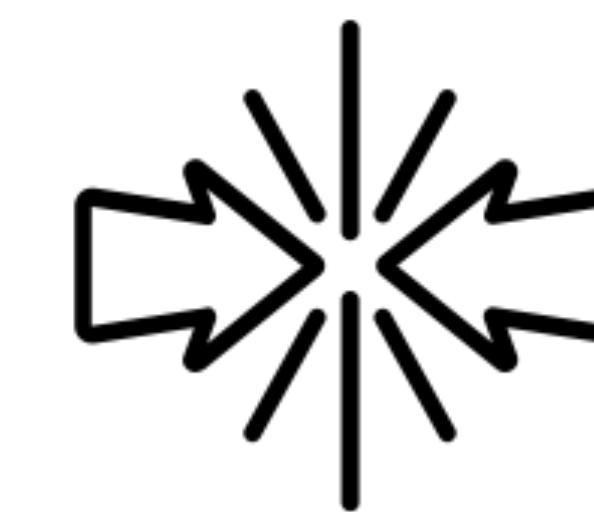
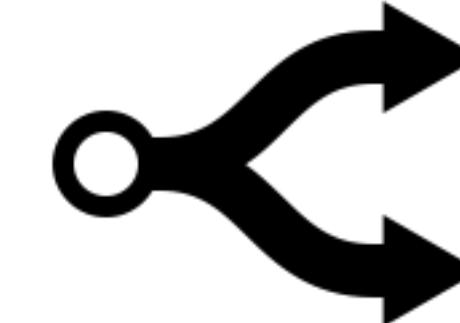
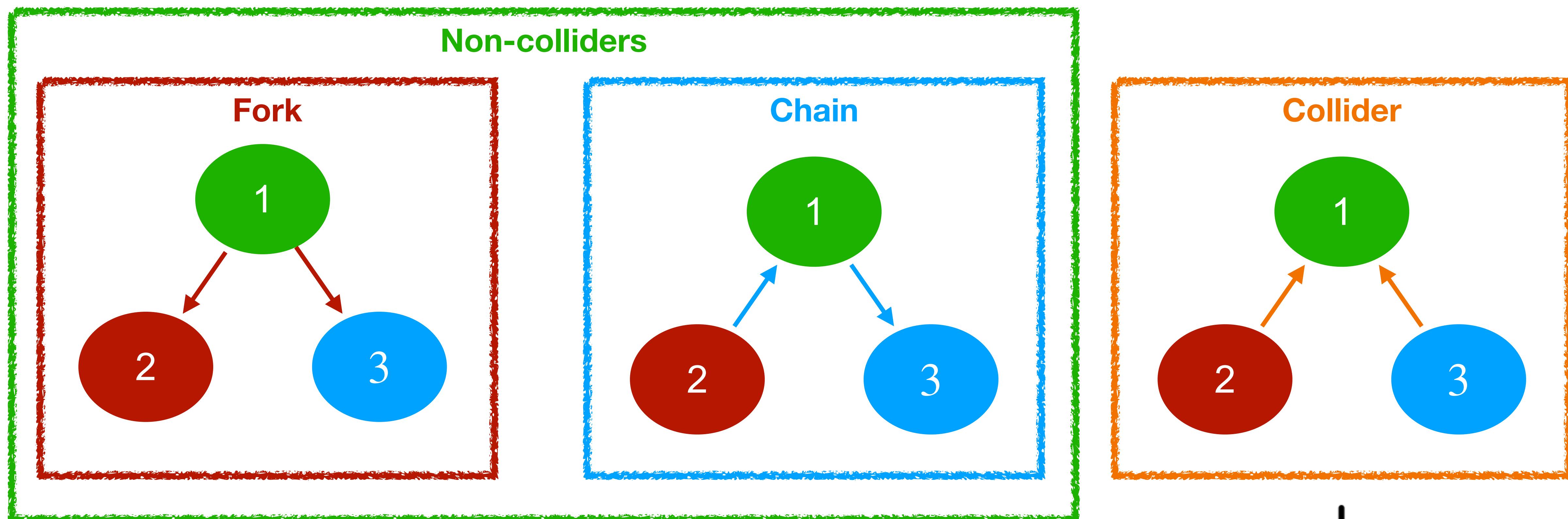
- **d-separations** that can be read purely from a graph imply **conditional independences** in the random variables and data generated by the graph
- The reverse implication is not true in general, but if it is **p is faithful to G**

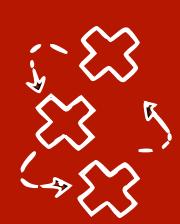
$$A \perp_d B | C \iff X_A \perp\!\!\!\perp X_B | X_C$$

We will usually assume both assumptions hold

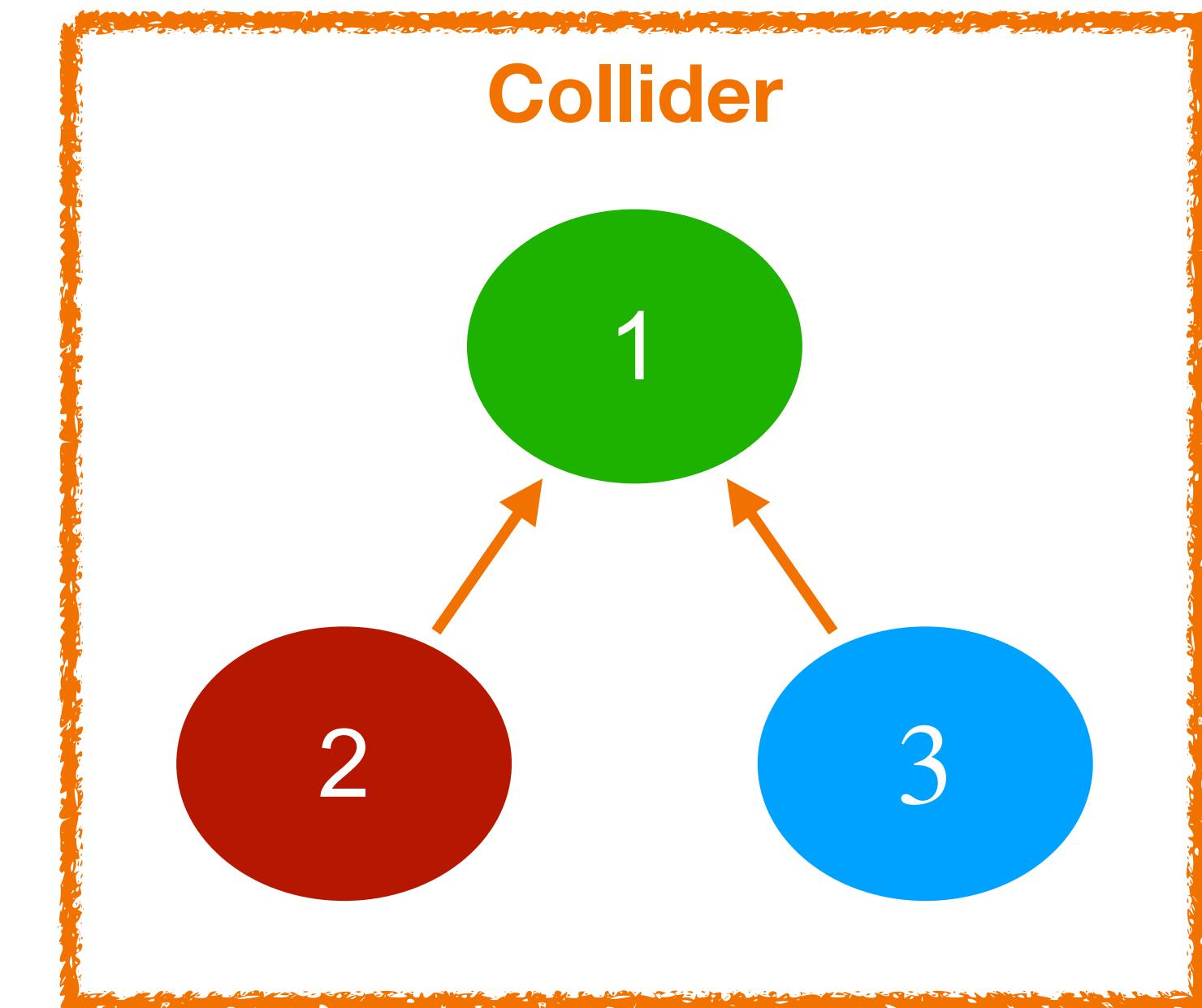
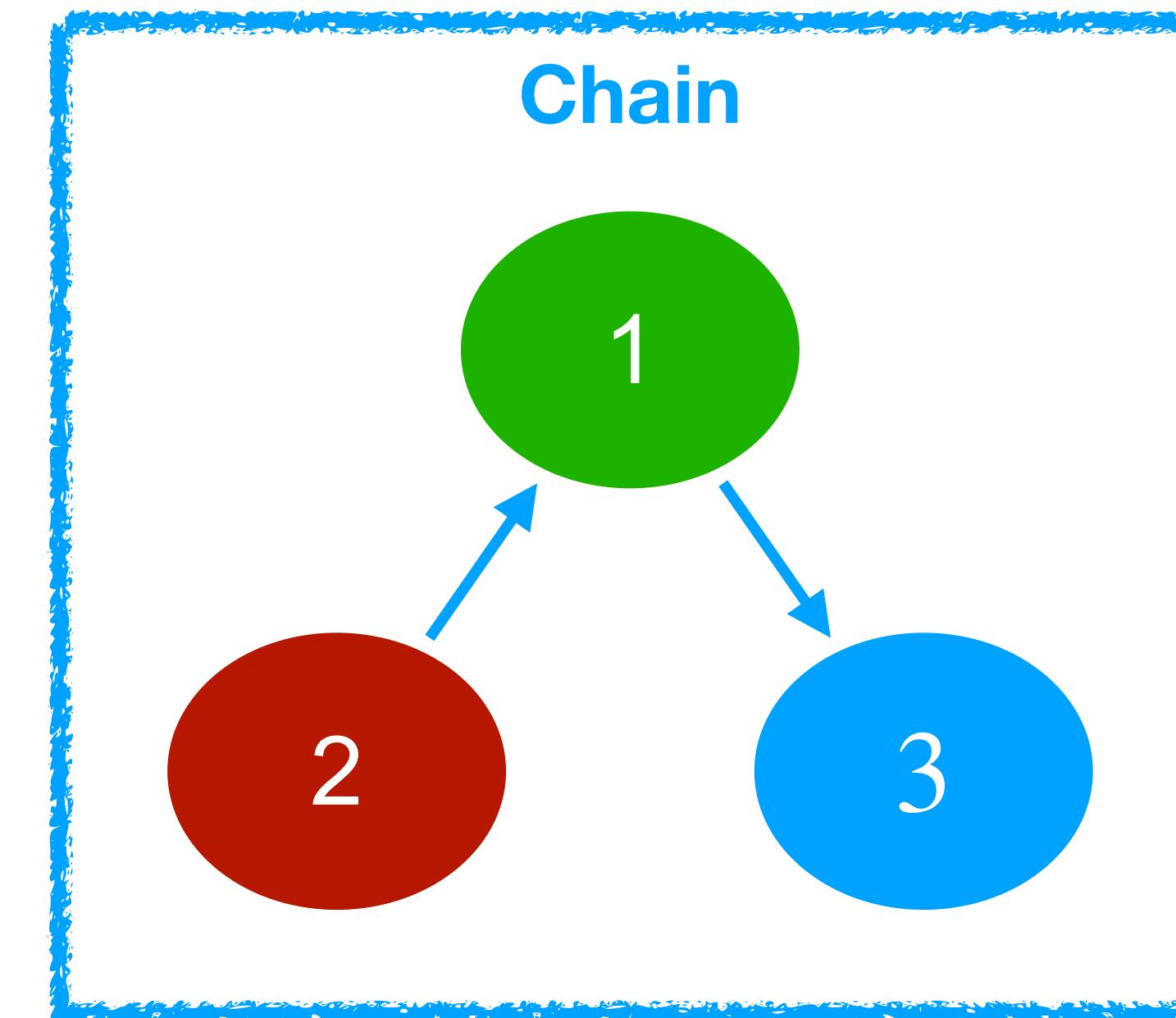
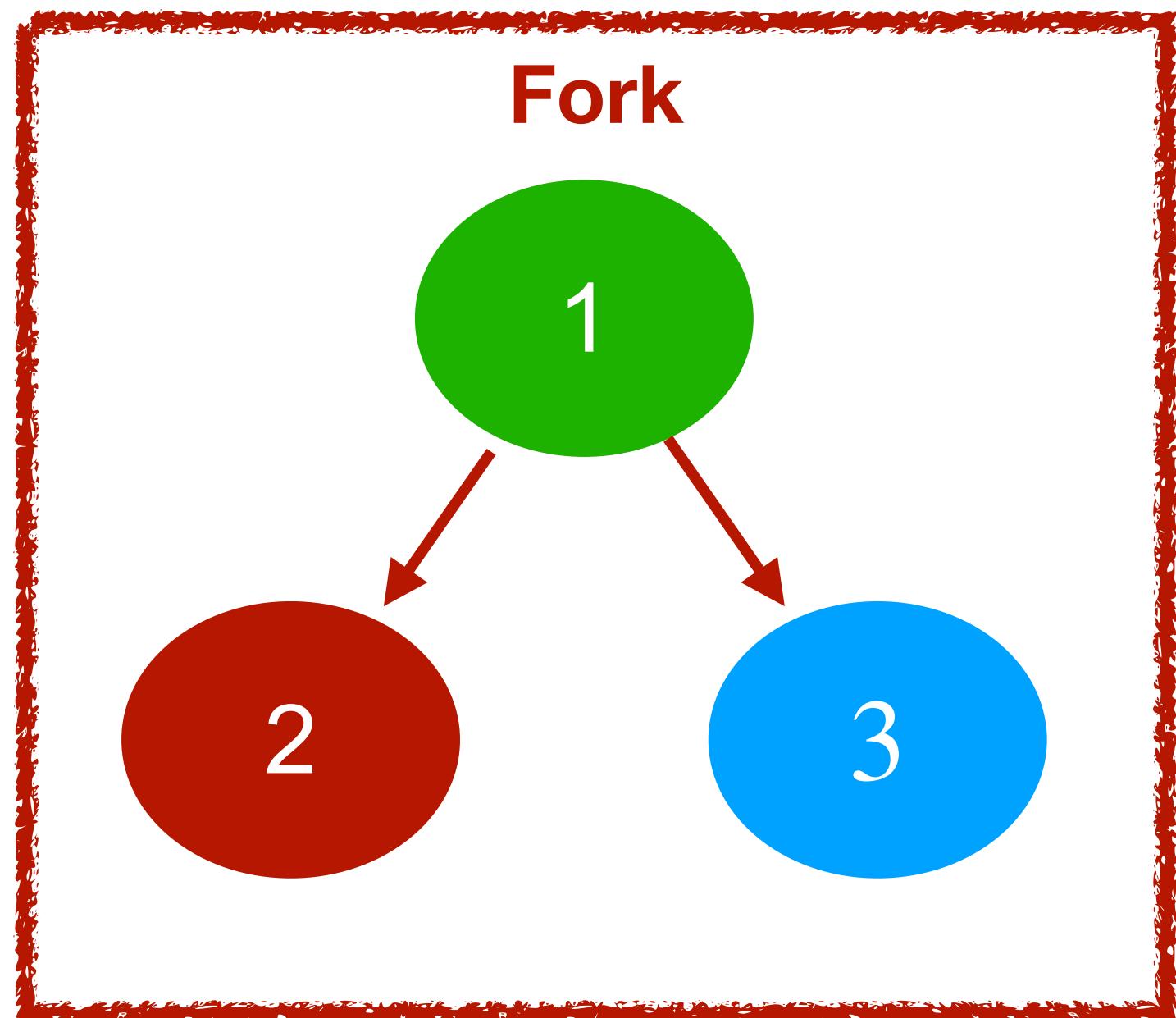


# D-separation (reprise): three basic patterns





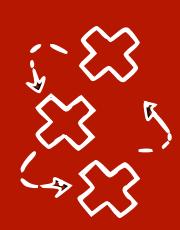
# D-separation (reprise): three basic patterns



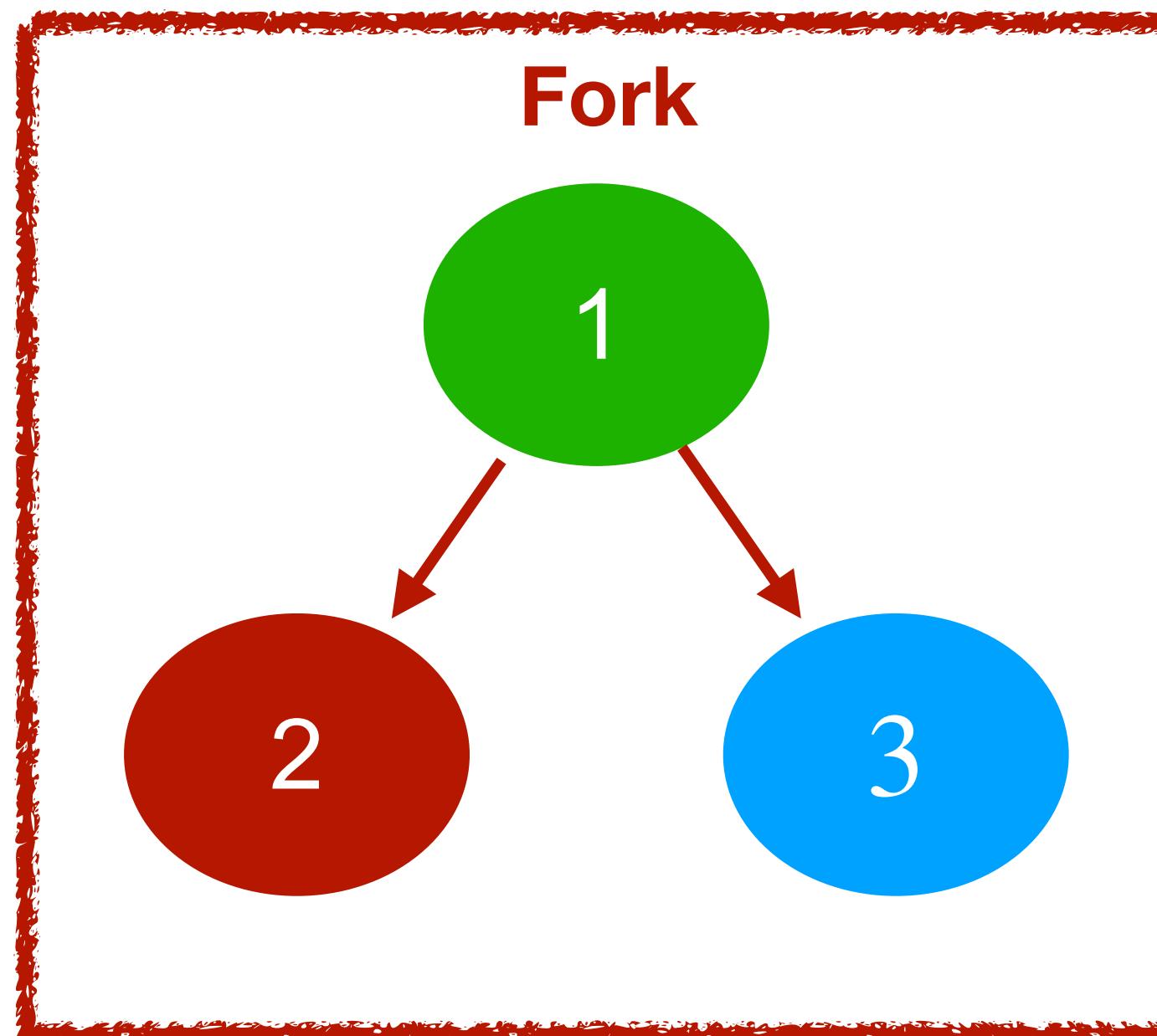
$$P(X_1)P(X_2 | X_1)P(X_3 | X_1)$$

$$P(X_2)P(X_1 | X_2)P(X_3 | X_1)$$

$$P(X_2)P(X_3)P(X_1 | X_2, X_3)$$



# Fork ( shoe size <- age -> height)



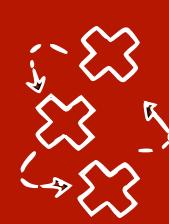
$$P(X_1, X_2, X_3) = P(X_1)P(X_2 | X_1)P(X_3 | X_1, X_2)$$

$$3 \perp_d 2 | 1 \implies X_3 \perp\!\!\!\perp X_2 | X_1$$

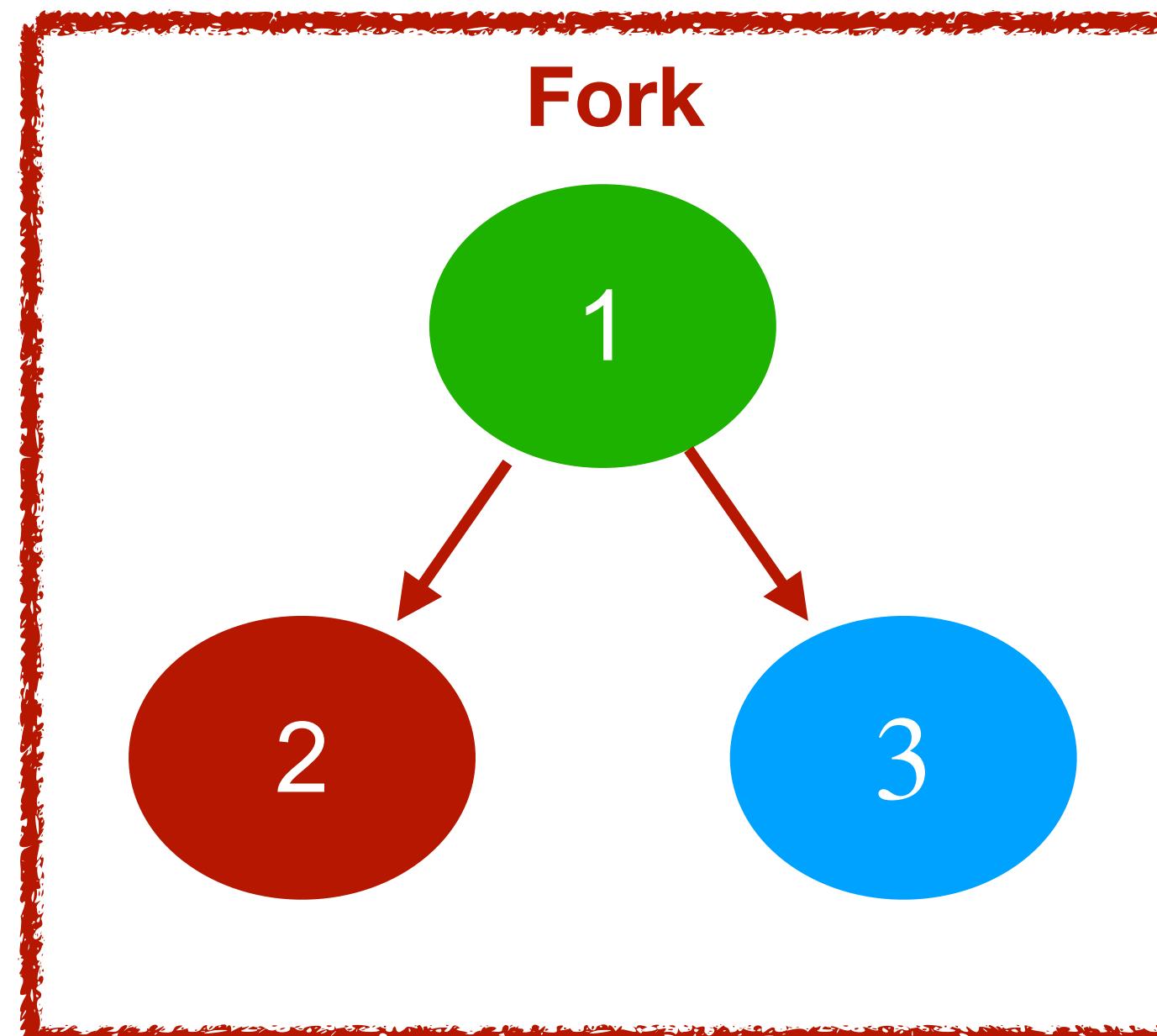
$\downarrow$

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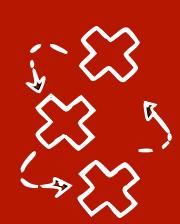
$$= P(X_1)P(X_2|X_1)P(X_3|X_1)$$

In general  $2 \perp_d 3 \not\implies X_2 \perp\!\!\!\perp X_3$ , because

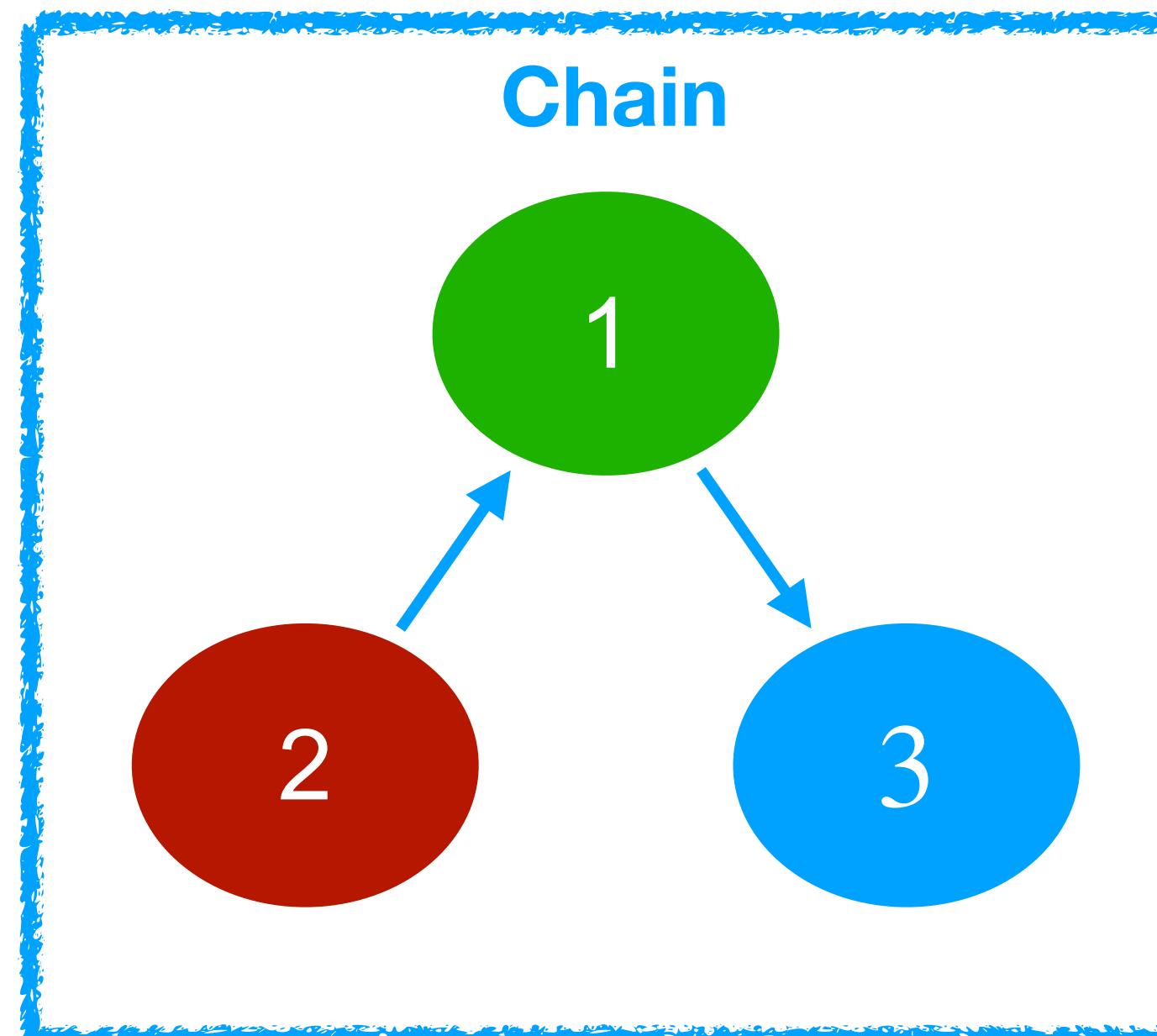
$$A \perp_d B | C \implies X_A \perp\!\!\!\perp X_B | X_C$$

In practice, for faithful graphs  $2 \perp_d 3 \iff X_2 \perp\!\!\!\perp X_3$

$$P(X_1)P(X_2|X_1)P(X_3|X_1)$$



# Chain (fire -> smoke -> alarm)



$$P(X_1, X_2, X_3) = P(X_2)P(X_1 | X_2)P(X_3 | X_1, X_2)$$

$$3 \perp_d 2 | 1 \implies X_3 \perp X_2 | X_1$$

$\downarrow$

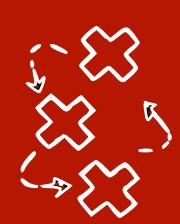
$$= P(X_2)P(X_1 | X_2)P(X_3 | X_1)$$

In general  $2 \perp_d 3 \not\implies X_2 \perp X_3$ , because

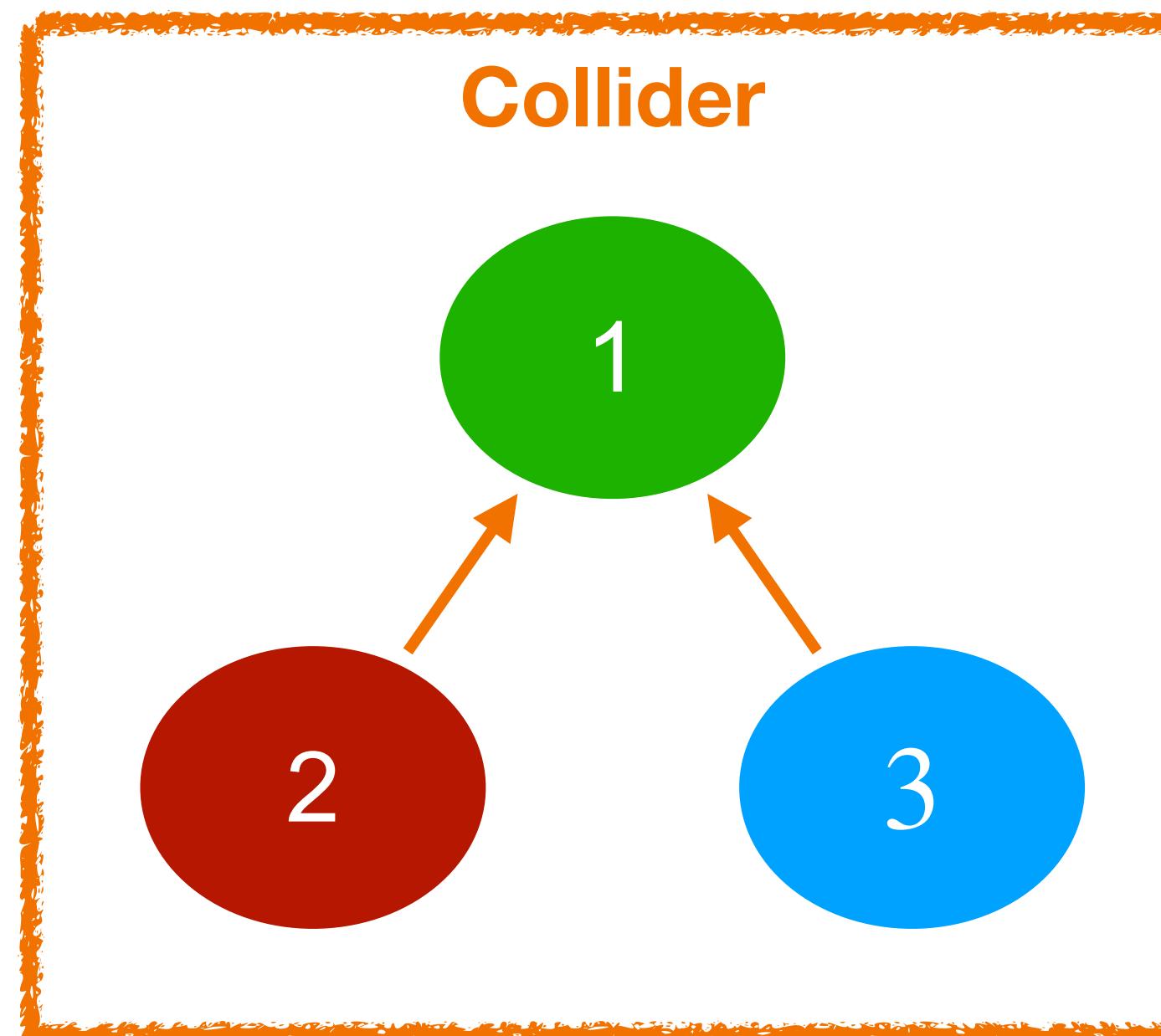
$$A \perp_d B | C \implies X_A \perp X_B | X_C$$

In practice, for faithful graphs  $2 \perp_d 3 \iff X_2 \perp X_3$

$$P(X_2)P(X_1 | X_2)P(X_3 | X_1)$$



# Collider (Math skills -> admission <- English skills)



$$P(X_1, X_2, X_3) = P(X_2)P(X_3 | X_2)P(X_1 | X_2, X_3)$$

$$3 \perp_d 2 \implies X_3 \perp X_2$$

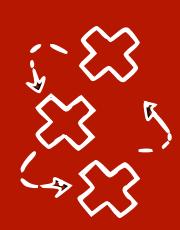
$\downarrow$

$$= P(X_2)P(X_3)P(X_1 | X_2, X_3)$$

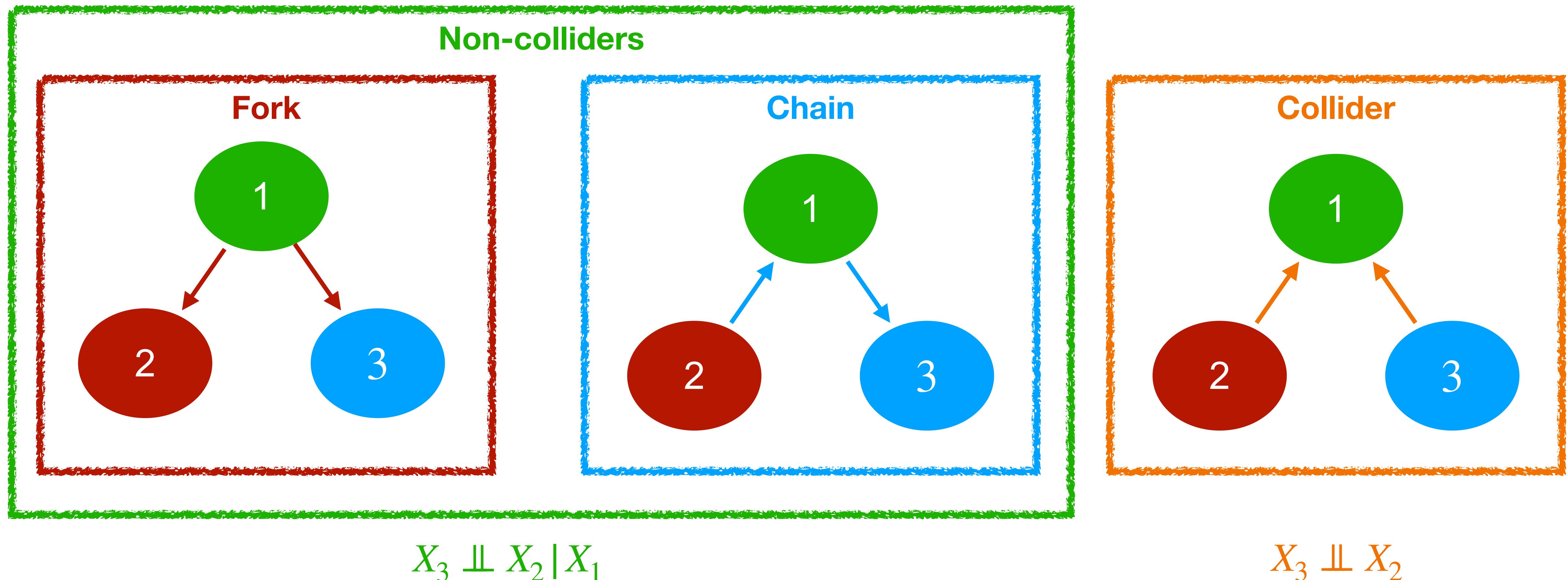
In general  $2 \perp_d 3 | 1 \not\implies X_2 \perp X_3 | X_1$

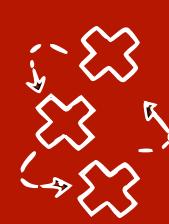
$$P(X_2)P(X_3)P(X_1 | X_2, X_3)$$

For faithful graphs  $2 \perp_d 3 | 1 \iff X_2 \perp X_3 | X_1$



# Three basic patterns





# Why should we care about Bayesian networks?

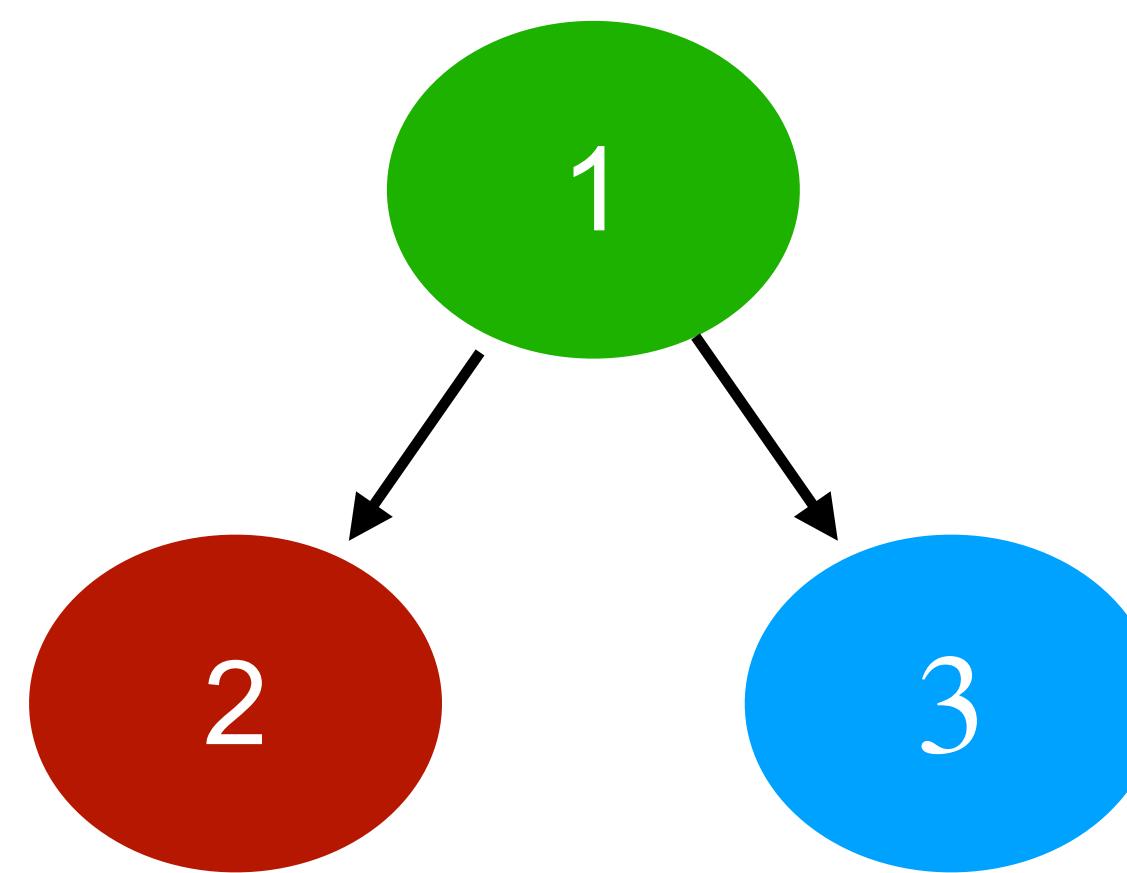
- We have a set of random variables  $X_1, \dots, X_p$  with joint  $p(X_1, \dots, X_p)$
- We have a DAG  $G$ , s.t. **each random variable  $X_i$  is represented by node  $i$**
- We then say  $p(X_1, \dots, X_p)$  **factorizes over  $G$**  if

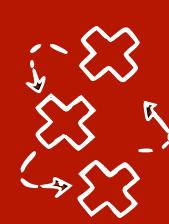
$$p(X_1, \dots, X_p) = \prod_{i \in V} p(X_i | \mathbf{X}_{\text{pa}(i)})$$

They can help simplify the factorisation

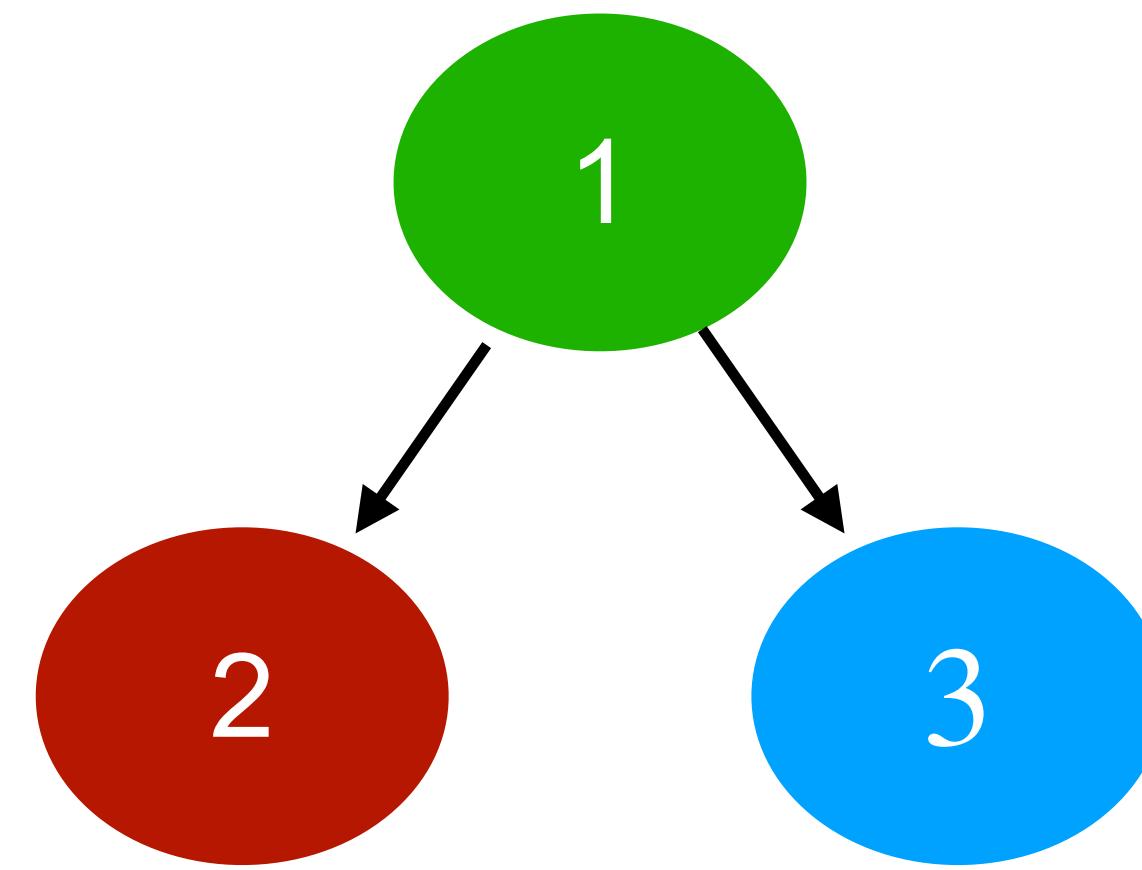
We can easily read conditional independences

They can represent causal models



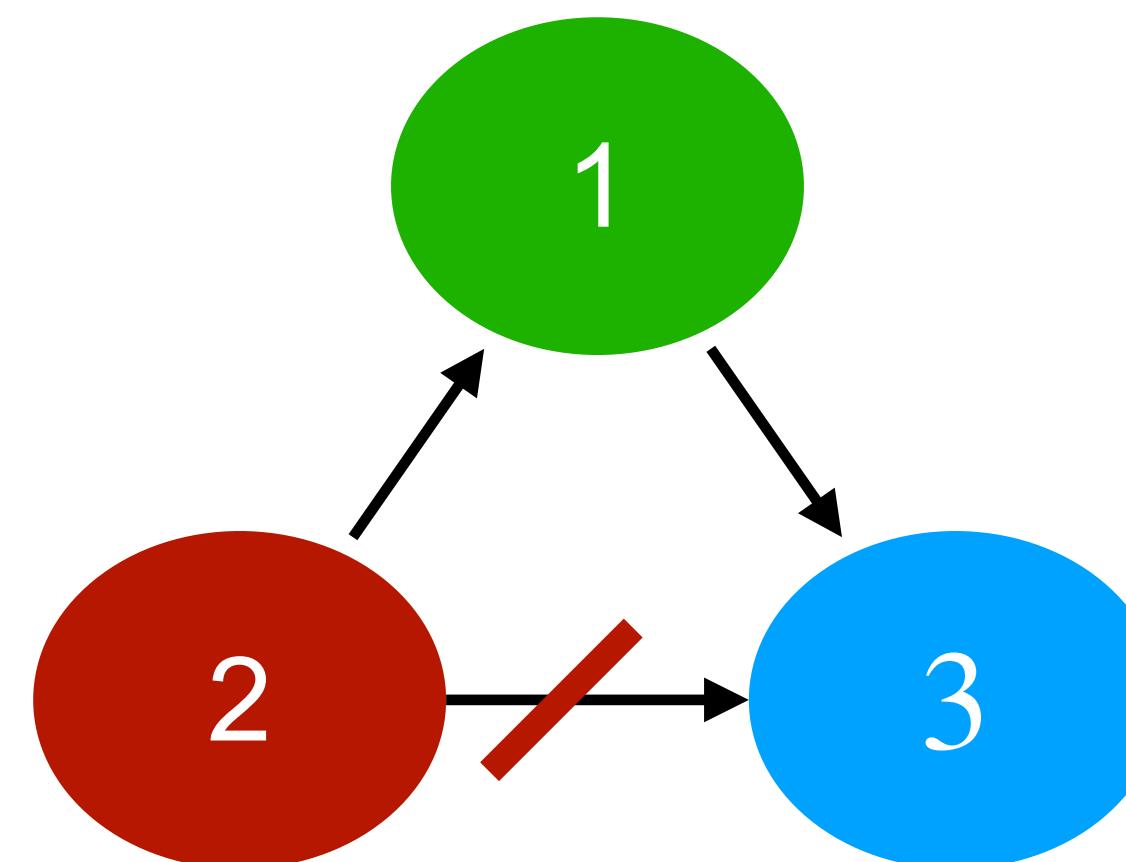


# Example Bayesian networks

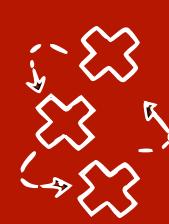


$$p(X_1, X_2, X_3) = p(X_1 | X_{\text{Pa}(1)})p(X_2 | X_{\text{Pa}(2)})p(X_3 | X_{\text{Pa}(3)})$$
$$p(X_1) \quad p(X_2 | X_1) \quad p(X_3 | X_1)$$

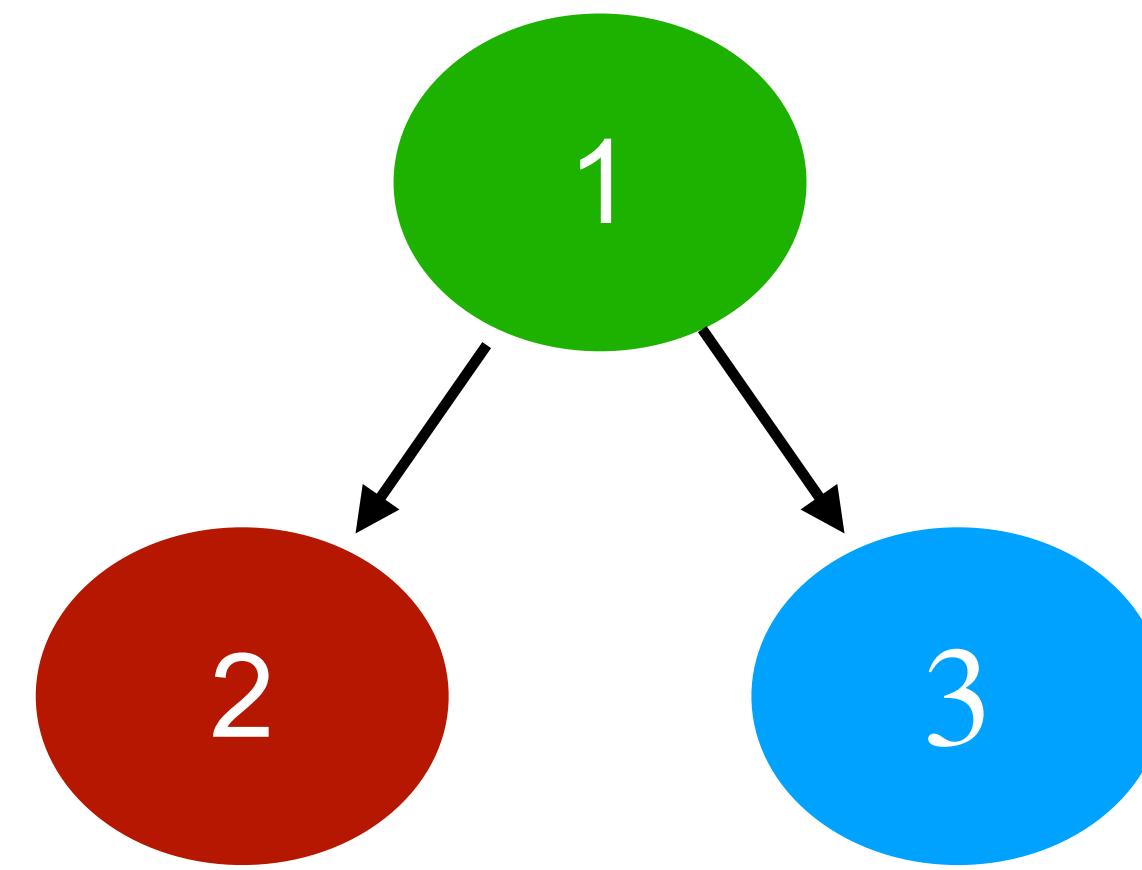
$$3 \perp_d 2 | 1 \implies X_3 \perp\!\!\!\perp X_2 | X_1$$



$$p(X_1, X_2, X_3) = p(X_2)p(X_1 | X_2)p(X_3 | X_1, \cancel{X_2})$$

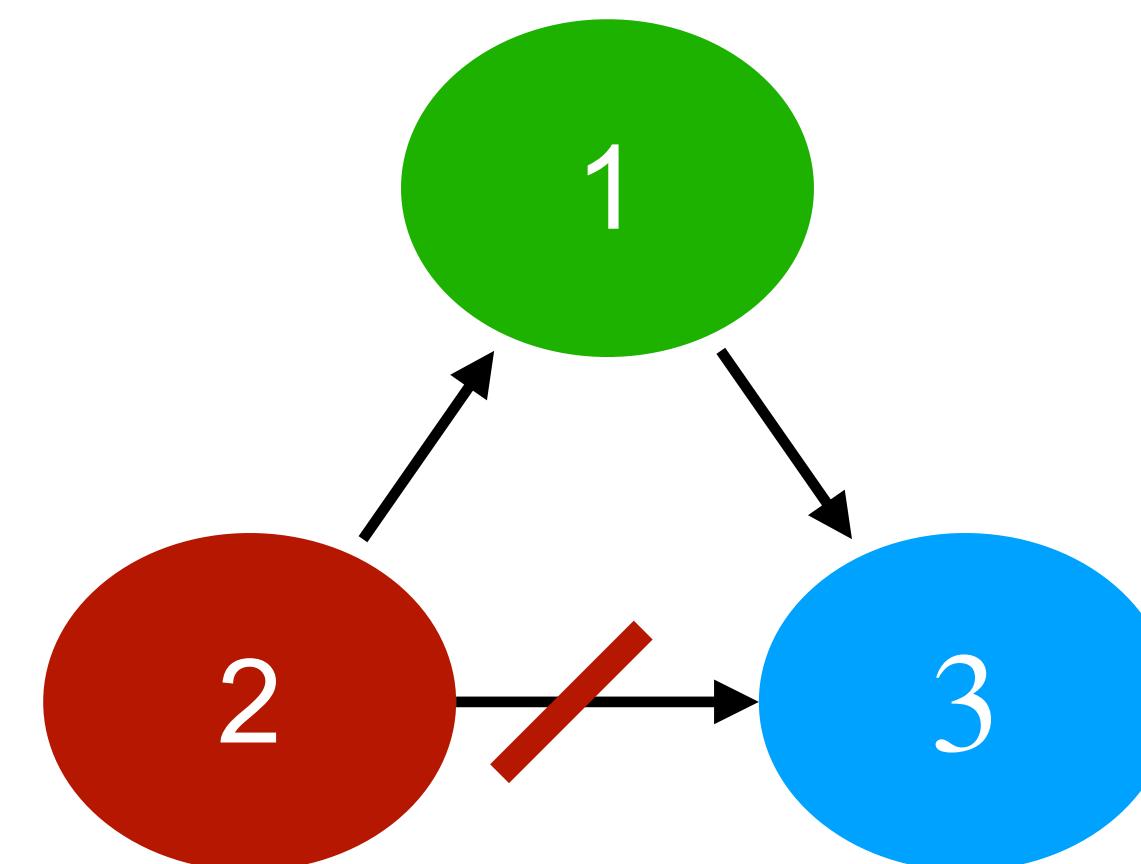


# Example Bayesian networks



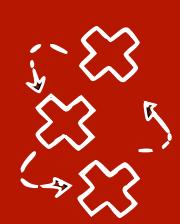
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$$3 \perp_d 2 | 1 \implies X_3 \perp\!\!\!\perp X_2 | X_1$$



$$p(X_1, X_2, X_3) = p(X_2)p(X_1 | X_2)p(X_3 | X_1, \cancel{X_2})$$

We have multiple “minimal” graphs for a given distribution, which is the causal one?

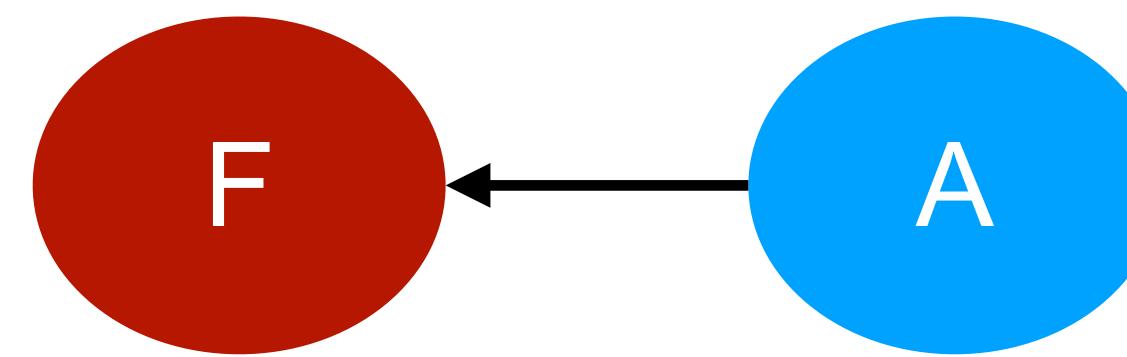
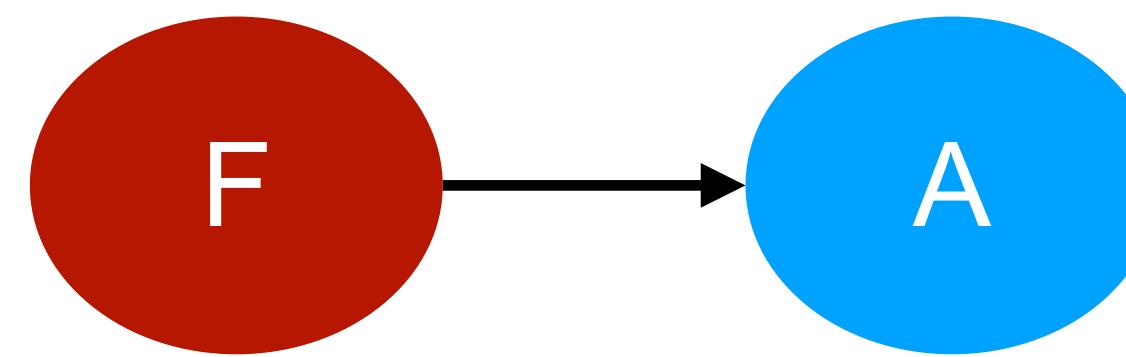


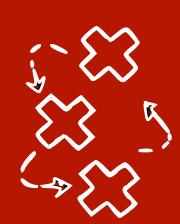
# Next time: BNs vs causal BNs

- Fire (F) and Alarm (A) with  $p(F, A)$  and  $A \not\perp\!\!\!\perp F$  can be factorized as:

$$p(F, A) = p(F) p(A|F)$$

$$p(F, A) = p(A) p(F|A)$$

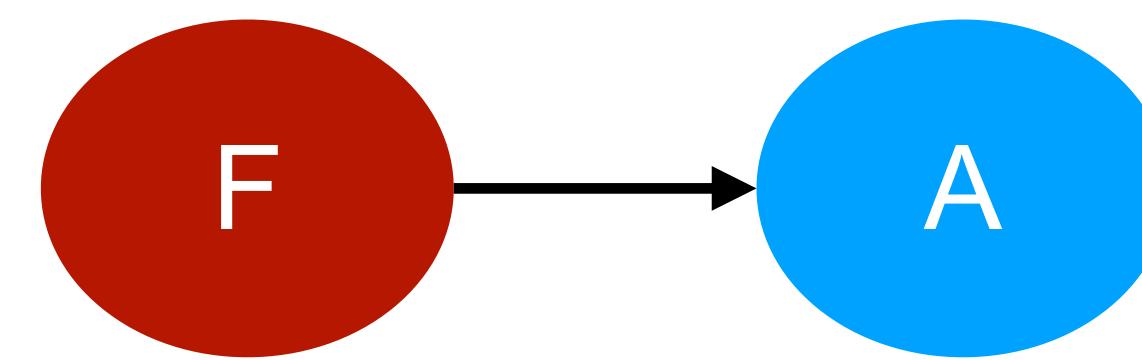




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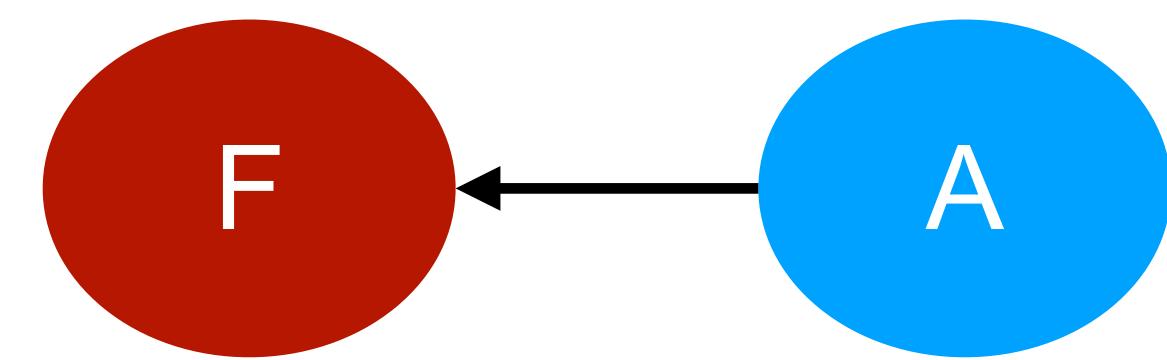
$$p(F, A) = p(F) p(A|F)$$



CAUSAL

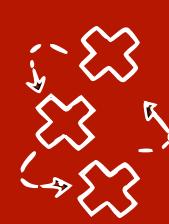
(lighting a fire triggers alarm)

$$p(F, A) = p(A) p(F|A)$$



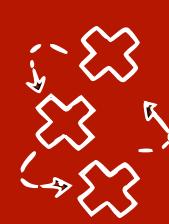
NOT-CAUSAL

(triggering alarm does not light a fire)



# Next time: BNs vs causal BNs

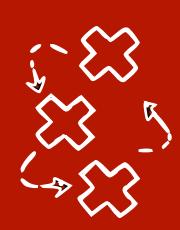
|    |  |
|----|--|
| 1  | Introduction   |
| 2  | Probability recap  |
| 3  | Graphical models, d-separation                                   |
| 4  | Causal graphs, Interventions, SCMs                               |
| 5  | Covariate adjustment: backdoor criterion                         |
| 6  | Covariate Frontdoor criterion, Instrumental variables            |
| 7  | Counterfactuals, potential outcomes, estimating causal effects 1 |
| 8  | Estimating causal effects 2 (matching, IPW)                      |
| 9  | Constraint based structure learning                              |
| 10 | Score based structure learning, restricted models                |
| 11 | Do-calculus, transportability, Joint Causal Inference            |
| 12 | Causality-inspired ML, recap of the course                       |



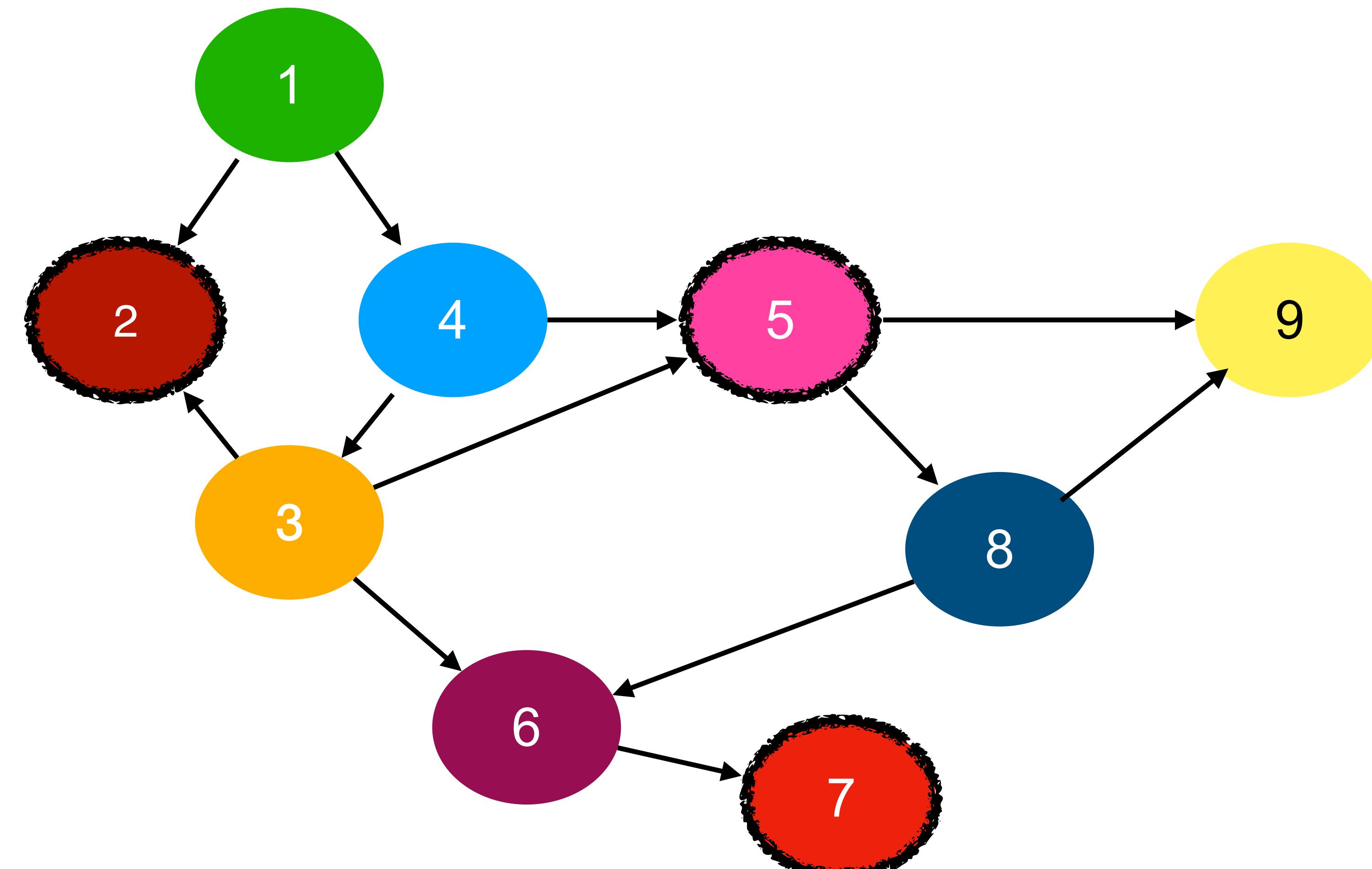
# We will talk in detail how to learn the causal graph here:

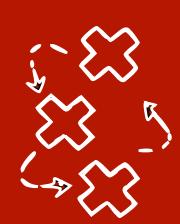
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What happens if the graph is unknown?

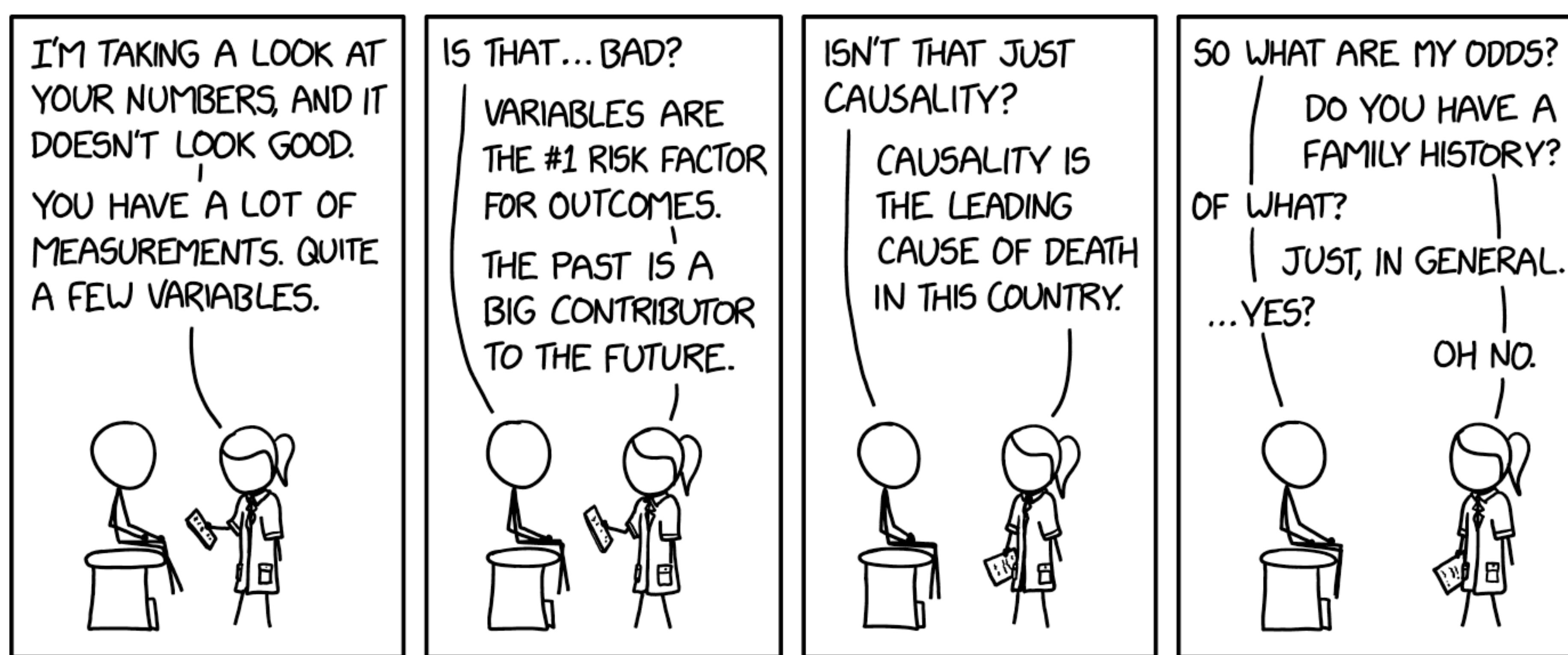


# Optional exercise in Canvas: d-separation 2





# Questions??



<https://xkcd.com/2620/>