Given an SCM and the realization for a specific unit, compute unit-level counterfactuals and explain every step

Question:

Given an SCM below and the realization for a specific unit i, compute unit-level counterfactuals of $X \leftarrow 1$ and $Y \leftarrow 1$ separately, and explain every step

$$\begin{aligned}
X &\leftarrow \epsilon_X & \epsilon_X &\sim \mathcal{N}(0,1) \\
Y &\leftarrow 3 + \epsilon_Y & \epsilon_Y &\sim \mathcal{N}(0,0.5) \\
Z &\leftarrow X + 2Y - 2 + \epsilon_Z & \epsilon_Z &\sim \mathcal{N}(0,1) \\
V &\leftarrow X + Y - Z + \epsilon_V & \epsilon_V &\sim \mathcal{N}(0,2)
\end{aligned}$$

Realization for a specific unit i:

$$(x^i = 0.25, \quad y^i = 2.9, \quad z^i = 4.1, \quad v^i = -1)$$

Answer:

Counterfactual of $X \leftarrow 1$

1. Abduction:

$$\begin{aligned}
\epsilon_X^i &= 0.25 - 0 & &= 0.25 \\
\epsilon_Y^i &= 2.9 - 3 & &= -0.1 \\
\epsilon_Z^i &= 4.1 - (0.25 + 2 * 2.9 - 2) & &= 0.05 \\
\epsilon_V^i &= -1 - (0.25 + 2.9 - 4.1) & &= -0.05
\end{aligned}$$

2. Action: $X^{c,i} \leftarrow 1$

3. Prediction:

$$\begin{aligned}
x^{c,i} & & &= 1 \\
y^{c,i} &= 3 - 0.1 & &= 2.9 \\
z^{c,i} &= 1 + 2 * 2.9 - 2 + 0.05 & &= 4.85 \\
v^{c,i} &= 1 + 2.9 - 4.85 - 0.05 & &= -1
\end{aligned}$$

Counterfactual of $Y \leftarrow 1$

1. Abduction (same as before):

$$\begin{aligned}
\epsilon_X^i &= 0.25 - 0 & &= 0.25 \\
\epsilon_Y^i &= 2.9 - 3 & &= -0.1 \\
\epsilon_Z^i &= 4.1 - (0.25 + 2 * 2.9 - 2) & &= 0.05 \\
\epsilon_V^i &= -1 - (0.25 + 2.9 - 4.1) & &= -0.05
\end{aligned}$$

2. Action: $Y^{c,i} \leftarrow 1$

3. Prediction:

$$\begin{aligned}
x^{c,i} & & &= 0.25 \\
y^{c,i} & & &= 1 \\
z^{c,i} &= 0.25 + 2 * 1 - 2 + 0.05 & &= 0.3 \\
v^{c,i} &= 0.25 + 1 - 0.3 - 0.05 & &= 0.9
\end{aligned}$$

Apply SGS on this set of conditional independences and show the results of each phase, explaining why you remove or orient edges at each step

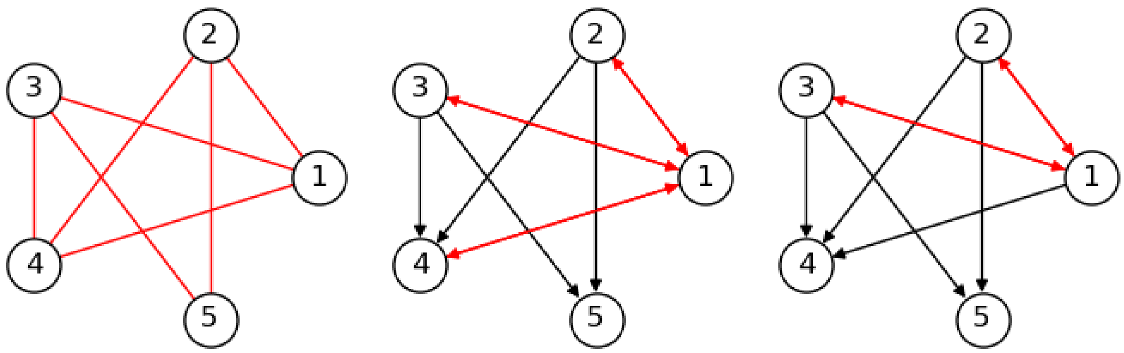
Question:

Apply SGS on the set of conditional independences below and show the results of each phase, explaining why you remove or orient edges at each step.

$$
\begin{aligned}
&2 \perp\!\!\!\perp 3|1 \\
&1 \perp\!\!\!\perp 5|2,3 \\
&1 \perp\!\!\!\perp 5|2,3,4 \\
&4 \perp\!\!\!\perp 5|2,3 \\
&4 \perp\!\!\!\perp 5|1,2,3
\end{aligned}
$$

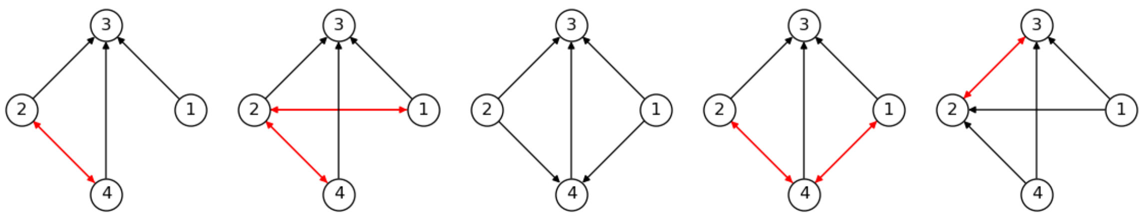
Answer:

1. Skeleton learning, with $|S| = k \in \{0, 1, 2, 3\}$, results in the graph shown in Figure 1a.
   - $k = 0$: No such conditional independence
   - $k = 1$: $2 \perp\!\!\!\perp 3|1 \implies$ remove edge $2 - 3$
   - $k = 2$:
     - $1 \perp\!\!\!\perp 5|2,3 \implies$ remove edge $1 - 5$
     - $4 \perp\!\!\!\perp 5|2,3 \implies$ remove edge $4 - 5$
   - $k = 3$: No such conditional independence/ Already done (two more CIs)
2. After identifying v-structures, we get the graph shown in Figure 1b.
   - $1 - 2 - 5$: not a v-structure because of $1 \perp\!\!\!\perp 5|2,3$ ($S = \{3\}$)
   - $1 - 3 - 5$: not a v-structure because of $1 \perp\!\!\!\perp 5|2,3$ ($S = \{2\}$)
   - $2 - 1 - 3$: not a v-structure because of $2 \perp\!\!\!\perp 3|1$ ($S = \emptyset$)
   - $2 - 4 - 3$: v-structure because only $2 \perp\!\!\!\perp 3|1$ holds and $4 \notin \{1\}$
   - $2 - 5 - 3$: v-structure because only $2 \perp\!\!\!\perp 3|1$ holds and $5 \notin \{1\}$
   - $4 - 2 - 5$: not a v-structure because of $4 \perp\!\!\!\perp 5|2,3$ ($S = \{3\}$)
   - $4 - 3 - 5$: not a v-structure because of $4 \perp\!\!\!\perp 5|2,3$ ($S = \{2\}$)
3. We can apply R3 to orient $1 \rightarrow 4$, which results in the final graph given in Figure 1c.



(a) Skeleton (b) V-structures (c) Meek's rules

Figure 1: Graphs for SGS in exam 2.



(a) Start (b) (c) (d) (e)

Figure 2: CPDAGs for GES in exam 2.
(b): Given $2 \to 4$ add $2 \to 1$ or $1 \to 2$, or given $4 \to 2$ add $2 \to 1$.
(c): Given $2 \to 4$ add $1 \to 4$. 3-4 SHOULD BE UNDIRECTED!!
(d): Given $2 \to 4$ add $4 \to 1$, or given $4 \to 2$ add $1 \to 4$ or $4 \to 1$.
(e): Given $4 \to 2$ add $1 \to 2$.

Starting from this CPDAG, which are the other CPDAGs in the phase 1 neighbours of GES of this CPDAG?

Question:

Starting from the CPDAG in Figure 2a, which are the other CPDAGs in the phase 1 neighbours of GES of this CPDAG?

Answer:

The MEC of the starting CPDAG in Figure 2a contains two DAGs with edges $2 \to 4$ or $4 \to 2$. In each case there are two-two possible edges to add between $(1, 2)$ and $(1, 4)$ each:

- Given $2 \to 4$, add $1 \to 2$ or $2 \to 1$: The resulting CPDAG is shown in Figure 2b.
- Given $2 \to 4$, add $1 \to 4$: The resulting CPDAG is shown in Figure 2c.
- Given $2 \to 4$, add $4 \to 1$: The resulting CPDAG is shown in Figure 2d.
- Given $4 \to 2$, add $1 \to 2$: The resulting CPDAG is shown in Figure 2e.
- Given $4 \to 2$, add $1 \to 4$ or $4 \to 1$: The resulting CPDAG is shown in Figure 2d.
  Same as adding $4 \to 1$ given $2 \to 4$.
- Given $4 \to 2$, add $2 \to 1$: The resulting CPDAG is shown in Figure 2b.
  Same as adding $1 \to 2$ or $2 \to 1$ given $2 \to 4$.