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# Multimodal multi-objective optimization: Comparative study of the state-of-the-art

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A B S T R A C T

Multimodal multi-objective problems (MMOPs) commonly arise in the real world where distant solutions in decision space correspond to very similar objective values. To obtain more Pareto optimal solutions for MMOPs, many multimodal multi-objective evolutionary algorithms (MMEAs) have been proposed. For now, few studies have encompassed most of the representative MMEAs and made a comparative comparison. In this study, we first review the related works during the last two decades. Then, we choose 15 state-of-the-art algorithms that utilize different diversity-maintaining techniques and compared their performance on different types of the existing test suites. Experimental results indicate the strengths and weaknesses of different techniques on different types of MMOPs, thus providing guidance on how to select/design MMEAs in specific scenarios.

## 1. Introduction

Many real-world engineering problems consider optimizing more than one objective. In general, there is a conflict between objectives which means no solution can obtain the best performance on all objectives. Such problems are recognized as multi-objective optimization problems (MOPs) [1–4]. Without loss of generality, a MOP can be expressed as follows:

$$\begin{aligned} \text{Min} \quad & F(\mathbf{x}) = \{f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_m(\mathbf{x})\} \\ \text{s.t.} \quad & \mathbf{x} = (x_1, x_2, \dots, x_n) \in \Omega \end{aligned} \tag{1}$$

where $\Omega$ denotes the search space, $m$ is the number of objectives, and $\mathbf{x}$ is a decision vector that consists of $n$ decision variables $x_i$. A solution, $\mathbf{x_a}$, is considered to Pareto dominate another solution, $\mathbf{x_b}$, $iff$ $\forall i = 1, 2, \dots, m, f_i(\mathbf{x_a}) \leq f_i(\mathbf{x_b})$ and $\exists j = 1, 2, \dots, m, f_j(\mathbf{x_a}) < f_j(\mathbf{x_b})$. Furthermore, a Pareto optimal solution is a solution that is not Pareto dominated by any other solution. The set of Pareto optimal solutions is called a Pareto set (PS). The image of the PS is known as the Pareto front (PF).

To address MOPs, many multi-objective evolutionary algorithms (MOEAs) have been proposed and verified over many wide-acceptable benchmark problems. In general, the aim of MOEAs is to obtain a solution set that approximates the known true Pareto front. This aim contains two parts: the convergence to true PF and the uniformity of distribution in the objective space. In order to address these issues, most of the existing MOEAs adopt the convergence-first strategy and use the crowding distance in the objective space as the second-selection criteria to select a new population after offspring generation, which is also known as the environmental selection strategy.

In real-world engineering optimization, there arises a kind of MOP in that multiple different solutions share the same or similar objective values, termed multimodal multi-objective problems (MMOPs). Fig. 1 shows a two-variable two-objective MMOP, where $A$ and $B$ are distant in the decision space but share the same objective values. The aim of solving MMOPs is to obtain as many Pareto optimal (global and local) solutions as possible. The benefits of solving MMOPs are listed as follows: (1) Finding multiple local or global optimal solutions can help reveal the underlying nature of the problem, thus helping decision-makers (DMs) better understand the problem and conduct analysis of the problem [5]. (2) Multiple alternative solutions can provide DMs with more choices [6]. For manufacturers, multiple alternative solutions mean multiple different scenarios. (3) Finding multiple alternative solutions helps to find robust solutions [7]. (4) Fast switching between multiple candidate solutions helps to solve dynamic optimization problems [8]. (5) Retaining multiple optimal solutions can increase the diversity of solutions and help the algorithm jump out of the local optimal area [9,10].

Fig. 2 shows the research trends of research in MMOPs from 2001 to 2021, from which we can see that research in this field raises more and more attention. Many approaches have been proposed in the recent three years. Compared to MOPs, MMOPs are much more challenging.

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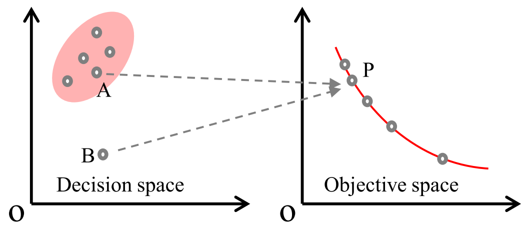


**Fig. 1.** Illustration of MMOPs and the difficulties in solving MMOPs.

To be specific, for traditional MOEAs, the convergence-first strategy weakens the diversity of solutions in the decision space and thus dents the exploration ability of the MOEA. For the problem with more than one global PSs, the PS locates on the steep landscape is likely to be removed during the evolution. Another challenge is to balance the distribution of solutions both in the objective and decision spaces. Overall, traditional MOEAs face serious challenges in solving MMOPs, and developing novel approaches is important for this field.

Over the last two decades, a number of multi-objective multi-modal evolutionary algorithms (MMEAs) have been proposed to solve MMOPs. In 2019, Tanabe et al. [5] made a review of evolutionary multimodal multi-objective optimization (MMO) and discussed several open issues about the existing test suites and the performance metrics. In addition, Yue et al. [11] made a recent review in 2021 that mainly focuses on difficulties in dealing with MMOPs and properties of the existing diversity-maintaining methods, which was written in Chinese.

As we can see from Fig. 2, many pieces of work have been conducted during the last three years. Moreover, to our best knowledge, there is no work that systematically made a comparative comparison of the existing MMEAs. To this end, in this paper, we first review the existing MMOPs test suites and discuss their features. Then, we choose 15 representative MMEAs with different diversity-maintaining mechanisms and compare their performances on different test suites in detail. These comprehensive comparative results show not only the advantages and disadvantages of different diversity-maintaining techniques but also the challenges that existed in different test suites. The contributions of this work can be summarized as follows:

- A comprehensive review of the existing MMOP test suites is conducted. We compare their properties and make a discussion about the open issues and future studies.
- We first conduct a comparative study on the existing representative MMEAs. 15 state-of-the-art MMEAs and 62 test problems (in four groups) are chosen for comparison. The performances in terms of $IGD$, $IGDX$, and $RPSP$ are analyzed. The diversity-maintaining techniques, searching behaviors, and computational complexity are studied.
- Based on the experimental results, the strengths and weaknesses of different techniques, limitations of the indicators, and suggestions for further studies are discussed in this work. Researchers can find suggestions for selecting and designing MMEAs.

The rest of this paper is organized as follows. Section 2 describes existing MMOPs test suites and performance metrics for MMEAs, followed by Section 3 that introduces 15 state-of-the-art MMEAs in detail. Section 4 illustrates the experimental settings, followed by Section 5, which analyzes the experimental results in detail. In Section 6, we further show the overall performance comparison results of the 15 algorithms over all the test suites and analyze the limitations of existing test suites. Section 7 presents our conclusions and some possible paths for future research.

## 2. Related study

It is reported in [5] that, although MMOPs have been addressed for more than ten years, the definition of an MMOP is still controversial. By combining definitions from the existing work, we define an MMOP as follows:

**Definition 1.** For a Pareto optimal solution $\mathbf{x}$ of an MOP, if there exists a distant solution $\mathbf{y}$ satisfying $\|f(\mathbf{x}) - f(\mathbf{y})\| \leq \delta$ ($\delta$ is a small positive value), then the MOP is called as an MMOP.

**Definition 2.** Two solutions $\mathbf{x}$ and $\mathbf{y}$ are said as distant solutions if $\|\mathbf{x} - \mathbf{y}\| \geq \theta$ ($\theta$ is a positive value provided by decision maker).

To be specific, most primitive works consider the situation when $\delta = 0$, which means there is no local PS but several global PSs corresponding to the same global PF. $\delta > 0$ indicates that there exists local PS and local PF, termed multimodal multi-objective problems with local Pareto fronts (MMOPLs) [12].

### 2.1. Multimodal multi-objective evolutionary algorithms

Over the last two decades, many works have been proposed that focus on obtaining as many PSs as possible for MMOPs. Most of the primitive MMEAs perform the environmental selection by calculating the crowding distance in the decision space to enhance the diversity of solutions, e.g., alternative crowding distance used in Omni-optimizer [13,14], active diversity strategy used in MMPICEAg [15].

Another popular way is to utilize the niching mechanism, e.g., niching-covariance matrix adaptation (CMA) approach [16], double-niched evolutionary algorithm (DNEA) [17], decision space-based niching NSGA-II (DN-NSGAII) [18], multi-objective particle swarm optimization using ring topology and special crowding distance (MO_Ring_PSO_SCD) [19] and multi-objective evolution strategy for MMO (MMO-MOES) [20]. To be specific, the clustering-based method is popular to form stable niches, e.g., the k-means clustering method in GSMPSO-MM [21] and clustering-based special crowding distance in MMODE_CSCD [22]. In [23], parallel offspring generation mechanisms are utilized to generate solutions with different requirements. Such methods can be also seen as parallelization techniques, which are to divide the population into several sub-population and evolve them independently [24].

In [25], Liu et al. proposed the imbalanced distance minimization problem (IDMP) test suite, which is harder to be solved since obtaining different PSs needs different numbers of function evaluations. Experimental results show that the previous works are unable to obtain all PSs on IDMPs. Then, many enhanced diversity-maintaining strategies were proposed. Liu et al. [25] proposed a convergence-penalized density (CPD) method to lead the population to search the entire decision space, where solutions will not prematurely converge to some PSs. Motivated by CPDEA, Li et al. [26] proposed a weighted-indicator and resulting in MMEA-WI, where an individual's fitness is related to its neighbors. This method is competitive in dealing with problems with many decision variables. In addition, the CPD-based k-means clustering method and the identical k-means clustering based on distance are proposed to divide the population into multiple subpopulations to deal with the imbalanced characteristic, resulting in TS_MMOEAC [27]. Fan et al. [28] figured out a zoning search with the adaptive resource allocating (ZS-ARA) method to divide the entire search space into many subspaces and got high performance on IDMPs.

Moreover, it is reported in [12,29,30] that MMOPLs are more common in real-world problems and normal MMOPs are special cases of MMOPLs. To address this issue, several approaches were proposed, like $\epsilon$-dominance in $P_{Q,\epsilon}$-MOEA [31] and multifront archive update method in DNEA-L [32]. Lin et al. [33] proposed a dual clustering method in both the objective and decision spaces to obtain the local

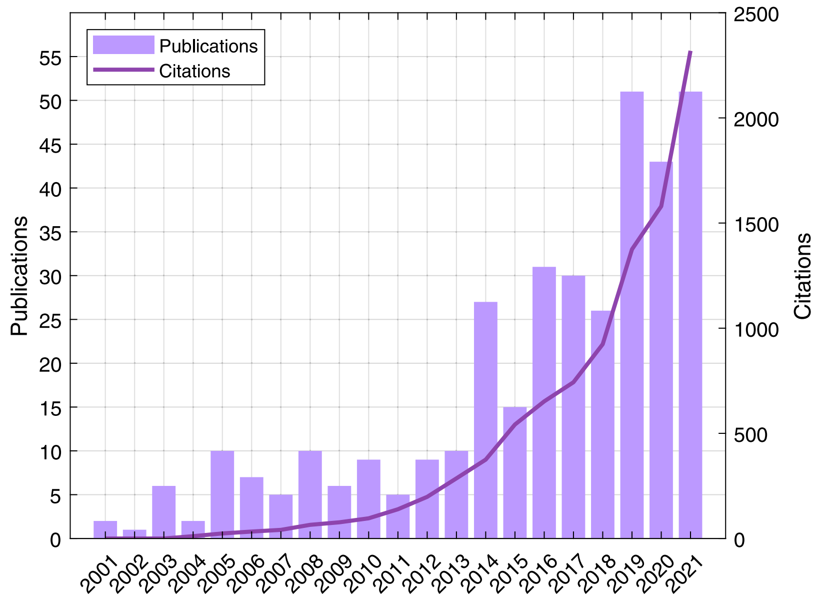


**Fig. 2.** Times cited and publications of **multimodal multi-objective optimization** over time from the Web of Science Core Collection from 2001 to 2021.

PSs with acceptable quality. In addition, Wang et al. [34] proposed a clearing-based evolutionary algorithm with a layer-to-layer evolution strategy (CEA-LES) to solve MMOPLs, where the clearing-based niching technique is used to remove inferior local Pareto optimal solutions. Recently, a hierarchy ranking method in HREA [12] is proposed, where the quality of the obtained local PF can be controlled through a user-predefined parameter.

As we can see, many MMEAs have been proposed to better solve the MMOPs with different diversity-maintaining strategies. Therefore, a comprehensive comparison between these techniques is needed to be conducted.

### 2.2. Benchamrk test suits

To examine the performance of MMEAs, a number of MMOP test suites have been designed. Deb [35] proposed Omni-test problems, where the number of decision variables and the number of PSs is adjustable. After that, SYM-PART test suites [36] were proposed by rotating and distortion operation. In addition, TWO-ON-ONE [37], SSUF problems [18], HPS problems [38] and Polygon problems [39] are also proposed to exam the diversity-maintaining performance of algorithms. However, the number of decision variables is small and not scalable. To this end, the Multi-polygon problems [40,41] are proposed with adjustable dimensions of decision variables and objectives.

In 2017, Yue et al. [19] designed eight MMF test problems by translation and symmetry operation, which are widely adopted as the benchmark problems for MMOPs. Based on the same idea, a novel scalable test suite is proposed [42] that contains both local and global PSs, which is also used as the benchmark for the 2019 IEEE CEC MMO competition [29]. In addition, Liang et al. [43] extended this suite and proposed the CEC 2020 test suite, which contains both MMOPs and MMOPLs with adjustable decision variables and objectives. In 2018, Liu et al. [44] designed the MMMOP test suite, which is scalable both in the objective and decision space. In 2021, Tian et al. [45] proposed a large-scale sparse MMOP test suite motivated by the optimal architecture design problem of the convolutional neural network (CNN).

Most of the above-mentioned test suites assume that the difficulties in searching for different PSs are the same. However, this may not be true for real-world problems, where the landscape may be complex and irregular. To this end, Liu et al. [25] designed the IDMP test suite with 2–4 objectives and variables. The main property of IDMP is that for a point on the PF, solutions close to one equivalent Pareto optimal solution are more likely to dominate solutions close to another equivalent Pareto optimal solution. Experimental results show that the primitive MMEAs are unable to obtain all PSs on IDMPs. Based on IDMP, Li et al. [12] designed the IDMP_e test suite by introducing other single-objective multimodal functions, which contain several local PSs and PFs with adjustable quality.

**Table 1**
Properties of MMOP test suites, where D, M, and P denote the number of decision variables, objectives, and equivalent PSs, respectively. Local indicates if there are local PSs.

| Test suites | D | M | P | Local | PF shape |
|---|---|---|---|---|---|
| Omni-test [35] | Any | 2 | Any | No | Convex |
| SYM-PART [36] | 2 | 2 | 9 | No | Convex |
| TWO-ON-ONE [37] | 2 | 2 | 2 | No | Convex |
| Polygon [39] | 2 | Any | Any | No | Convex |
| Multi-polygon [40] | Any | Any | Any | No | Convex |
| MMF [19] | 2 | 2 | 2 | No | Both |
| Problems in [42] | Any | Any | Any | Yes | Both |
| CEC 2020 test suite [43] | Any | Any | Any | Yes | Both |
| MMMOP [44] | Any | 2 | 4 | No | Concave |
| SMMOP [45] | Any | Any | Any | No | Convex |
| IDMP [25] | 2–4 | 2–4 | 2–4 | No | Convex |
| IDMP_e [12] | 2–3 | 2–3 | Any | Yes | Convex |
| Location planning [39] | 2 | 4 | – | No | Convex |
| Feature selection [46] | Any | 2 | – | Yes | Convex |
| Path planning [43] | – | 2–7 | – | No | Convex |

As for the real-world problems, Ishibuchi et al. [39] generated a multimodal four-objective location planning problem from a real-world map considering the distances to the nearest elementary school, junior high school, railway station, and convenience store. However, there are only two decision variables. In addition, Yue et al. [46] found that the multi-objective feature selection problem is a typical MMOP. Moreover, Liang et al. [43] proposed a multimodal multiobjective path planning test suite and launch a competition [6,47]. However, since the encoded length of a path is discrete and indeterminate, the existing MMEAs cannot be directly used to solve these problems. Recently, Liang et al. [48] made some efforts in identifying personalized biomarkers for disease prediction and proposed a multi-modal personalized dynamic network biomarkers (MMPDNB) model.

Generally speaking, a comprehensive MMOP test suite should have the following properties: the dimension of the decision space can be extended; the number of objectives can be adjusted; the true PSs and

**Table 2**
Overview of the compared MMEAs (in chronological order).

| Algorithm | Framework | Local | Mechanism/Strategy |
|---|---|---|---|
| Omni-optimizer [13] | GA | No | Latin hypercube sampling-based population; restricted mating selection and alternative crowding distance |
| DN-NSGAII [18] | GA | No | Niching in the decision space; crowding distance in the decision space |
| MO_Ring_PSO_SCD [19] | PSO | No | Ring topology; special crowding distance |
| MO_PSO_MM [9] | PSO | No | Self-organizing method to find the neighborhood relation; special crowding distance |
| DNEA [17] | GA | No | Niche sharing method in both the objective and decision spaces |
| Tri-MOEA&TAR [44] | Decompose-based | No | Diversity archive and convergence archive; clustering method and niching; decision variable analysis method |
| DNEA-L [32] | GA | Yes | Niche sharing method in both the objective and decision spaces; multi-front archive |
| CPDEA [25] | One-by-one | No | Convergence-penalized density method; double k-nearest neighbor method |
| SS-MOPSO [51] | PSO | No | Self-organized speciation method; special crowding distance techniques |
| MMODE_CSCD [22] | DE | No | Clustering-based special crowding distance (CSCD); distance-based elite selection mechanism (DBESM) |
| MP-MMEA [45] | GA | No | Guide multiple subpopulations via adaptively updated guiding vectors; binary tournament selection; subpopulation similarity |
| MMOEA/DC [33] | GA | Yes | Neighborhood-based clustering method in the decision space; hierarchical clustering method in the objective space; harmonic averaged distance |
| MMODE_ICD [52] | DE | No | Adaptive selection method for selecting parents; improved crowding distance; embed the nondominated rank into SCD |
| MMEA-WI [26] | Indicator-based | No | Weighted-indicator; convergence archive |
| HREA [12] | GA | Yes | Local convergence quality; hierarchy ranking method; convergence archive |

PFs are known; the PSs and PFs have various shapes; the number of PSs is scalable; the local PSs and the global PSs coexist. To be specific, the overall properties of the existing MMOP test suites are listed in Table 1. As we can see, the test suite proposed in [42] is considered a comprehensive benchmark for now, which is extended to form the CEC 2019 test suite and CEC 2020 test suite. It is scalable both in objectives and decision variables with local PSs. However, many existing test problems are relatively simple and the required function evaluations for these problems are small compared to real-world problems. For the existing MMEAs, there is no suitable benchmark problem that can both examine the abilities to maintain diversity and converge to the PF. In addition, many real-world problems are discrete or mix-integer. It is hard to check the performance of the existing MMEAs on such optimization problems so far.

### 2.3. Performance metrics

For traditional MOPs, obtaining well-distributed solutions with good convergence is the most important target. Therefore, indicators such as Inverted Generation Distance ($IGD$) [49] and HyperVolume ($HV$) are usually utilized to measure the quality of the obtained solutions. Different from traditional MOPs, the aim of solving MMOPs is to obtain as many Pareto optimal solutions as possible. Thus, the diversity of solutions in the decision space is important. To this end, motivated by $IGD$, the Inverted Generation Distance in the Decision Space ($IGDX$) [50] is proposed. Specifically, for an obtained solution set $\mathbf{X}$, the $IGD$ and $IGDX$ can be calculated as:

$$IGD(\mathbf{X}) = \frac{1}{|\mathbf{X}^*|}\sum_{\mathbf{y}\in\mathbf{X}^*}\min_{\mathbf{x}\in\mathbf{X}}\{ED(f(\mathbf{x}), f(\mathbf{y}))\}, \tag{2}$$

$$IGDX(\mathbf{X}) = \frac{1}{|\mathbf{X}^*|}\sum_{\mathbf{y}\in\mathbf{X}^*}\min_{\mathbf{x}\in\mathbf{X}}\{ED(\mathbf{x}, \mathbf{y})\}, \tag{3}$$

where $ED(\mathbf{x}, \mathbf{y})$ is the Euclidean distance between $\mathbf{x}$ and $\mathbf{y}$. $\mathbf{X}$ and $\mathbf{X}^*$ denote the obtained solution set and a set of a finite number of Pareto optimal solutions uniformly sampled from the true PS, respectively.

$IGDX$ is a representative and widely used metric that evaluates the performance of an MMEA in finding solutions with high convergence quality and good distribution in the decision space. However, $IGDX$ cannot measure the diversity in the objective space. Thus, many works use both $IGD$ and $IGDX$ to overall evaluate the performance of an MMEA. In addition, Cover Rate ($CR$) and Pareto Sets Proximity ($PSP$) [19] are proposed to reflect the similarity between the obtained PSs and the true PSs, where $CR$ is a modification of the maximum spread ($MS$) [53], $PSP$ is a combination of $CR$ and $IGDX$, which can be expressed as:

$$PSP(\mathbf{X}) = \frac{CR(\mathbf{X})}{IGDX(\mathbf{X})}, \tag{4}$$

$$CR(\mathbf{X}) = (\prod_{i=1}^{D}\sigma_i)^{\frac{1}{2D}}, \tag{5}$$

$$\sigma_i = (\frac{\min(x_i^{*,max}, x_i^{max}) - \max(x_i^{*,min}, x_i^{min})}{x_i^{*,max} - x_i^{*,min}})^2, \tag{6}$$

where $x_i^{*,max}$ and $x_i^{max}$ are the maximum values of the $i$th variable in the PS and the obtained solutions respectively. $\sigma_i = 1$ when $x_i^{*,max} = x_i^{*,min}$; $\sigma_i = 0$ when $x_i^{*,max} \leq x_i^{min}$ or $x_i^{max} \leq x_i^{*,min}$.

The larger $PSP$ is the better performance of the solution set. Since the best $PSP$ value is infinitely large, it is hard to evaluate the distance between the evaluated solution set and the reference solution set. Then in [42], $RPSP = 1/PSP$ is used as a new indicator.

The above-mentioned metrics evaluate the solutions' quality only in the decision space. To this end, the Inverted Generational Distance-Multi-modal ($IGDM$) [44] is proposed, which can measure not only the convergence performance but also the diversity performances both in the objective and the decision spaces. However, it needs a parameter defined by the user.

To sum up, the convergence and diversity quality in the decision space of a solution set can be well evaluated by $IGDX$ and $RPSP$. However, the quality in the objective space is not evaluated properly. Thus, $IGD$ and $HV$ are also popular indicators for evaluating the performance of MMEAs. For now, the existing performance metrics need information on the true PF and PS, which is hard for real-world problems. Moreover, a parameter-free indicator that can measure the diversity and convergence quality of a solution set both in the objective and the decision space is needed.

## 3. Compared multimodal multi-objective evolutionary algorithms

In this part, the detailed information of the 15 chosen MMEAs is introduced, namely, Omni-optimizer [13,14], DN-NSGAII [18], MO_Ring_PSO_SCD [19], MO_PSO_MM [9], DNEA [17], Tri-MOEA&TAR [44], DNEA-L [32], CPDEA [25], SS-MOPSO [51], MMODE_CSCD [22], MP-MMEA [45], MMOEA/DC [33], MMODE_ICD [52], MMEA-WI [26] and HREA [12]. It is worth mentioning that, the selection of comparison algorithms is subjective, and different researchers may have great differences in the selection over many proposed MMEAs. The purpose of this paper is to compare as many diversity-maintaining strategies in solving MMOPs as possible. Therefore, we select the above-mentioned MMEAs mainly according to the diversity-maintaining mechanisms (summarized in Table 2), attention to the works (mainly refers to the citations for previously published algorithms and attention degree for later proposed algorithms), and the period of the paper publication.

**Omni-optimizer** is considered one of the most representative MMEAs. It is proposed to obtain multi-optima both for single and multi-objective problems. There are several strategies utilized in Omni-optimizer. First, the Latin hypercube is used to uniformly generate the initial population. Second, a nearest neighbor-based strategy is proposed to choose two individuals that take part in tournament selection (known as restricted selection). Third, a two-tier fitness assignment scheme is adopted in which the primary fitness is computed using the phenotypes (objectives and constraint values). And the secondary fitness is computed using both phenotypes and genotypes (decision variables).

**DN-NSGAII** introduces a niching method and a selection operator to create the mating pool and select offspring, respectively. The procedure of the niching method can be roughly described as follow. A solution and a constant number of solutions are randomly chosen. Then, the current solution and the solution with the smallest Euclidean distance to the current solution are selected. Repeat the above steps until the mating pool is full. In the original paper, the authors also proposed SS-UF1 and S-UF3 test problems.

**MO_Ring_PSO_SCD** is another representative MMEA that received much attention. The personal best archive (PBA) and the neighborhood best archive (NBA) are first established. Then, ring topology [54] is used to induce multiple niches. In addition, motivated by Omni-optimizer, special crowding distance (SCD) is proposed to maintain the diversity of solutions both in the decision and objective space. In this paper, the MMF test problems and the $PSP$ indicator are proposed to examine the performance of MMEAs.

**MO_PSO_MM** introduced a self-organizing mechanism, which is updated simultaneously during the evolution, to find the distribution structure of the population and build the neighborhood in the decision space. Then, the solutions which are similar to each other can be mapped into the same neighborhood. In addition, special crowding distance [19] is utilized to maintain the diversity of solutions. The effectiveness of MO_PSO_MM is mainly verified by the MMF test suite.

**DNEA** introduced a niche-sharing method to diversify the solution set in both the objective and decision spaces. Through the double-niching method, a solution that is very close to others in the objective (decision) space but far away from others in the decision (objective) space still has a chance to be selected. The performance and behavior of DNEA were verified by Polygon-based problems.

**Tri-MOEA&TAR** proposed to use two archives (convergence and diversity archives) and the recombination strategy to solve MMOPs. To be specific, the decision variable analysis method is first performed to find the convergence-related and diversity-related decision variables. Then, this information is passed to the two archives to ensure better convergence and diversity quality. Finally, the recombination strategy is used to obtain a large number of multiple Pareto optimal solutions. The MMMOP test suite was proposed in this work.

**DNEA-L** is proposed to solve MMOPLs based on DNEA [17]. A multi-front archive update method is utilized to obtain both global and local PSs. To be specific, a neighborhood size is first calculated. Then, solutions dominated by their neighbors are removed. The sizes of the first $K$ fronts are maintained by the double-sharing function proposed in DNEA. The effectiveness of DNEA-L is verified by Polygon-based problems with local Pareto optima.

**CPDEA** is the first work that considers the imbalanced searching difficulties for different PSs. Convergence-penalized density method is proposed to help explore the whole decision space during the evolution. Thus, the population will not prematurely converge to some equivalent Pareto optimal solutions which are easy to find. In addition, the double $k$-nearest neighbor method is utilized to further improve the distribution both in the decision and objective space. Moreover, the IDMP test suite was proposed to better examine the ability of MMEAs in maintaining diversity.

**SS-MOPSO** introduced a self-organized speciation strategy to form stable niches, which is used to seek the neighborhood structure of the population. To be specific, during the selection process, SS-MOPSO first selects the species seed according to the non-dominated ranks. Then, the seed will directly grow to form its own species according to a preset parameter radius. Therefore, the efficiency and the performance of species formulation could be well enhanced with no individuals overlapping. In addition, the multi-objective particle swarm optimizer is utilized with special crowding distance techniques. The performance of SS-MOPSO is verified through the MMF test suite and Omni-test problems.

**MMODE_CSCD** proposed a clustering-based special crowding distance (CSCD) method that takes the crowding degree in the decision and the objective spaces into account. Specifically, the authors used the clustering algorithm to divide the solutions with the same non-dominated ranks into multiple classes, which can be seen as different PSs. Then, a distance-based elite selection mechanism (DBESM) is adopted to get evenly distributed solutions. CEC 2019 MMO benchmark suite [29] is utilized to examine the performance and the searching behaviors of MMODE_CSCD.

**MP-MMEA** first considers solving large-scale MMOPs. Multiple subpopulations are introduced to obtain equivalent PSs, which are maintained according to the proposed subpopulation similarities. Thus, for each subpopulation, the diversity in the decision space does not need to be measured. An adaptively updated guiding vector is utilized to distinguish the search directions of different subpopulations. In addition, the proposed benchmark problems SMMOP1-SMMOP8 are adopted to verify the performance of MP-MMEA.

**MMOEA/DC** considered obtaining both global and local PSs by utilizing the double clustering method, namely, the neighborhood-based clustering method (NCM) in the decision space and the hierarchical clustering method (HCM) in the objective space. In addition, the harmonic averaged distance is utilized to evaluate the crowding distance of solutions in the decision space. The performance of MMOEA/DC is verified by the novel MMF test suites [42].

**MMODE_ICD** proposed several techniques to obtain multiple PSs. First, based on a differential evolution algorithm, three methods are adaptively used to pick parents for offspring generation. Then, the authors proposed to embed the non-dominated rank information into the special crowding distance which is replaced with the weighted sum of Euclidean distances to its neighbors. Finally, a novel environmental selection method is proposed so that only a certain percent of solutions in front ranks can be selected.

**MMEA-WI** adopted a weighted-indicator to evaluate the potential convergence quality of a solution, which is derived from IBEA [55]. The weighted-indicator for a solution is calculated by summing the fitness of other solutions according to the distance. Then, solutions will gather around the PS. In addition, a convergence archive is introduced to improve the convergence ability and maintain the uniformity of solutions. The effectiveness of MMEA-WI is mainly verified by IDMP test suites.

**HREA** adopted a local convergence indicator to evaluate the local convergence of solutions. Then, both global and local PSs can be preserved during the evolution. To control the quality of the obtained local PF, a hierarchy ranking method is proposed that can balance the convergence and diversity of solutions. In addition, based on IDMP, IDMP_e is proposed which has an adjustable number of global and local PSs.

Overall, the main differences and similarities of the 15 selected algorithms can be summarized in Table 2, which lists the framework and diversity-maintaining mechanisms. As we can see, most of the MMEAs adopt crowding distance (or variants) in the decision

**Table 3**
Specific parameters setting for the compared twelve MMEAs.

| Algorithm | Parameter setting |
|---|---|
| Omni-optimizer [13] | $\epsilon = 0.001$ |
| DN-NSGAII [18] | – |
| MO_Ring_PSO_SCD [19] | $C_1 = C_2 = 2.05,\ W = 0.7298$ |
| MO_PSO_MM [9] | $C_1 = C_2 = 2.05,\ W = 0.7298,\ \eta_0 = 0.7$ |
| DNEA [17] | $\sigma_{obj} = 0.06,\ \sigma_{var} = 0.02$ |
| Tri-MOEA&TAR [44] | $\epsilon_{peak} = 0.01,\ \sigma = 0.05,\ p_{con} = 0.2$ |
| DNEA-L [32] | $nb = 3,\ K = 3,\ N_{ns} = 50$ |
| CPDEA [25] | $\eta = 2,\ K = 3$ |
| SS-MOPSO [51] | $C_1 = C_2 = 2.05,\ W = 0.7298$, species radius is set to 1/20 of the variable range |
| MMODE_CSCD [22] | $FF = 0.8,\ Cr = 1,\ n = 10$ for clustering |
| MP-MMEA [45] | – |
| MMOEA/DC [33] | $\lambda = 0.1,\ \beta = 5$ |
| MMODE_ICD [52] | $F = 0.5,\ Cr = 0.1$, neighborhood size is set to 12 |
| MMEA-WI [26] | $p = 0.4$ |
| HREA [12] | $\epsilon = 0.2$ for all test problems and $\epsilon = 0.5$ for IMDPM3_e |

space as the mating selection and/or second-selection criteria. Primitive works considered more diversity in the decision space. However, since MMEAs belong to MOEAs, it is also important to maintain the distribution of solutions in the objective space. Thus, more and more works take the crowding distance both in the decision and objective spaces into account. Genetic algorithm (GA) is the most popular framework/optimizer, while particle swarm optimizer (PSO), differential evolution (DE), decomposed-based method, and indicator-based method are also successfully utilized.

For the three MMEAs that focus on obtaining local PSs, niching techniques are still popular, which is to compare solutions in a small region instead of the whole decision space. The performance of such methods is strongly dependent on correctly dividing solutions into their species. To fulfill this goal, DEAN-L adopted the niche sharing method, MMOEA/DC utilized the clustering approach, and HREA proposed a local convergence quality indicator. The use of niching can somehow improve the diversity of solutions in the search space. However, since the convergence qualities of solutions are compared with their neighbors instead of the whole population, the convergence ability of the algorithm is certainly worse than normal MOEAs. Therefore, the performance of the proposed MMEAs in solving normal MMOPs should be further studied.

## 4. Setting of computational experiments

### 4.1. Benchmark and parameter specifications

This work aims to comprehensively compare the performance of existing MMEAs. Therefore, according to our previous discussion, the CEC 2020 test suite (13 problems), IDMP (12 problems), Polygon-based problems (20 problems), and problems with local PFs (IDMP_e and some of the CEC 2020 test suite, 17 problems) are selected as the test problems (62 problems in total). Specifically, the CEC 2020 test suite has complex PF shapes and is considered representative MMOPs, IDMP is used to test the diversity-maintaining ability since there are different difficulties in finding different PSs, and Polygon-based problems are adopted to examine the ability in solving many-objective and many-decision-variable MMOPs, where the number of objectives $M$ and decision variables $D$ are set to {3, 4, 6, 8} and {2, 4, 8, 10, 20}. Finally, IDMP_e and some of the CEC 2020 test suite is chosen to examine the MMEAs' ability in obtaining local PSs.

As for the algorithms' parameters, each algorithm has its preferred setting for common parameters (population size $N$ and function evaluations $FEs$), e.g., there is no need for Tri-MOEA&TAR to set a large population size. Therefore, the performance of algorithms may vary on the parameter setting. For a widely-accepted and fair comparison, according to the previous works [29], we set $N = 100 * D$ and $FEs = 5000 * D$ respectively, where $D$ is the number of decision variables. For Polygon-based problems, we set $N$ to 200, 300, 400, and 400 when $M$ is 3, 4, 6, and 8, respectively. The simulated binary crossover (SBX) and polynomial mutation (PM) operators are employed to generate offspring except for MO_Ring_PSO_SCD, MO_PSO_MM, MMODE_CSCD, and MMODE_ICD. In addition, other specific parameters are set according to the original papers, which are listed in Table 3. All experiments are conducted on a PC configured with an Intel i9-9900X @ 3.50 GHz and 64G RAM. PlatEMO [56] and framework proposed in IEEE CEC 2019 competition[1] [29] are adopted. For the convenience of researchers in the MMOP field, we collect the existing source code of MMEAs and MMOP test suites and keep updating.[2]

### 4.2. Performance evaluation

As we discussed in Section 2.3, $IGDX$ and $RPSP$ can well evaluate the distribution of solutions in the decision space while $IGD$ can evaluate solution quality in the objective space. Other performance metrics like $HV$ and $IGDM$ are similar to $IGD$ and $IGDX$. Specifically, for MMOPLs, $HV$ is inaccurate to evaluate the performance of the obtained solutions. $HV$ aims to calculate the contributions of each solution, where solutions located in the global PF contribute more than that in the local PF. Therefore, we select $IGD$, $IGDX$, and $RPSP$ as the performance metrics to comprehensively compare the performance of all MMEAs. In addition, to evaluate the overall performances, the non-parametric statistical test Friedman test [57] is adopted. Specifically, for each test problem, the results of 30 times independent runs of all MMEAs are used to calculate the average ranks ($r_i^j$, where $i$ and $j$ are indexes of algorithms and test problems) by the Friedman test. Then, for a test suite, these ranks are summed to calculate the overall average ranks, which indicate the performances of MMEAs in the specific test suite, shown as follows:

$$R_i = \frac{\sum_{j=1}^{J} r_i^j}{J} \tag{7}$$

where $J$ is the number of test problems, e.g., $J = 12$ for IDMP test suite. The smaller the $R_i$ is, the better performance of the $i$th MMEA.

## 5. Results and discussion

To comprehensively compared the performances of MMEAs, four groups of MMOP test problems are adopted. All experiments are independently executed 30 times and the average results are used to present and analyze. However, due to page limitations, the detailed values of the running result are provided in the supplementary file, e.g., $IGD$, $IGDX$, and $RPSP$. Each table (Table S-III to Table S-XIV) shows the

[1] The source codes and results can be found in http://www5.zzu.edu.cn/ecilab/info/1036/1211.htm.

[2] The source code of algorithms and MMOP test suites used in this work can be accessed in https://github.com/Wenhua-Li/ComparativeStudyofMMOP.

**Table 4**
Average ranks $R$ of IGD, IGDX and RPSP for 15 compared MMEAs on four different MMOP test suites, where the best rank is highlighted with gray background.

| Problems | Indicators | Omni-optimizer | DN-NSGAII | MO_Ring_PSO_SCD | MO_PSO_MM | DNEA | Tri-MOEA&TAR | DNEA-L | CPDEA | SS-MOPSO | MMODE_CSCD | MP-MMEA | MMOEA/DC | MMODE_ICD | MMEA-WI | HREA |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| MMF | IGD | 8.71 | 11.53 | 10.88 | 6.10 | 3.10 | 11.29 | 12.36 | 3.83 | 8.61 | 4.10 | 10.49 | 5.69 | 5.08 | 9.40 | 8.83 |
| | IGDX | 13.73 | 14.05 | 8.59 | 5.45 | 8.21 | 10.23 | 8.84 | 2.86 | 6.29 | 5.23 | 12.53 | 6.59 | 7.53 | 6.49 | 3.35 |
| | RPSP | 13.71 | 14.02 | 8.63 | 5.51 | 8.24 | 10.32 | 8.84 | 2.80 | 6.37 | 5.23 | 12.51 | 6.54 | 7.48 | 6.45 | 3.35 |
| IDMP | IGD | 9.40 | 10.04 | 13.84 | 13.95 | 1.11 | 10.36 | 5.42 | 4.57 | 13.60 | 4.58 | 7.51 | 9.71 | 3.68 | 7.49 | 4.75 |
| | IGDX | 11.53 | 11.68 | 8.72 | 7.66 | 11.90 | 13.04 | 5.24 | 5.19 | 7.47 | 8.32 | 9.72 | 2.96 | 8.39 | 4.29 | 3.88 |
| | RPSP | 11.03 | 11.18 | 8.48 | 7.44 | 12.41 | 13.34 | 5.22 | 5.30 | 7.28 | 8.39 | 9.97 | 2.94 | 8.70 | 4.37 | 3.95 |
| MMOPL | IGD | 11.22 | 12.27 | 8.70 | 8.26 | 8.12 | 13.97 | 3.67 | 7.25 | 8.10 | 6.62 | 7.87 | 1.67 | 11.49 | 9.40 | 1.39 |
| | IGDX | 11.89 | 11.68 | 7.54 | 7.91 | 10.56 | 12.42 | 3.49 | 8.50 | 7.45 | 7.33 | 8.46 | 1.90 | 10.50 | 9.14 | 1.23 |
| | RPSP | 10.80 | 10.44 | 6.53 | 6.92 | 11.97 | 13.74 | 3.43 | 9.24 | 6.52 | 7.61 | 8.46 | 1.91 | 11.94 | 9.27 | 1.23 |
| Polygon | IGD | 13.49 | 13.31 | 10.86 | 11.25 | 2.33 | 8.52 | 6.07 | 8.67 | 13.01 | 4.11 | 8.19 | 7.27 | 4.35 | 3.55 | 5.05 |
| | IGDX | 14.02 | 13.65 | 9.78 | 10.48 | 5.25 | 9.13 | 5.31 | 9.35 | 12.83 | 4.25 | 9.37 | 4.43 | 4.25 | 3.05 | 4.88 |
| | RPSP | 13.17 | 12.97 | 9.89 | 10.93 | 5.90 | 10.15 | 4.18 | 8.92 | 12.45 | 4.68 | 9.07 | 3.54 | 4.29 | 3.75 | 6.11 |
| Overall | IGD | 11.07 | 12.02 | 10.85 | 9.87 | 3.84 | 10.95 | 6.61 | 6.47 | 10.85 | 4.89 | 8.45 | 5.88 | 6.33 | 7.14 | 4.78 |
| | IGDX | 12.89 | 12.82 | 8.71 | 8.17 | 8.61 | 11.02 | 5.54 | 6.95 | 8.95 | 6.09 | 9.85 | 3.90 | 7.45 | 5.68 | 3.36 |
| | RPSP | 12.22 | 12.15 | 8.43 | 8.02 | 9.32 | 11.79 | 5.15 | 7.02 | 8.55 | 6.32 | 9.80 | 3.61 | 7.91 | 5.95 | 3.78 |

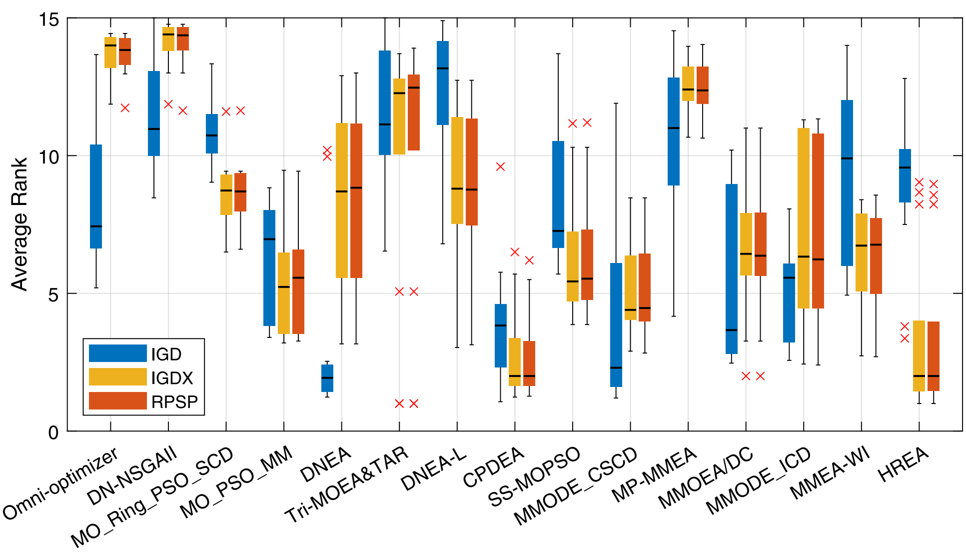


**Fig. 3.** The box plot of Friedman ranks for all compared MMEAs on part of the CEC 2020 test suite in terms of $IGD$, $IGDX$, and $RPSP$, where the black lines and red X points are average ranks and outliers, respectively. The 25% and 75% quantiles are presented with boxes.

average and variance over 30 runs with the best results highlighted. In addition, all experimental results can be obtained online. Table 4 lists the detailed average rank values of IGD, IGDX, and RPSP for all algorithms. In addition, the intuitive exhibitions are provided with bar plots in the following discussion.

## 5.1. Performance comparison on CEC 2020 test problems

Fig. 3 presents the overall Friedman rank of all compared MMEAs on the CEC 2020 test suite, from which we can see that the average ranks of $IGDX$ and $RPSP$ are very similar. Although there are minor differences in the values of these two indicators, they are highly consistent in the ranking of algorithm performance. Therefore, in subsequent content, we only discuss the results of $IGD$ and $IGDX$. In terms of $IGD$, DNEA, CPDEA, and MMODE_CSCD perform better than other compared algorithms. As we can see from Table S-III, which lists the $IGD$ results, DNEA, CPDEA, and MMODE_CSCD win 6, 2, and 5 instances respectively. Another competitive MMEA is MMODE_ICD, which performs a bit worse than DNEA. As for algorithms that consider local PSs (DNEA-L, MMOEA/DC, and HREA), their performances are apparently worse than other MMEAs in terms of $IGD$. Further study shows that some of the obtained solutions cannot reach the true PF. The aim to obtain local PS needs to simultaneously explore the whole decision space, which may be the reason for bad performance in terms of $IGD$.

As for $IGDX$ and $RPSP$, CPDEA and HREA outperform other algorithms. As shown in Table S-IV and Table S-V, HREA and CPDEA win 7 and 1 instances over 13 problems. CPDEA performs well on CEC 2020 test problems both in terms of $IGD$ and $IGDX$. The double $k$-nearest neighbor method used in CPDEA can measure the crowding distance in both the decision and objective spaces. HREA performs a bit worse than CPDEA. It adopts the local convergence quality to maintain diversity which is effective in dealing with MMOPs. Although MO_PSO_MM and MMODE_CSCD cannot obtain the best result for any test problem, their overall performance is relatively strong and stable. The self-organizing method can help form a stable neighbor relationship that can help maintain diversity. Omni-optimizer, DN-NSGAII, Tri-MOEA&TAR, and MP-MMEA are the worst four MMEAs in terms of $IGDX$. To be specific, Omni-optimizer and DN-NSGAII are two primitive and representative MMEAs. Although the diversity-maintaining strategies are relatively weak, many later works are motivated by them. Tri-MOEA&TAR used a decision variable analysis method, which may be inapplicable to the MMF test suite. MP-MMEA is designed especially for large-scale sparse MMOPs. Thus, its performance on two-variable simple MMOPs is relatively weak.

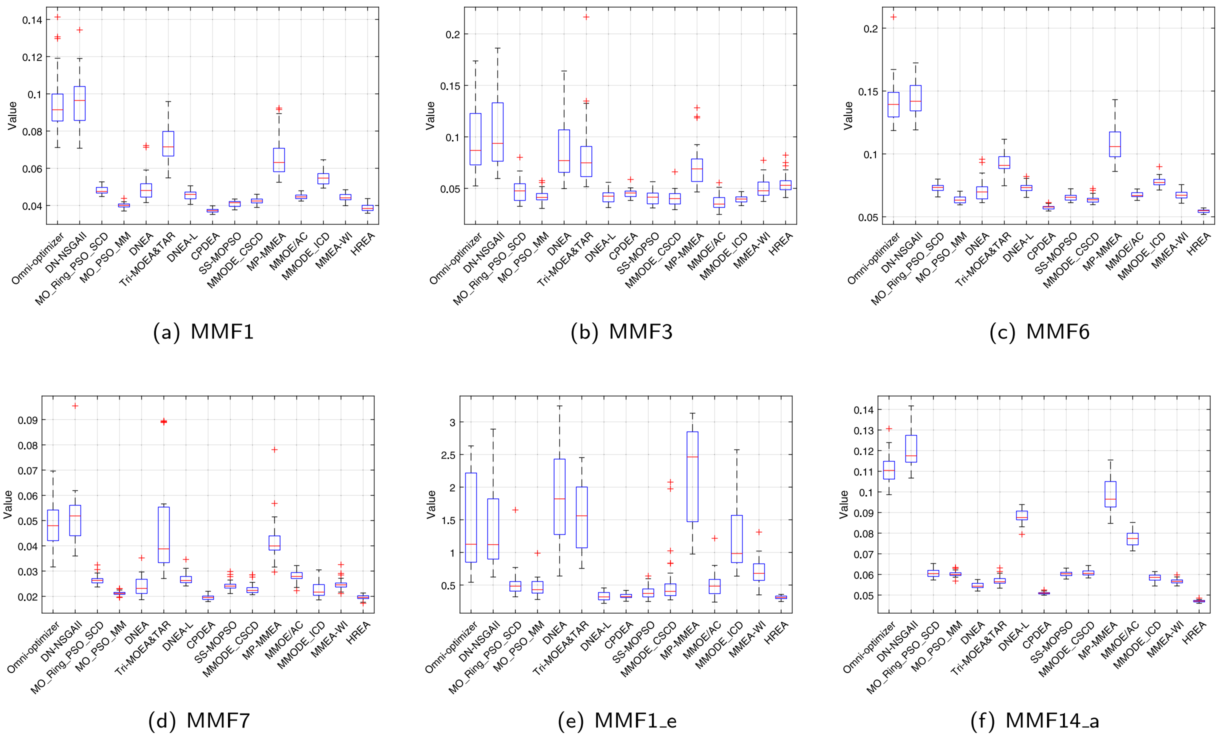


**Fig. 4.** The box plots of $IGDX$ over 30 independent runs obtained by all algorithms on part of the CEC 2020 test problems.

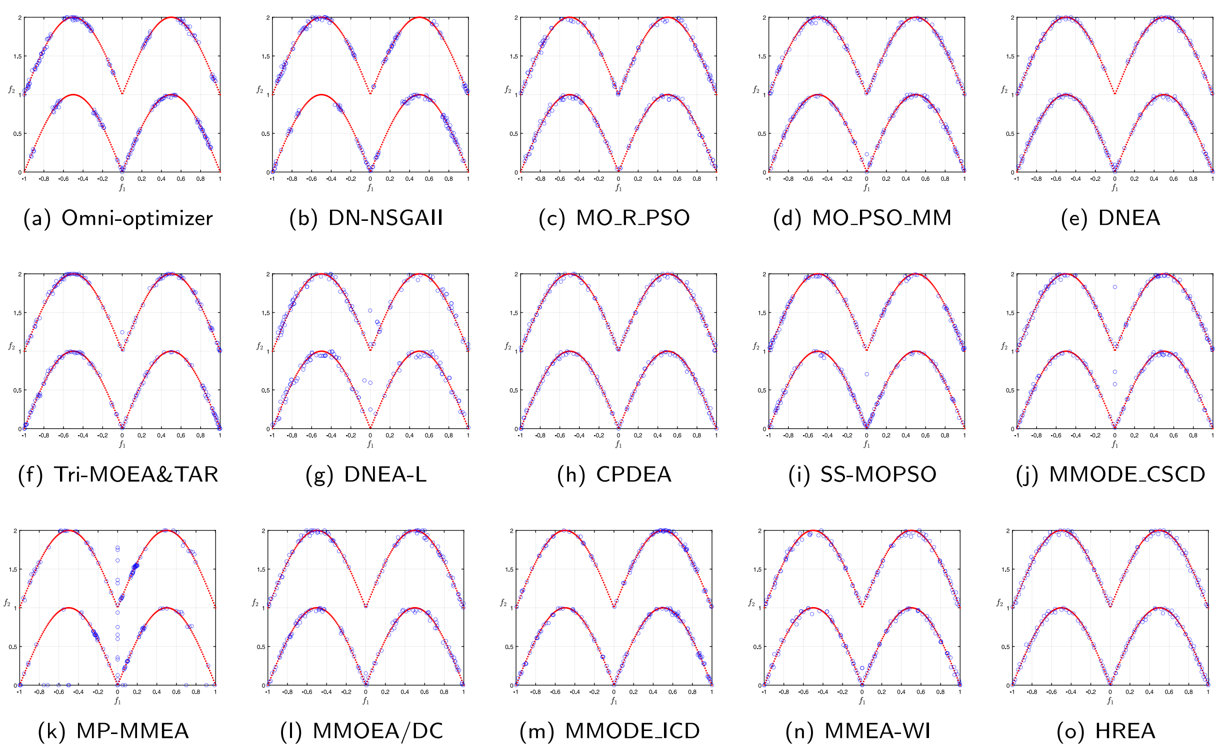


**Fig. 5.** The distribution of solutions obtained by all algorithms (MO_R_PSO is the short name for MO_Ring_PSO_SCD) in the decision spaces on MMF4, where the red points and blue circles are true PS and obtained solutions respectively.

Fig. 4 shows the box plots of these algorithms on some of the test problems in terms of $IGDX$. As we can see, MO_Ring_PSO_SCD, MO_PSO_MM, DNEA-L, CPDEA, SS-MOPSO, MMODE_CSCD, MMOEA/DC, MMEA-WI, and HREA are stable in dealing with CEC 2020 problems, while Omni-optimizer, DN-NSGAII, and MP-MMEA are unstable.

To further analyze the performance of all algorithms, Fig. 5 shows the final distribution of solutions on MMF4. As we can see, solutions obtained by MO_PSO_MM, CPDEA, MMEA-WI, and HREA are more evenly distributed in the decision space. The same situation can be observed in other test problems. Readers can find more information

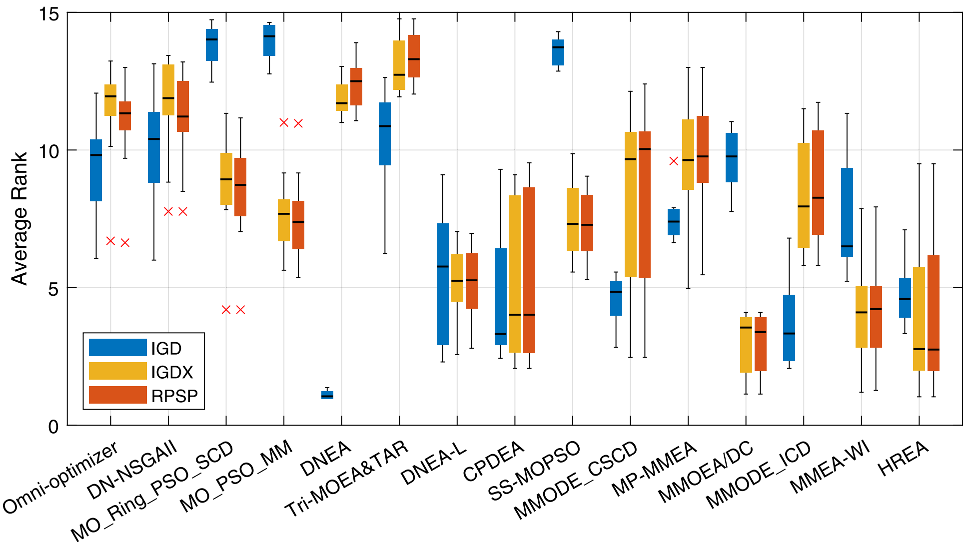


**Fig. 6.** The box plot of Friedman ranks for all compared MMEAs on IDMP test suite in terms of $IGD$, $IGDX$, and $RPSP$, where the black lines and red X points are average ranks and outliers, respectively. The 25% and 75% quantiles are presented with boxes.

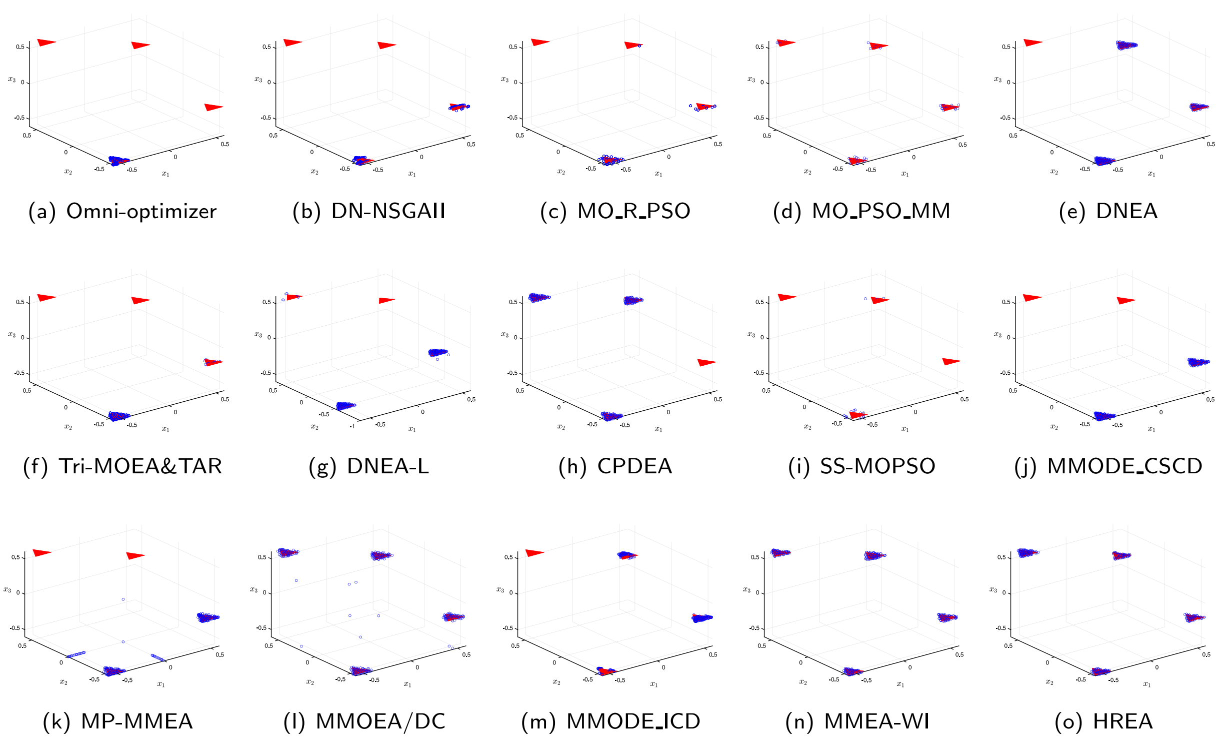


**Fig. 7.** The distribution of solutions obtained by all algorithms (MO_R_PSO is the short name for MO_Ring_PSO_SCD) in the decision spaces on IDMPM3T4, where the red points and blue circles are true PS and obtained solutions respectively.

in the supplementary file. To sum up, CPDEA is the best algorithm for the selected CEC 2020 test problems, while HREA, MMODE_CSCD, and MO_PSO_MM are competitive.

### 5.2. Performance comparison on IDMP problems

The main property of the IDMP test suite is that the difficulties of searching different PSs are different. Thus, normal MOEAs are more likely to converge to the easy-searching PS. Therefore, IDMP is a more accurate test suite to examine the diversity-maintaining ability of algorithms.

The overall performance rank is presented in Fig. 6, from which we can see that in terms of $IGDX$ and $RPSP$, MMOEA/DC, HREA and MMEA-WI are in the first echelon, followed by DNEA-L and CPDEA. As we can see, algorithms considering local PSs (DNEA-L, MMOEA/DC, and HREA) show overwhelming superiority in such problems. They can easily obtain all PSs for most of the IDMP test problems except for some problems with 4 objectives, while other primitive algorithms can only obtain some of the PSs. In general, algorithms proposed before DNEA-L perform poorly on the IDMP test suite. Primitive works did not consider the situation that different difficulties in searching for different PSs. The same conclusion can be found in [25]. Table S-VI,

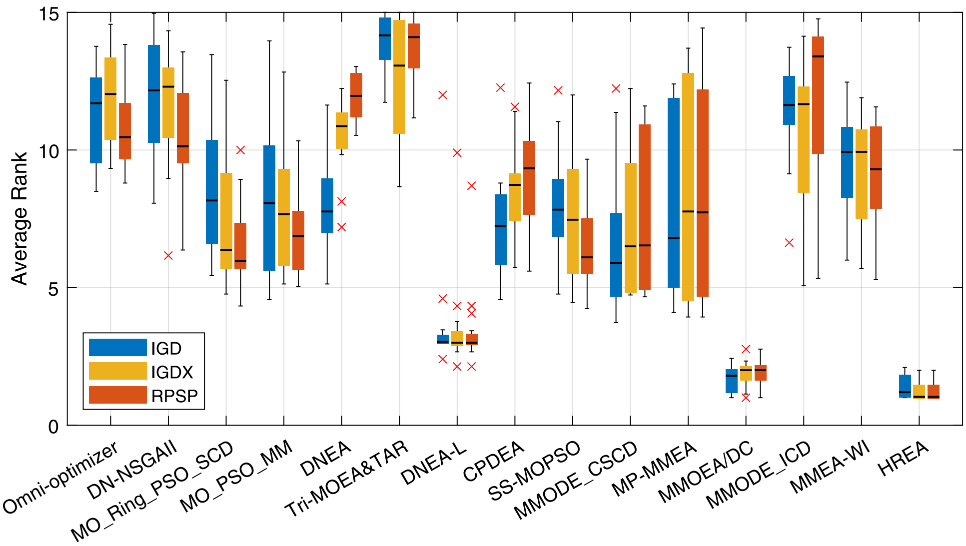


**Fig. 8.** The box plot of Friedman ranks for all compared MMEAs on MMOPL test suite in terms of $IGD$, $IGDX$, and $RPSP$, where the black lines and red X points are average ranks and outliers, respectively. The 25% and 75% quantiles are present with boxes.

Table S-VII, and Table S-VIII list the detailed $IGD$, $IGDX$, and $RPSP$ results of all MMEAs on IDMP. In terms of $IGDX$ and $RPSP$, HREA, MMOEA/DC, MMEA-WI, and MO_PSO_MM win 5, 5, 1, and 1 instances respectively. It is interesting that MO_PSO_MM shows a great result on IDMPM2T4. Compared to other IDMP test problems, the difference in difficulty between searching for different PSs is smaller in IDMPM2T4. Thus, MO_PSO_MM can stably obtain all PSs on IDMPM2T4.

As for $IGD$, DNEA shows its dominance since it wins all 12 test problems. That really means DNEA is an effective and competitive MOEA for solving IDMP problems. As a comparison, MO_PSO_MM, MO_Ring_PSO_SCD, and SS-MOPSO receive the worst average ranks. Results in Table S-V show that these three algorithms perform the worst on almost all test problems. The diversity in the decision space is overemphasized for these algorithms. Another two poorly performing algorithms are Tri-MOEA&TAR and MP-MMEA, in which the variable analysis method is adopted to improve the convergence ability.

Fig. 7 presents the final distribution of solutions on IDMPM3T4 in the decision space. Intuitively, solutions obtained by MMOEA/DC, MMEA-WI, and HREA distribute more evenly. To sum up, MMOEA/DC, HREA, and MMEA-WI are the three best algorithms for IDMP and primitive proposed algorithms are unable to obtain all PSs. Algorithms with a local-PS-maintaining strategy have a stronger ability in finding and obtaining PSs.

### 5.3. Performance comparison on MMOPLs

In this part, the performance of MMEAs on MMOPLs is discussed. To be specific, IDMP_e and some of the CEC 2020 test problems (MMF10-MMF13, MMF15, MMF15_a, MMF16_l1-MMF16_l3) are chosen as the test problems. Since there is no performance indicator designed for MMOPLs, we regard both global and local PSs as the true PS for calculating $IGD$, $IGDX$, and $RPSP$.

Since only DNEA-L, MMOEA/DC, and HREA are designed for MMOPLs, in this part, we mainly focus on their performance. The average rank of all algorithms on MMOPLs is presented in Fig. 8. As indicated in Fig. 8, HREA obtains significant best results on the chosen benchmark problems, followed by MMOEA/DC and DNEA-L. The overall average ranks for these three algorithms in terms of IGDX are 1.23, 1.90, and 3.49 respectively. HREA wins 14 instances for all 17 problems in terms of $IGDX$ and $RPSP$ as shown in Table S-X and Table S-XI. In addition, Fig. 9 presents the final distribution of solutions in the objective and decision spaces on IDMPM2T4_e. HREA can stably obtain all two global PSs and five local PSs, while MMOEA/DC and DNEA-L can obtain some of the global and local PSs. For DNEA-L, a parameter $K$ (set to 3 in this study) is introduced to get the first $K$-layer PFs. However, for IDMPM2T4_e, it can only obtain the first two PF layers. For MMOEA/DC, the double clustering method is proposed based on DBSCAN [58] to form evenly distributed solutions in each PS area. Experimental results show this method performs well on many MMOPLs. The main problem is that the clustering could be inaccurate and cause unstable convergence. For HREA, the local convergence quality is effective in finding and obtaining local PSs. A parameter $\epsilon$ is introduced to control the quality of the obtained local PSs. For DMs who are not familiar with MMOP, the setting of this parameter could be an obstacle, e.g., the authors suggested setting $\epsilon = 0.3$ for most problems. However, $\epsilon$ should be set to 0.5 for IDMPM2T4_e to obtain all PSs. Since IDMP_e is derived from IDMP, it is still hard for primitive MMEAs to find all global PSs.

### 5.4. Performance comparison on Multi-polygon problems

To study the performance of MMEAs on problems with many objectives and many decision variables, Multi-polygon problems are chosen as the benchmark, where $M$ is set to 3, 4, 6, 8, and $D$ is set to 2, 4, 8, 10,20 respectively.

Fig. 10 shows the overall performance ranking of all compared MMEAs, which indicates that MMEA-WI, MMOEA/DC, MMODE_ICD, and MMODE_CSCD are competitive. No algorithm shows overwhelmingly better performance on Multi-polygon problems. In addition, different from other MMOP test suites, the average ranks in terms of $IGDX$ and $RPSP$ vary for different algorithms. As shown in Table S-XIV, there are many $Inf$ values, which is the reason for the above issue. As we can see from Eq. (4), when the maximum value of the $i$th decision variable in the obtained solutions is less than the minimum value of the $i$th decision variable in the PS, then $PSP = 0$ and $RPSP = Inf$, which means that it fails in obtaining the true PS.

From Table S-XIV which shows the $RPSP$, only DNEA, DNEA-L, MMOEA/DC, MMEA-WI and HREA can find the true PS when $D$ is larger than 4. As for $IGDX$, MMODE_CSCD, MMOEA/DC, HREA, MMEA-WI, MMODE_ICD, and DNEA-L win 6, 5, 4, 2, 2, and 1 instances respectively. To be specific, HREA wins all instances when $D = 2$. HREA shows the best performance on low-dimension problems but very poor ability in dealing with high-dimension problems. The use of local convergence quality becomes an obstacle in converging to the true PSs.

Figs. 11 and 12 present the obtained solutions for Multi-polygon with $M = 8, D = 2$ and $M = 3, D = 8$, respectively. For the first two

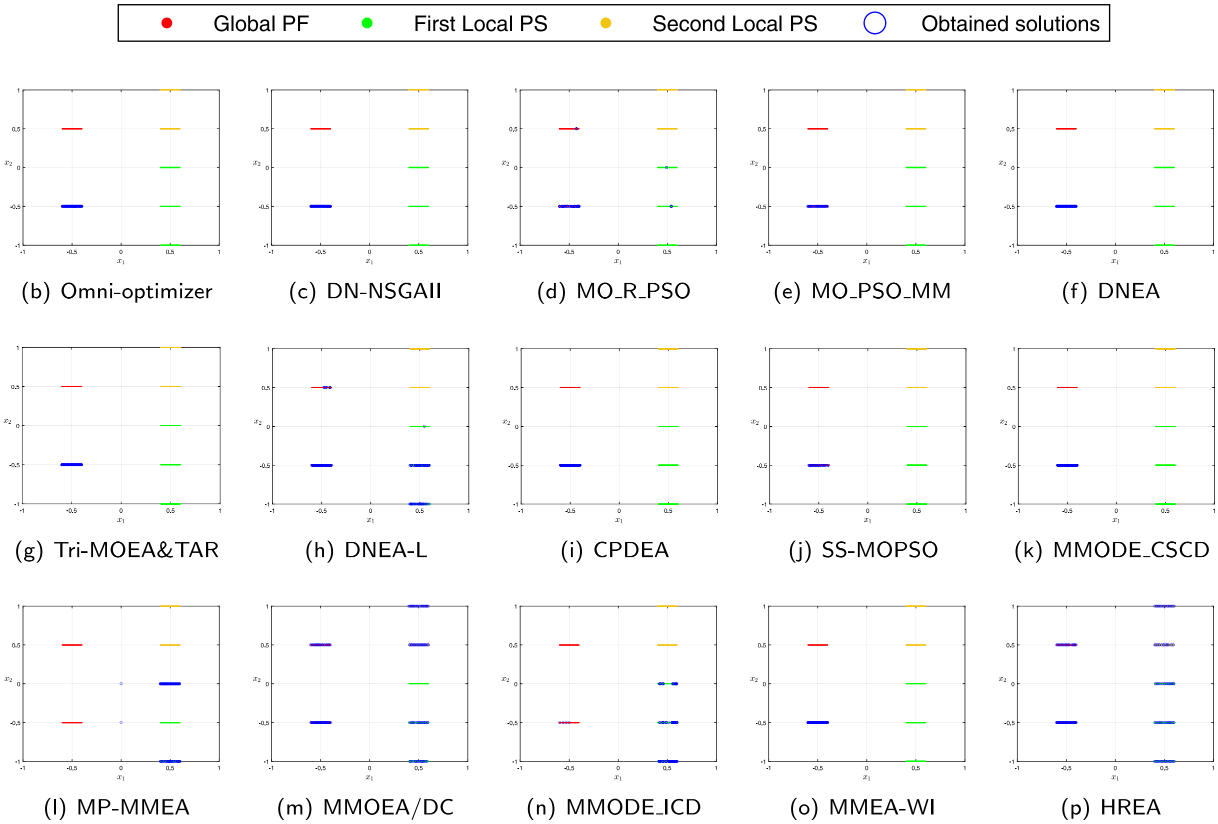


**Fig. 9.** The distribution of solutions obtained by all algorithms (MO_R_PSO is the short name for MO_Ring_PSO_SCD) in the decision space on IDMPM2T4_e, where there are two global PSs and five local PSs.

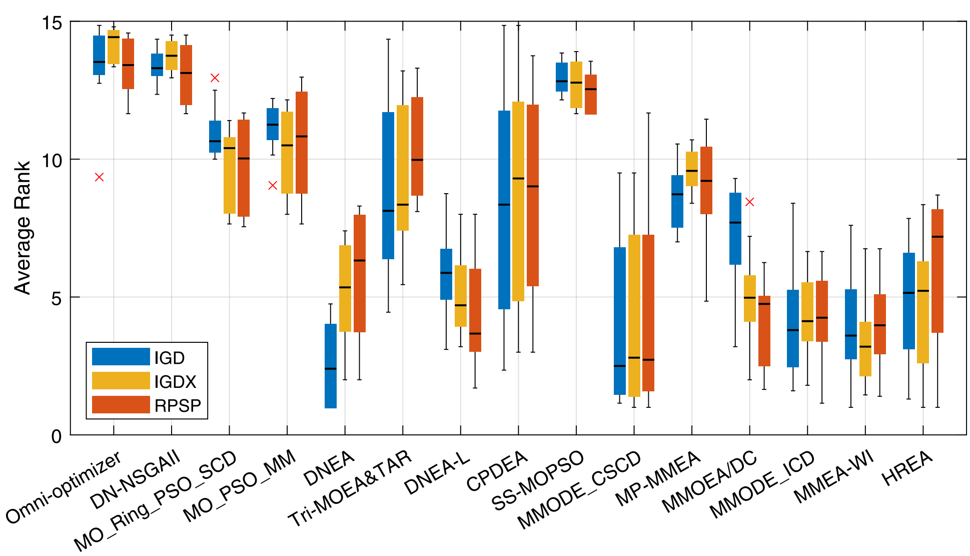


**Fig. 10.** The box plot of Friedman ranks for all compared MMEAs on Multi-polygon in terms of $IGD$, $IGDX$, and $RPSP$, where the black lines and red X points are average ranks and outliers, respectively. The 25% and 75% quantiles are presented with boxes.

rows, almost all MMEAs can obtain well-distributed solutions except Omni-optimizer, DN-NSGAII, and Tri-MOEA&TAR. That is, the existing MMEAs can well handle low-dimension and many-objective problems. For problems with 8 decision variables, only MMOEA/DC and MMEA-WI can obtain all PSs, and MMEA-WI has better performance. However, for Multi-polygon with more than 10 decision variables, all MMEAs fail to obtain all PSs (see supplementary material). It means that the existing MMEAs show poor performance when the number of decision variables is large. Although MMEA-WI performs best on high-dimension problems, it cannot obtain all PSs. In the primitive studies of MMOP, the test suites and MMEAs needed to consider intuitively presenting the effect of obtaining several different PSs. Therefore, almost all test suites are designed with 2 or 3 decision variables. The experimental results tell us that existing MMEAs may face great challenges in dealing with high-dimension problems. Thus, an MMOP test suite with intuitively different PSs for many decision variables is needed.

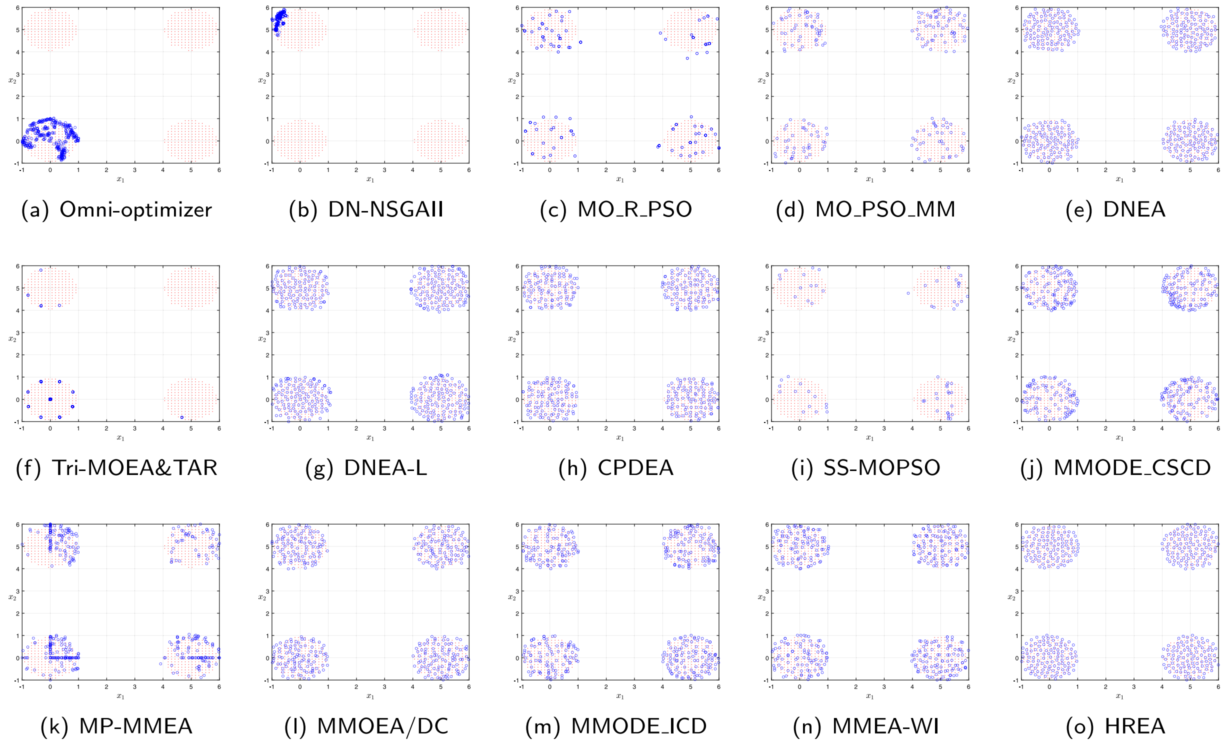


**Fig. 11.** The distribution of solutions obtained by all algorithms (MO_R_PSO is the short name for MO_Ring_PSO_SCD) in the decision space on Multi-polygon problems with 8 objectives and 2 decision variables.

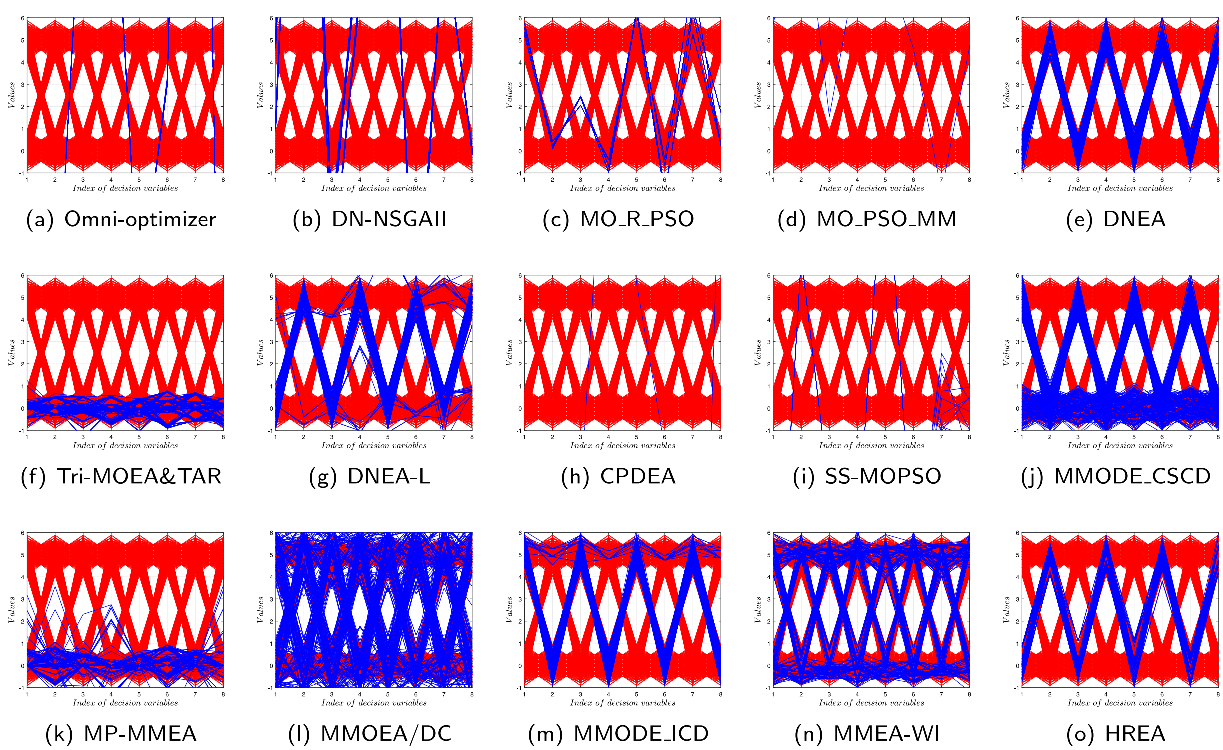


**Fig. 12.** The distribution of solutions obtained by all algorithms (MO_R_PSO is the short name for MO_Ring_PSO_SCD) in the decision space on Multi-polygon problems with 3 objectives and 8 decision variables.

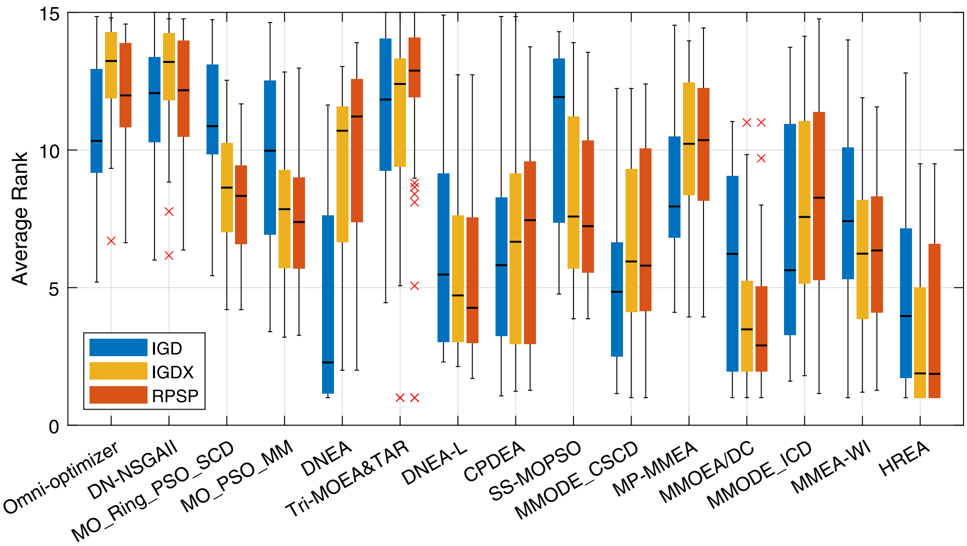


**Fig. 13.** The average rank of all compared MMEAs on all test problems in terms of $IGD$, $IGDX$, and $RPSP$.

## 6. Further discussions

### 6.1. Overall performance of all algorithms

In the previous parts, we discussed the performance of all compared MMEAs in solving different types of MMOPs in detail. Fig. 13 presents the overall performance comparison of all selected MMEAs on all test problems. In addition, to further study the critical difference of Friedman ranks over all MMEAs, we draw them in the supplementary material.

As indicated in Fig. 13, HREA and MMOEA/DC are two competitive MMEAs for now, which rank 3.36 and 3.90 in terms of $IGDX$ respectively. In the second echelon, DNEA-L, MMODE_CSCD, MMEA-WI, and CPDEA shine on some test problems, e.g., CPDEA performs best on MMF and MMEA-WI performs best on Multi-polygon. In the third echelon, MO_Ring_PSO_SCD, MO_PSO_MM, and MMODE_ICD can obtain all PSs for primitive proposed test suites like MMF but show poor performance on complex test suites like IDMP. Omni-optimizer, DN-NSGAII, Tri-MOEA&TAR, and MP-MMEA are the four algorithms that receive the worst results. To be specific, Omni-optimizer and DN-NSGAII are representative MMEAs in the early stage. They made a positive exploration of the MMOP community and motivated many later studies although their diversity-maintaining strategies are simple. Tri-MOEA&TAR and MP-MMEA are proposed for special problems, e.g., Tri-MOEA&TAR used the decision variable analysis method to better solve the MMMOP test suite and MP-MMEA is designed for large-scale sparse problems. Therefore, although they show poor performance on the selected test problems, they can well solve certain MMOPs. It is worth mentioning that, there is no algorithm that performs the best on all test suites. Most of them are verified through some of the proposed benchmarks. Thus, designing a robust algorithm is still the aim of the MMOP community.

In addition, many MMEAs have discussed their computational complexity theoretically and empirically. However, the overall comparison has not been made. In this part, we make an experimental computational complexity comparison. To be specific, the average run time over 30 times independent runs is collected. To better analyze the run time over the different number of objectives and decision variables, Multi-polygon problems are selected as the benchmark. In addition, we set $N = 200$ and $FEs = 20\,000$ for all experiments in this part.

Fig. 14 and Table S-XV show the average running time of all algorithms on Multi-polygon problems with the different numbers of decision variables and objectives. As we can observe, in terms of computational complexity, there is no significant difference between the different numbers of objectives. That is, almost all of the MMEAs are not sensitive to the number of objectives. As for the effect of decision variable number, SS-MOPSO, MP-MMEA, Tri-MOEA&TAR, and MMODE_CSCD are not sensitive. As the decision variable number increases, the running time changes slightly. In addition, there is an apparent increase in the runtime as the decision number grows for MO_Ring_PSO_SCD, MO_PSO_MM, MMODE_ICD, DN-NSGAII, and Omni-optimizer. The special crowding distance used in these algorithms is sensitive to the number of decision variables. SS-MOPSO is the most time-consuming algorithm over all compared MMEAs. The Non_domination_scd_sort method used in SS-MOPSO needs to iteratively find the subsequent fronts, which is complex in the calculation.

An interesting thing is that, for many algorithms like DNEA, MMOEA/DC, MMEA-WI, and DNEA-L, the running time decreases drastically when the decision variable number increases. Further study shows that, for these algorithms, niching techniques are utilized to improve the diversity of solutions. For problems with small decision variables, niches can be easily and stably formed, which needs huge time consumption. When the number of decision variables is large, niches are hard to form and the calculation for this process becomes time-saving. Another weird thing is that, for HREA and CPDEA, the runtime first decreases when the number of decision variables is less than 4 and increases after that. For these algorithms, the convergence archives and the co-evolution strategy are introduced, which are used to store the non-dominated solutions found so far. As a result, the runtime of these algorithms is somehow related to the number of solutions in the convergence archive. Therefore, for problems that are uneasy to quickly converge, like Multi-polygon with more than 4 decision variables, the runtime decrease as the number of decision variables grows from 2 to 4. After that, the major time-complexity factor is the calculation of crowding distance, which lead to a time increase as the number of decision variables increase.

To sum up, the running time of all MMEAs is not sensitive to the number of objectives while most of the MMEAs show intuitively time increases when the number of decision variables increases. DNEA, Tri-MOEA&TAR, and MP-MMEA are time-saving in dealing with MMOPs, while SS-MOPSO, MO_Ring_PSO_SCD, MO_PSO_MM, and MMODE_ICD are the top four time-consuming algorithms. There is a 30 to 100 times difference in terms of running time for the compared MMEAs.

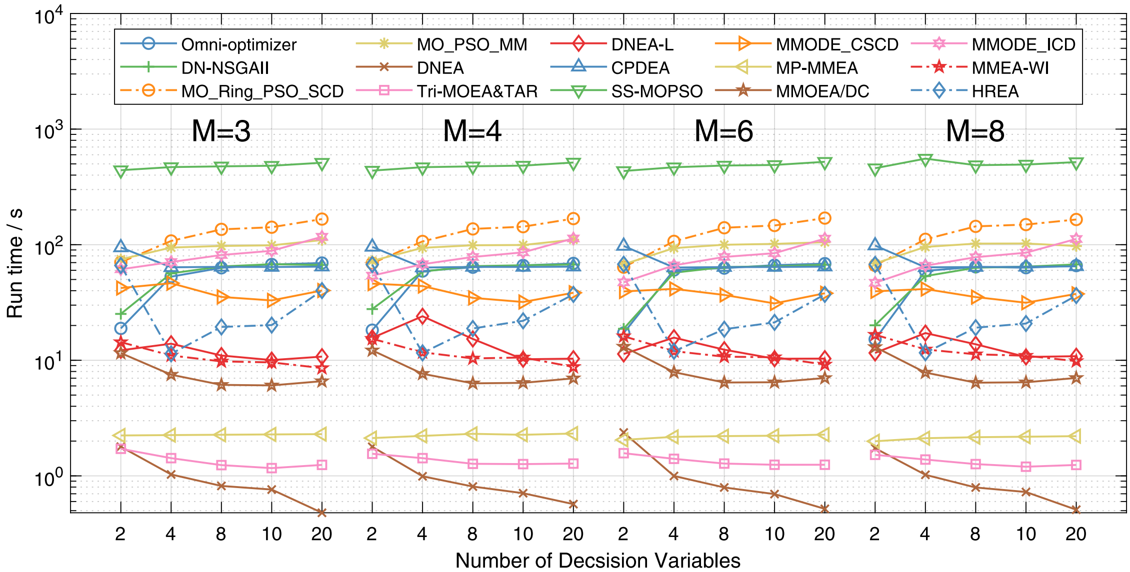


**Fig. 14.** The average running time of all algorithms on Multi-polygon problems with different numbers of objectives and decision variables.

## 6.2. Open issues and future study

(1) Through the analysis of the experimental results, we can see that most of the existing MMEAs can obtain a good result on normal MMOPs except for IDMPs and MMOPLs. Considering local PSs can significantly enhance the algorithms' ability on maintaining diversity, e.g., DNEA-L, MMOEA/DC, and HREA. However, there are few works considering obtaining local PSs for now. The IEEE CEC 2019 and CEC 2020 multimodal optimization competitions made some efforts on raising the attention of MMOPLs. After that, MMOEA/DC and HREA were proposed. We think that developing algorithms to obtain the local PSs is a more practical and general direction in the MMO community.

(2) Another important issue is that there lack of a multi-modality detection method for a given problem. That is, for DMs to deal with a certain real-world problem, there is no information on whether this problem is an MMOP or not. According to some previous works, the convergence ability of the MMEAs is worse than the state-of-the-art MOEAs. Thus, MMEAs will not be the first choice. Developing an effective and efficient tool or method to detect the multi-modality of a MOP is significantly important and urgent. On the other hand, since MMEAs focus on improving the diversity of solutions in the decision space, the convergence ability in the objective space is certainly worse than the normal MOEAs. Therefore, for a problem that is not sure if it is an MMOP, DMs will lack the confidence to choose MMEAs. So far, balancing convergence and diversity is still a challenging and urgent research topic for the MMO community.

(3) As we discussed in Section 2.2, limitations exist for the proposed benchmarks. So far, all of the famous benchmark problems are continuous, and few studies analyzed the discrete or mix-decision-variable MMOPs. Yue et al. [46] studied the multimodal multi-objective feature selection problems, which are encoded as binary optimization problems. The CEC 2021 multimodal multi-objective path planning optimization [43] had also made some effort in enriching the field of discrete optimization. For many real-world problems, the type of decision variables is usually various, e.g., continuous, discrete, and binary. Although it is easy to transform the existing test suites into discrete optimization problems, there is no work that systematically analyzes the performance of the existing MMEAs on mix-decision-variable problems.

(4) In addition, as results in Section 5.4 show, the existing MMEAs face huge challenges in solving MMOPs with many decision variables. Many proposed MMOPs are relatively simple to be solved, e.g., MMF1-8. An important reason is that multiple PSs cannot be directly observed for multi-dimension problems. Therefore, the searching ability and efficiency cannot be accurately evaluated by the existing test suites. Moreover, the drawbacks of utilizing the diversity-maintaining technique as the first-selection strategy have not been well studied yet. A comprehensively MMOP test suite that has difficulty in searching PF is needed for the MMOP community.

(5) As for the performance metrics, there lacks a comprehensive indicator that can measure both diversity and convergence in the objective and decision spaces, which are both important for solving an MMOP [5,12,33]. Fig. 15 presents the relationship between the diversity in the objective space and the diversity in the decision space of a two-objective two-decision-variable MMOP, where two different PSs correspond to the same PF. The left figure indicates the situation for primitive MMEAs, where the distribution in the objective is not considered. The middle figure presents the distribution of solutions for normal MOEAs, which only consider the objective space. The aim of MMEAs is shown in the right figure, which takes two space diversity into account. However, most of the existing performance metrics for MMEAs only consider solution quality in the decision space. $IGDM$ made some efforts to comprehensively evaluate the quality. Intuitively, to overall evaluate the quality of the two spaces, weights should be given, which is hard for the DMs to balance. In addition, true PS and PF data are necessary for the existing performance metrics, which is hard for real-world problems.

## 7. Conclusion

Multimodal multi-objective optimization problems are common in the real world and receive more and more attention. In this work, we first reviewed the proposed MMOP test suites and discussed their properties. Then, we introduced several state-of-the-art MMEAs with different diversity-maintaining techniques. Next, we comprehensively compared the performance of 15 popular MMEAs on the chosen benchmark problems. Our experimental results indicate that there is no algorithm that performs more overwhelmingly than all other compared MMEAs on all test suites. However, considering to obtain local PSs is apparently effective in dealing with most of the MMOPs. In addition, through the experimental results, several open issues and conclusions have been discussed to give suggestions and guidance for researchers in selecting/designing MMEAs.

Many multi-objective real-world optimization problems show multi-modality characteristics, e.g., the configuration of hybrid renewable

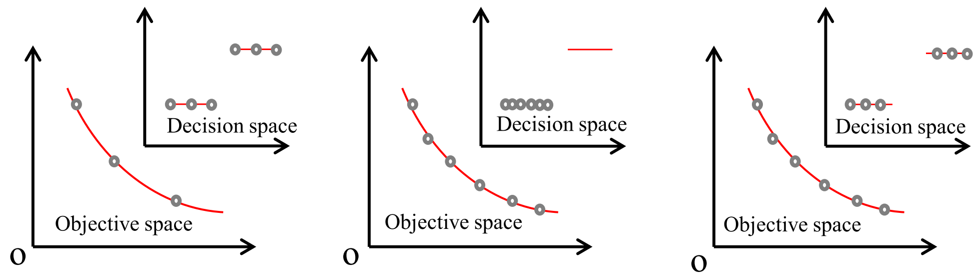


**Fig. 15.** Illustration of solution distributions in the objective and decision spaces, where the red line and gray circles indicate the true PF(PS) and the solutions respectively. Notably, two different PSs correspond to the same PF.

energy system [59], job-shop scheduling problem [60], satellite mission planning problems [61], etc. However, few of them choose to utilize MMEAs to obtain multiple solutions. For problems with or without multi-modality, obtaining all global and local optimal solutions can help DMs understand the implicit properties. Therefore, future works include applying MMEAs to more real-world problems. In addition, research on general diversity-maintaining techniques can significantly improve the ability to jump out of the local optimal region. Recently, Liang et al. [62] made a preliminary study on this research and proposed a constrained multimodal multi-objective differential evolution algorithm with speciation mechanism (CMMODE) to obtain all PSs for constrained MMOPs. Thus, embedding multimodal techniques into other MOEAs to help enhance the performance could be a useful research topic, e.g., enhancing the existing constraint multi-objective evolutionary algorithms (CMOEAs) [63–65] by improving the diversity of solutions.

## CRediT authorship contribution statement

**Wenhua Li:** Conceived and designed the analysis, Collected the data, Contributed data or analysis tools, Performed the analysis, Wrote the paper. **Tao Zhang:** Conceived and designed the analysis, Performed the analysis, Wrote the paper. **Rui Wang:** Conceived and designed the analysis, Contributed data or analysis tools, Performed the analysis, Wrote the paper. **Shengjun Huang:** Collected the data, Performed the analysis,. **Jing Liang:** Contributed data or analysis tools, Performed the analysis.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data availability

Data will be made available on request.

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## Appendix A. Supplementary data

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