这个单元格给助教，请忽略!

Score:

Comment:

请实现每个 function 内容，确保最终提交的notebook是可以运行的。

每一题除了必须要报告的 输出/图表，可以添加解释（中文即可）。此外可以自定义其他 function / 变量，自由添加单元格，但请确保题目中给出的 function（如第一题的 Print_values）可以正常调用。

Collaboration:
TA-jiaming Zhang explained to me what is asked in problem set 2

Zhaohan Li (my roommate) explained to me what is asked in problem set 5

map和split函数学习https://blog.csdn.net/fhy567888/article/details/136822735?ops_request_misc=%257B%2522request%255Fid%2522%253A%2522B5A17EDF-B048-466C-B019-DB97F512F6BC%2522%252C%2522scm%2522%253A%252220140713.130102334..%2522%257D&request_id=B5A17EDF-B048-466C-B019-DB97F512F6BC&biz_id=0&utm_medium=distribute.pc_search_result.none-task-blog-2~all~sobaiduend~default-1-136822735-null-null.142%5Ev100%5Epc_search_result_base8&utm_term=map%E5%92%8Csplit&spm=1018.2226.3001.4187

zip函数学习https://blog.csdn.net/Seven_Cloud/article/details/133359544?ops_request_misc=%257B%2522request%255Fid%2522%253A%25225173F2C3-0BF7-4CA7-87AE-74054FB32B5F%2522%252C%2522scm%2522%253A%252220140713.130102334..%2522%257D&request_id=5173F2C3-0BF7-4CA7-87AE-74054FB32B5F&biz_id=0&utm_medium=distribute.pc_search_result.none-task-blog-2~all~top_positive~default-2-133359544-null-null.142%5Ev100%5Epc_search_result_base8&utm_term=zip%E5%87%BD%E6%95%B0python%E4%BD%9C%E7%94%A8&spm=1018.2226.3001.4187

1. Flowchart

Write a function `Print_values` with arguments `a`, `b`, and `c` to reflect the following flowchart. Here the purple parallelogram operator on a list `[x, y, z]` is to compute and print `x+y-10z`. Try your output with some random `a`, `b`, and `c` values. Report your output when `a = 10, b = 5, c = 1`.

In [ ]:
```python
# 1, Flowchart
#输入数据
value_a= float(input("Please input a value_a: "))
value_b= float(input("Please input a value_b: "))
value_c= float(input("Please input a value_c: "))
#检查输入的值
print("your enter value is a="+str(value_a)+",b="+str(value_b)+",c="+str(value_c))
if value_a > value_b :  #逻辑判断
    if value_b > value_c:
        print(value_a+value_b-10*value_c)
    else:
        if value_a > value_c:
            print(value_a+value_c-10*value_b )
        else:
            print(value_c+value_a-10*value_b   )
else:
    if value_b > value_c:
        if value_a > value_c:
            print(value_b+value_a-10*value_c  )
        else:
            print(value_b+value_c-10*value_a )
    else:
        print(value_c+value_b-10*value_a)
```

Report your output when `a = 10, b = 5, c = 1`: 5

输入数据，然后先比较a,b。然后比较c,d。输入a = 10, b = 5, c = 1，输出结果为5。

2. Continuous ceiling function

Given a list with `N` positive integers. For every element `x` of the list, find the value of continuous ceiling function defined as `F(x) = F(ceil(x/3)) + 2x`, where `F(1) = 1`.

In [4]:
```python
#2. Continuous ceiling function   TA-jiaming Zhang explained to me what is asked in problem set 2
#定义函数
import numpy as np
def F(x):
    if x==1:
        return 1
    else:
        return F(np.ceil(x/3))+2*x
```

```
#输入数据
user_input= input("enter a number list(数字由英文逗号隔开) ")
#转换成列表
numbers = list(map(int, user_input.split(',')))

# 计算每个元素 x 对应的 F(x) 并存储结果
results = [F(x) for x in numbers]

# 输出结果
for x, result in zip(numbers, results):
    print(f"F({x}) = {result}")
```

```
F(32) = 99.0
F(46) = 141.0
F(54) = 161.0
```

First, give a function F(x) and use the if loop. Then," numbers = list(map(int, user_input.split(',')))" uses the map and split function to transfer user input to the list so that we can run F(x) one by one. when input (32,46,54) ,result is F(32)=99,F(46)=141,F(54)=161.

3. Dice rolling

3.1 Given `10` dice each with `6` faces, numbered from `1` to `6`. Write a function `Find_number_of_ways` to find the number of ways to get sum `x`, defined as the sum of values on each face when all the dice are thrown.

In [5]:
```
# 3.1 Dice rolling
# 定义函数找到和为x的方式数量
def Find_number_of_ways(dice,  target_sum):
    if dice == 0:
        if target_sum == 0:  #如果和为0, 筛子数为0, 则返回1, 有一种情况
            return 1
        else:     #如果和不为0, 筛子数为0, 则返回0, 没有这种情况
            0
    if target_sum < dice or target_sum > dice * 6:    #如果和小于筛子数或者和大于筛子数与最大面数乘积, 返回0, 也不存在这种情况
        return 0

    total_ways = 0
    for i in range(1, 7):
        total_ways += Find_number_of_ways(dice - 1, target_sum - i)   #递归函数, Find_number_of_ways (0, target_sum)是递归终止。
    return total_ways
```

```
[1, 10, 55, 220, 715, 2002, 4995, 11340, 23760, 46420, 85228, 147940, 243925, 383470, 576565, 831204, 1151370, 1535040, 1972630, 2446300, 29
30455, 3393610, 3801535, 4121260, 4325310, 4395456, 4325310, 4121260, 3801535, 3393610, 2930455, 2446300, 1972630, 1535040, 1151370, 831204,
576565, 383470, 243925, 147940, 85228, 46420, 23760, 11340, 4995, 2002, 715, 220, 55, 10, 1]
最大的方式数量是 4395456, 对应的和是 35
```

3.2 Count the number of ways for any `x` from `10` to `60`, assign the number of ways to a list called `Number_of_ways`, so which `x` yields the maximum of `Number_of_ways`?

In [ ]:
```
# 3.2
# 计算从10到60的方式数量
dice = 10   # 骰子的数量
Number_of_ways = []

for x in range(10, 61):  # 遍历 x 从 10 到 60
    ways = Find_number_of_ways(dice,  x)
    Number_of_ways.append(ways)
print(Number_of_ways)
# 找到最大方式数量以及对应的x
max_ways = max(Number_of_ways)    #找最大值
x_with_max_ways = 10 + Number_of_ways.index(max_ways)  # +10 是因为 x 从 10 开始计算

print(f"最大的方式数量是 {max_ways}, 对应的和是 {x_with_max_ways}")
```

运行结果为最大的方式数量是 4395456，对应的和是 35

解释：3.1：定义函数，采用迭代计算的方法，依次从1到6中减少一个数字并且在target_sum中减小对应的值。然后进行第二次迭代，第三次迭代…直到当dice（骰子数）为0，判断target_sum是都恰好为0，若是，则发现一种方法；若不是，则返回0，这条路走不通。如果target_sum小于骰子数或者大于6乘以骰子数（即最大值）都不行，返回0.

3.2：对10到60的target_sum进行遍历，带入方程中寻找对应的路径数，然后用max函数找到这些路径数中的最大值与对应的x。

4. Dynamic programming

4.1 [5 points] Write a function `Random_integer` to fill an array of `N` elements by randomly selecting integers from `0` to `10`.

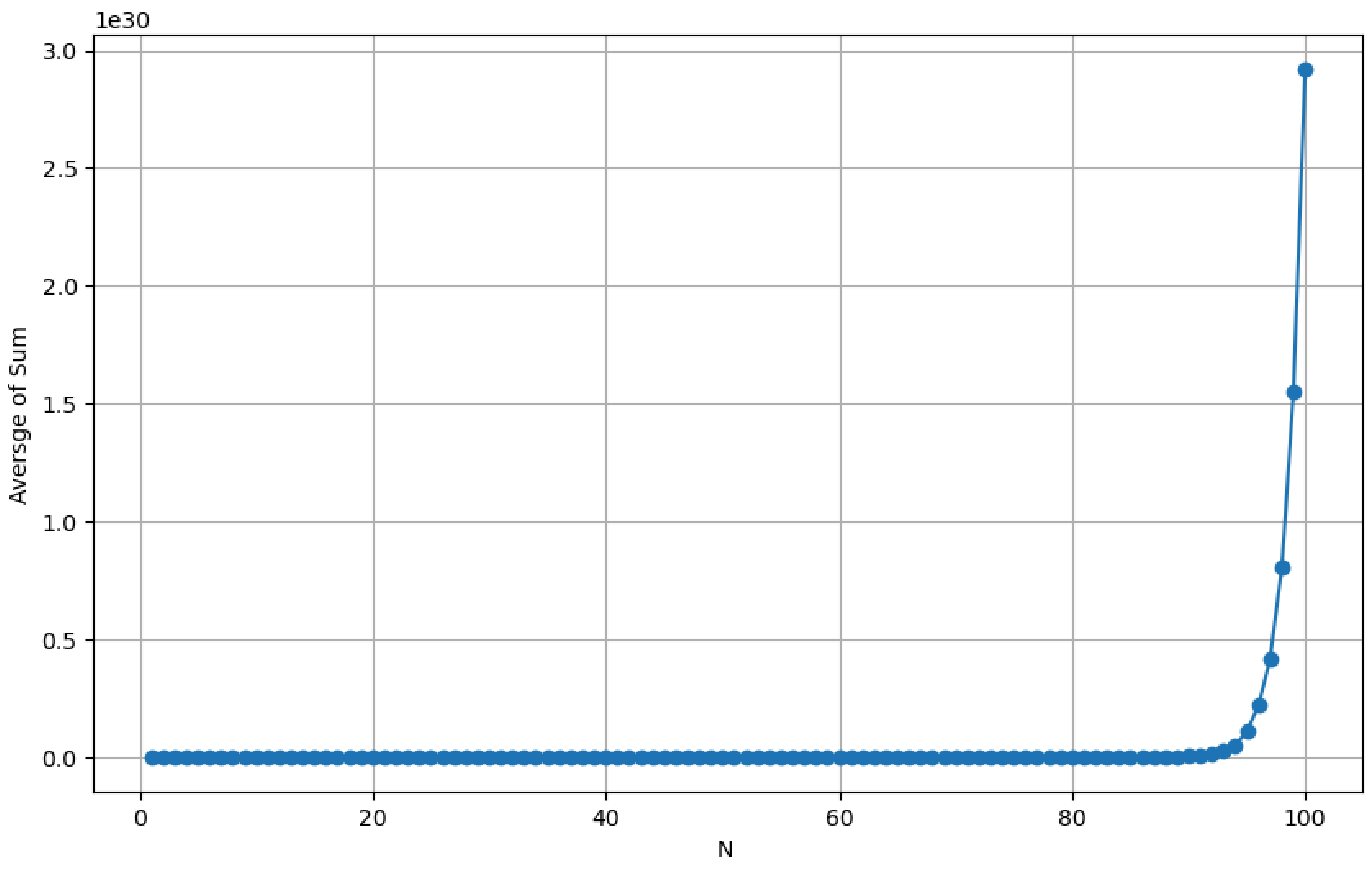
In [ ]:
```
#4. Dynamic programming（第三题代码为完整代码，方便老师或TA运行）
import random
import matplotlib.pyplot as plt

# 生成包含 N 个随机整数的数组, 范围从 0 到 10
def Random_integer(N):
    return [random.randint(0, 10) for _ in range(N)]    #下划线代表是个虚值, 只需要重复N次就可以了
```

4.2 [15 points] Write a function `Sum_averages` to compute the sum of the average of all subsets of the array. For example, given an array of `[1, 2, 3]`, you `Sum_averages` function should compute the sum of: average of `[1]`, average of `[2]`, average of `[3]`, average of `[1, 2]`, average of `[1, 3]`, average of `[2, 3]`, and average of `[1, 2, 3]`.

In [ ]:
```
# 优化的 Sum_averages，直接计算所有子集平均值和
def Sum_averages(array):
    N = len(array)
    if N == 0:
        return 0
    # 计算数组元素的总和
    total_sum = sum(array)
    # 计算所有子集的平均值和
    return (total_sum * (2**(N-1))) / N     #运算子集平均值的和的公式
```

4.3 [5 points] Call `Sum_averages` with `N` increasing from `1` to `100`, assign the output to a list called `Total_sum_averages`. Plot `Total_sum_averages`, describe what you see.

In [17]:
```
#4.3 Dynamic programming
import random
import matplotlib.pyplot as plt

# 生成包含 N 个随机整数的数组，范围从 0 到 10
def Random_integer(N):
    return [random.randint(0, 10) for _ in range(N)]    #下划线代表是个虚值，只需要重复N次就可以了


# 优化的 Sum_averages，直接计算所有子集平均值和
def Sum_averages(array):
    N = len(array)
    if N == 0:
        return 0
    # 计算数组元素的总和
    total_sum = sum(array)
    # 计算所有子集的平均值和
    return (total_sum * (2**(N-1))) / N     #运算子集平均值的和的公式

# 存储每个 N 对应的 Sum_averages 结果
Total_sum_averages = []

# 调用 Sum_averages，N 从 1 到 100
for N in range(1, 101):
    random_array = Random_integer(N)
    total_sum = Sum_averages(random_array)
    Total_sum_averages.append(total_sum)


# 绘制结果
plt.figure(figsize=(10, 6))
plt.plot(range(1, 101), Total_sum_averages, marker='o')
plt.xlabel('N ')
plt.ylabel('Aversge of Sum')
plt.grid()
plt.show()
```



平均值在0-80都比较稳定，接近0，但是从85开始陡峭上升，100时停留在3.00左右。这与公式中的2**（N-1)有关。

5. Path counting

5.1 [5 points] Create a matrix with `N` rows and `M` columns, fill the right-bottom corner and top-left corner cells with `1`, and randomly fill the rest of matrix with integer `0` or `1`.

In [ ]:
```
#5. Path counting       Zhaohan Li (my roommate) explained to me what is asked in problem set 5（第三题代码为完整代码，方便老师或TA运行）
import random
def create_matrix(N, M):

    matrix = [[0 for _ in range(M)]for _ in range(N)]   #建立一个空的二维矩阵
    for i in range(N):
        for j in range(M):
            matrix[i][j]=random.randint(0,1)    #在每行进行0，1的随机填充


    matrix[0][0] = 1
    matrix[N-1][M-1] = 1               # 设置左上角和右下角为 1

    return matrix

print(" 实验(10, 8)的结果  " + str(create_matrix(10,8)) )   #试验
```

5.2 [25 points] Consider a cell marked with `0` as a blockage or dead-end, and a cell marked with `1` is good to go. Write a function `Count_path` to count the total number of paths to reach the right-bottom corner cell from the top-left corner cell.

Notice: for a given cell, you are **only allowed** to move either rightward or downward.

In [ ]:
```
def Count_path(matrix): #定义函数
    N = len(matrix)
    M = len(matrix[0])
    # 创建一个与矩阵大小相同的路径计数矩阵，下划线表示虚值
    dp = [[0] * M for _ in range(N)]

    # 初始化左上角
    dp[0][0] = 1

    # 填充第一列的路径数
    for i in range(1, N):
        if matrix[i][0] == 1:
            dp[i][0] = dp[i-1][0]

    # 填充第一行的路径数
    for j in range(1, M):
        if matrix[0][j] == 1:
            dp[0][j] = dp[0][j-1]

    # 填充其余位置的路径数
    for i in range(1, N):
        for j in range(1, M):
            if matrix[i][j] == 1:
                dp[i][j] = dp[i-1][j] + dp[i][j-1]

    # 右下角的值就是到达右下角的所有路径数
    return dp[N-1][M-1]
```

5.3 [5 points] Let `N = 10, M = 8`, run `Count_path` for `1000` times, each time the matrix (except the right-bottom corner and top-left corner cells, which remain being `1`) is re-filled with integer `0` or `1` randomly, report the mean of total number of paths from the `1000` runs.

In [3]:
```
#5. Path counting       Zhaohan Li (my roommate) explained to me what is asked in problem set 5（第三题代码为完整代码，方便老师或TA运行）
import random
def create_matrix(N, M):

    matrix = [[0 for _ in range(M)]for _ in range(N)]   #建立一个空的二维矩阵
    for i in range(N):
        for j in range(M):
            matrix[i][j]=random.randint(0,1)    #在每行进行0，1的随机填充


    matrix[0][0] = 1
    matrix[N-1][M-1] = 1               # 设置左上角和右下角为 1

    return matrix

print(" 实验(10, 8)的结果  " + str(create_matrix(10,8)) )   #试验


def Count_path(matrix): #定义函数
    N = len(matrix)
    M = len(matrix[0])
    # 创建一个与矩阵大小相同的路径计数矩阵，下划线表示虚值
    dp = [[0] * M for _ in range(N)]

    # 初始化左上角
    dp[0][0] = 1
    # 填充第一列的路径数
    for i in range(1, N):
        if matrix[i][0] == 1:
            dp[i][0] = dp[i-1][0]

    # 填充第一行的路径数
    for j in range(1, M):
        if matrix[0][j] == 1:
            dp[0][j] = dp[0][j-1]

    # 填充其余位置的路径数
    for i in range(1, N):
        for j in range(1, M):
            if matrix[i][j] == 1:
                dp[i][j] = dp[i-1][j] + dp[i][j-1]

    # 右下角的值就是到达右下角的所有路径数
    return dp[N-1][M-1]
#5.3
average_paths = 0
for _ in range(1001):  #运行一千次
    matrix=create_matrix(10,8)
    paths = Count_path(matrix)
    average_paths +=paths   #计算总和
print("your paths 1000 times , get the answer " ,average_paths/1000)
```

```
 实验(10, 8)的结果  [[1, 1, 0, 1, 0, 0, 0, 1], [1, 1, 0, 1, 0, 0, 1, 1], [1, 1, 1, 1, 0, 1, 0, 1], [0, 0, 1, 1, 0, 0, 1, 0], [1, 1, 0, 1, 1,
0, 1, 0], [1, 0, 0, 1, 0, 0, 0, 1], [0, 0, 0, 0, 1, 0, 1, 1], [0, 1, 0, 0, 0, 1, 1, 1], [1, 1, 1, 1, 0, 0, 1, 1], [0, 1, 0, 0, 0, 0, 0, 1]]
your paths 1000 times , get the answer  0.338
```

实验(10，8)的结果 [[1, 1, 0, 1, 0, 0, 0, 1], [1, 1, 0, 1, 0, 0, 1, 1], [1, 1, 1, 1, 0, 1, 0, 1], [0, 0, 1, 1, 0, 0, 1, 0], [1, 1, 0, 1, 1, 0, 1, 0], [1, 0, 0, 1, 0, 0, 0, 1], [0, 0, 0, 0, 1, 0, 1, 1], [0, 1, 0, 0, 0, 1, 1, 1], [1, 1, 1, 1, 0, 0, 1, 1], [0, 1, 0, 0, 0, 0, 0, 1]] your paths 1000 times , get the answer 0.338

解释：5.1，先创造一个空的二位矩阵，然后用一个两层for循环来填充随机数。最后把左上和右下改成1. 5.2，创建一个与矩阵大小相同的路径计数矩阵，依次填充第一列和第一行的路经数。最后改变其他位置的路径书， 5.3，运行1000次后，获得的平均值为0.338，每次运行的结果不一样，因为随机数列的存在。