ECSE 597: Circuit Simulations and Modeling

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1 Question 1

Matrix A is given as following:

$$A = \begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 4 & 9 & 14 & 8 & 8 \\ 2 & 6 & 12 & 12 & 21 \\ 4 & 10 & 20 & 17 & 32 \\ 8 & 17 & 30 & 18 & 34 \end{bmatrix}.$$

1.a Doolittle's Algorithm

To perform the LU decomposition using Doolittle's algorithm, first row of U and first column of L can be calculated by:

$$u_{1k} = a_{1k}, \quad k = 1, ..., n \tag{1}$$

and

$$l_{k1} = a_{k1}/u_{11}, \quad k = 2, ..., n \tag{2}$$

The following equations will be applied to compute the LU decomposition using Doolittle's algorithm:

$$u_{ik} = a_{ik} - \sum_{m=1}^{i-1} l_{im}u_{mk}, \quad k = i, ..., n \tag{3}$$

$$l_{kj} = (a_{kj} - \sum_{m=1}^{j-1} l_{km}u_{mj})/u_{jj}, \quad k = (j+1), ..., n \tag{4}$$

From Equation 1 and 2, and define that the diagonal entries of L are 1, we calculate the ***first row*** of U and the ***first column*** of L:

$$U = \begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 0 & u_{22} & u_{23} & u_{24} & u_{25} \\ 0 & 0 & u_{33} & u_{34} & u_{35} \\ 0 & 0 & 0 & u_{44} & u_{45} \\ 0 & 0 & 0 & 0 & u_{55} \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 1 & l_{32} & 1 & 0 & 0 \\ 2 & l_{42} & l_{43} & 1 & 0 \\ 4 & l_{52} & l_{53} & l_{54} & 1 \end{bmatrix}$$

Use Equation 3 and 4, and from the U and L obtained above, calculate the rest entries of L and U:
Second Step: Use Equation 3, u_{22} can be calculated as such:

$$\begin{aligned} u_{22} &= a_{22} - \sum_{m=1}^{1} l_{2m}u_{12} \\ &= 9 - 2 \times 4 = 1 \end{aligned}$$

$$\begin{aligned} l_{32} &= (a_{32} - \sum_{m=1}^{1} l_{3m}u_{m2})/u_{22} \\ &= (6 - 1 \times 4)/1 = 2 \end{aligned}$$

Use the similar techniques, the ***second row and column*** of U and L are now updated as such:

$$U = \begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 0 & 1 & 2 & 4 & 6 \\ 0 & 0 & u_{33} & u_{34} & u_{35} \\ 0 & 0 & 0 & u_{44} & u_{45} \\ 0 & 0 & 0 & 0 & u_{55} \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 2 & 2 & l_{43} & 1 & 0 \\ 4 & 1 & l_{53} & l_{54} & 1 \end{bmatrix}$$

Continue using the same equations, and obtain the ***third row and column*** of U and L:

$$U = \begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 0 & 1 & 2 & 4 & 6 \\ 0 & 0 & 2 & 2 & 8 \\ 0 & 0 & 0 & u_{44} & u_{45} \\ 0 & 0 & 0 & 0 & u_{55} \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 2 & 2 & 2 & 1 & 0 \\ 4 & 1 & 2 & l_{54} & 1 \end{bmatrix}$$

Continue using the same equations, and obtain the ***forth row and column*** of U and L:

$$U = \begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 0 & 1 & 2 & 4 & 6 \\ 0 & 0 & 2 & 2 & 8 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & u_{55} \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 2 & 2 & 2 & 1 & 0 \\ 4 & 1 & 2 & 2 & 1 \end{bmatrix}$$

And finally, compute u_{55} to obtain the ***complete*** decomposition of A:

$$U = \begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 0 & 1 & 2 & 4 & 6 \\ 0 & 0 & 2 & 2 & 8 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 2 & 2 & 2 & 1 & 0 \\ 4 & 1 & 2 & 2 & 1 \end{bmatrix}$$

Verify this result by doing a cross product, and it is verified that

$$L \times U = A.$$

1.b Gaussian Version of Doolittle's Algorithm

Use Gaussion steps to perform the Doolittle's algorithm.

Choose a_{11} as the pivot as the first step, and divide every entry of A below the pivot by the pivot, we obtain that

$$A = \begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 4 & 9 & 14 & 8 & 8 \\ 2 & 6 & 12 & 12 & 21 \\ 4 & 10 & 20 & 17 & 32 \\ 8 & 17 & 30 & 18 & 34 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ \mathbf{2} & 9 & 14 & 8 & 8 \\ \mathbf{1} & 6 & 12 & 12 & 21 \\ \mathbf{2} & 10 & 20 & 17 & 32 \\ \mathbf{4} & 17 & 30 & 18 & 34 \end{bmatrix}$$

After this step, overwrite each entry from a_{22} to a_{55}, such that

$$a_{ij} \rightarrow a_{ij} - a_{ik} \times a_{kj}$$

where k stands for the row index of the pivot.

After this step, we obtain a new A matrix as shown below:

$$A = \begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 2 & 9 & 14 & 8 & 8 \\ 1 & 6 & 12 & 12 & 21 \\ 2 & 10 & 20 & 17 & 32 \\ 4 & 17 & 30 & 18 & 34 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 2 & 1 & 2 & 4 & 6 \\ 1 & 2 & 6 & 10 & 20 \\ 2 & 2 & 8 & 13 & 30 \\ 4 & 1 & 6 & 10 & 30 \end{bmatrix}$$

Move the pivot to a_{22}, and perform the same procedures like what is done above.

Step 2:

$$A = \begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 2 & 1 & 2 & 4 & 6 \\ 1 & 2 & 6 & 10 & 20 \\ 2 & 2 & 8 & 13 & 30 \\ 4 & 1 & 6 & 10 & 30 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 2 & 1 & 2 & 4 & 6 \\ 1 & 2 & 6 & 10 & 20 \\ 2 & 2 & 8 & 13 & 30 \\ 4 & 1 & 6 & 10 & 30 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 2 & 1 & 2 & 4 & 6 \\ 1 & 2 & 2 & 2 & 8 \\ 2 & 2 & 4 & 5 & 18 \\ 4 & 1 & 4 & 6 & 24 \end{bmatrix}$$

Step 3:

$$A = \begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 2 & 1 & 2 & 4 & 6 \\ 1 & 2 & \color{red}{2} & 2 & 8 \\ 2 & 2 & 4 & 5 & 18 \\ 4 & 1 & 4 & 6 & 24 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 2 & 1 & 2 & 4 & 6 \\ 1 & 2 & \color{red}{2} & 2 & 8 \\ 2 & 2 & 2 & 5 & 18 \\ 4 & 1 & 2 & 6 & 24 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 2 & 1 & 2 & 4 & 6 \\ 1 & 2 & \color{red}{2} & 2 & 8 \\ 2 & 2 & 2 & 1 & 2 \\ 4 & 1 & 2 & 2 & 8 \end{bmatrix}$$

Step 4:

$$A = \begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 2 & 1 & 2 & 4 & 6 \\ 1 & 2 & 2 & 2 & 8 \\ 2 & 2 & 2 & \color{red}{1} & 2 \\ 4 & 1 & 2 & 2 & 8 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 2 & 1 & 2 & 4 & 6 \\ 1 & 2 & 2 & 2 & 8 \\ 2 & 2 & 2 & \color{red}{1} & 2 \\ 4 & 1 & 2 & 2 & 8 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 2 & 1 & 2 & 4 & 6 \\ 1 & 2 & 2 & 2 & 8 \\ 2 & 2 & 2 & \color{red}{1} & 2 \\ 4 & 1 & 2 & 2 & 4 \end{bmatrix}$$

After these processes, we obtain the matrices L and U in separated format:

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 2 & 2 & 2 & 1 & 0 \\ 4 & 1 & 2 & 2 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 2 & 4 & 6 & 2 & 1 \\ 0 & 1 & 2 & 4 & 6 \\ 0 & 0 & 2 & 2 & 8 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

This result agrees with the results obtained by Doolittle's algorithm, and it is verified that

$$L \times U = A.$$