Truth Table

What is truth table?

- A diagram in rows and columns showing how the truth or falsity of a proposition varies with that of its components
- **List all possibilities**, **one row for each possible assignment** of True/False to variables
- By convention, usually use **1 represents TRUE** and **0 represents FALSE**

Examples 1:

Draw a truth table for statement "¬P"

- result of ¬P depends on "P"
- "P" has two possible result (True or False)
- Then we can draw a full truth table which list all possibilities

P	**¬P**
0(False)	**1(True)**
1(True)	**0(False)**

Truth Table Semantics

Why we need truth table?

- Propositional logic is not about the meaning of a sentence
- We use an example like “Socrates is bald” and replace it with the proposition letter, S
- That’s ok, but we don’t know if S is true or false, because it is just a proposition
- What we want to do is to define operations on these propositions by truth table

Truth Table Semantics

Why we need truth table (Continued)?

- So now, if we know S is true but G is false, then we can work out if S & G is true. But S can also be false or another combination

- S and G are just variables that can be assigned any truth value (i.e. true or false)

- They have no intrinsic truth value. They are just variables that can be assigned a value

- Therefore, if we want to look at all combinations of assignments of true or false to each letter, we will create a truth table showing all the consequence

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Examples 2:

Draw a truth table for statement “P ∧ Q”

- result of P ∧ Q depends on “P” and “Q”
- “P” has two possible result (True or False)
- “Q” has two possible result (True or False)
- Then we can draw a full truth table which list all possibilities

P	Q	P∧Q
0	0	**0**
0	1	**0**
1	0	**0**
1	1	**1**

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Examples 3:

Draw a truth table for statement “P V Q”

- result of P V Q depends on “P” and “Q”
- “P” has two possible result (True or False)
- “Q” has two possible result (True or False)
- Then we can draw a full truth table which list all possibilities

P	Q	PVQ
0	0	**0**
0	1	**1**
1	0	**1**
1	1	**1**

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Examples 4:

Draw a truth table for statement "P → Q"

- result of P → Q depends on "P" and "Q"
- "P" has two possible result (True or False)
- "Q" has two possible result (True or False)
- Recall: P → Q will be false iif Q is false but P is true
- Then we can draw a full truth table which list all possibilities

P	Q	**P → Q**
0	0	**1**
0	1	**1**
1	0	**0**
1	1	**1**

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Examples 5:

Draw a truth table for statement "P ↔ Q"

- result of P ↔ Q depends on "P" and "Q"
- Recall: P ↔ Q will be true iif P is false and Q is false or P is true and Q is true
- Then we can draw a full truth table which list all possibilities

P	Q	**P ↔ Q**
0	0	**1**
0	1	**0**
1	0	**0**
1	1	**1**

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Examples 6:

Draw a truth table for statement "(R ∧ S) → (¬R ∨ S)"

- result depends on "(R ∧ S)" and "(¬R ∨ S)"
- result of "(R ∧ S)" depends on "R" and "S"
- result of "(¬R ∨ S)" depends on "R" and "S"
- Recall: "(R ∧ S) → (¬R ∨ S)" is False iff (R ∧ S) is True but (¬R ∨ S) is False
- Then we can draw a full truth table which list all possibilities

R	S	¬R	(R ∧ S)	(¬R ∨ S)	**(R ∧ S) → (¬R ∨ S)**
0	0	1	0	1	**1**
0	1	1	0	1	**1**
1	0	0	0	0	**1**
1	1	0	1	1	**1**

Quiz Time!