

Propositions

What is proposition?

- Propositions are entities (facts or non-facts) that can be true or false
- It is a declarative sentence
- We cannot say a proposition is True or False just by looking at itself
- Its truth value (true or false) will be determined by given context or semantics

Examples:

- “The sky is blue” - the sky is blue (here and now).
- “Socrates is bald” (assumes ‘Socrates’, ‘bald’ are well defined)
- “The car is red” (requires ‘the car’ to be identified)

Propositional Letter

What is proposition Letter?

- Let Letters stand for “basic” propositions
- A shortcut way that let you do not need to write a long sentence

Examples:

- We can let B represents “The sky is blue” this proposition
- We can let Q represents “Socrates is bald” this proposition
- We can let G represents “Grass is green” this proposition
- We can let S represents “The car is red” this proposition
- We can let P represents “The plants will grow up” this proposition

Propositional Logic

What is proposition Logic?

- Propositional Letters stand for “basic” propositions
- Combine into more complex sentences using operators **not**, **and**, **or**, **implies**, **iff**

Propositional connectives:

\neg	represents	negation
\wedge	represents	conjunction
\vee	represents	disjunction
\rightarrow	represents	implication
\leftrightarrow	represents	bi-implication

$\neg P$ means “not P”

$P \wedge Q$ means “P and Q”

$P \vee Q$ means “P or Q”

$P \rightarrow Q$ means “If P then Q”

$P \leftrightarrow Q$ means “P if and only if Q”

From English to Propositional Logic

Recall from previous example:

- We can let B represents “The sky is blue”
- We can let Q represents “Socrates is bald”
- We can let G represents “Grass is green”
- We can let S represents “The car is red”
- We can let P represents “The plants will grow up”

Then:

- “It is not the case that the sky is blue”: $\neg B$ (alternatively “the sky is not blue”)
- “The sky is blue and the grass is green”: $B \wedge G$
- “Either the sky is blue or the grass is green”: $B \vee G$
- “If the sky is blue, then the grass is not green”: $B \rightarrow \neg G$
- “The sky is blue if and only if the grass is green”: $B \leftrightarrow G$
- “If the sky is blue, then if the grass is not green, the plants will not grow”: $B \rightarrow (\neg G \rightarrow \neg P)$

Quiz Time!