Key Definition (1)

- **Valid**: A sentence is **valid** if it is **True under all possible assignments** of True/False to its variables (e.g. $P \vee \neg P$)

- **Tautology**: A tautology is a valid sentence

- **Equivalent**: Two sentences are equivalent if they have the **same truth table**, e.g. $P \wedge Q$ and $Q \wedge P$. So P is equivalent to Q if and only if $P \leftrightarrow Q$ is valid

- **Satisfiable**: A sentence is satisfiable if there is **some assignment** of True/False to its variables for which **the sentence is True**

- **Unsatisfiable**: A sentence is unsatisfiable if it is not satisfiable (e.g. $P \wedge \neg P$). Sentence is **False** for **all assignments** of True/False to its variables

Tautology:

Examples 1:

Show statement “(R ∧ S) → (¬R ∨ S)” is tautology:

- Recall that tautology means “True” under all possible assignments

- Then we can draw a full truth table which list all possibilities to see if it is tautology

- Looking at the truth table, the all-possible assignments of R, S have truth of the statement. Then the statement is tautology

R	S	¬R	(R ∧ S)	(¬R ∨ S)	**(R ∧ S) → (¬R ∨ S)**
0	0	1	0	1	**1**
0	1	1	0	1	**1**
1	0	0	0	0	**1**
1	1	0	1	1	**1**

Tautology:

Examples 2:

Show statement “(P∧ Q)→ Q” is tautology:

- Recall that tautology means “True” under all possible assignments

- Then we can draw a full truth table which list all possibilities to see if it is tautology

- Looking at the truth table, the all-possible assignments of P, Q have truth of the statement. Then the statement is tautology

P	Q	(P ∧ Q)	**(P ∧ Q) → Q**
0	0	0	**1**
0	1	0	**1**
1	0	0	**1**
1	1	1	**1**

Equivalence:

Examples:

Show statement “¬(P ∧ Q) ↔ (¬P ∨¬Q)” (which is De Morgan theorem):

- Recall that equivalence requires left-hand side has the same truth table with the right-hand side
- Looking at the truth table below, ¬(P ∧ Q) is the same as (¬P ∨¬Q), then they are equivalent

P	Q	(P ∧ Q)	¬(P ∧ Q)
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

P	Q	¬P	¬Q	(¬P ∨¬Q)
0	0	1	1	1
0	1	1	0	1
1	0	0	1	1
1	1	0	0	0

Satisfiable vs Unsatisfiable:

Examples:

Is statement ¬(P ∧ Q) satisfiable or unsatisfiable? What about (P ∧ Q) ∧ ¬Q ?

- Recall that satisfiable means at least one possible assignment makes the statement true
- Unsatisfiable means all possible assignments make the statement false

P	Q	(P ∧ Q)	**¬(P ∧ Q)**
0	0	0	**1**
0	1	0	**1**
1	0	0	**1**
1	1	1	**0**

(Satisfiable)

P	Q	(P ∧ Q)	¬Q	**(P ∧ Q) ∧ ¬Q**
0	0	0	1	**0**
0	1	0	0	**0**
1	0	0	1	**0**
1	1	1	0	**0**

(Unsatisfiable)

Quiz Time!