# Propositional Logic: Implication

**→ : Implication**

- used to suggests a causal connection between antecedent (P) and consequent (Q)
- P → Q, has the meaning “If P is true then implies Q is true”

**Examples:**

**Let P = “It is raining” , Q = “the floor now is wet”, then let’s consider 4 different cases:**

- Case 1: “It is raining, and the floor is wet”
- Case 2: “It is raining, and the floor is not wet”
- Case 3: “It is not raining, and the floor is wet”
- Case 4: “It is not raining, and the floor is not wet”

**Question**: Which case is impossible to happen?

**Answer**: Case 2 is impossible to happen. It’s impossible that “It is raining now but the floor is dry”

But it’s possible that ”the floor is wet whilst it is not raining now” (It might because raining yesterday)

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**Examples (Continue from previous slide):**

**Let P = "It is raining" , Q = "the floor now is wet":**

- We found that: only when Q is true P will impossible be true, which is "if the floor is wet it is impossible raining now"
- It means: **P → Q is false if and only if when Q is false but P is true**
- In all other cases, the result of P → Q is true

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**More Examples :**

**Assume P = "I won the first prize in the lottery" , Q = "I become a richman" are True:**

- Is P → Q  True or False? – True ("If I won the lottery then I will be a richman" is possible)

- Is ¬P → Q True or False? – True ("If didn't win the lottery then I am a richman" is still possible, it might because my father give me a million dollors.)

- Is P → ¬Q True or False? – False (It is impossible that "If I won the lottery then I am not a richman")

- Is ¬P → ¬Q True or False? – True ("If I didn't win the lottery then I am not a richman" is possible)

**Note: P → Q evaluates to FALSE iif when consequent is false but antecedent is true**

# Propositional Logic: Bi-implication

**↔ : Bi-implication**

- used to suggests a bi-implication or equivalence
- P → Q and Q → P, which is "P is true if and only if Q is true"
- P ↔ Q, has the meaning "P is equivalent to Q"

**Examples:**

**Assume P: "It's raining", Q: "I will bring my umbrella"**

- if it's not raining, I won't bring my umbrella,
- if I don't bring my umbrella, it means it's not raining.
- So, both propositions are either both true or both false at the same time.

# Operator Precedence

When an expression exists connectives, what is the order to evaluate it?

- If there exists parentheses, then calculate **parentheses first**
- Otherwise, Precedence **from highest to lowest** is: **¬, ∧, ∨, →, ↔**
- All binary operators are left associative, Eg: P ∧ Q ∧ R = (P ∧ Q) ∧ R

**Examples:**

**Let P = "2 + 3 = 5" , Q = "2 + 5 = 5", R = "Grass is green", S = "Socrates is bald"**

- (P ∨ Q) ∧ R is true since we will do (P ∨ Q) first then evaluate it with "∧ R"
- P ∨ Q ∧ R is false since we will do "Q ∧ R" first then evaluate it with "P ∨"
- ¬P ∨ Q ∧ R → S is False since we will evaluate ¬P (False) first then Q ∧ R (False), then evaluate ¬P ∨ Q ∧ R (False), finally use the result to evaluate ¬P ∨ Q ∧ R → S (False)

# Quiz Time!