

1. Solve Example 3.2.1 using **fem1d** with 4 linear, 2 quadratic and 4 quadratic elements. Compare the results with exact solution given in Table 3.2.1. Plot $u(x)$ vs x and du/dx vs x .

With 4 linear elements (4L),

The solution of nodal value is,

U1 (0)	U2 (0.25)	U3 (0.50)	U4 (0.75)	U5 (1)
0	-0.023233	-0.040519	-0.039191	0

With 2 quadratic elements (2Q),

The solution of nodal value is,

U1 (0)	U2 (0.25)	U3 (0.50)	U4 (0.75)	U5 (1)
0	-0.023447E	-0.040779E	-0.039473	0

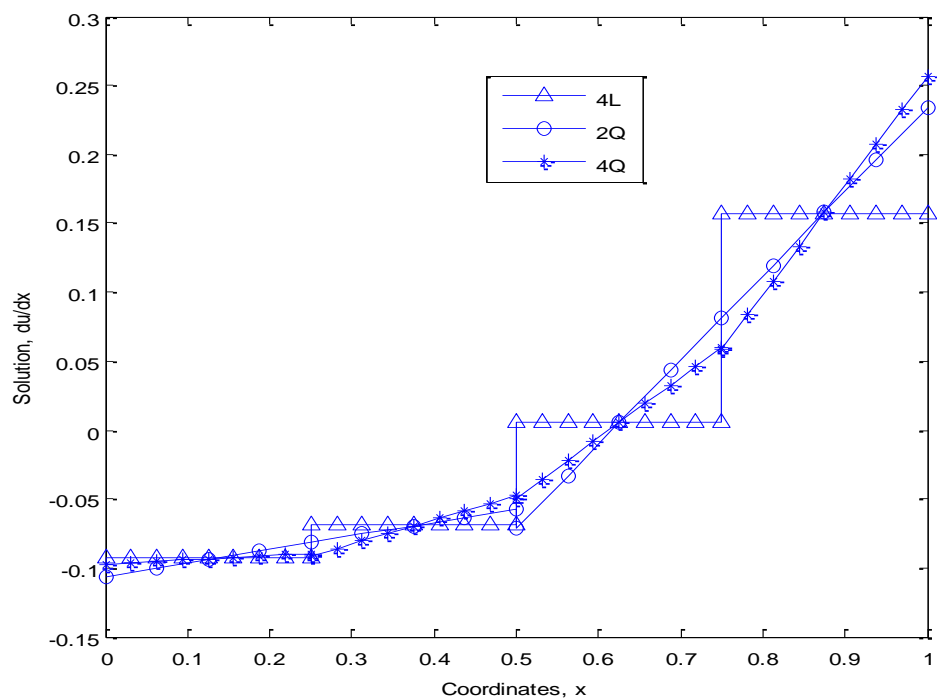
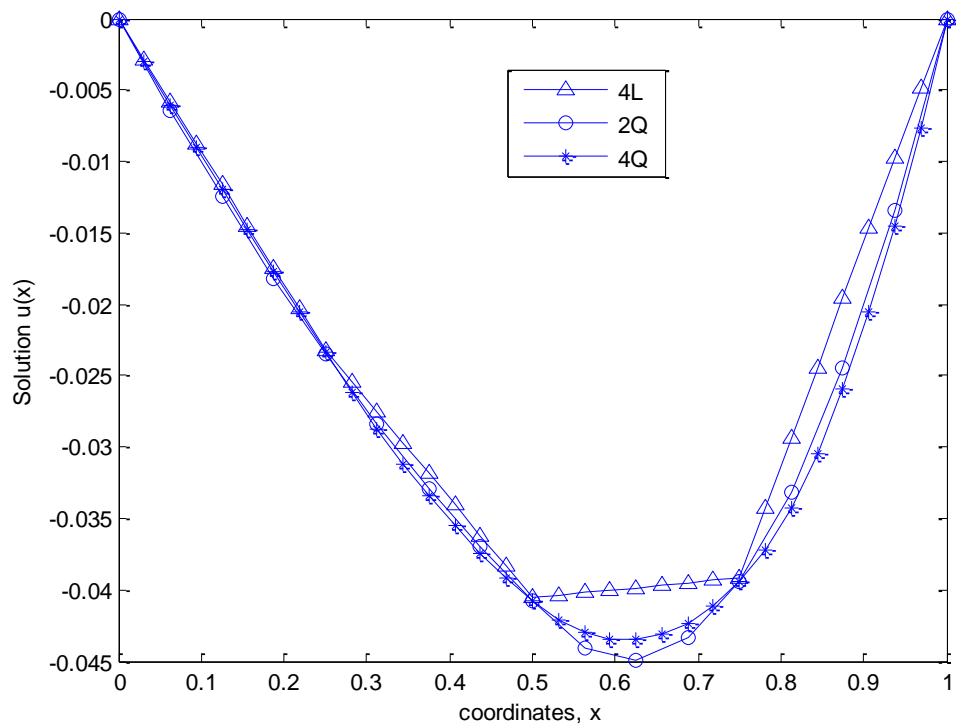
With 4 quadratic elements (4Q),

The solution of nodal value is,

U1 (0)	U2 (0.125)	U3 (0.250)	U4(0.375)	U5 (0.5)	U6(0.625)	U7(0.75)	U8(0.875)	U9(1)
0	-0.011927	-0.023375	-0.033450	-0.040760	-0.0435	-0.039417	-0.025907	0

Comparison of the finite element results with the exact solution (**-10u**) is shown below.

x	FEM solution			Exact Solution
	4L	2Q	4Q	
0	0	0	0	0
0.0625	0.058084	0.06435	0.060232	0.0598
0.125	0.11617	0.12488	0.11927	0.1192
0.1875	0.17425	0.18158	0.17711	0.1775
0.25	0.23233	0.23447	0.23375	0.2337
0.3125	0.27555	0.28353	0.28758	0.2866
0.375	0.31876	0.32877	0.3345	0.3345
0.4375	0.36198	0.37019	0.37451	0.3755
0.5	0.40519	0.40779	0.4076	0.4076
0.5625	0.40187	0.44031	0.42983	0.4283
0.625	0.39855	0.44897	0.435	0.435
0.6875	0.39523	0.43377	0.42311	0.4246
0.75	0.39191	0.39473	0.39417	0.3942
0.8125	0.29393	0.33182	0.34212	0.3402
0.875	0.19595	0.24507	0.25907	0.259
0.9375	0.097977	0.13446	0.14503	0.147
1	0	0	0	0



Sample data and output:

*** ECHO OF THE INPUT DATA STARTS ***

Example 3.2.1: Solution of a differential equation (4L)

```
1 0 0          MODEL, NTYPE, ITEM
1 4           IELEM, NEM
1 0           ICONT, NPRNT
0.0 0.25 0.25 0.25 0.25  DX(I)
1.0 0.0       AX0, AX1
0.0 0.0       BX0, BX1
-1.0 0.0      CX0, CX1
0.0 0.0 -1.0  FX0, FX1, FX2
2            NSPV
1 1 0.0       ISPV(1,1), ISPV(1,2), VSPV(1)
5 1 0.0       ISPV(1,1), ISPV(1,2), VSPV(1)
0            NSSV
0            NNBC
0            NMPC
```

**** ECHO OF THE INPUT DATA ENDS ****

OUTPUT from program FEM1D by J N REDDY

Example 3.2.1: Solution of a differential equation (4L)

*** ANALYSIS OF MODEL 1, AND TYPE 0 PROBLEM ***
(see the code below)

MODEL=1,NTYPE=0: A problem described by MODEL EQ. 1
MODEL=1,NTYPE=1: A circular DISK (PLANE STRESS)
MODEL=1,NTYPE>1: A circular DISK (PLANE STRAIN)
MODEL=2,NTYPE=0: A Timoshenko BEAM (RIE) problem
MODEL=2,NTYPE=1: A Timoshenko PLATE (RIE) problem
MODEL=2,NTYPE=2: A Timoshenko BEAM (CIE) problem
MODEL=2,NTYPE>2: A Timoshenko PLATE (CIE) problem
MODEL=3,NTYPE=0: A Euler-Bernoulli BEAM problem
MODEL=3,NTYPE>0: A Euler-Bernoulli Circular plate
MODEL=4,NTYPE=0: A plane TRUSS problem
MODEL=4,NTYPE=1: A Euler-Bernoulli FRAME problem
MODEL=4,NTYPE=2: A Timoshenko (CIE) FRAME problem

Element type (0, Hermite,>0, Lagrange)..= 1
No. of deg. of freedom per node, NDF....= 1
No. of elements in the mesh, NEM.....= 4

No. of total DOF in the model, NEQ.....= 5
Half bandwidth of matrix [GLK], NHBW ...= 2
No. of specified primary DOF, NSPV.....= 2
No. of specified secondary DOF, NSSV.....= 0
No. of specified Newton B. C.: NNBC.....= 0
No. of speci. multi-pt. cond.: NMPC.....= 0

Boundary information on primary variables:

1 1 0.00000E+00
5 1 0.00000E+00

Global coordinates of the nodes, {GLX}:

0.00000E+00 0.25000E+00 0.50000E+00 0.75000E+00 0.10000E+01

Coefficients of the differential equation:

AX0 = 0.1000E+01 AX1 = 0.0000E+00
BX0 = 0.0000E+00 BX1 = 0.0000E+00
CX0 = -0.1000E+01 CX1 = 0.0000E+00
FX0 = 0.0000E+00 FX1 = 0.0000E+00 FX2 = -0.1000E+01

SOLUTION (values of PVs) at the NODES:

0.00000E+00 -0.23233E-01 -0.40519E-01 -0.39191E-01 0.00000E+00

x is the global coord. if ICONT=1 and it is the local coord. if ICONT=0

x P. Variable S. Variable

0.00000E+00 0.00000E+00 -0.92934E-01
0.31250E-01 -0.29042E-02 -0.92934E-01
0.62500E-01 -0.58084E-02 -0.92934E-01
0.93750E-01 -0.87125E-02 -0.92934E-01
0.12500E+00 -0.11617E-01 -0.92934E-01
0.15625E+00 -0.14521E-01 -0.92934E-01
0.18750E+00 -0.17425E-01 -0.92934E-01
0.21875E+00 -0.20329E-01 -0.92934E-01
0.25000E+00 -0.23233E-01 -0.92934E-01
0.25000E+00 -0.23233E-01 -0.69144E-01
0.28125E+00 -0.25394E-01 -0.69144E-01
0.31250E+00 -0.27555E-01 -0.69144E-01
0.34375E+00 -0.29716E-01 -0.69144E-01
0.37500E+00 -0.31876E-01 -0.69144E-01
0.40625E+00 -0.34037E-01 -0.69144E-01

```

0.43750E+00 -0.36198E-01 -0.69144E-01
0.46875E+00 -0.38359E-01 -0.69144E-01
0.50000E+00 -0.40519E-01 -0.69144E-01
0.50000E+00 -0.40519E-01 0.53143E-02
0.53125E+00 -0.40353E-01 0.53143E-02
0.56250E+00 -0.40187E-01 0.53143E-02
0.59375E+00 -0.40021E-01 0.53143E-02
0.62500E+00 -0.39855E-01 0.53143E-02
0.65625E+00 -0.39689E-01 0.53143E-02
0.68750E+00 -0.39523E-01 0.53143E-02
0.71875E+00 -0.39357E-01 0.53143E-02
0.75000E+00 -0.39191E-01 0.53143E-02
0.75000E+00 -0.39191E-01 0.15676E+00
0.78125E+00 -0.34292E-01 0.15676E+00
0.81250E+00 -0.29393E-01 0.15676E+00
0.84375E+00 -0.24494E-01 0.15676E+00
0.87500E+00 -0.19595E-01 0.15676E+00
0.90625E+00 -0.14697E-01 0.15676E+00
0.93750E+00 -0.97977E-02 0.15676E+00
0.96875E+00 -0.48989E-02 0.15676E+00
0.10000E+01 0.00000E+00 0.15676E+00

```

Stop - Program terminated.

*** ECHO OF THE INPUT DATA STARTS ***

Example 3.2.1: Solution of a differential equation (2Q)

```

1 0 0          MODEL, NTYPE, ITEM
2 2          IELEM, NEM
1 0          ICONT, NPRNT
0.0 0.5 0.5    DX(I)
1.0 0.0        AX0, AX1
0.0 0.0        BX0, BX1
-1.0 0.0       CX0, CX1
0.0 0.0 -1.0   FX0, FX1, FX2
2            NSPV
1 1 0.0        ISPV(1,1), ISPV(1,2), VSPV(1)
5 1 0.0        ISPV(1,1), ISPV(1,2), VSPV(1)
0            NSSV
0            NNBC
0            NMPC

```

**** ECHO OF THE INPUT DATA ENDS ****

OUTPUT from program FEM1D by J N REDDY

Example 3.2.1: Solution of a differential equation (2Q)

*** ANALYSIS OF MODEL 1, AND TYPE 0 PROBLEM ***
(see the code below)

MODEL=1,NTYPE=0: A problem described by MODEL EQ. 1
MODEL=1,NTYPE=1: A circular DISK (PLANE STRESS)
MODEL=1,NTYPE>1: A circular DISK (PLANE STRAIN)
MODEL=2,NTYPE=0: A Timoshenko BEAM (RIE) problem
MODEL=2,NTYPE=1: A Timoshenko PLATE (RIE) problem
MODEL=2,NTYPE=2: A Timoshenko BEAM (CIE) problem
MODEL=2,NTYPE>2: A Timoshenko PLATE (CIE) problem
MODEL=3,NTYPE=0: A Euler-Bernoulli BEAM problem
MODEL=3,NTYPE>0: A Euler-Bernoulli Circular plate
MODEL=4,NTYPE=0: A plane TRUSS problem
MODEL=4,NTYPE=1: A Euler-Bernoulli FRAME problem
MODEL=4,NTYPE=2: A Timoshenko (CIE) FRAME problem

Element type (0, Hermite,>0, Lagrange)..= 2
No. of deg. of freedom per node, NDF.....= 1
No. of elements in the mesh, NEM.....= 2
No. of total DOF in the model, NEQ.....= 5
Half bandwidth of matrix [GLK], NHBW ...= 3
No. of specified primary DOF, NSPV.....= 2
No. of specified secondary DOF, NSSV.....= 0
No. of specified Newton B. C.: NNBC.....= 0
No. of speci. multi-pt. cond.: NMPC.....= 0

Boundary information on primary variables:

1 1 0.00000E+00
5 1 0.00000E+00

Global coordinates of the nodes, {GLX}:

0.00000E+00 0.25000E+00 0.50000E+00 0.75000E+00 0.10000E+01

Coefficients of the differential equation:

AX0 = 0.1000E+01 AX1 = 0.0000E+00
BX0 = 0.0000E+00 BX1 = 0.0000E+00
CX0 = -0.1000E+01 CX1 = 0.0000E+00
FX0 = 0.0000E+00 FX1 = 0.0000E+00 FX2 = -0.1000E+01

SOLUTION (values of PVs) at the NODES:

0.00000E+00 -0.23447E-01 -0.40779E-01 -0.39473E-01 0.00000E+00

x is the global coord. if ICONT=1 and it is the local coord. if ICONT=0

x P. Variable S. Variable

0.00000E+00 0.00000E+00 -0.10602E+00
0.62500E-01 -0.64350E-02 -0.99902E-01
0.12500E+00 -0.12488E-01 -0.93788E-01
0.18750E+00 -0.18158E-01 -0.87673E-01
0.25000E+00 -0.23447E-01 -0.81558E-01
0.31250E+00 -0.28353E-01 -0.75443E-01
0.37500E+00 -0.32877E-01 -0.69329E-01
0.43750E+00 -0.37019E-01 -0.63214E-01
0.50000E+00 -0.40779E-01 -0.57099E-01
0.50000E+00 -0.40779E-01 -0.71106E-01
0.56250E+00 -0.44031E-01 -0.32940E-01
0.62500E+00 -0.44897E-01 0.52262E-02
0.68750E+00 -0.43377E-01 0.43392E-01
0.75000E+00 -0.39473E-01 0.81558E-01
0.81250E+00 -0.33182E-01 0.11972E+00
0.87500E+00 -0.24507E-01 0.15789E+00
0.93750E+00 -0.13446E-01 0.19606E+00
0.10000E+01 0.00000E+00 0.23422E+00

Stop - Program terminated.

*** ECHO OF THE INPUT DATA STARTS ***

Example 3.2.1: Solution of a differential equation (4Q)

1 0 0 MODEL, NTYPE, ITEM
2 4 IELEM, NEM
1 0 ICONT, NPRNT
0.0 0.25 0.25 0.25 0.25 DX(I)
1.0 0.0 AX0, AX1
0.0 0.0 BX0, BX1
-1.0 0.0 CX0, CX1
0.0 0.0 -1.0 FX0, FX1, FX2
2 NSPV
1 1 0.0 ISPV(1,1), ISPV(1,2), VSPV(1)
9 1 0.0 ISPV(1,1), ISPV(1,2), VSPV(1)
0 NSSV
0 NNBC
0 NMPC

**** ECHO OF THE INPUT DATA ENDS ****

Example 3.2.1: Solution of a differential equation (4Q)

*** ANALYSIS OF MODEL 1, AND TYPE 0 PROBLEM ***
(see the code below)

MODEL=1,NTYPE=0: A problem described by MODEL EQ. 1
MODEL=1,NTYPE=1: A circular DISK (PLANE STRESS)
MODEL=1,NTYPE>1: A circular DISK (PLANE STRAIN)
MODEL=2,NTYPE=0: A Timoshenko BEAM (RIE) problem
MODEL=2,NTYPE=1: A Timoshenko PLATE (RIE) problem
MODEL=2,NTYPE=2: A Timoshenko BEAM (CIE) problem
MODEL=2,NTYPE>2: A Timoshenko PLATE (CIE) problem
MODEL=3,NTYPE=0: A Euler-Bernoulli BEAM problem
MODEL=3,NTYPE>0: A Euler-Bernoulli Circular plate
MODEL=4,NTYPE=0: A plane TRUSS problem
MODEL=4,NTYPE=1: A Euler-Bernoulli FRAME problem
MODEL=4,NTYPE=2: A Timoshenko (CIE) FRAME problem

Element type (0, Hermite,>0, Lagrange)..= 2
No. of deg. of freedom per node, NDF....= 1
No. of elements in the mesh, NEM.....= 4
No. of total DOF in the model, NEQ.....= 9
Half bandwidth of matrix [GLK], NHBW ...= 3
No. of specified primary DOF, NSPV.....= 2
No. of specified secondary DOF, NSSV.....= 0
No. of specified Newton B. C.: NNBC.....= 0
No. of speci. multi-pt. cond.: NMPC.....= 0

Boundary information on primary variables:

1 1 0.00000E+00
9 1 0.00000E+00

Global coordinates of the nodes, {GLX}:

0.00000E+00 0.12500E+00 0.25000E+00 0.37500E+00 0.50000E+00
0.62500E+00 0.75000E+00 0.87500E+00 0.10000E+01

Coefficients of the differential equation:

AX0 = 0.1000E+01 AX1 = 0.0000E+00
BX0 = 0.0000E+00 BX1 = 0.0000E+00
CX0 = -0.1000E+01 CX1 = 0.0000E+00

FX0 = 0.0000E+00 FX1 = 0.0000E+00 FX2 = -0.1000E+01

SOLUTION (values of PVs) at the NODES:

0.00000E+00 -0.11927E-01 -0.23375E-01 -0.33450E-01 -0.40760E-01
-0.43500E-01 -0.39417E-01 -0.25907E-01 0.00000E+00

x is the global coord. if ICONT=1 and it is the local coord. if ICONT=0
x P. Variable S. Variable

0.00000E+00 0.00000E+00 -0.97329E-01
0.31250E-01 -0.30266E-02 -0.96371E-01
0.62500E-01 -0.60232E-02 -0.95414E-01
0.93750E-01 -0.89900E-02 -0.94457E-01
0.12500E+00 -0.11927E-01 -0.93500E-01
0.15625E+00 -0.14834E-01 -0.92543E-01
0.18750E+00 -0.17711E-01 -0.91586E-01
0.21875E+00 -0.20558E-01 -0.90629E-01
0.25000E+00 -0.23375E-01 -0.89671E-01
0.25000E+00 -0.23375E-01 -0.91657E-01
0.28125E+00 -0.26153E-01 -0.86128E-01
0.31250E+00 -0.28758E-01 -0.80599E-01
0.34375E+00 -0.31190E-01 -0.75070E-01
0.37500E+00 -0.33450E-01 -0.69542E-01
0.40625E+00 -0.35537E-01 -0.64013E-01
0.43750E+00 -0.37451E-01 -0.58484E-01
0.46875E+00 -0.39192E-01 -0.52955E-01
0.50000E+00 -0.40760E-01 -0.47426E-01
0.50000E+00 -0.40760E-01 -0.49199E-01
0.53125E+00 -0.42085E-01 -0.35556E-01
0.56250E+00 -0.42983E-01 -0.21914E-01
0.59375E+00 -0.43454E-01 -0.82709E-02
0.62500E+00 -0.43500E-01 0.53718E-02
0.65625E+00 -0.43119E-01 0.19015E-01
0.68750E+00 -0.42311E-01 0.32657E-01
0.71875E+00 -0.41077E-01 0.46300E-01
0.75000E+00 -0.39417E-01 0.59943E-01
0.75000E+00 -0.39417E-01 0.58492E-01
0.78125E+00 -0.37202E-01 0.83287E-01
0.81250E+00 -0.34212E-01 0.10808E+00
0.84375E+00 -0.30447E-01 0.13288E+00
0.87500E+00 -0.25907E-01 0.15767E+00
0.90625E+00 -0.20593E-01 0.18246E+00
0.93750E+00 -0.14503E-01 0.20726E+00
0.96875E+00 -0.76391E-02 0.23205E+00

0.10000E+01 0.00000E+00 0.25685E+00

Stop - Program terminated.

2. Solve Example 4.3.1 using **fem1d** with 3 linear elements and compare the results with those obtained using hand calculations.

Solution at nodes:

U1 = 200°C U2 = 199.58°C U3 = 198.67°C U4 = 195.76°C

The FEM solution matches well with the results of Example 4.3.1.

Sample data and output:

*** ECHO OF THE INPUT DATA STARTS ***

Example 4.3.1: Heat transfer in a composite wall

```
1 0 0          MODEL, NTYPE, ITEM
1 3           IELEM, NEM
0 0          ICONT, NPRNT
4           NNM
1  2  0.02     NOD(1,J), GLX(1)
70.0 0.0      AX0, AX1  | Data for
0.0 0.0      BX0, BX1  | Element 1
0.0 0.0      CX0, CX1  |
0.0 0.0 0.0   FX0,FX1,FX2 |

2  3  0.025    NOD(2,J), GLX(2)
40.0 0.0      AX0, AX1  | Data for
0.0 0.0      BX0, BX1  | Element 2
0.0 0.0      CX0, CX1  |
0.0 0.0 0.0   FX0,FX1,FX2 |

3  4  0.04     NOD(2,J), GLX(3)
20.0 0.0      AX0, AX1  | Data for
0.0 0.0      BX0, BX1  | Element 3
0.0 0.0      CX0, CX1  |
0.0 0.0 0.0   FX0,FX1,FX2 |

1           NSPV
1  1  200.0    ISPV(1,1), ISPV(1,2), VSPV(1)

0           NSSV

1           NNBC (with convection)
```

```
4 1 10.0 50.0          INBC(1,1),INBC(1,2),VNBC(1),UREF(1)
0                      NMPC
```

```
**** ECHO OF THE INPUT DATA ENDS ****
```

OUTPUT from program FEM1D by J N REDDY

Example 4.3.1: Heat transfer in a composite wall

```
*** ANALYSIS OF MODEL 1, AND TYPE 0 PROBLEM ***
      (see the code below)
```

```
MODEL=1,NTYPE=0: A problem described by MODEL EQ. 1
MODEL=1,NTYPE=1: A circular DISK (PLANE STRESS)
MODEL=1,NTYPE>1: A circular DISK (PLANE STRAIN)
MODEL=2,NTYPE=0: A Timoshenko BEAM (RIE) problem
MODEL=2,NTYPE=1: A Timoshenko PLATE (RIE) problem
MODEL=2,NTYPE=2: A Timoshenko BEAM (CIE) problem
MODEL=2,NTYPE>2: A Timoshenko PLATE (CIE) problem
MODEL=3,NTYPE=0: A Euler-Bernoulli BEAM problem
MODEL=3,NTYPE>0: A Euler-Bernoulli Circular plate
MODEL=4,NTYPE=0: A plane TRUSS problem
MODEL=4,NTYPE=1: A Euler-Bernoulli FRAME problem
MODEL=4,NTYPE=2: A Timoshenko (CIE) FRAME problem
```

```
Element type (0, Hermite,>0, Lagrange)..= 1
No. of deg. of freedom per node, NDF.....= 1
No. of elements in the mesh, NEM.....= 3
No. of total DOF in the model, NEQ.....= 4
Half bandwidth of matrix [GLK], NHBW ...= 2
No. of specified primary DOF, NSPV.....= 1
No. of specified secondary DOF, NSSV.....= 0
No. of specified Newton B. C.: NNBC.....= 1
No. of speci. multi-pt. cond.: NMPC.....= 0
```

Boundary information on primary variables:

```
1 1 0.20000E+03
```

Boundary information on mixed boundary cond.:

```
4 1 0.10000E+02 0.50000E+02
```

Properties of Element = 1

Element length, H = 0.2000E-01
AX0 = 0.7000E+02 AX1 = 0.0000E+00
BX0 = 0.0000E+00 BX1 = 0.0000E+00
CX0 = 0.0000E+00 CX1 = 0.0000E+00
FX0 = 0.0000E+00 FX1 = 0.0000E+00 FX2 = 0.0000E+00

Properties of Element = 2

Element length, H = 0.2500E-01
AX0 = 0.4000E+02 AX1 = 0.0000E+00
BX0 = 0.0000E+00 BX1 = 0.0000E+00
CX0 = 0.0000E+00 CX1 = 0.0000E+00
FX0 = 0.0000E+00 FX1 = 0.0000E+00 FX2 = 0.0000E+00

Properties of Element = 3

Element length, H = 0.4000E-01
AX0 = 0.2000E+02 AX1 = 0.0000E+00
BX0 = 0.0000E+00 BX1 = 0.0000E+00
CX0 = 0.0000E+00 CX1 = 0.0000E+00
FX0 = 0.0000E+00 FX1 = 0.0000E+00 FX2 = 0.0000E+00

SOLUTION (values of PVs) at the NODES:

0.20000E+03 0.19958E+03 0.19867E+03 0.19576E+03

x is the global coord. if ICONT=1 and it is the local coord. if ICONT=0
x P. Variable S. Variable

0.00000E+00 0.20000E+03 -0.14576E+04
0.25000E-02 0.19995E+03 -0.14576E+04
0.50000E-02 0.19990E+03 -0.14576E+04
0.75000E-02 0.19984E+03 -0.14576E+04
0.10000E-01 0.19979E+03 -0.14576E+04
0.12500E-01 0.19974E+03 -0.14576E+04
0.15000E-01 0.19969E+03 -0.14576E+04
0.17500E-01 0.19964E+03 -0.14576E+04
0.20000E-01 0.19958E+03 -0.14576E+04
0.00000E+00 0.19958E+03 -0.14576E+04
0.31250E-02 0.19947E+03 -0.14576E+04
0.62500E-02 0.19936E+03 -0.14576E+04
0.93750E-02 0.19924E+03 -0.14576E+04

0.12500E-01	0.19913E+03	-0.14576E+04
0.15625E-01	0.19901E+03	-0.14576E+04
0.18750E-01	0.19890E+03	-0.14576E+04
0.21875E-01	0.19879E+03	-0.14576E+04
0.25000E-01	0.19867E+03	-0.14576E+04
0.00000E+00	0.19867E+03	-0.14576E+04
0.50000E-02	0.19831E+03	-0.14576E+04
0.10000E-01	0.19794E+03	-0.14576E+04
0.15000E-01	0.19758E+03	-0.14576E+04
0.20000E-01	0.19721E+03	-0.14576E+04
0.25000E-01	0.19685E+03	-0.14576E+04
0.30000E-01	0.19649E+03	-0.14576E+04
0.35000E-01	0.19612E+03	-0.14576E+04
0.40000E-01	0.19576E+03	-0.14576E+04

Stop - Program terminated.