COMPUTER PROJECT #2

ME/AE 6212 Advanced Finite Element Analysis

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Statement of the problem:

Consider plates with circular, elliptical, and rectangular holes at the center, subjected to tensile stresses. The material to be used has a modulus of elasticity E = 200 GPa, Poisson's ratio ν = 0.3. Use ABAQUS to analyze the plane stress problem.

(a) Use a full plate model (for the three cases), with 3 node triangular elements. Take plate thickness 0.02 m. Plot the deformed shape and von-Mises stress distribution.

(b) Use a symmetric quarter plate model (for the three cases), with 3 node triangular elements. Plot the deformed shape and von-Mises stress distribution.

(c) Compare and tabulate the maximum displacements and the maximum von-Mises stresses for cases (a) and (b). Perform the convergence study using at least three different element sizes (coarse to fine).

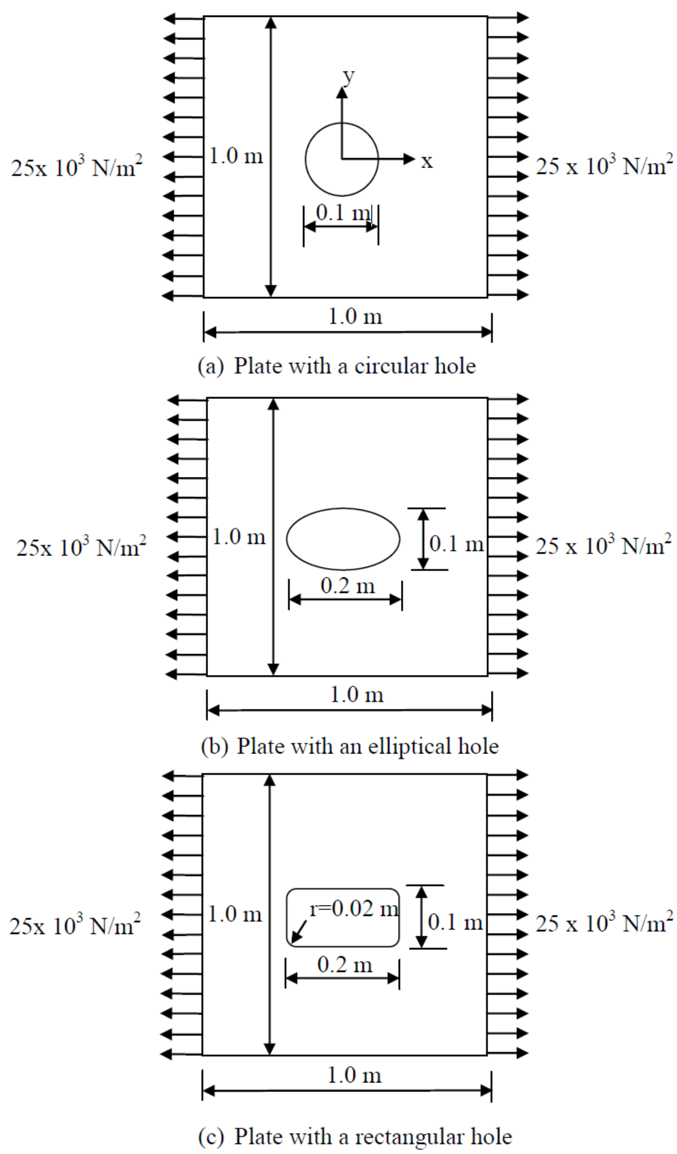


Figure 1. Plates with circular, elliptical, and rectangular holes at the center

Procedures:

FE Model and boundary conditions:
The 3D deformable shell model with thickness of 0.02m was applied to solve the above problems. The pressure boundary condition is $25\text{x}10^3\text{N/m}^2$ on both sides.
Then the shell edge load can be obtained as,

$$25\text{x}10^3\text{N/m}^2 \text{ x } 0.02\text{m} = 500\text{N/m}$$

Given the symmetry of the full plate model, two additional boundary conditions of displacement constraints were applied to the vertical and horizontal midlines, as shown in the following figure (U1 = 0 for the vertical midline, U2 = 0 for the horizontal midline).

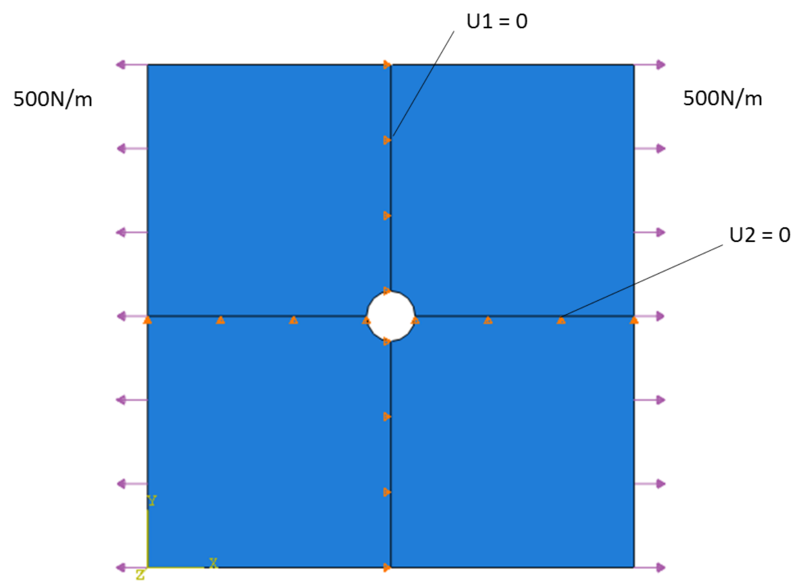


Figure 2. Boundary conditions of the full plate model

For the quarter plate model, the boundary conditions are shown in the following figure.

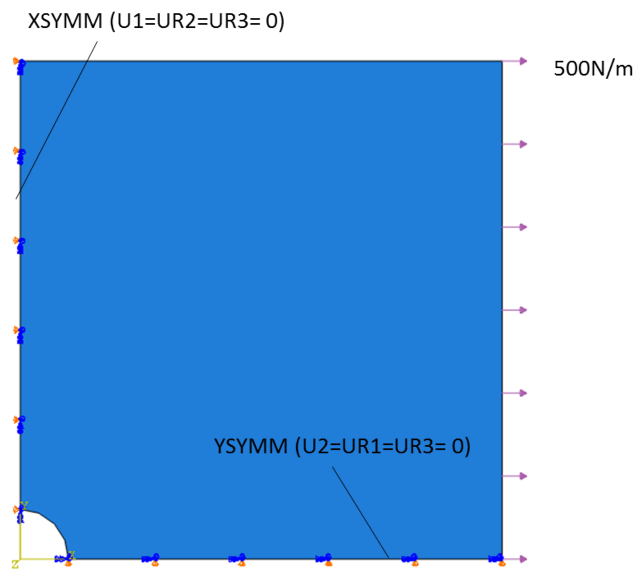
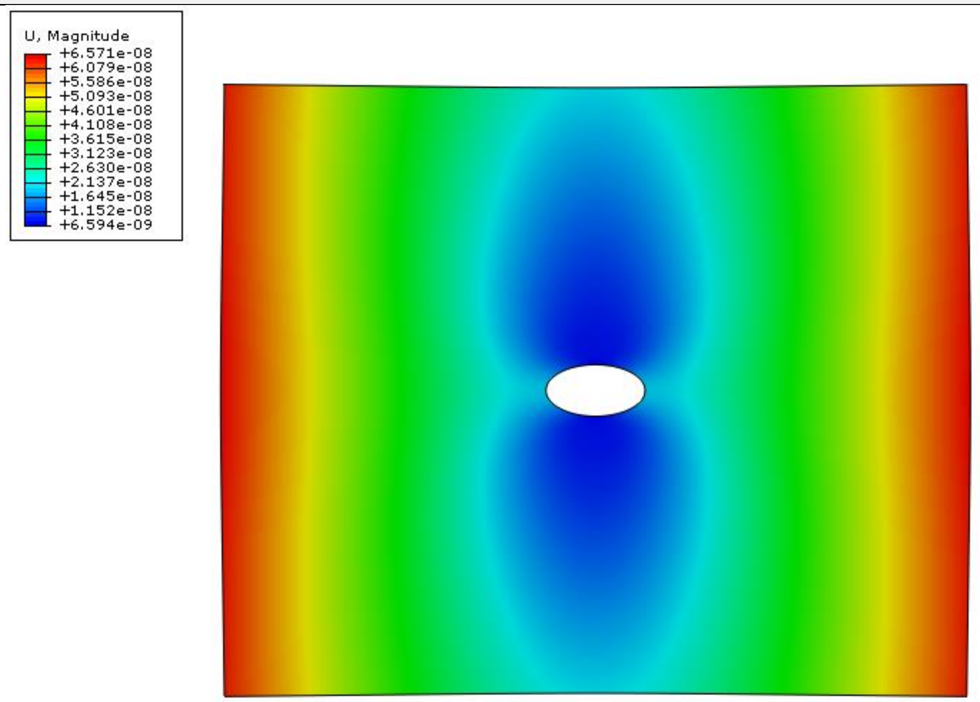
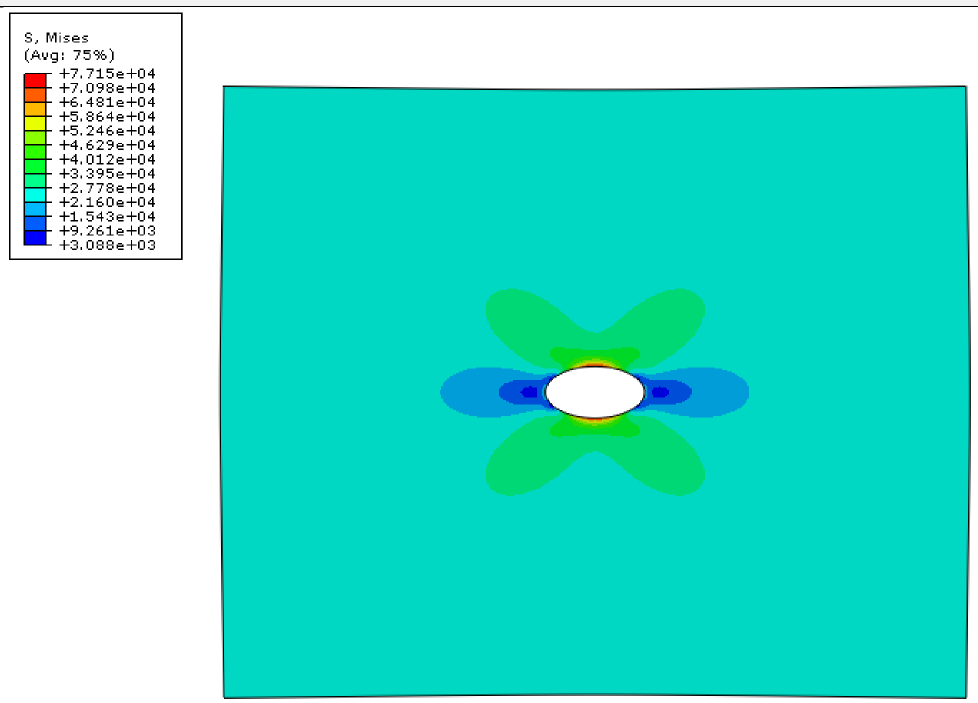
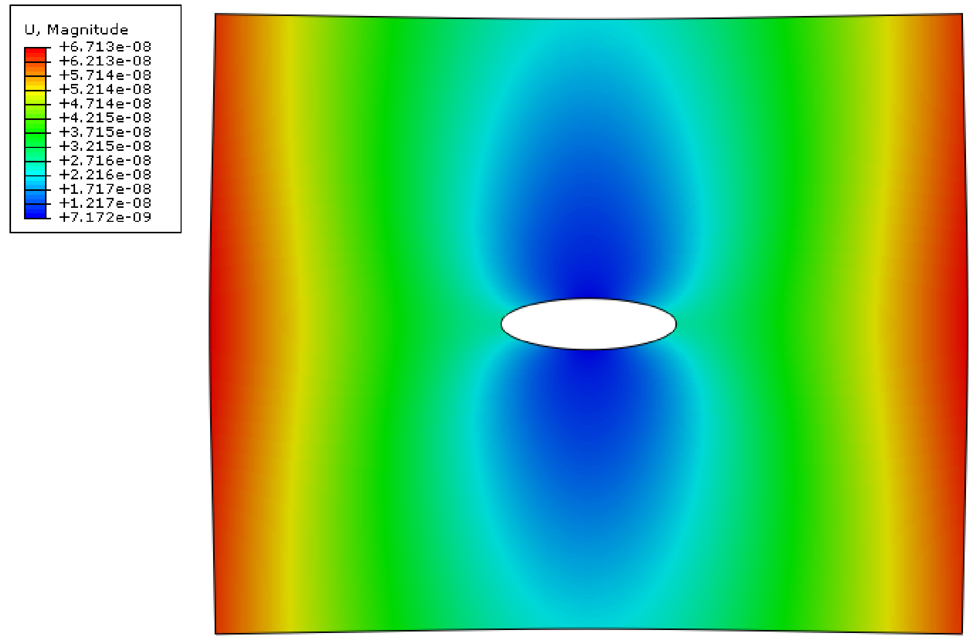
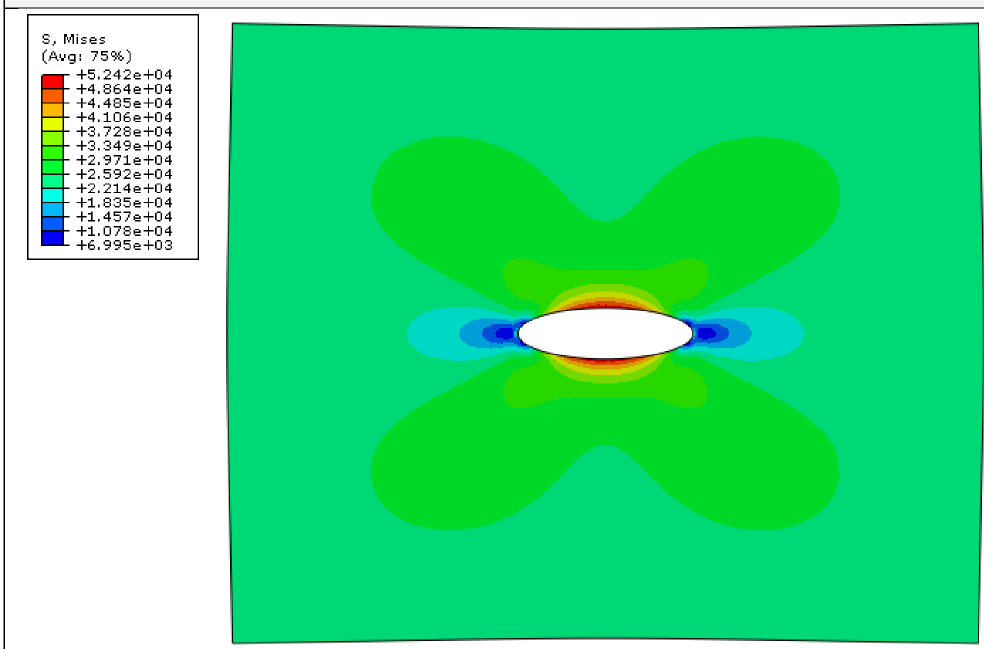


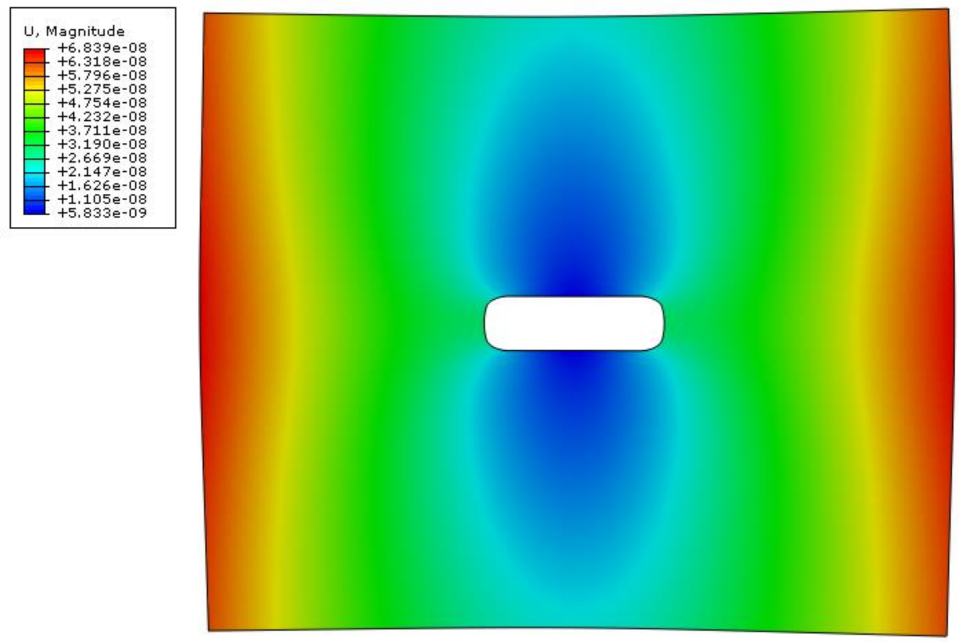
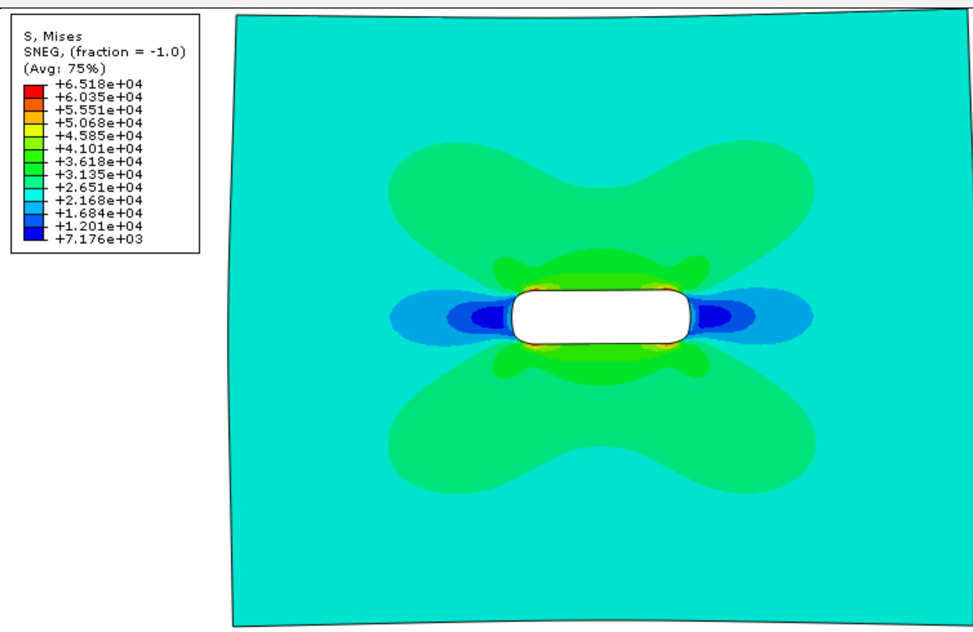
Figure 3. Boundary conditions of the quarter plate model

Results:

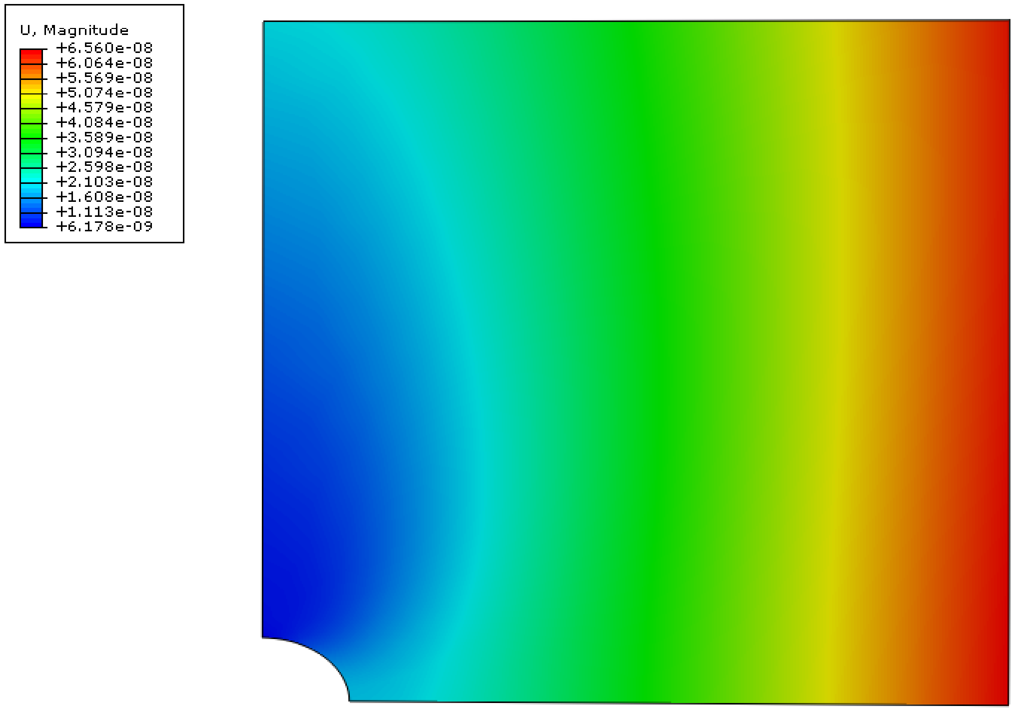
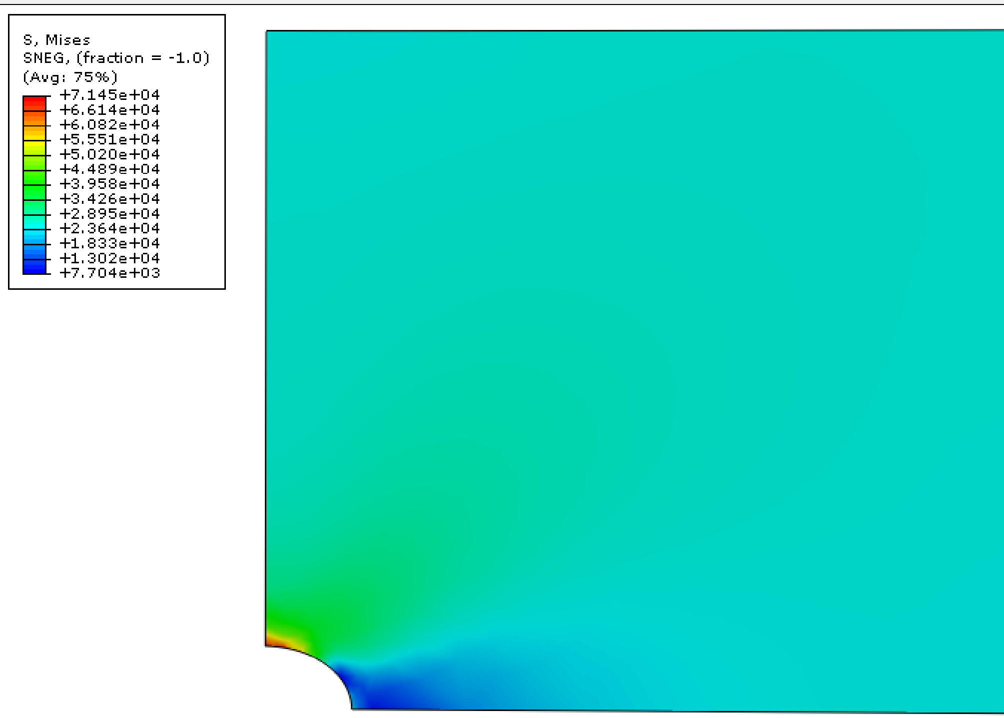
(a) Using a full plate model with 3 node triangular elements. The deformed shape and von-Mises stress distribution of the three cases are plotted below.

<table>
<tr><td rowspan="4">Case 1, full plate model</td><td>Deformed shape</td></tr>
<tr><td>
</td></tr>
<tr><td>von-Mises stress distribution</td></tr>
<tr><td>
</td></tr>
</table>

<table>
<tr><td rowspan="4">Case 2, full plate model</td><td>Deformed shape</td></tr>
<tr><td>
</td></tr>
<tr><td>von-Mises stress distribution</td></tr>
<tr><td>
</td></tr>
</table>

<table>
<tr><td rowspan="4">Case 3, full plate model</td><td>Deformed shape</td></tr>
<tr><td>
</td></tr>
<tr><td>von-Mises stress distribution</td></tr>
<tr><td>
</td></tr>
</table>

(b) Using a symmetric quarter plate model with 3 node triangular elements. The deformed shape and von-Mises stress distribution of the three cases are plotted below.

<table>
<tr><td rowspan="4">Case 1, quarter plate model</td><td>Deformed shape</td></tr>
<tr><td>
</td></tr>
<tr><td>von-Mises stress distribution</td></tr>
<tr><td>
</td></tr>
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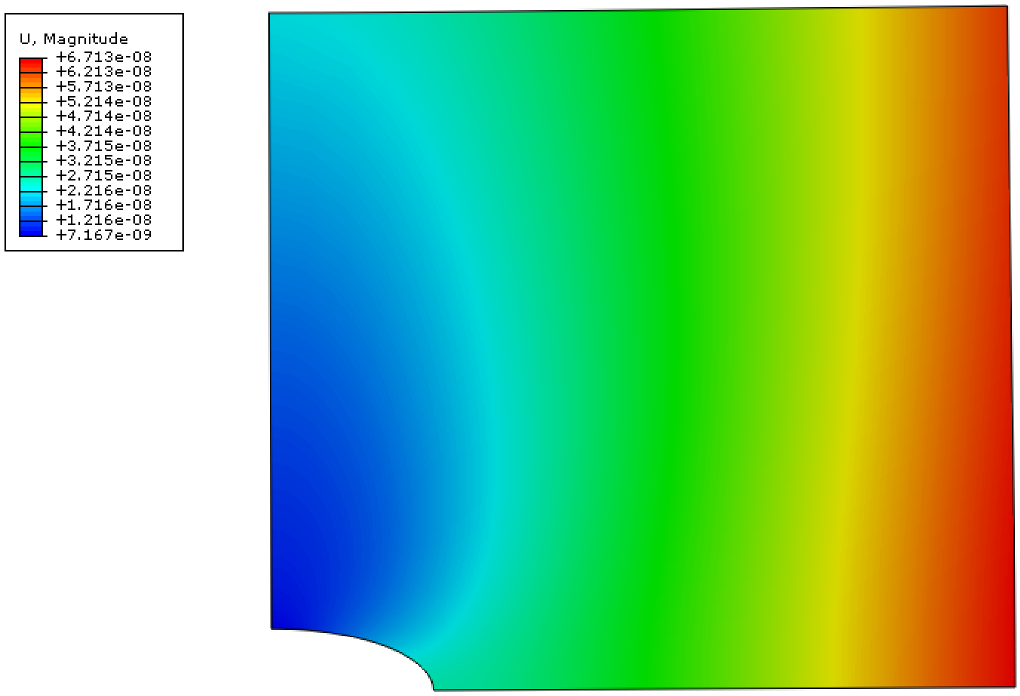
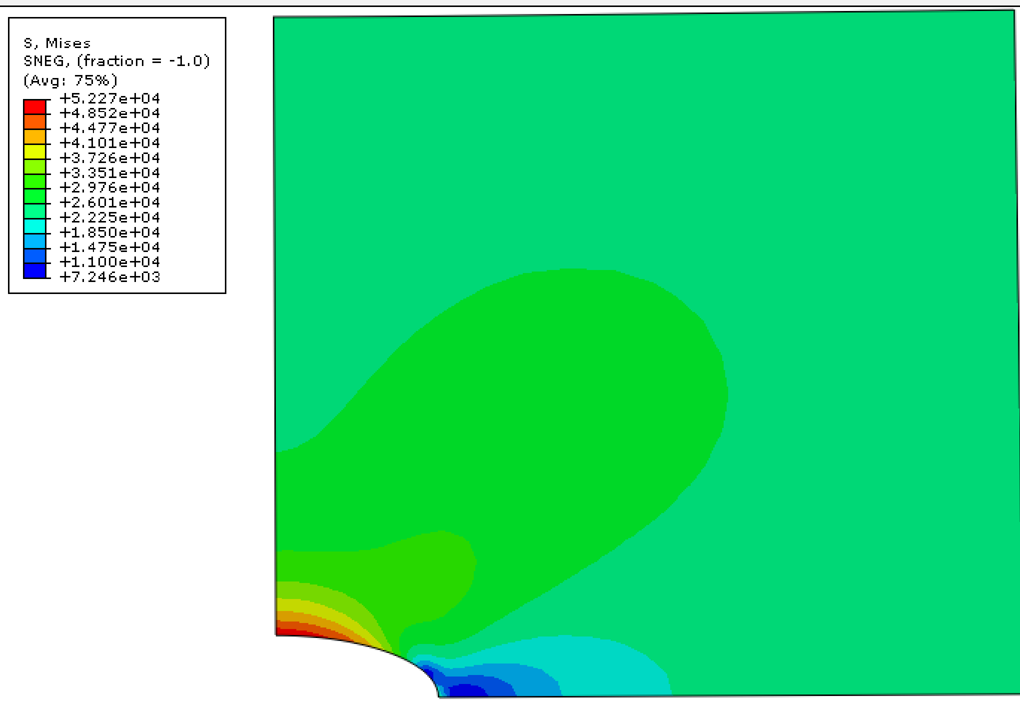
<table>
<tr><td rowspan="4">Case 2, quarter plate model</td><td>Deformed shape</td></tr>
<tr><td>
</td></tr>
<tr><td>von-Mises stress distribution</td></tr>
<tr><td>
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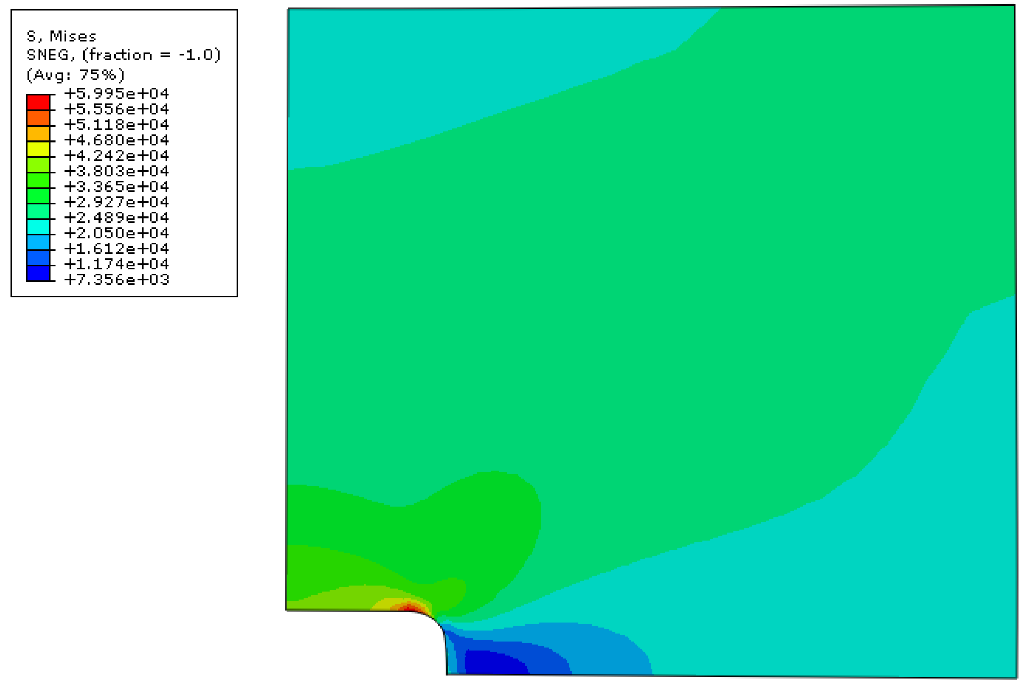
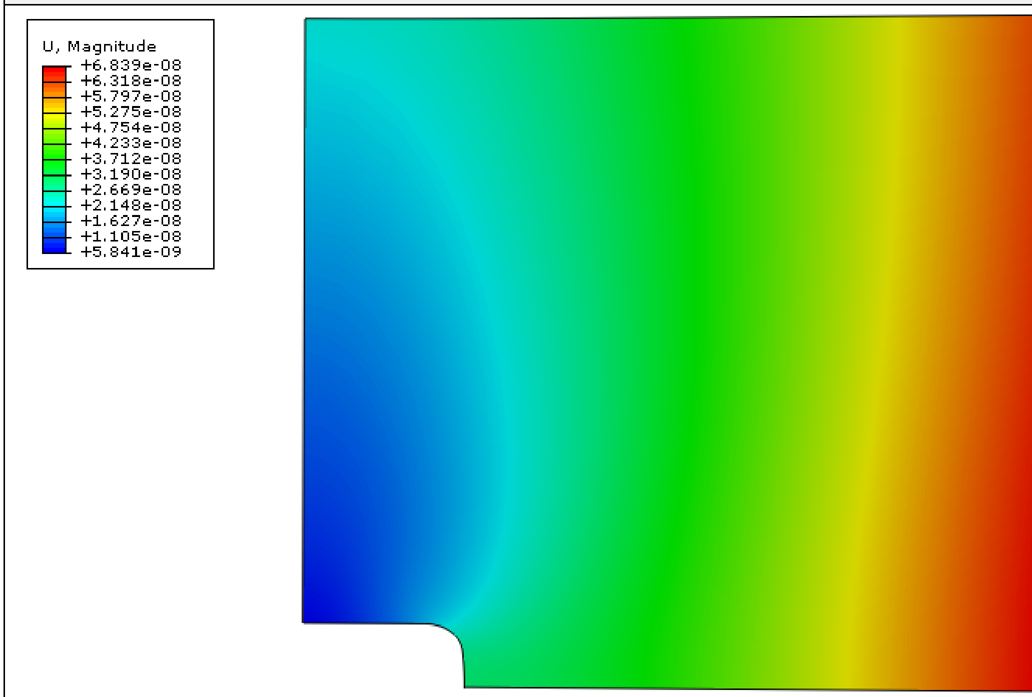
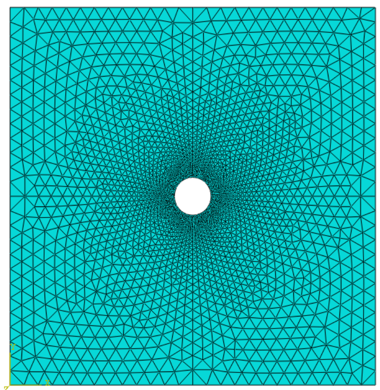
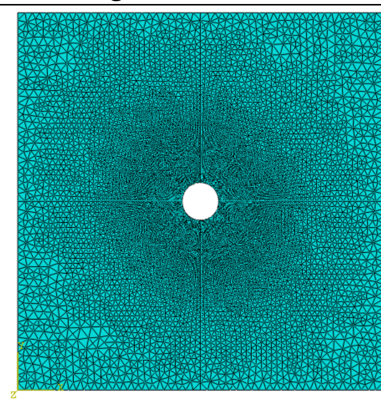
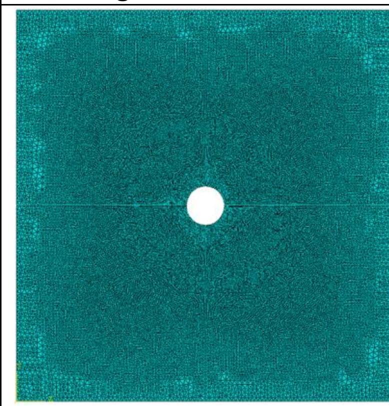
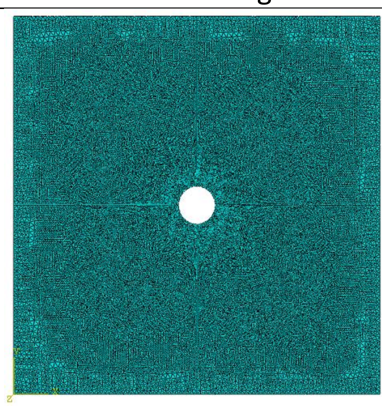
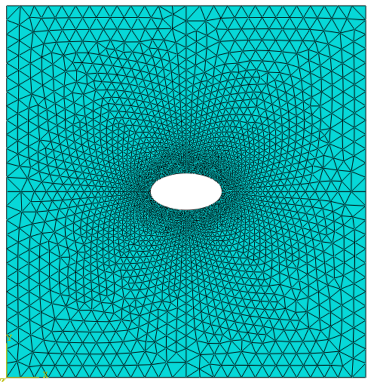
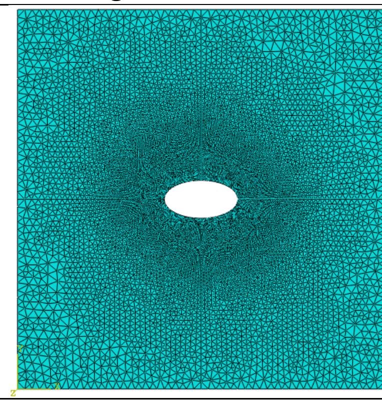
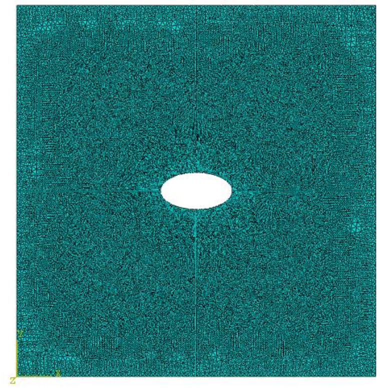
<table>
<tr><td rowspan="4">Case 3, quarter plate model</td><td>Deformed shape</td></tr>
<tr><td>
</td></tr>
<tr><td>von-Mises stress distribution</td></tr>
<tr><td>
</td></tr>
</table>

(c)

Convergence study: In order to acquire enough accuracy and acceptable computing time, at least three meshing size were implemented, as listed below.

<table>
<tr><td colspan="4">Case 1, full plate model</td></tr>
<tr><td>Mesh size (m)</td><td>Outer edge: 0.04
Hole edge: 0.004</td><td>Outer edge: 0.02
Hole edge: 0.002</td><td>Outer edge: 0.01
Hole edge: 0.001</td></tr>
<tr><td colspan="2"></td><td></td><td></td></tr>
<tr><td colspan="2">Outer edge: 0.01
Hole edge: 0.0005</td><td></td><td></td></tr>
<tr><td colspan="2"></td><td></td><td></td></tr>
<tr><td colspan="4">Results:</td></tr>
<tr><td colspan="2">Maximum displacements (m)</td><td>Maximum von-Mises stress (Pa)</td><td></td></tr>
<tr><td colspan="2">6.568e-8</td><td>7.676e4</td><td></td></tr>
<tr><td colspan="2">6.572e-8</td><td>7.689e4</td><td></td></tr>
<tr><td colspan="2">6.573e-8</td><td>7.770e4</td><td></td></tr>
<tr><td colspan="2">6.573e-8</td><td>7.756e4</td><td></td></tr>
</table>

<table>
<tr><th colspan="4">Case 2, full plate model</th></tr>
<tr><td>Mesh size (m)</td><td>Outer edge: 0.04
Hole edge: 0.004</td><td>Outer edge: 0.02
Hole edge: 0.002</td><td>Outer edge: 0.01
Hole edge: 0.001</td></tr>
<tr><td colspan="2"></td><td></td><td></td></tr>
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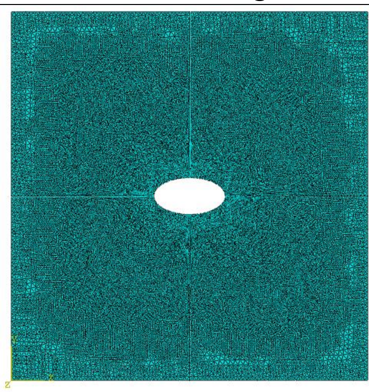
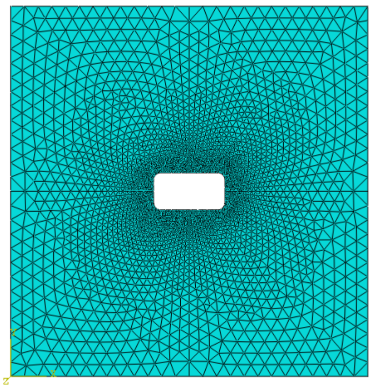
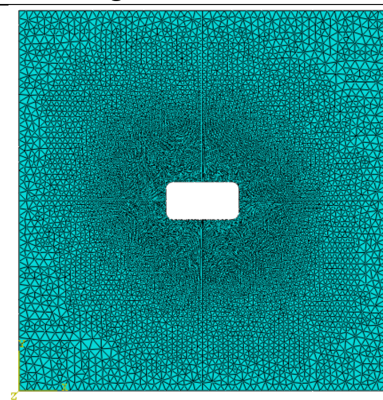
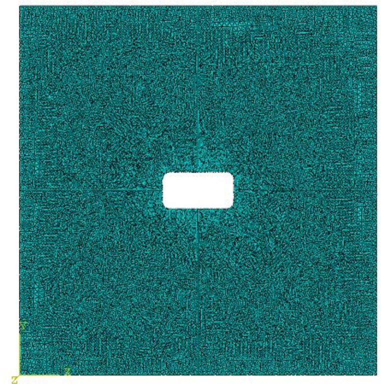
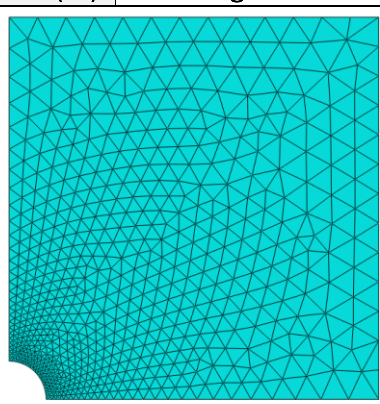
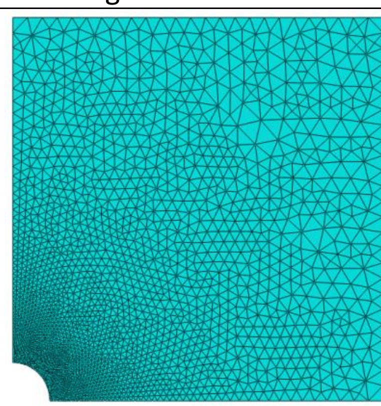
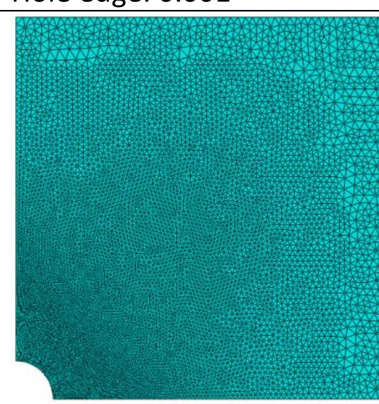
<tr><td colspan="2"></td><td></td><td></td></tr>
<tr><td colspan="4">Results:</td></tr>
</table>

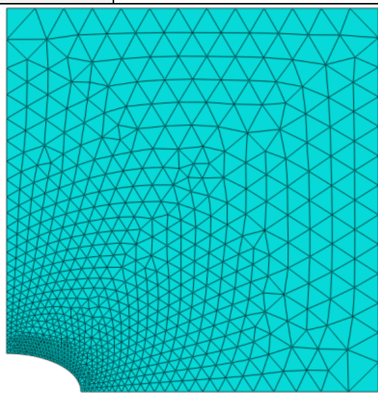
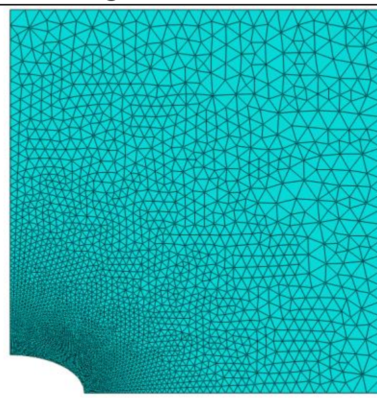
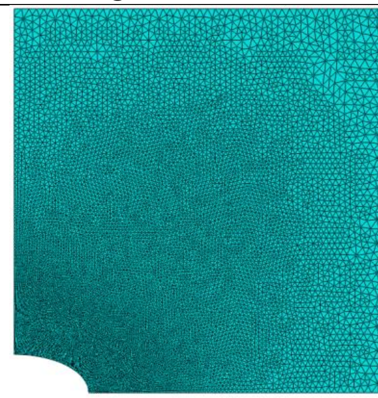
Maximum displacements (m)	Maximum von-Mises stress (Pa)
6.713e-8	5.242e4
6.717e-8	5.242e4
6.719e-8	5.248e4
6.719e-8	5.240e4

Case 3, full plate model			
Mesh size (m)	Outer edge: 0.04 Hole edge: 0.004	Outer edge: 0.02 Hole edge: 0.002	Outer edge: 0.01 Hole edge: 0.001
			
Results:			

Maximum displacements (m)	Maximum von-Mises stress (Pa)
6.839e-8	6.518e4
6.846e-8	7.282e4
6.848e-8	7.280e4

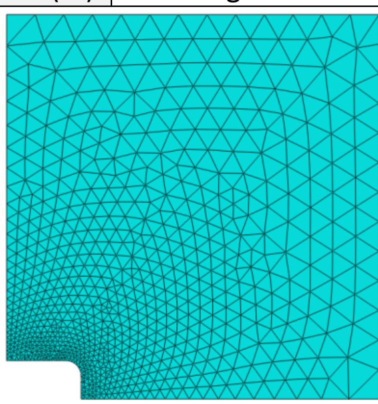
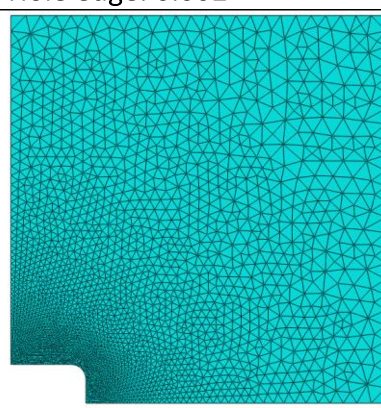
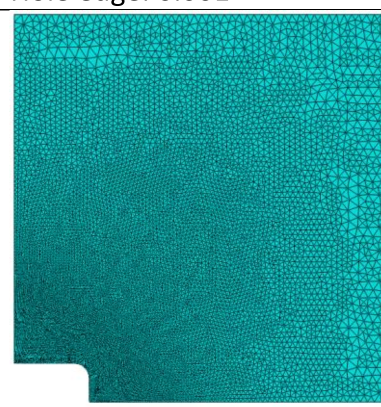
Case 1, quarter plate model			
Mesh size (m)	Outer edge: 0.04 Hole edge: 0.004	Outer edge: 0.02 Hole edge: 0.002	Outer edge: 0.01 Hole edge: 0.001
			
Results:			

Maximum displacements (m)	Maximum von-Mises stress (Pa)
6.570e-8	7.618e4
6.572e-8	7.707e4
6.573e-8	7.742e4

Case 2, quarter plate model			
Mesh size (m)	Outer edge: 0.04 Hole edge: 0.004	Outer edge: 0.02 Hole edge: 0.002	Outer edge: 0.01 Hole edge: 0.001
			

Results:

Maximum displacements (m)	Maximum von-Mises stress (Pa)
6.713e-8	5.227e4
6.717e-8	5.256e4
6.719e-8	5.247e4

Case 3, quarter plate model			
Mesh size (m)	Outer edge: 0.04 Hole edge: 0.004	Outer edge: 0.02 Hole edge: 0.002	Outer edge: 0.01 Hole edge: 0.001
			

Results:

Maximum displacements (m)	Maximum von-Mises stress (Pa)
6.839e-8	5.995e4
6.846e-8	6.759e4
6.848e-8	7.276e4

The comparison of the maximum displacements and the maximum von-Mises stresses for case (a) and (b) are summarized in the following table.

	Maximum displacements (m)		**Maximum von-Mises stress (MPa)**	
	Full plate model	Quarter plate model	Full plate model	Quarter plate model
Case 1	6.573e-8	6.573e-8	7.756e4	7.742e4
Case 2	6.719e-8	6.719e-8	5.240e4	5.247e4
Case 3	6.848e-8	6.848e-8	7.280e4	7.276e4

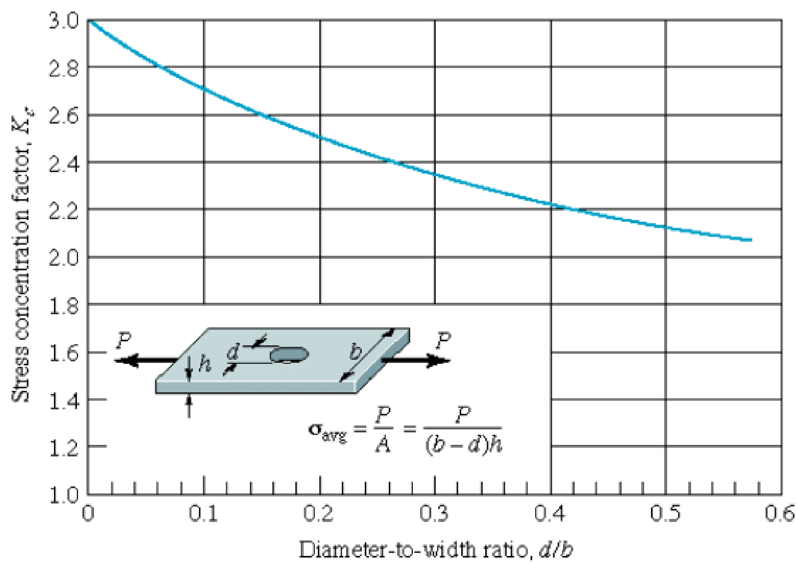
The results of case (a) and (b) match well which demonstrates the accuracy of the numerical results.

Theoretical stress concentration factor:

The theoretical stress concentration factor can be acquired from the handbook for these three cases.

Case 1:

For the plate with a circular hole, given the dimension in the problem statement and the following chart from literature. The stress concentration factor***, Kc ~2.7***

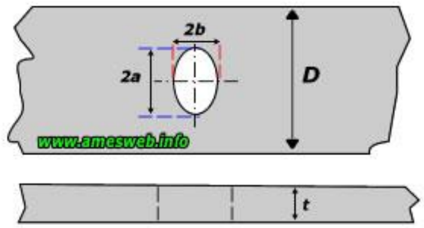


Reference: http://www.ux.uis.no/~hirpa/KdB/ME/stressconc.pdf

Case 2:

According to the given geometry specifications and the following chart, a=0.05, b=0.1, then the stress concentration factor can be approximately calculated as ***Kc = 1.8607***

Central single elliptical hole in finite-width plate

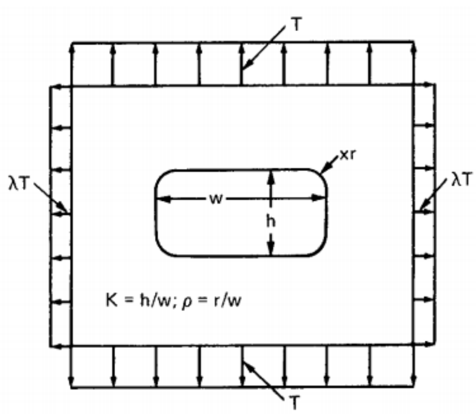


For $0.5 \le a/b \le 10.0$	
C_1	$1.000 - 0.000\sqrt{a/b} + 2.000a/b$
C_2	$-0.351 - 0.021\sqrt{a/b} - 2.483a/b$
C_3	$3.621 - 5.183\sqrt{a/b} + 4.494a/b$
C_4	$-2.270 + 5.204\sqrt{a/b} - 4.011a/b$
K_{tA}	$C_1 + C_2\frac{2a}{D} + C_3(\frac{2a}{D})^2 + C_4(\frac{2a}{D})^3$
σ_{nom}	$P/[(D-2a)t]$
σ_A	$K_{tA}\sigma_{nom}$

Reference: Young, W. C., Budynas, R. G.(2002). Roark's Formulas for Stress and Strain .7nd Edition McGraw-Hill

Case 3:

According to the given geometry specifications and the following chart, the stress concentration factor can be approximately taken as ***Kc ~ 2.6***



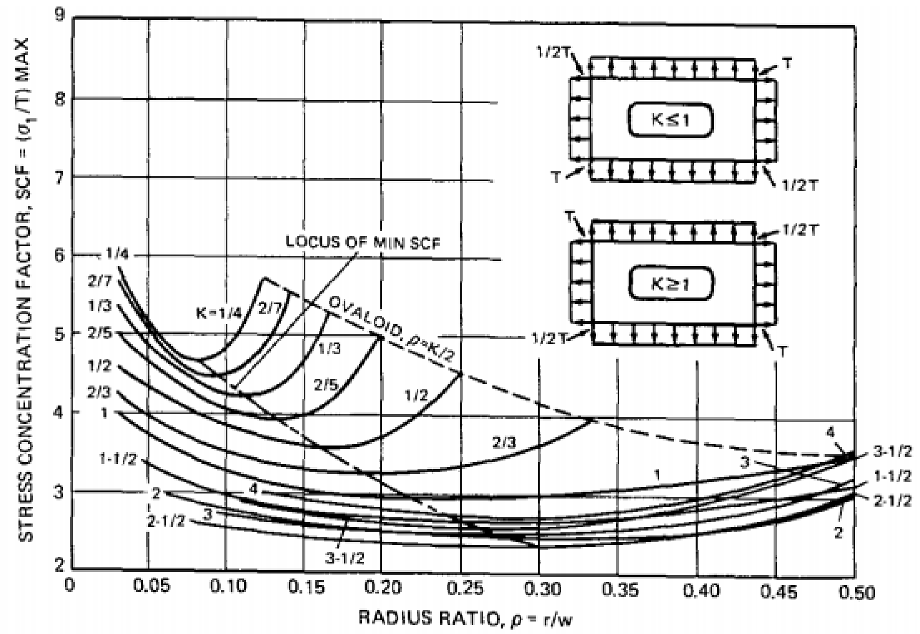


Figure 4c — $\lambda = -1/2$

Reference: http://www.dtic.mil/dtic/tr/fulltext/u2/758644.pdf

Nominal stress of the three cases can be obtained,

$$\sigma_n = \frac{P}{A} = \frac{25000N/m^2 \times 1m \times 0.02m}{(1m - 0.1m) \times 0.02m} = 2.7778e4\ Pa$$

Then the ratio of maximum stress obtained to nominal stress can be calculated, as listed in the following table, which match well respectively.

	Ratio of maximum stress to nominal stress		**Theoretical stress concentration factor**
	Full plate model	Quarter plate model	
Case 1: Circular hole	2.7922	2.7871	2.7
Case 2: Elliptical hole	1.8864	1.8889	1.8607
Case 3: Rectangular hole	2.6208	2.6194	2.6