Consider the two-variable second order system:

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = \{F\}$$
$$\{U_0\} = \{\dot{U}_0\} = 0$$

where:

$$[M] = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix};\ [C] = [K] = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix};\ \{F\} = \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$$

Substituting $\{U_0\}$ and $\{\dot{U}_0\}$ into the system, $\{\ddot{U}_0\}$ can be obtained:

$$\{\ddot{U}_0\} = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$$

Newmark: ($\Delta t = 0.5, \theta = 1, a = b = 0.5$)

$$[\bar{K}]\{U_{t+\Delta t}\} = \{R_{t+\Delta t}\}$$

$$[\bar{K}] = [M] + 0.25[C] + 0.0625[K] = \begin{bmatrix} 1.3125 & 1.3125 \\ 1.3125 & 2.9375 \end{bmatrix}$$

$$\{R_{t+\Delta t}\} = 0.0625\{F_{t+\Delta t}\} + [M]\left(\{U_t\} + 0.5\{\dot{U}_t\} + 0.0625\{\ddot{U}_t\}\right) + [C]\left(0.25\{U_t\} + 0.0625\{\dot{U}_t\}\right)$$

After each time step, $\{\ddot{U}_{t+\Delta t}\}$ and $\{\dot{U}_{t+\Delta t}\}$ can be obtained:

$$\{\ddot{U}_{t+\Delta t}\} = -\{\ddot{U}_t\} + 16\left(\{U_{t+\Delta t}\} - \{U_t\} - 0.5\{\dot{U}_t\}\right)$$
$$\{\dot{U}_{t+\Delta t}\} = \{\dot{U}_t\} + 0.25\left(\{\ddot{U}_t\} + \{\ddot{U}_{t+\Delta t}\}\right)$$

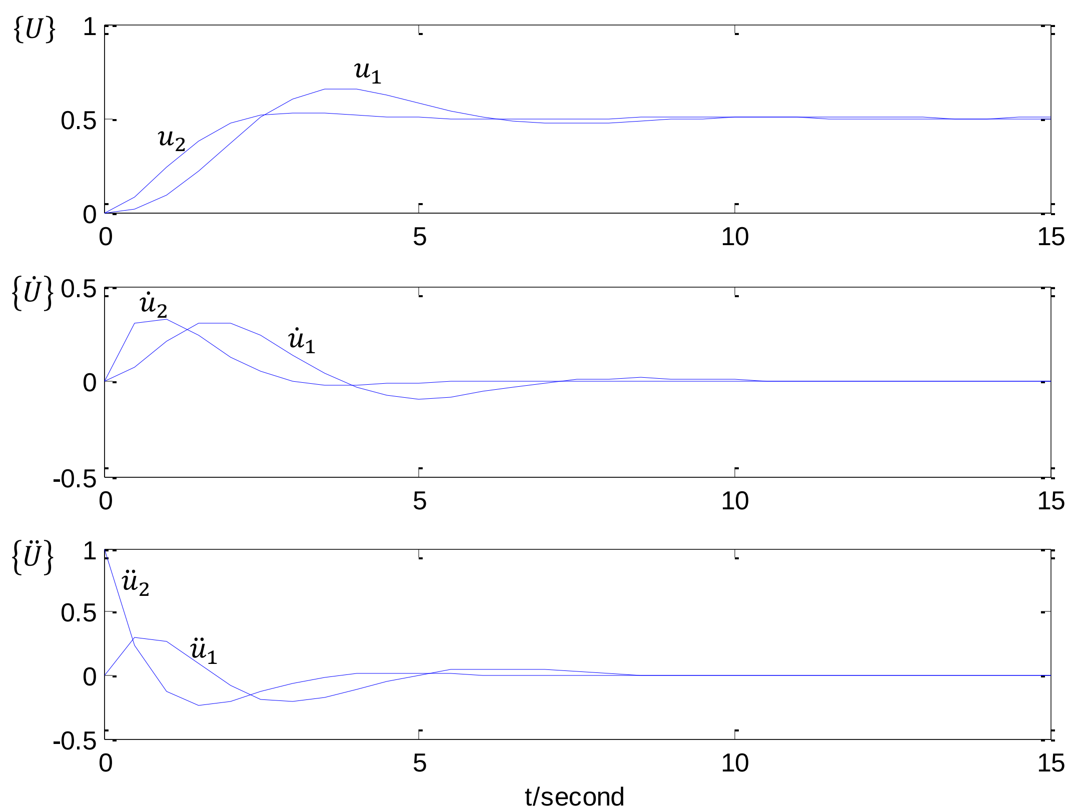
- **Summary**：

The numerical results are listed below:

t/s	u_1	u_2	$\dot{u}_1$	$\dot{u}_2$	$\ddot{u}_1$	$\ddot{u}_2$
0.0	0.000	0.000	0.000	0.000	0.000	1.000
0.5	0.018	0.077	0.073	0.308	0.293	0.231
1.0	0.090	0.237	0.213	0.331	0.265	-0.136
1.5	0.219	0.379	0.303	0.239	0.096	-0.235
2.0	0.371	0.471	0.307	0.129	-0.078	-0.201
2.5	0.509	0.515	0.241	0.048	-0.187	-0.126
3.0	0.604	0.528	0.141	0.002	-0.215	-0.058
3.5	0.649	0.524	0.041	-0.017	-0.183	-0.014
4.0	0.651	0.515	-0.035	-0.019	-0.120	0.007
4.5	0.623	0.507	-0.078	-0.014	-0.052	0.013
5.0	0.581	0.502	-0.090	-0.007	0.003	0.011
5.5	0.539	0.499	-0.080	-0.003	0.037	0.007
6.0	0.504	0.498	-0.057	0.000	0.051	0.003
6.5	0.482	0.499	-0.032	0.001	0.050	0.001
7.0	0.471	0.499	-0.010	0.001	0.039	0.000
7.5	0.470	0.500	0.006	0.001	0.024	-0.001
8.0	0.475	0.500	0.014	0.000	0.011	-0.001
8.5	0.483	0.500	0.017	0.000	0.000	0.000
9.0	0.491	0.500	0.015	0.000	-0.007	0.000
9.5	0.498	0.500	0.011	0.000	-0.009	0.000
10.0	0.503	0.500	0.007	0.000	-0.009	0.000

10.5	0.505	0.500	0.003	0.000	-0.007	0.000
11.0	0.505	0.500	-0.001	0.000	-0.005	0.000
11.5	0.505	0.500	-0.002	0.000	-0.002	0.000
12.0	0.503	0.500	-0.003	0.000	0.000	0.000
12.5	0.502	0.500	-0.003	0.000	0.001	0.000
13.0	0.501	0.500	-0.002	0.000	0.002	0.000
13.5	0.500	0.500	-0.001	0.000	0.002	0.000
14.0	0.499	0.500	-0.001	0.000	0.001	0.000
14.5	0.499	0.500	0.000	0.000	0.001	0.000
15.0	0.499	0.500	0.000	0.000	0.001	0.000

$\{U\}$, $\{\dot{U}\}$ and $\{\ddot{U}\}$ in terms of time *t* are illustrated in the following figure:



➢ **The MATLAB code is listed below,**

Code:

```
%%%%%%%%%%%%%%% for ME6212/Hw_4%%%%%%%%%%%%%%%
% Software: MATLAB R2014a
% Author: Wenjin Tao
% Missouri S&T
% Copyright(c) by Wenjin Tao
% Date: 02/29/2015

clc;clear;close all;
M = [1 1;1 2];
C = [1 1;1 3];
K = C;
```

```
F=[1;2];
K_bar = M + 0.25*C + 0.0625*K;
t=0:0.5:15;
U = zeros(2,31);
V = zeros(2,31);  % velocity
A = zeros(2,31);  % acceleration
A(:,1) = [0;1];
R = zeros(2,31);
for i = 2:31
    R(:,i) = 0.0625*F + M*( U(:,(i-1)) + 0.5*V(:,(i-1)) + 0.0625*A(:,(i-
1)) ) + C*( 0.25*U(:,(i-1)) + 0.0625*V(:,(i-1)) );
    U(:,i) = inv(K_bar)*R(:,i);
    A(:,i) = -A(:,i-1) + 16*( U(:,i) - U(:,i-1) - 0.5*V(:,i-1) );
    V(:,i) = V(:,(i-1)) + 0.25*( A(:,i-1)+A(:,i) );
end
r = [t',U',V',A'];  % result
subplot(3,1,1);
    plot(r(:,1),r(:,2)); hold on;
    plot(r(:,1),r(:,3)); hold on;
subplot(3,1,2);
    plot(r(:,1),r(:,4)); hold on;
    plot(r(:,1),r(:,5)); hold on;
subplot(3,1,3);
    plot(r(:,1),r(:,6)); hold on;
    plot(r(:,1),r(:,7)); hold on;
    xlabel('t/second');
```